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**Introduction to differential geometry**

**12hp Ph.D. course, spring 2016**

**The aim of the course:** to give a comprehensive introduction to the most important notions of differential geometry, to present some basic applications of differential geometry and to teach students how to perform simple differential-geometric calculations.

**Prerequisites:** elementary courses in linear algebra and calculus in several variables.

**Course book:** "A Course of Differential Geometry and Topology", by A. Mishchenko and A. Fomenko (Mir Publisher Moscow 1988). The book is well furnished with non-trivial examples that are more than helpful in understanding the text. Also, it widely uses the coordinate-dependent approach to vector and tensor fields, which makes it easier to perform calculations.

The students are expected to:

1) read carefully the indicated parts of the textbook
2) present some of these parts as minilectures to other participants
3) solve a number of problems that will be successively given by me
4) present the solutions during a series of meetings
5) handle in handwritten solutions for check and if necessary to correct them

**Content of the course:**

**Chapter 1 (Introduction to Differential Geometry):**

1.1 Curvilinear coordinate systems (all)
1.2 The length of curve in a Euclidean coordinate system: (all)

This part of the course is merely refreshment of the ideas from calculus in several variables. It also fixes some notation and prepares us to go beyond $R^n$.

**Chapter 2 (General Topology):**

2.1 Definition and basic properties of metric and topological spaces (all)
2.2 Connectedness. Separation axioms (all)
2.3 Compact spaces: 2.3.1, 2.3.2, Theorem 5 on p. 84.

This part should be read in order to understand, not to memorise the numerous definitions. Pay attention especially to the concepts of topology, Hausdorff topology, continuous mappings, homeomorphisms, compactness, and connectedness.

**Chapter 3 (Smooth Manifolds (General Theory)):**

3.1 The concept of a manifold (all). Observe that the same manifold can have many different atlases.

3.2 Definition of manifolds by equations (all). The most common way to write down a manifold.

3.3 Tangent vectors. Tangent space (all). Investigate carefully the alternative way of defining tangent vectors (par 3.3.4).

3.4 Submanifolds: pars 3.4.1, 3.4.2, Whitney theorem (thm 3 page 142) and the definition of the Riemannian metric on page 145.

**Chapter 4 (Smooth manifolds (Examples)):**

4.1 The theory of curves on a plane and in space (all)

4.2 Surfaces. First and second fundamental forms. (4.2.1, 4.2.2, 4.2.4)

**Chapter 5 (Tensor Analysis and Riemannian Geometry):**

5.1 The general concept of a tensor field on a manifold (all)

5.2 Simple tensor fields: 5.2.1, 5.2.2, 5.2.3 to the page 312.

5.3 Connection and covariant differentiation (all).

5.4 Parallel displacement. Geodesics (all).

5.5 Curvature tensor (all).

That's all! Bear in mind that sometimes you may be forced to read some isolated portions of the text not included in the above course description. This will depend also on your background. In any case, you can always consult the first volume of the book by M. Spivak, "Introduction to differential geometry" for further explanations. This is a very thorough book so probably any question you are going to have can be found explained in it.