Guarding Problems and k-Transmitter Watchman Routes

Christiane Schmidt

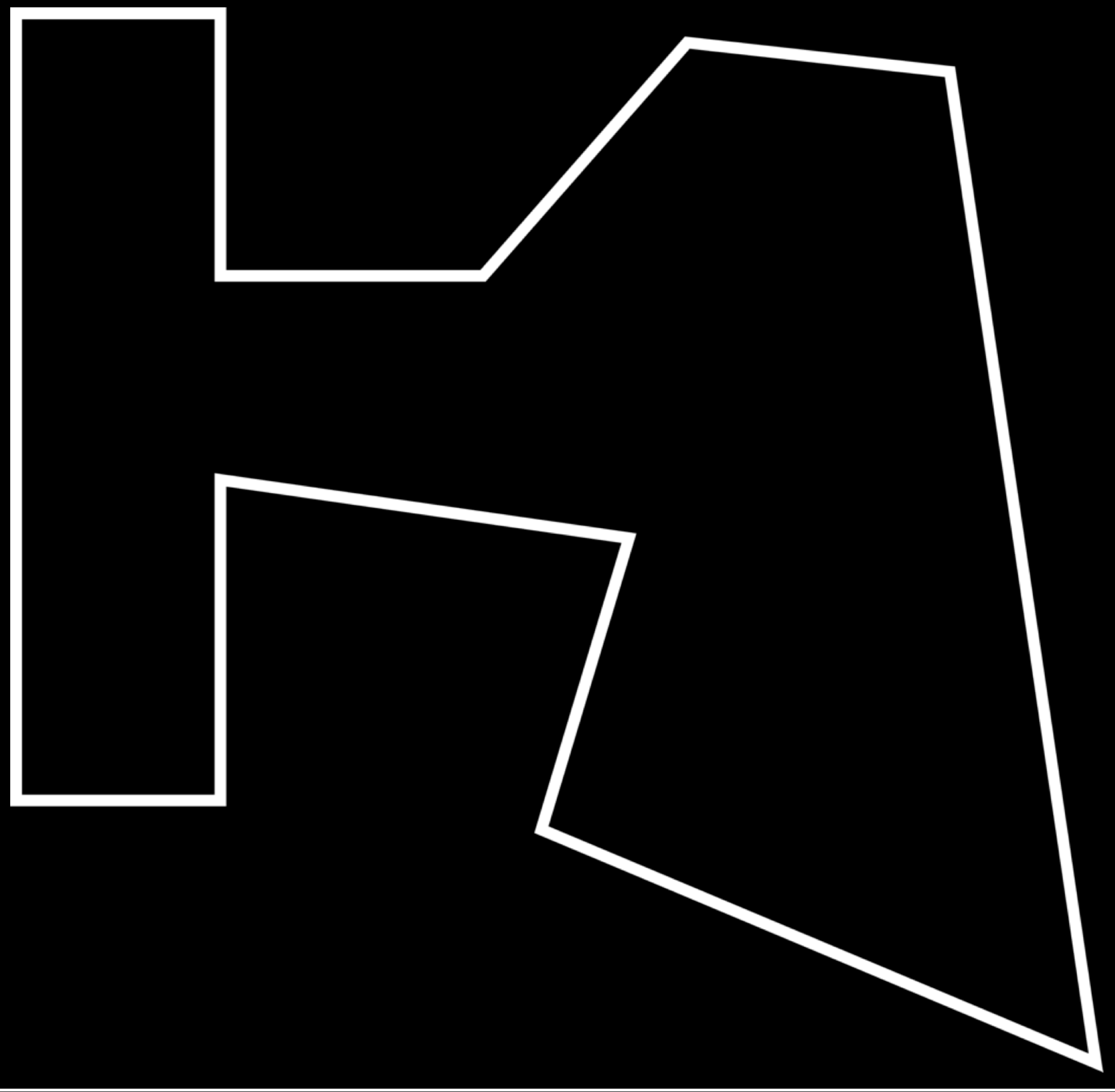
Colloquium @ The Open University Israel, January 11, 2023

LINKÖPING UNIVERSITY

Agenda

- The Art Gallery Problem and Its Variants
- *k*-Transmitters
- The Watchman Route Problem (WRP)
- *k*-Transmitter Watchman Routes
- Outlook

The Art Gallery Problem (AGP)



Given: Polygon P

How many guards do we need to monitor P?

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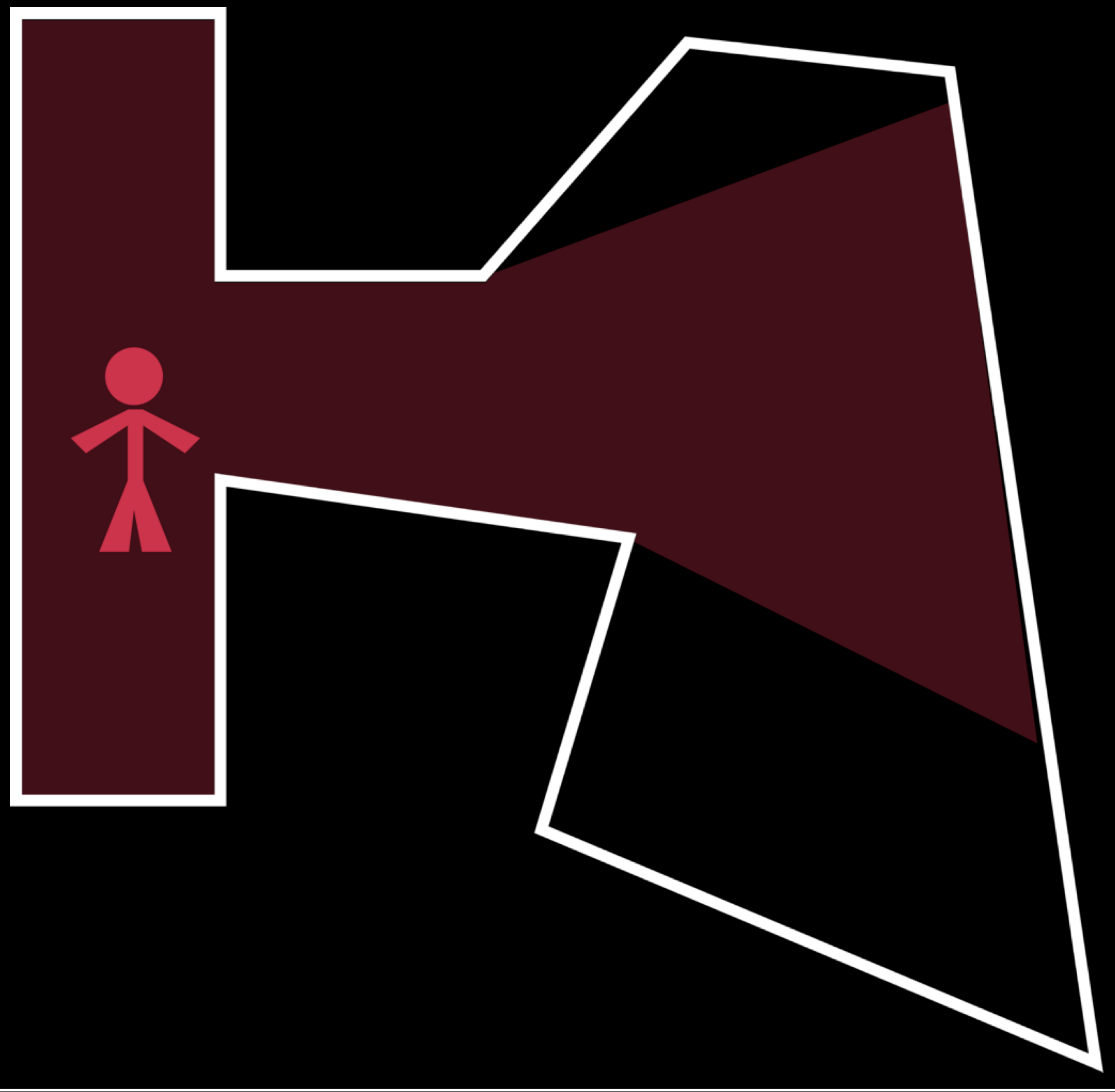

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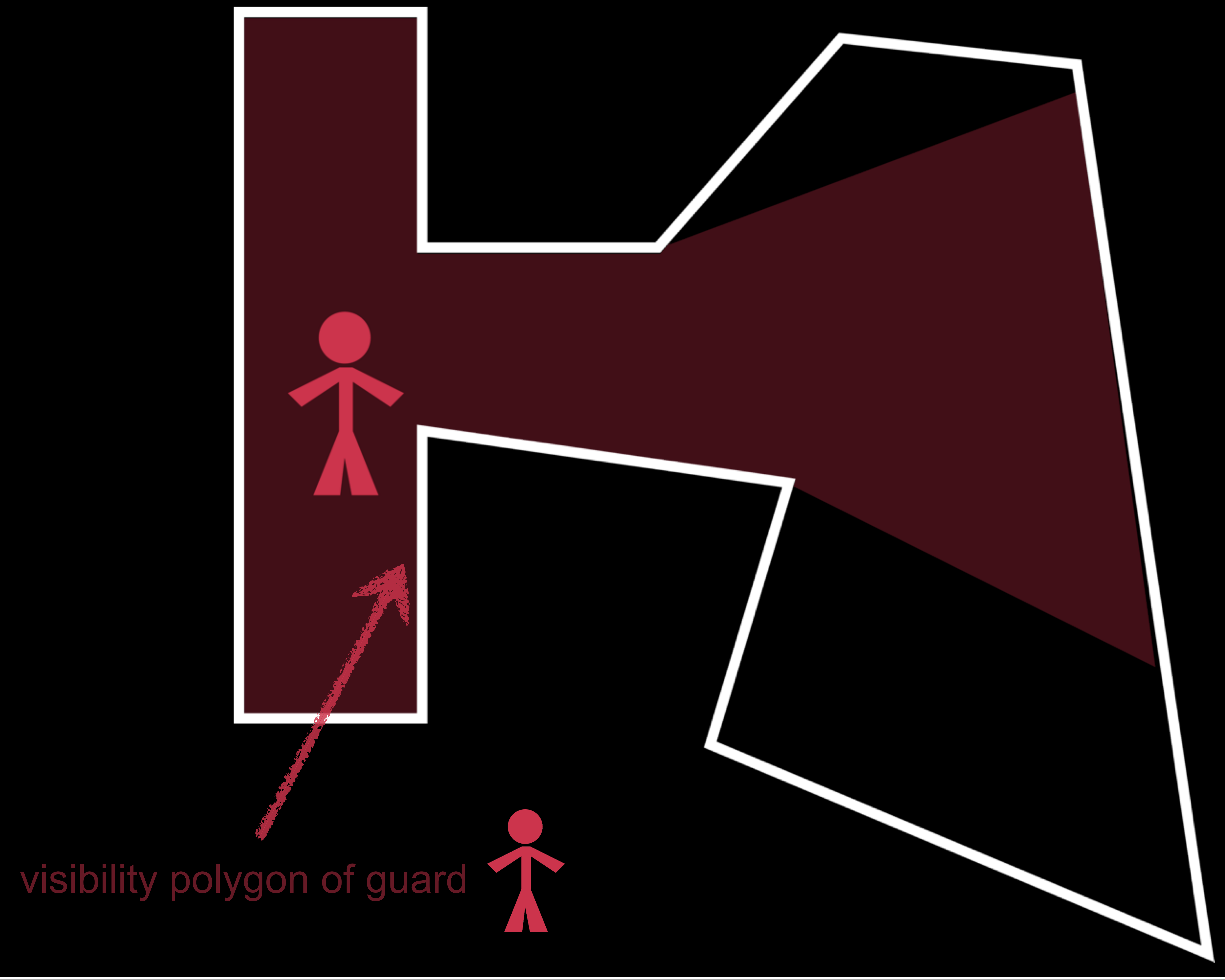

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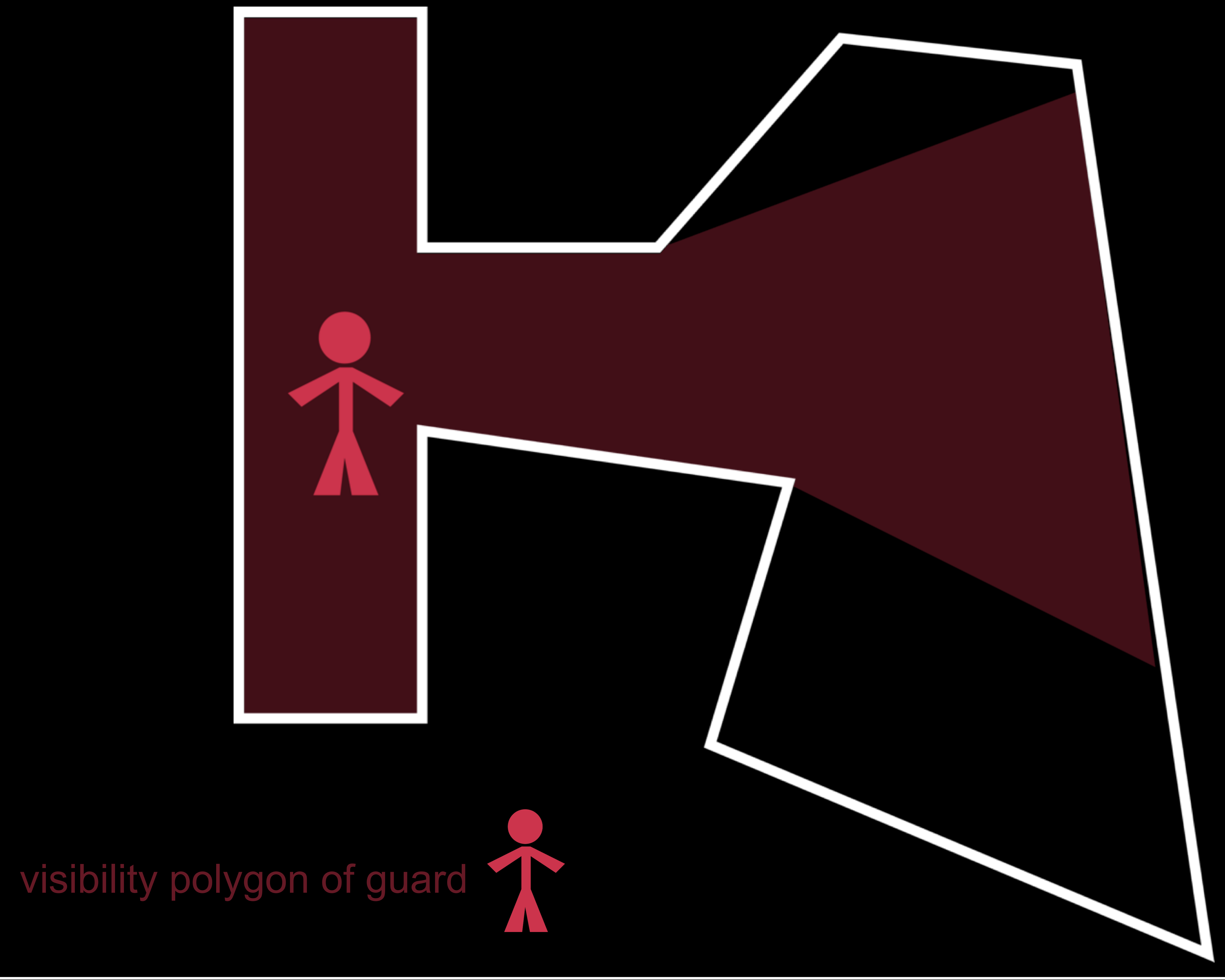

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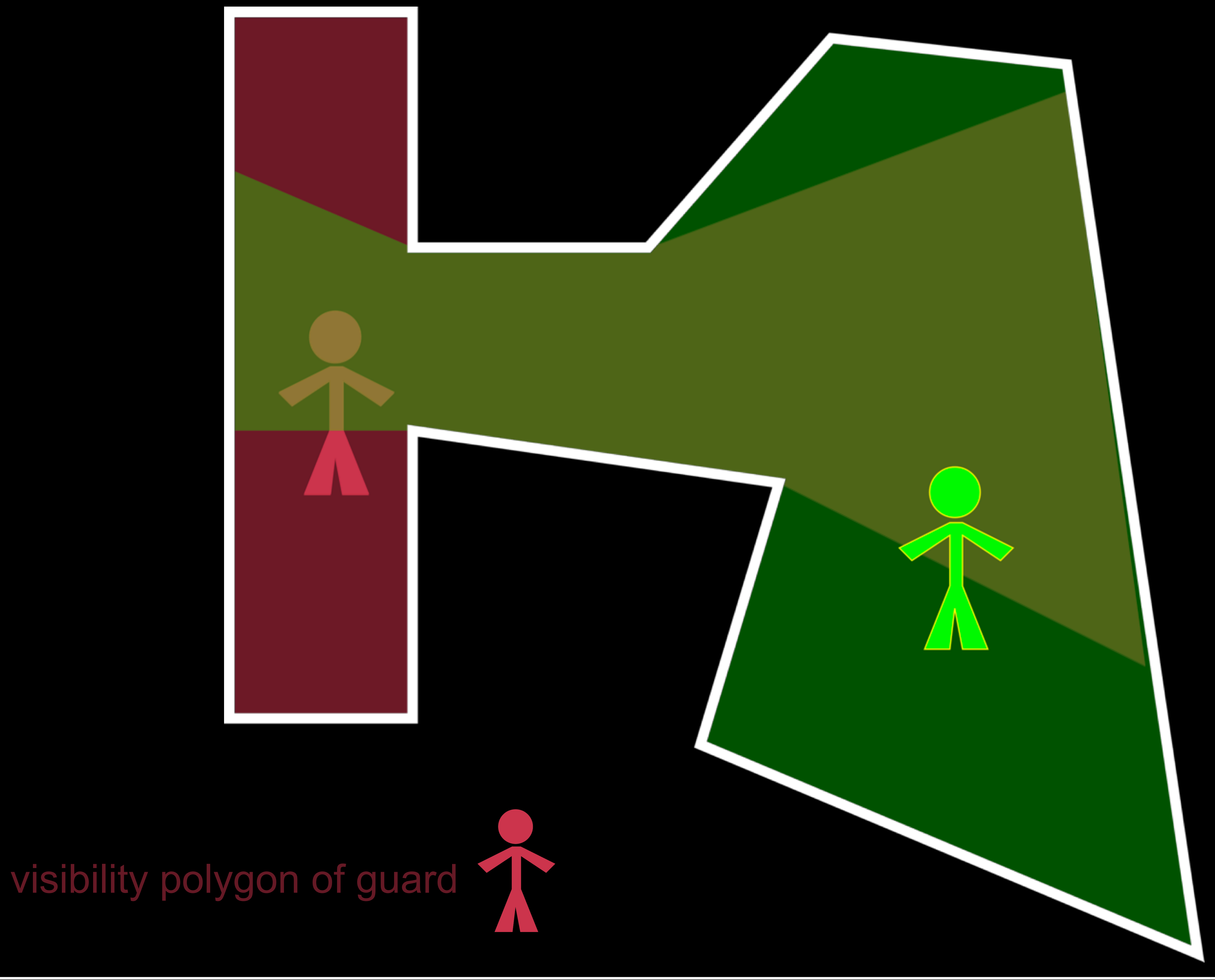

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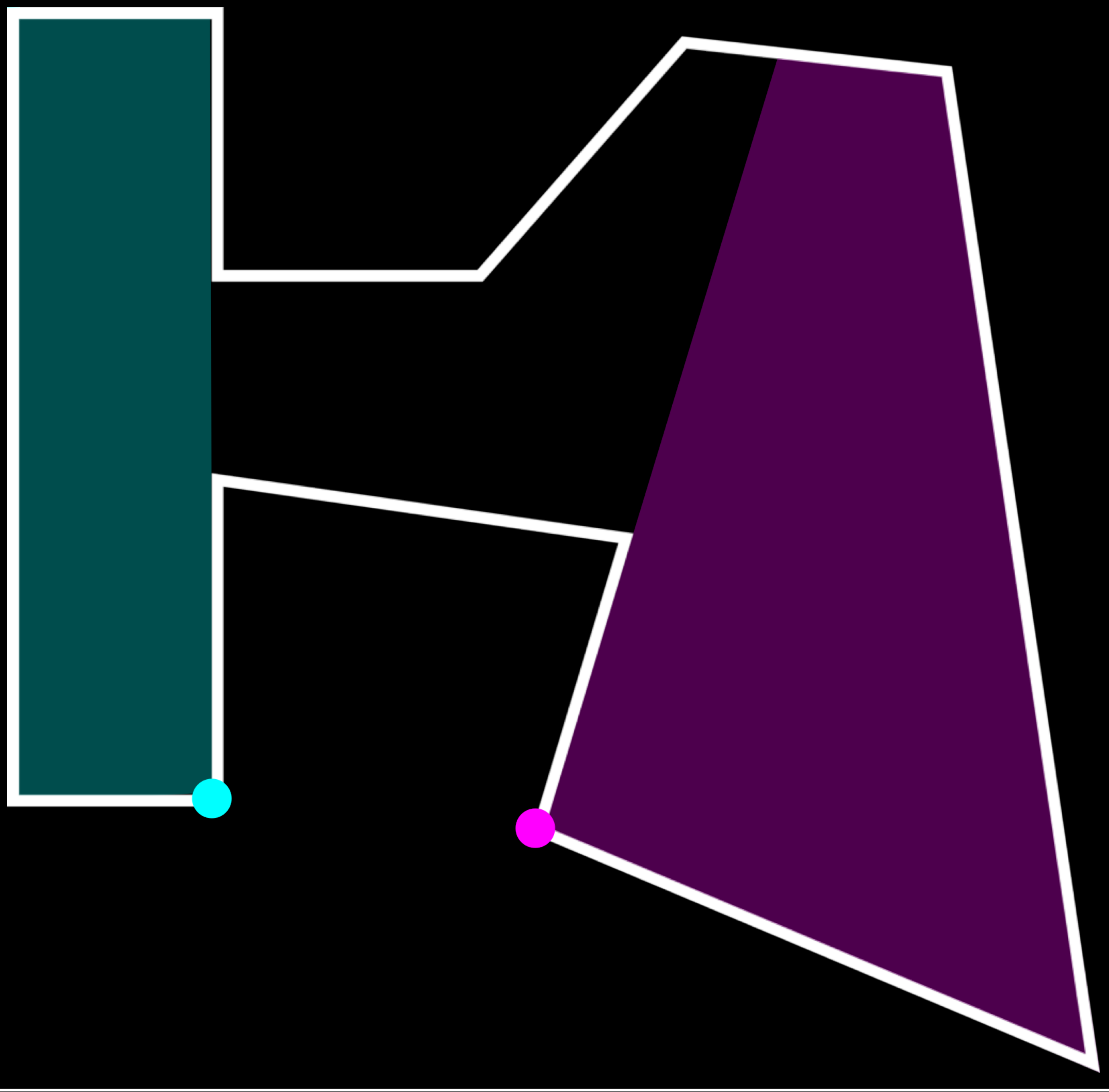

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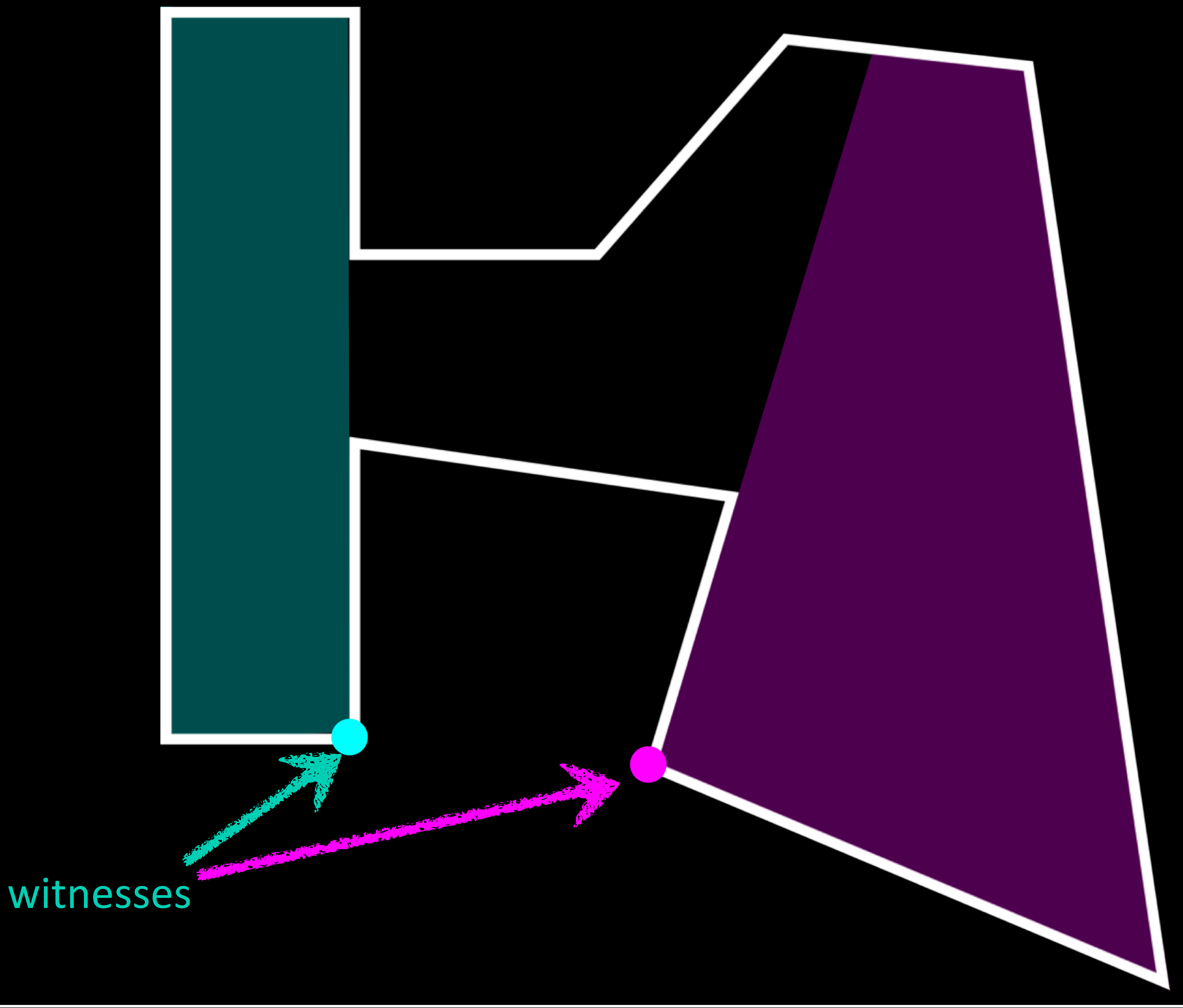

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➜ Lower bound of 2
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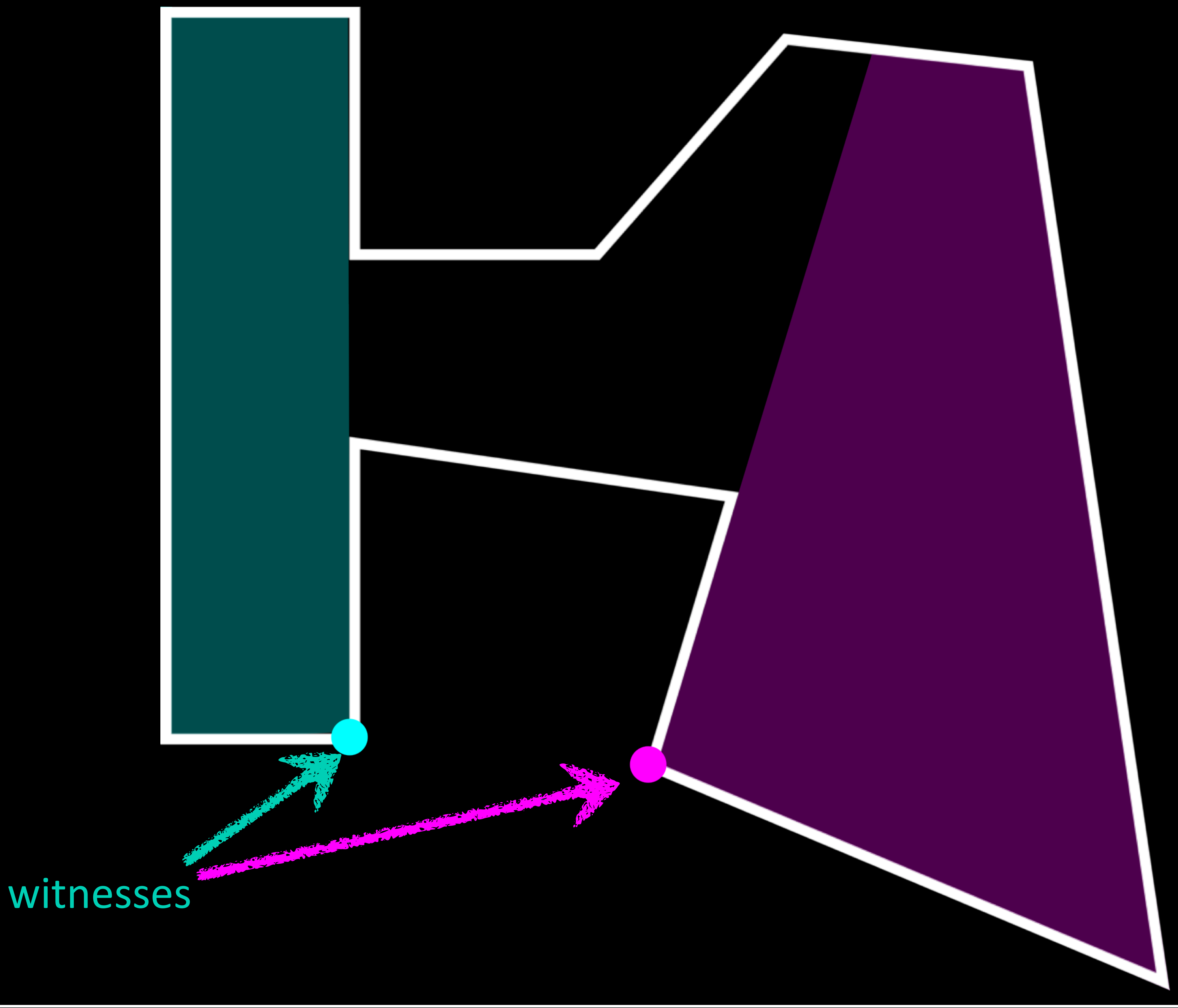

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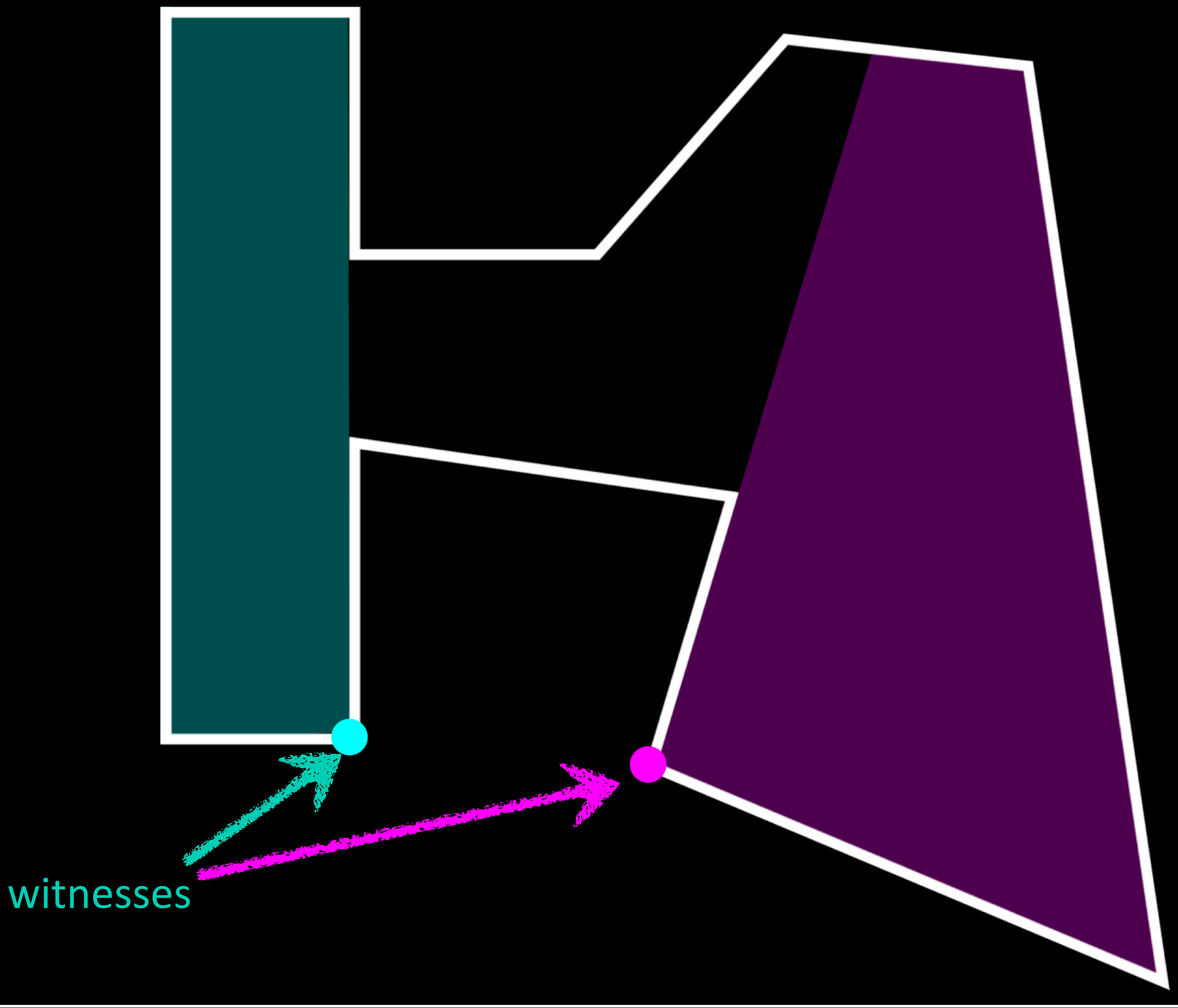

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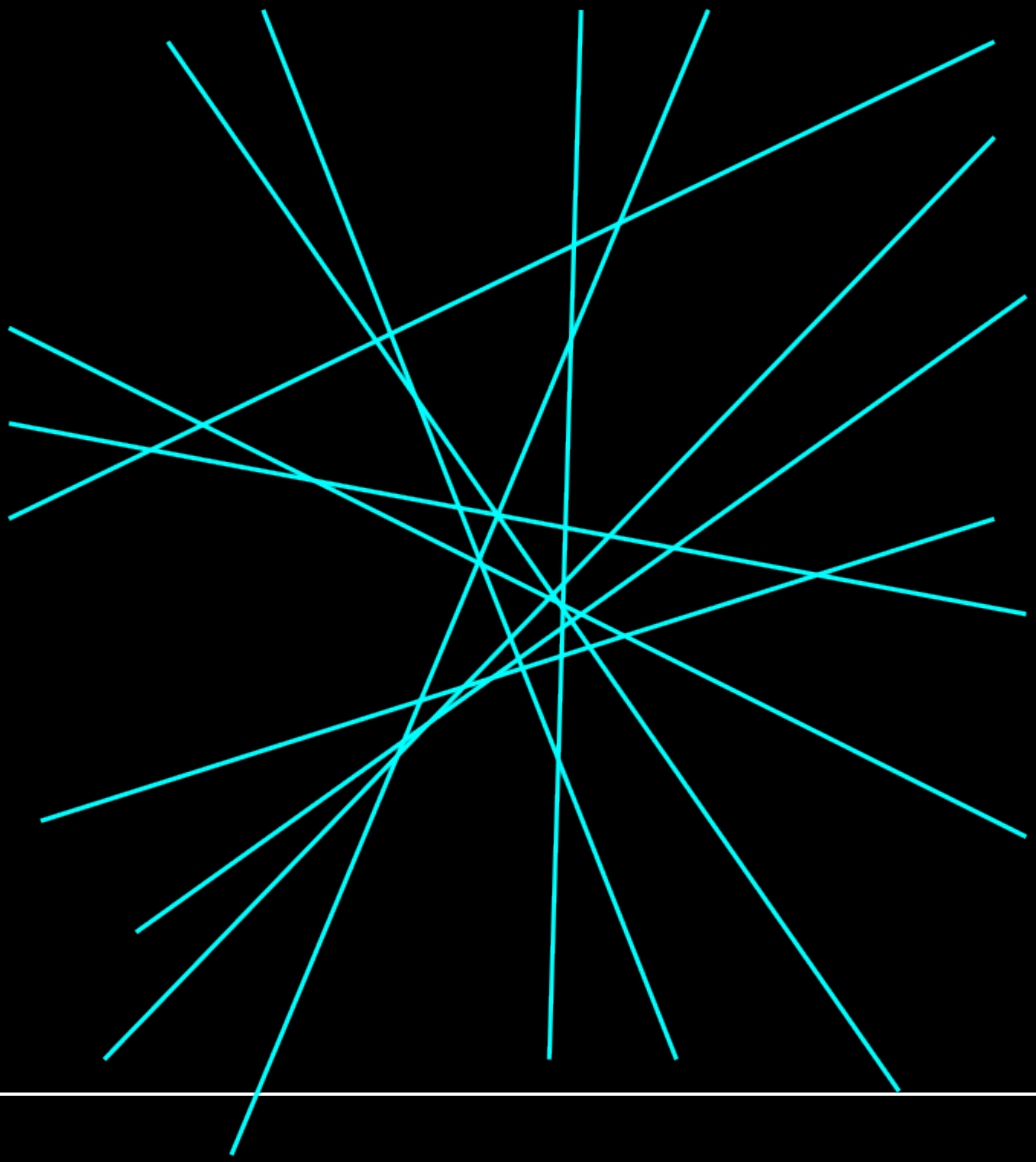

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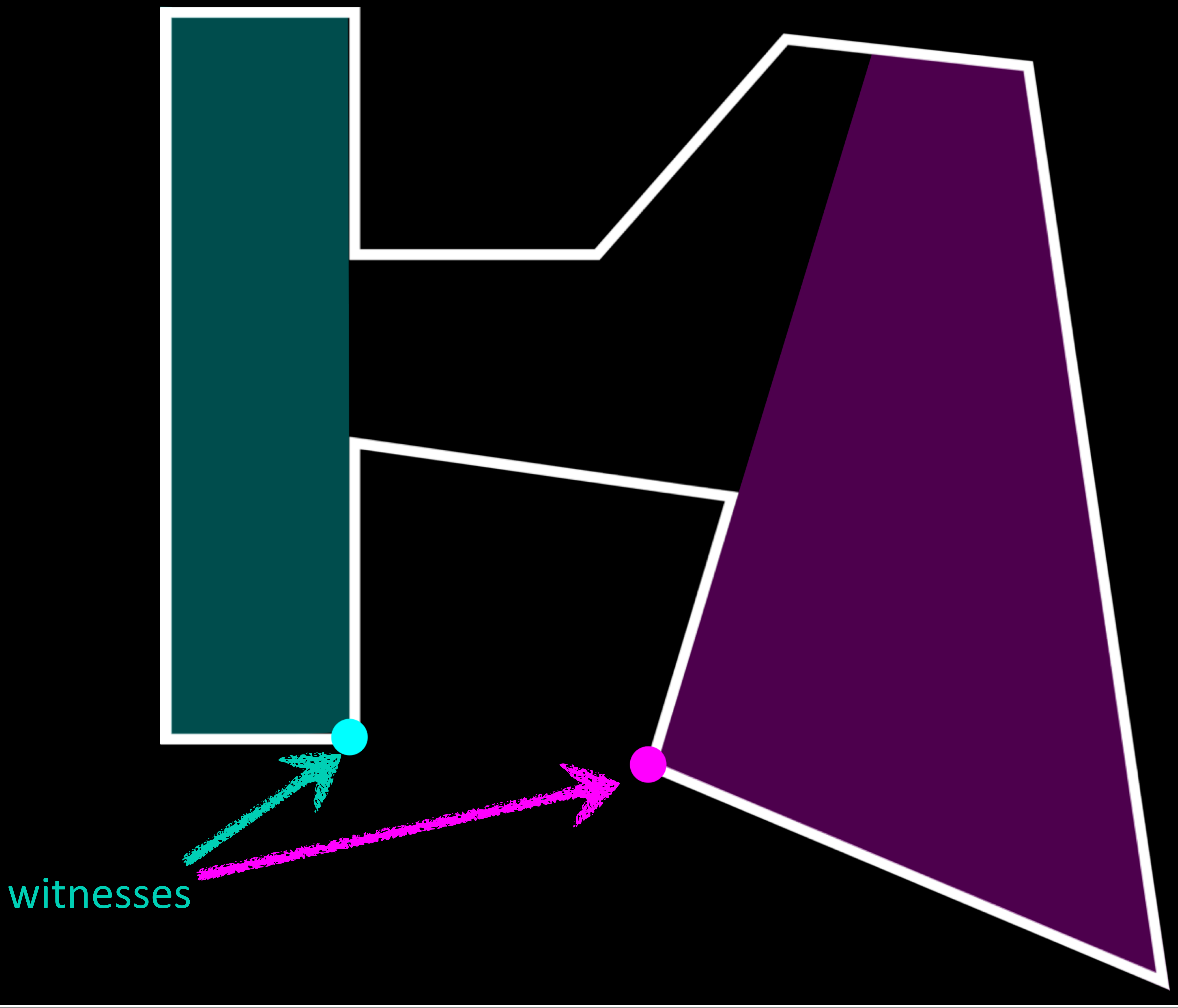

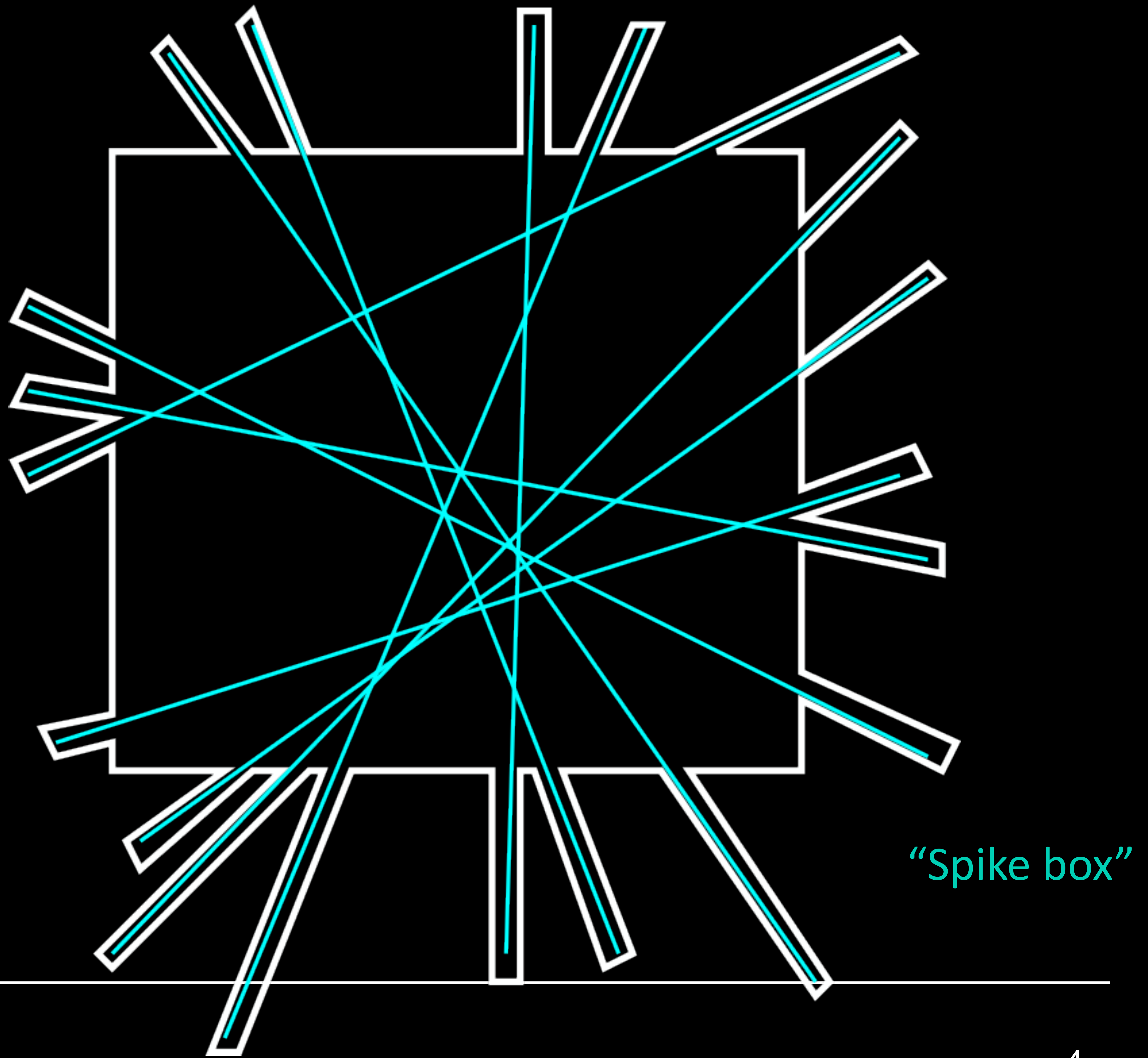

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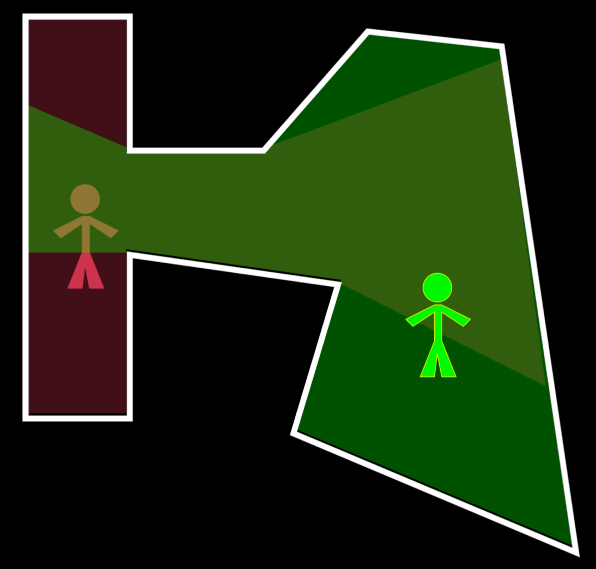

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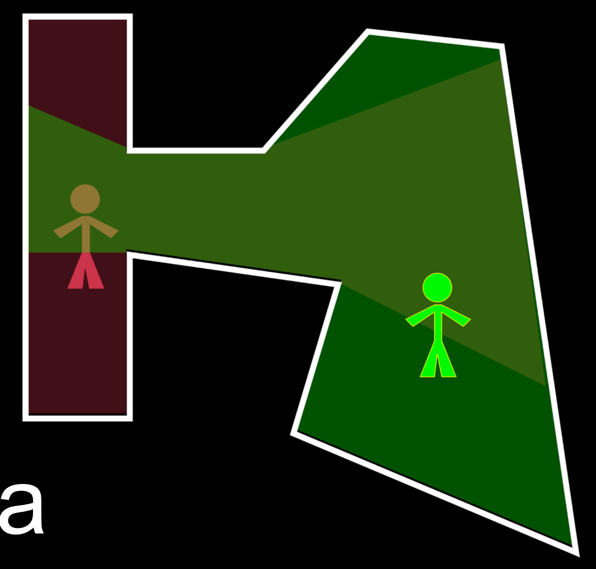

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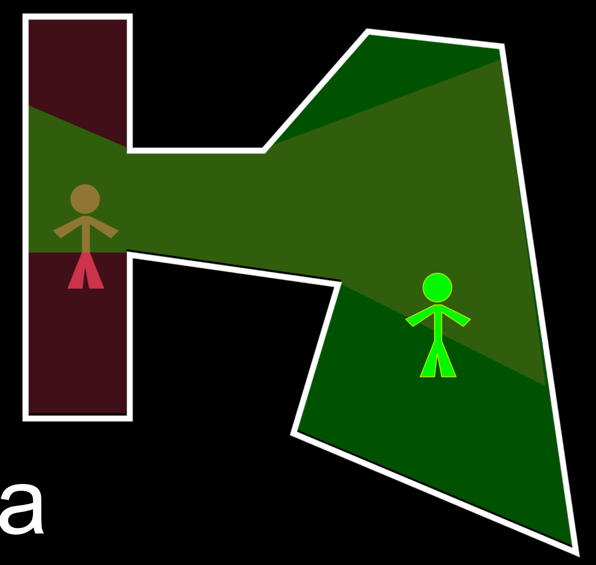

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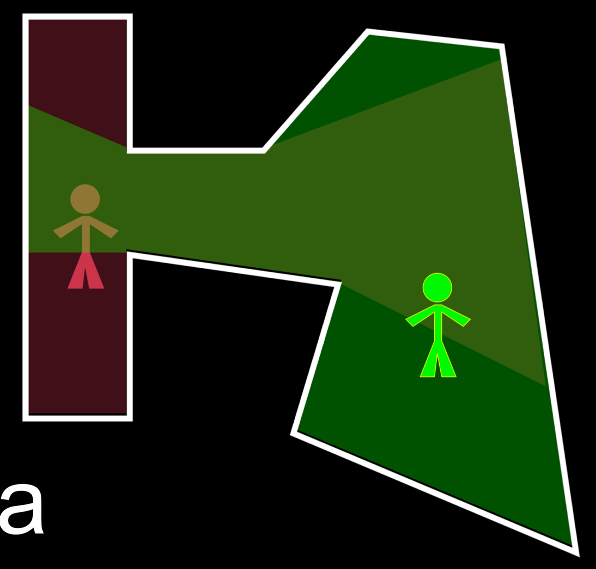

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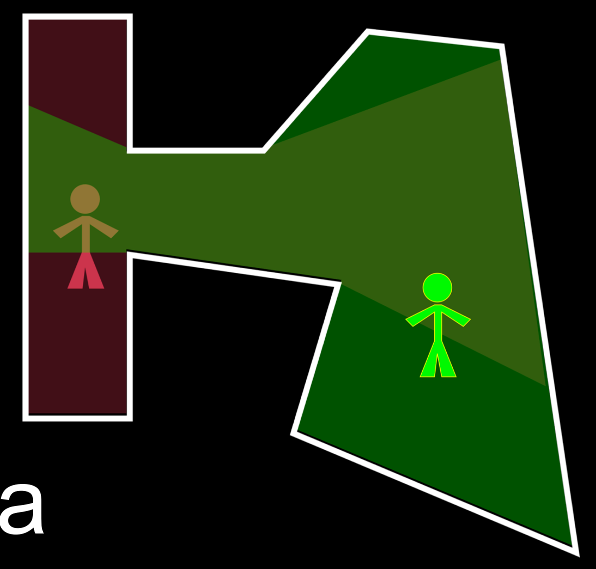

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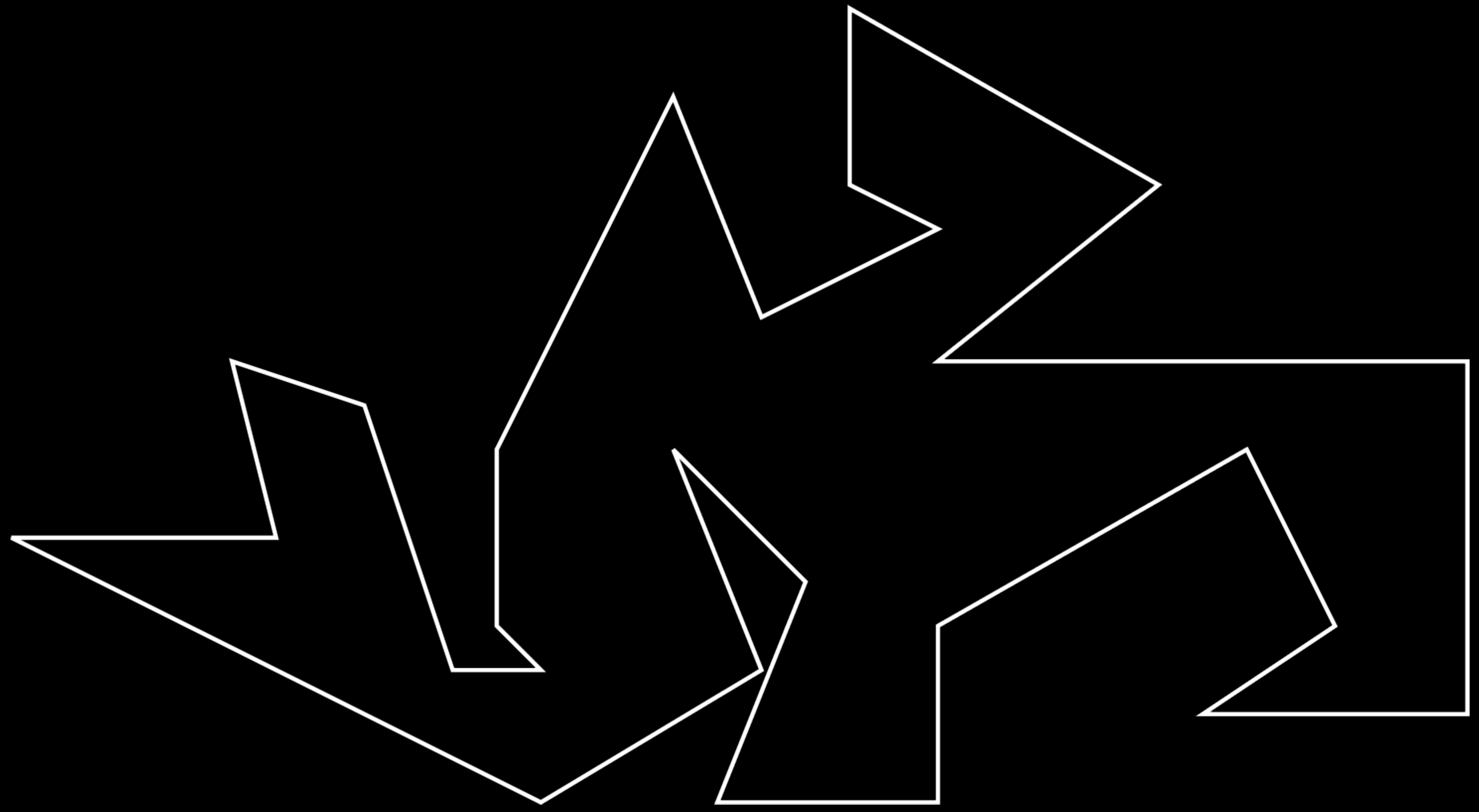

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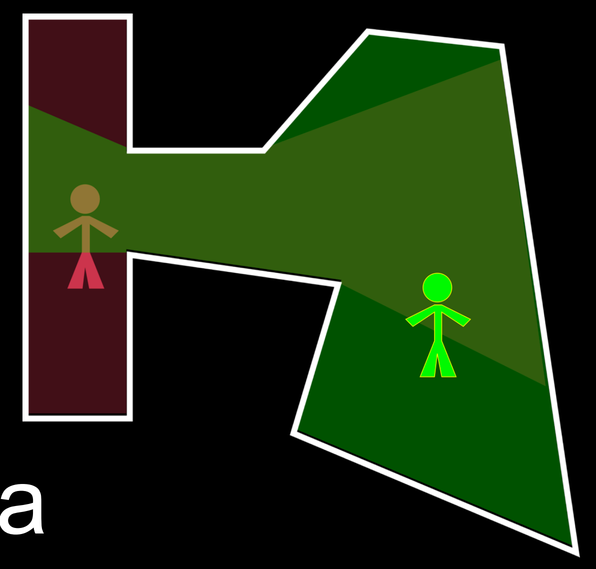

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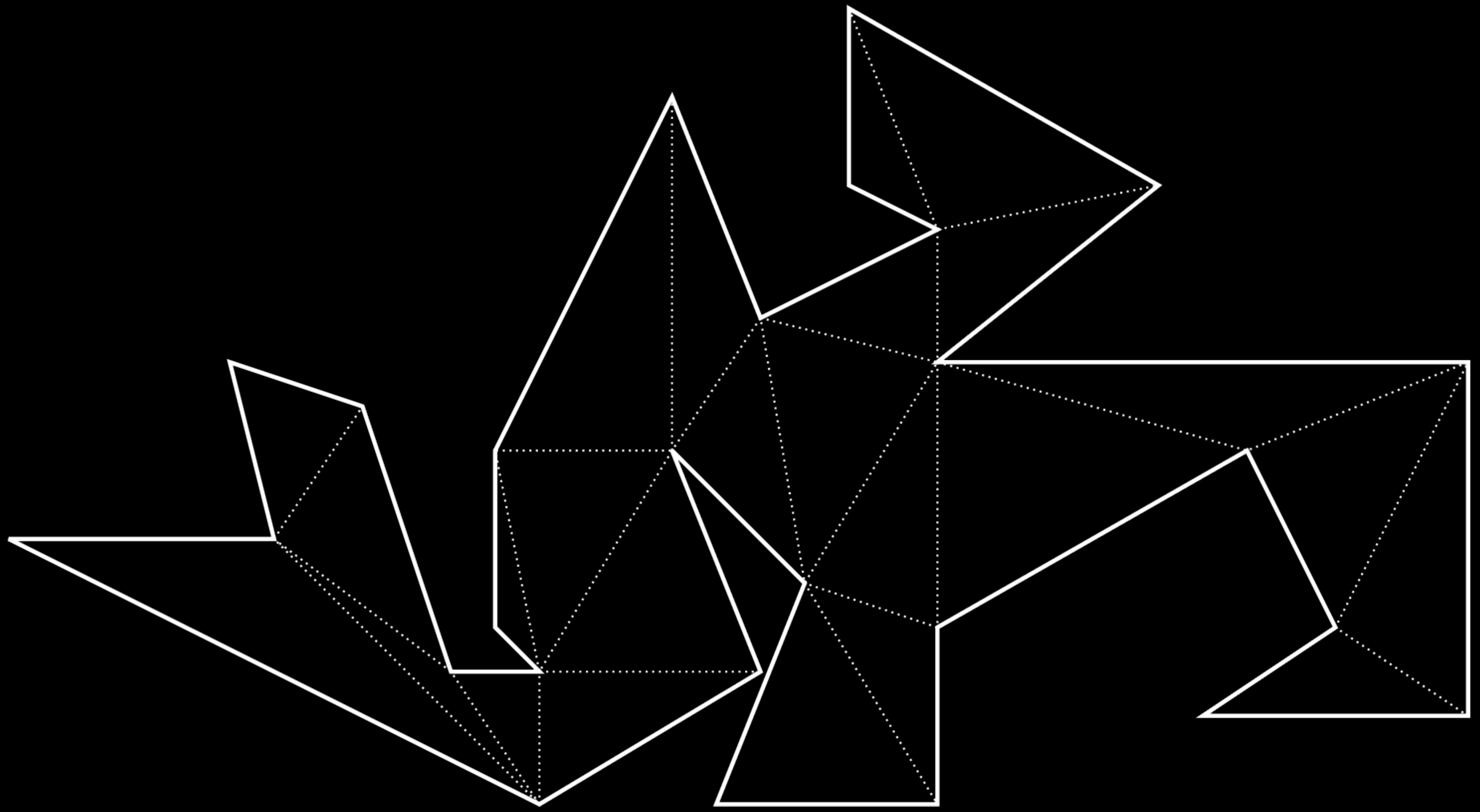

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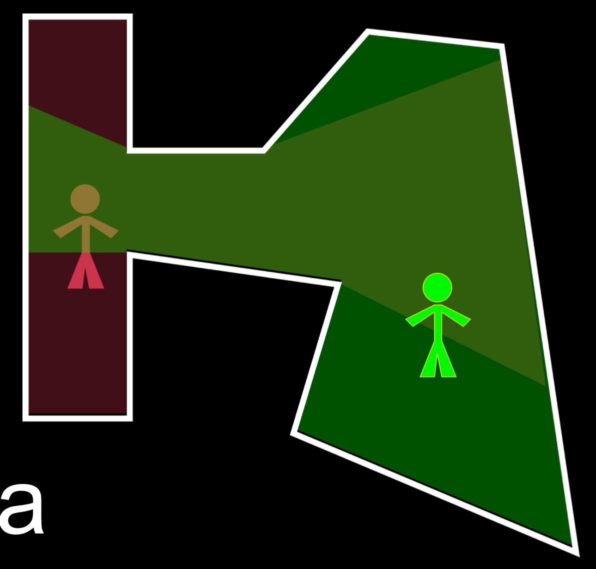

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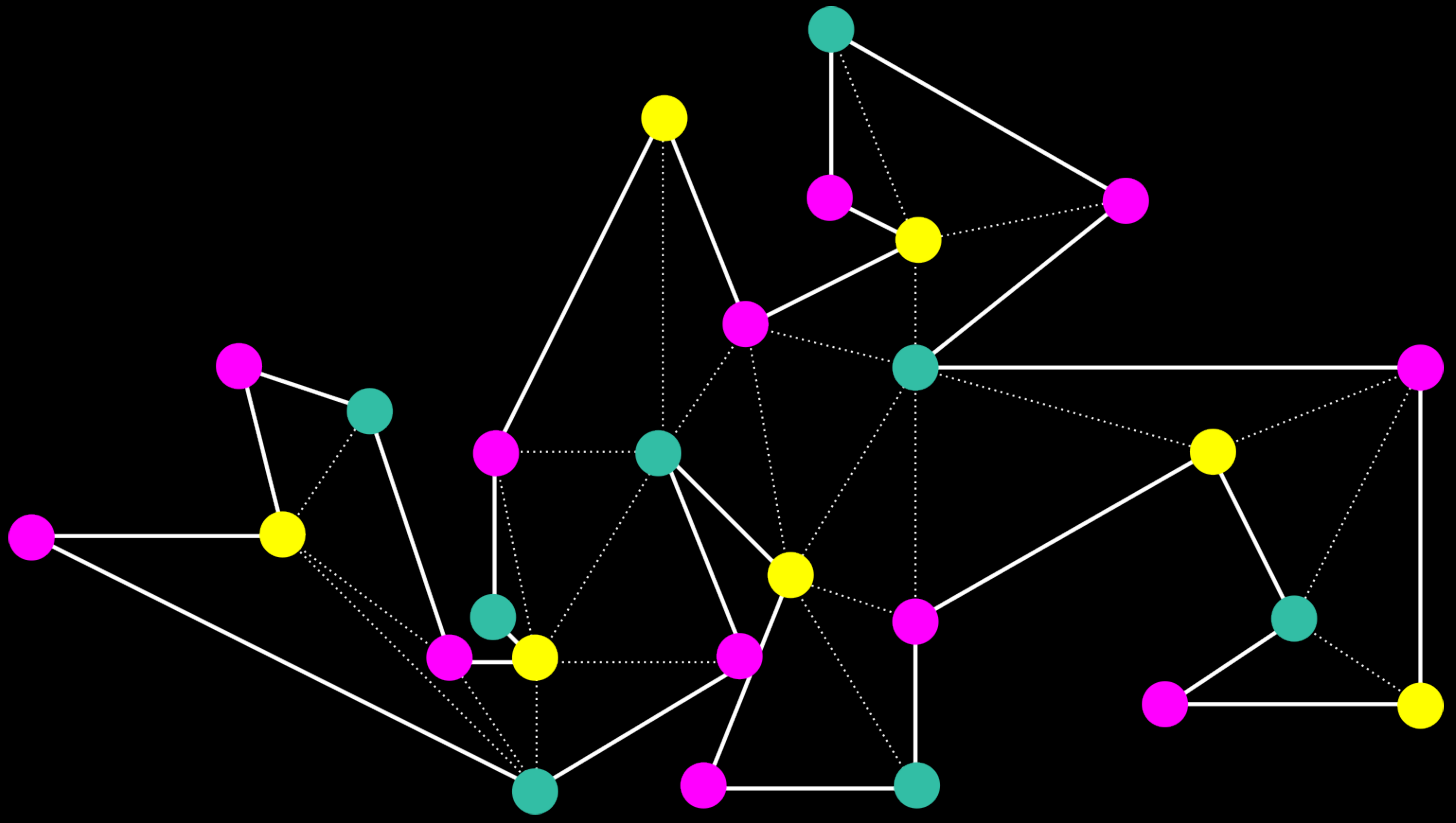

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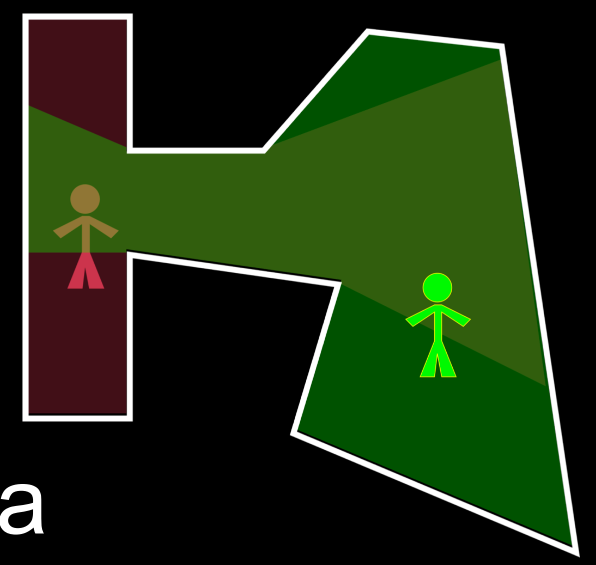

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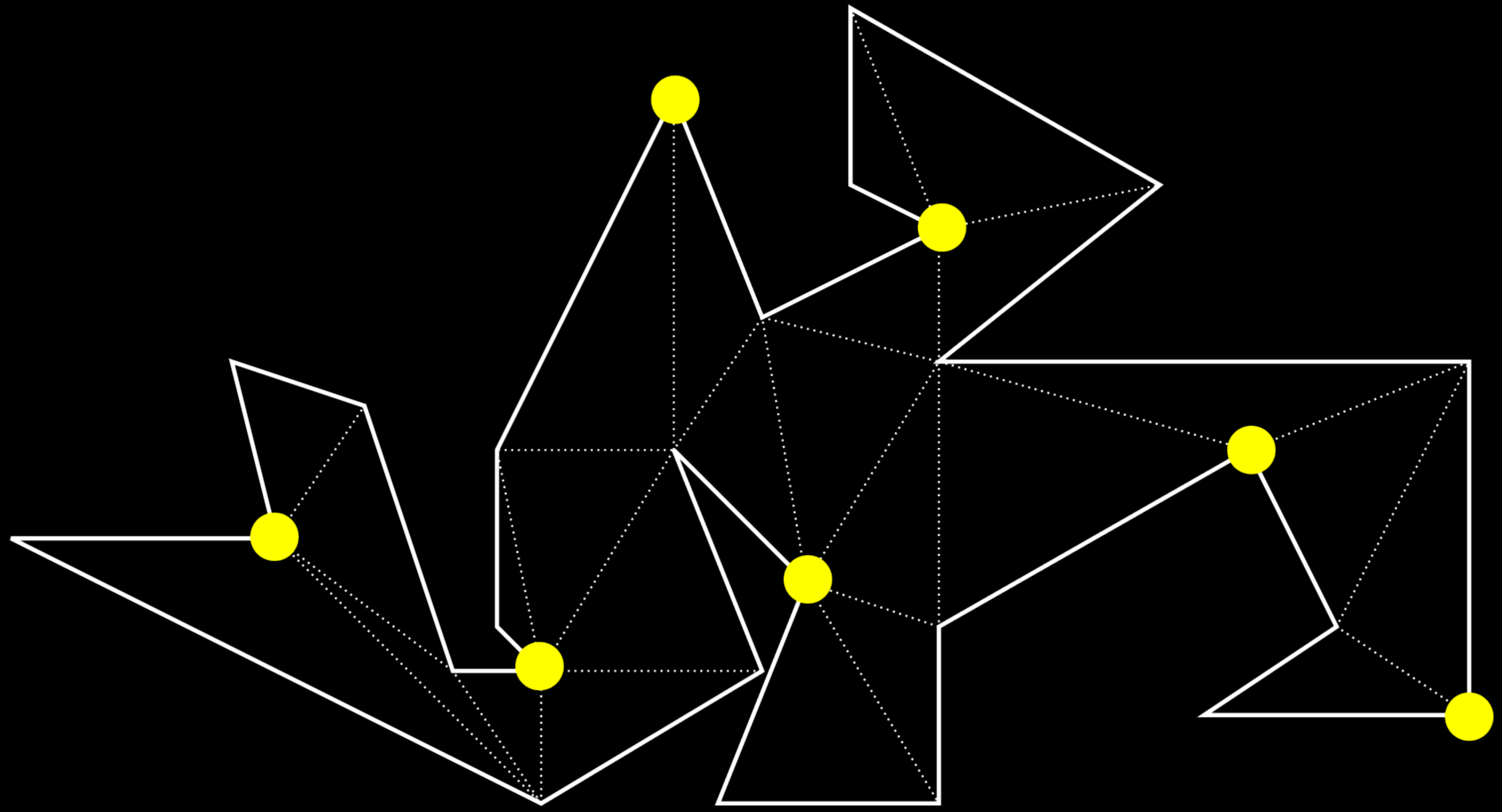

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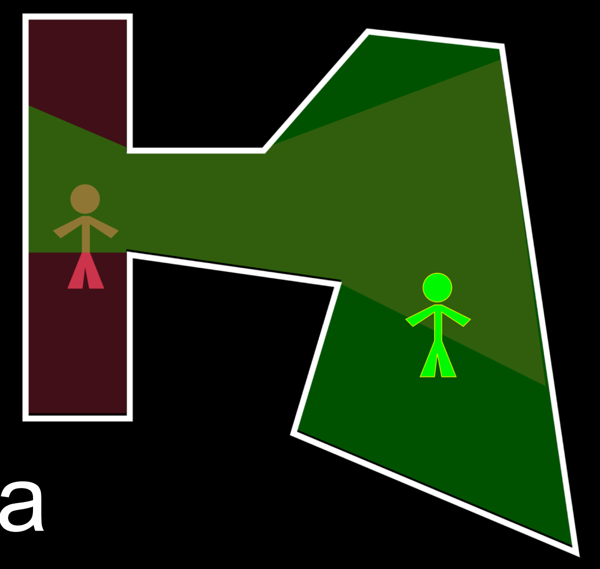

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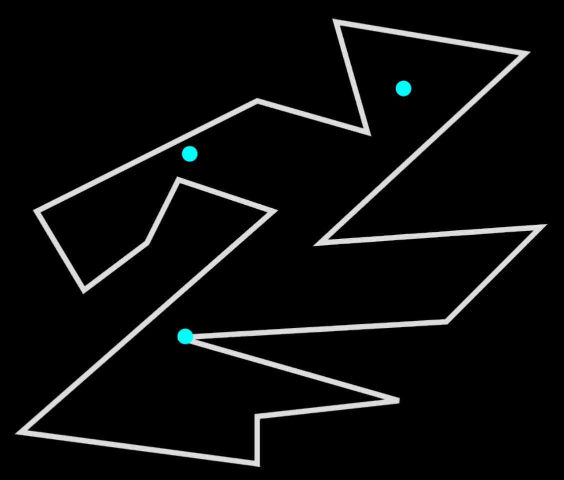
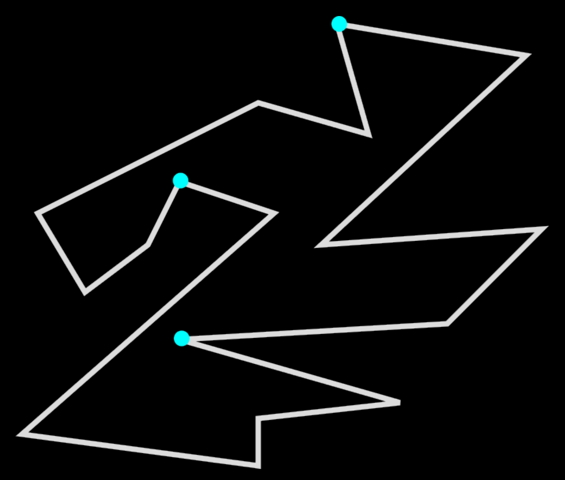
point guards





vertex guards

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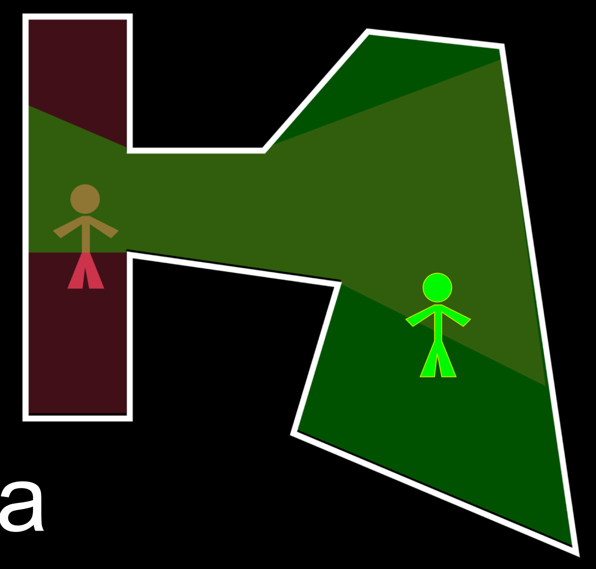

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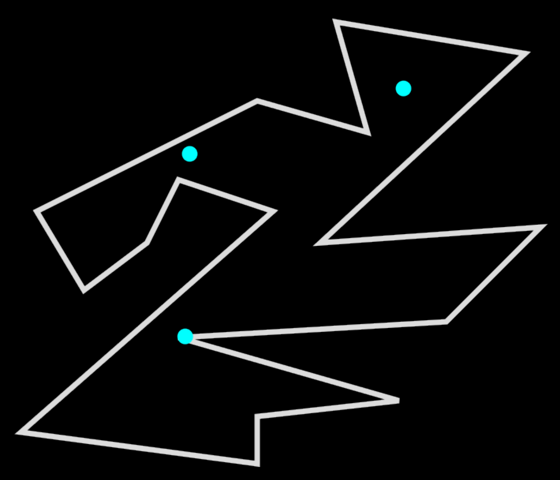
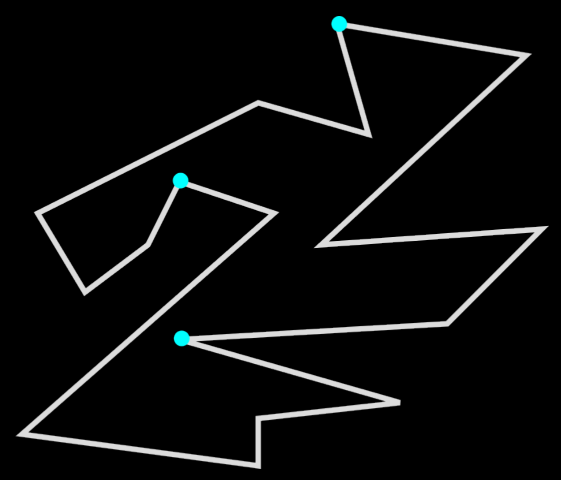
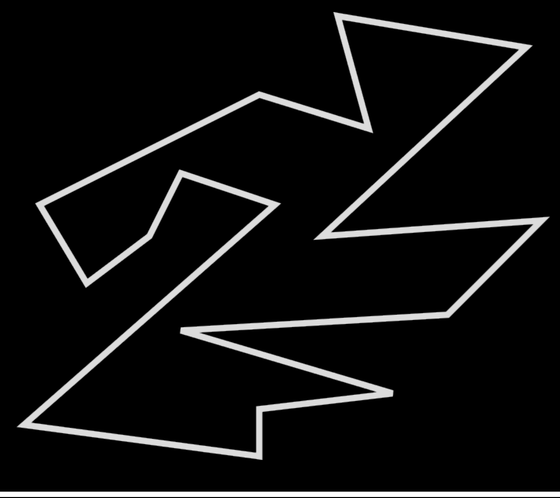




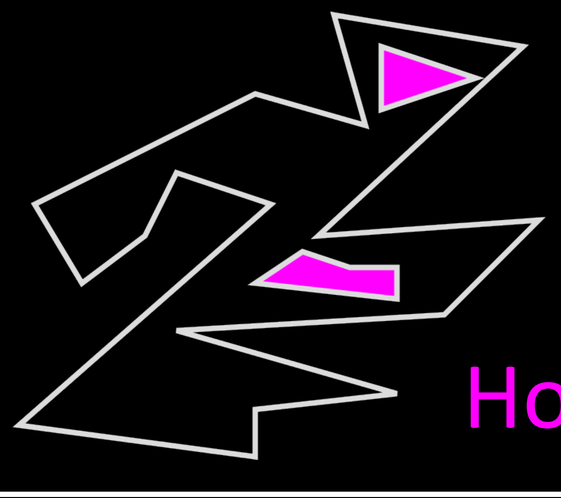
vertex guards

Simple polygon:

- Does not intersect itself
- No holes



$\Longleftrightarrow$



Holes

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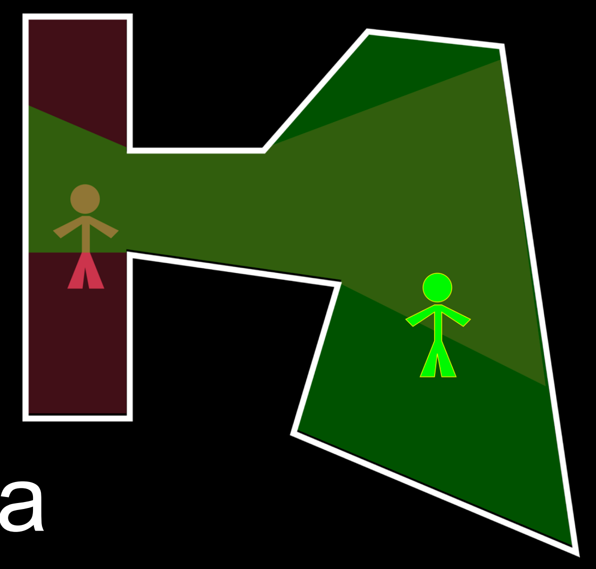

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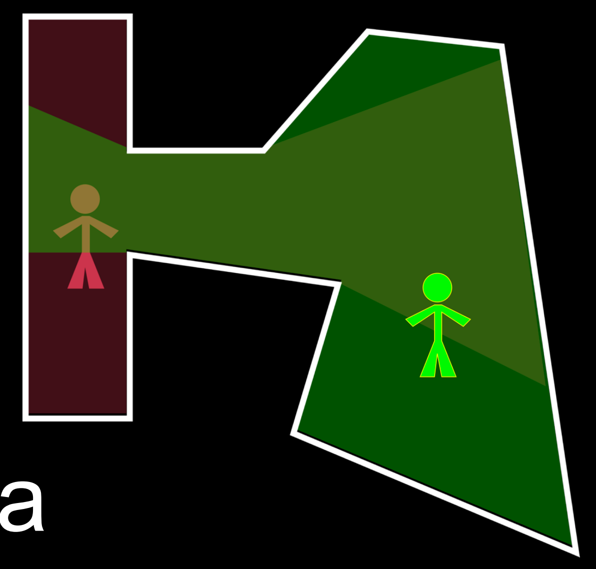

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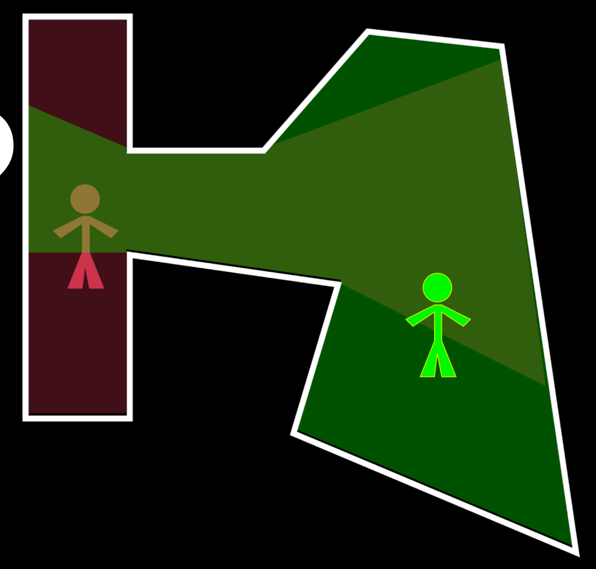
Other structural results

The Art Gallery Problem and Its Variants

The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

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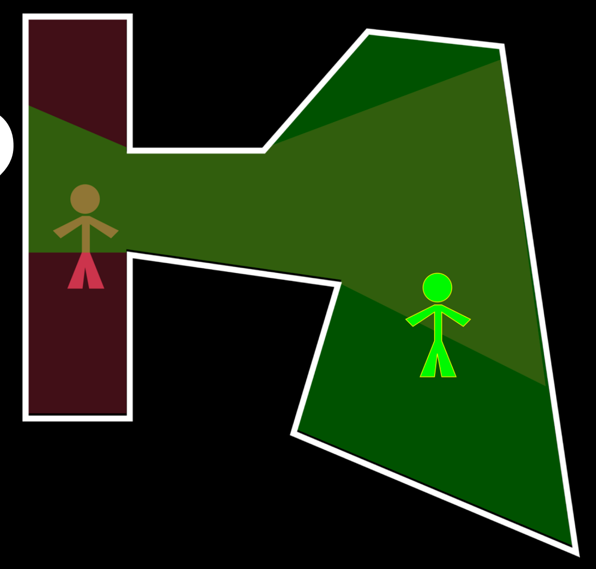

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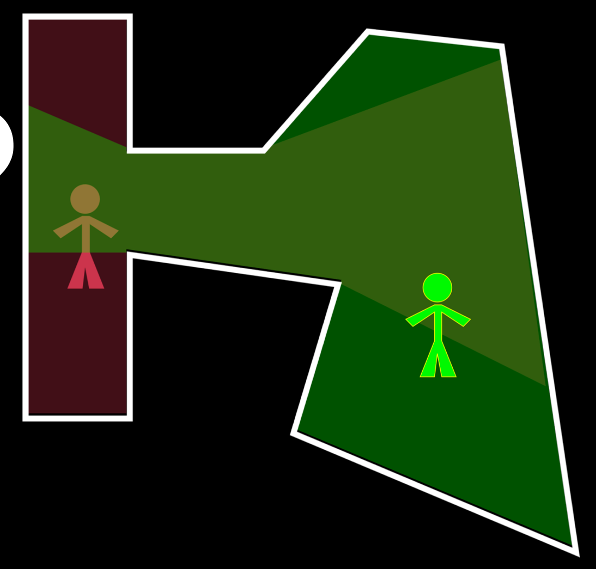

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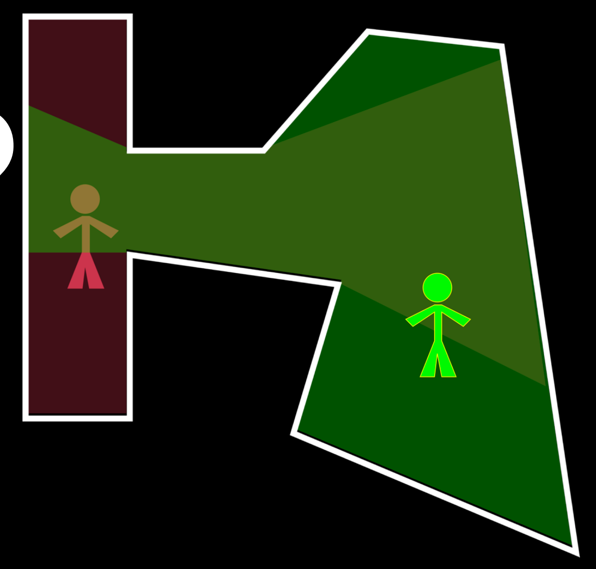

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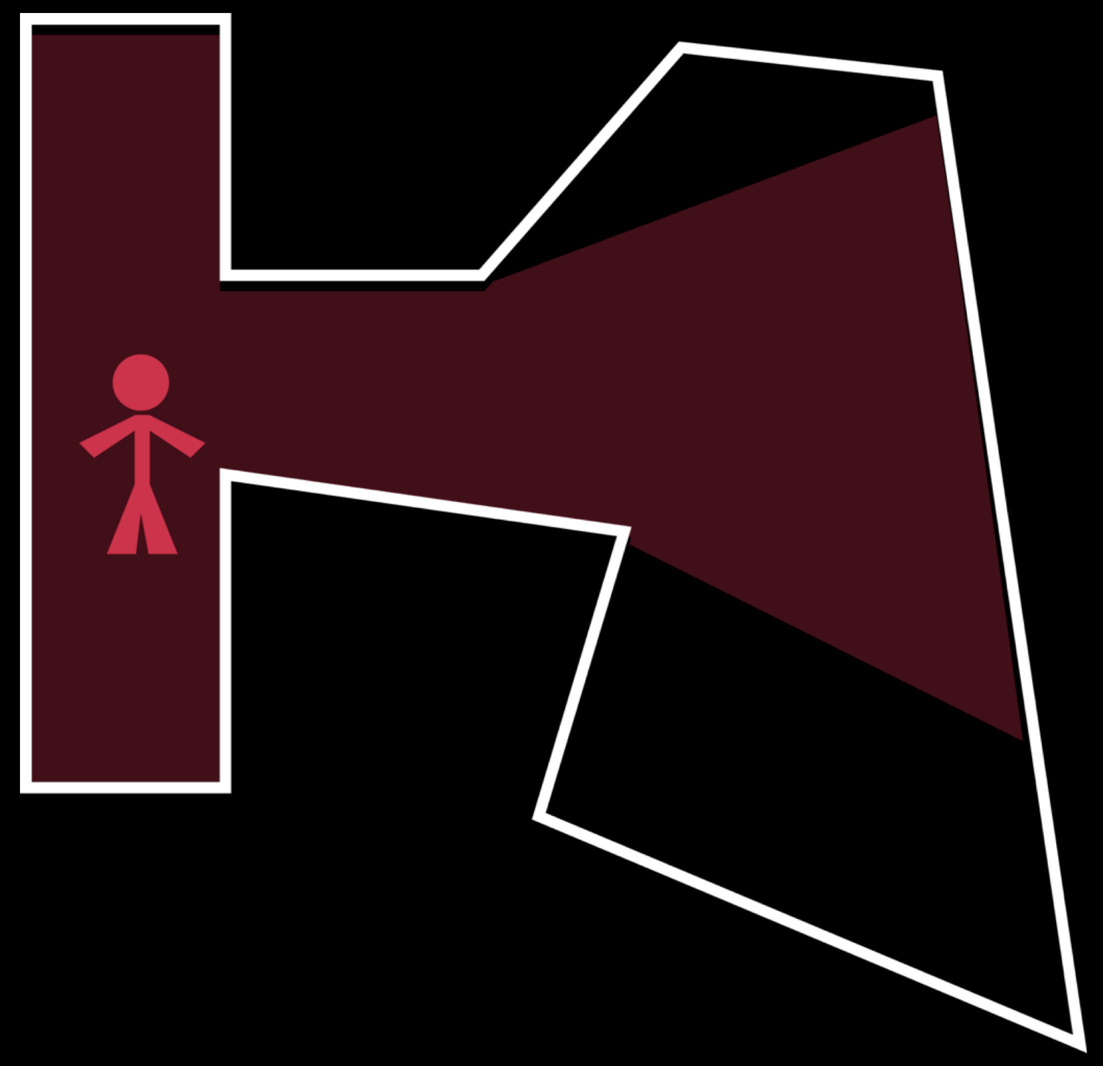

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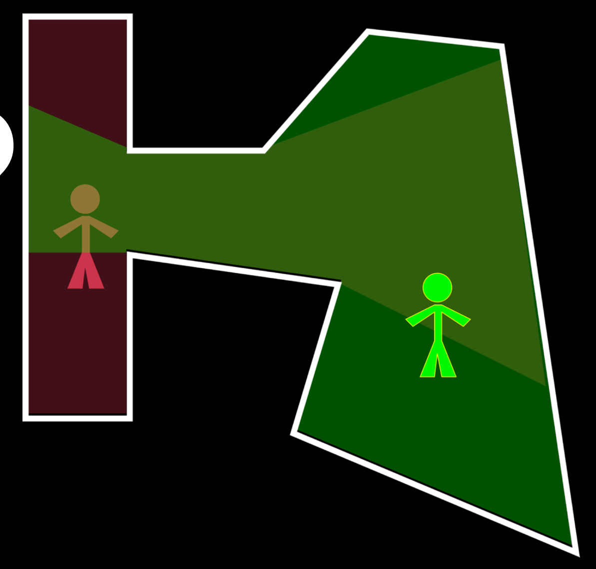

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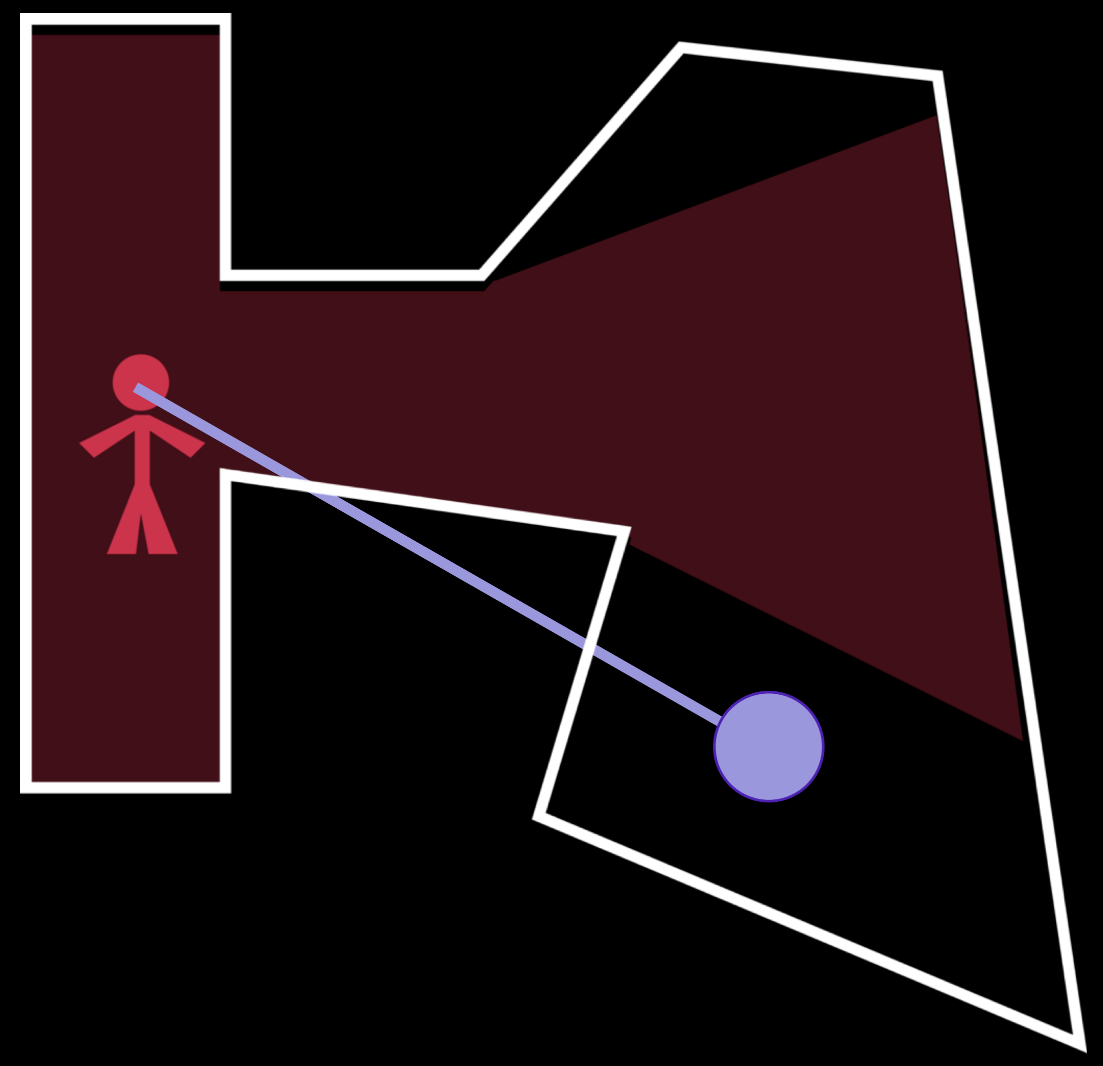

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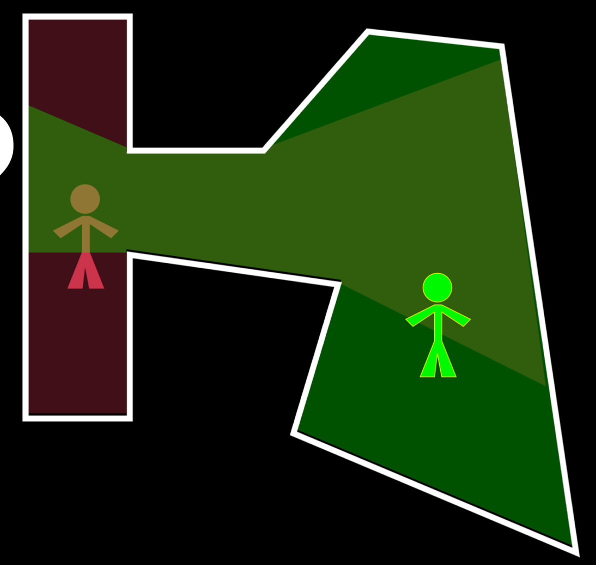

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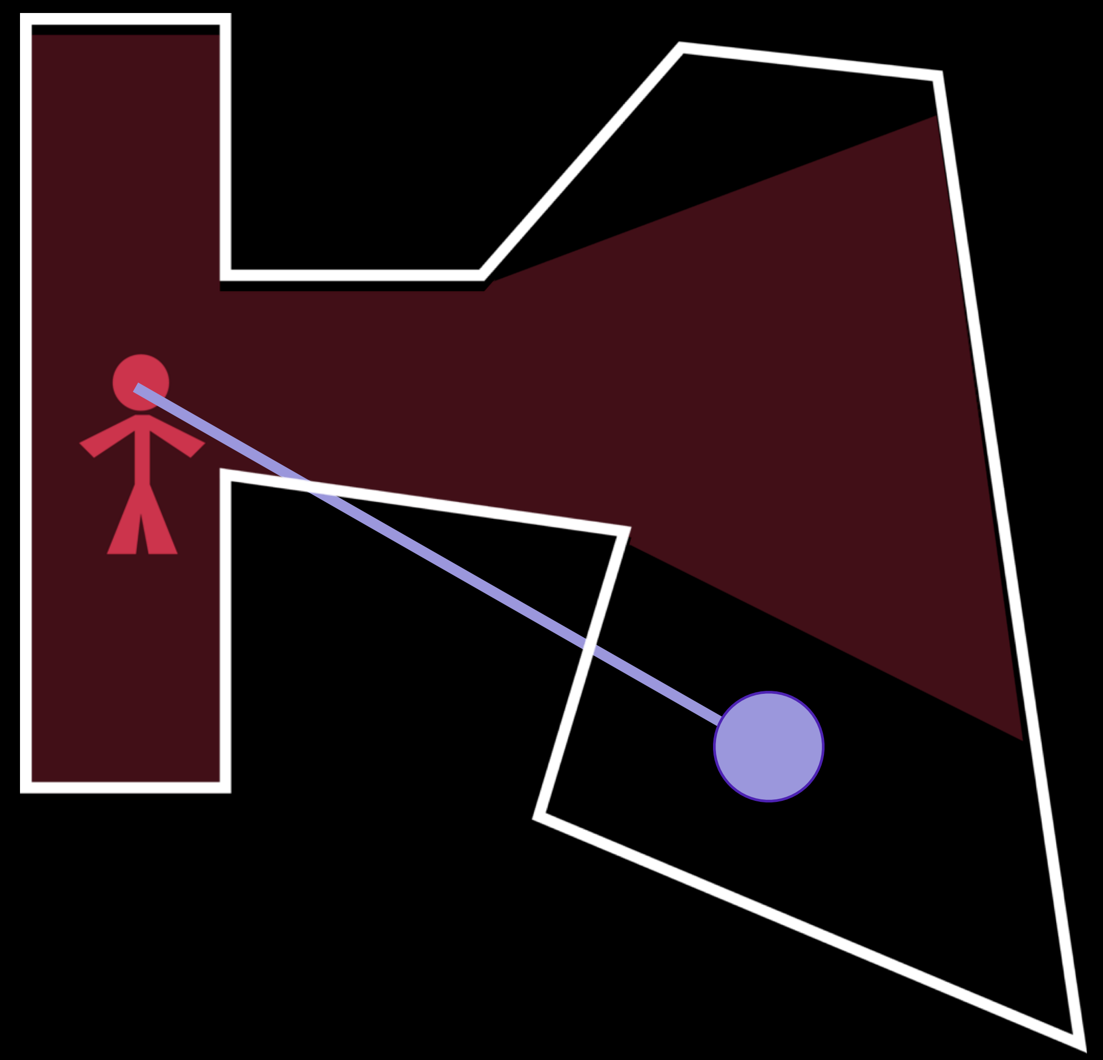

Line crosses at most 2 walls
⇒visible from the 2-transmitter

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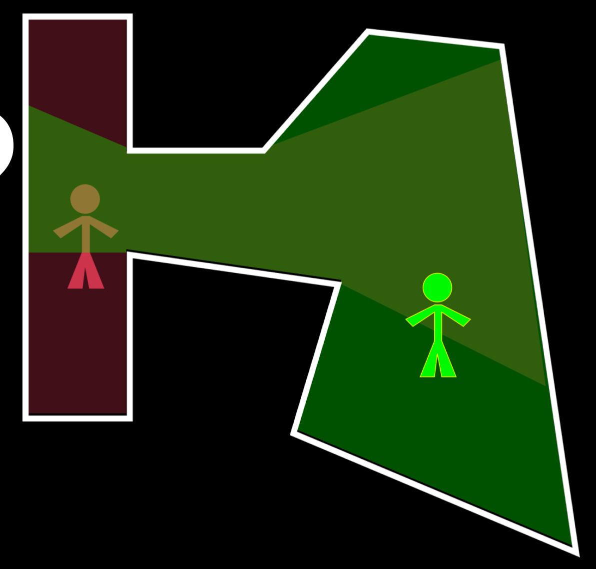

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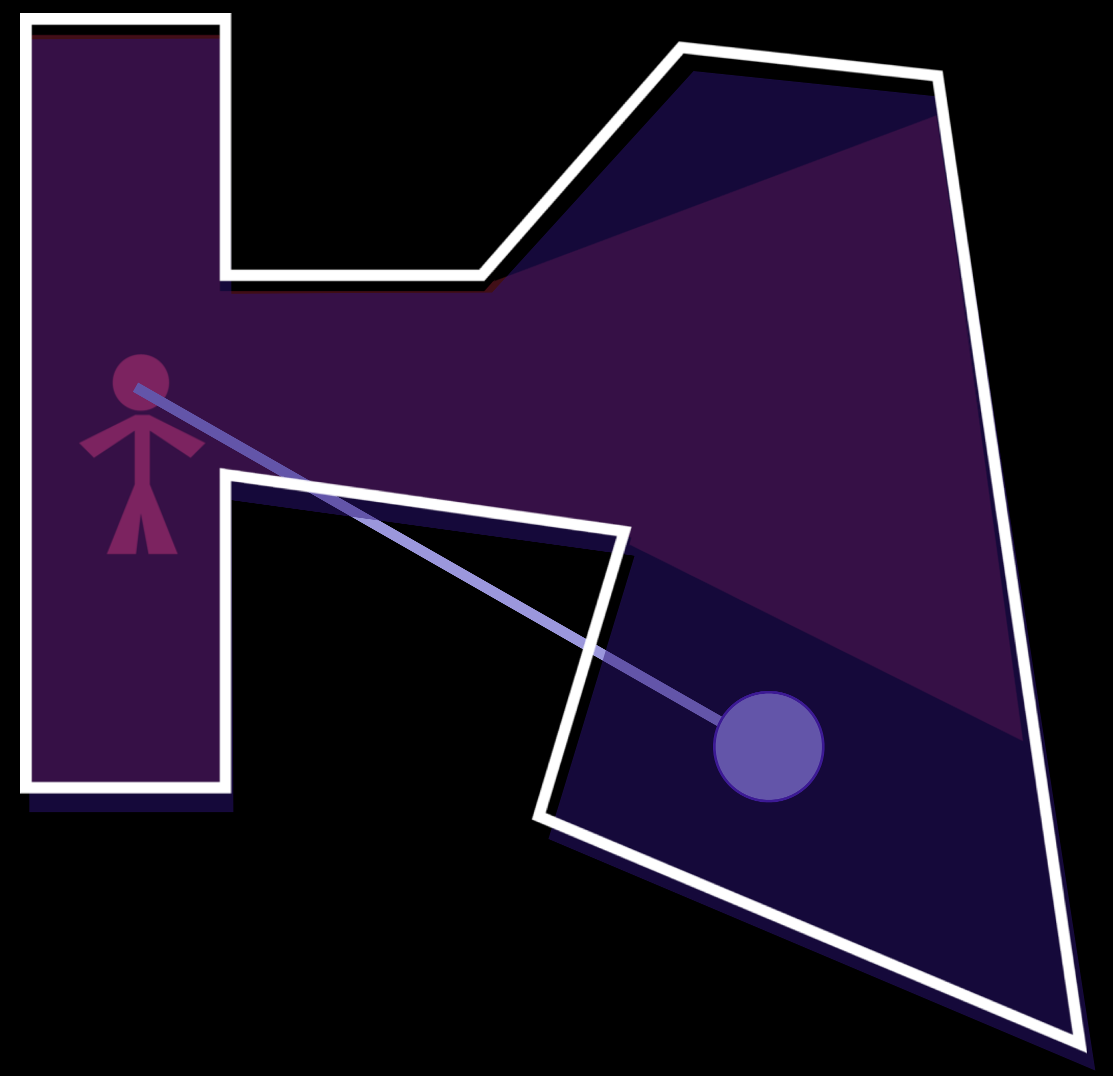

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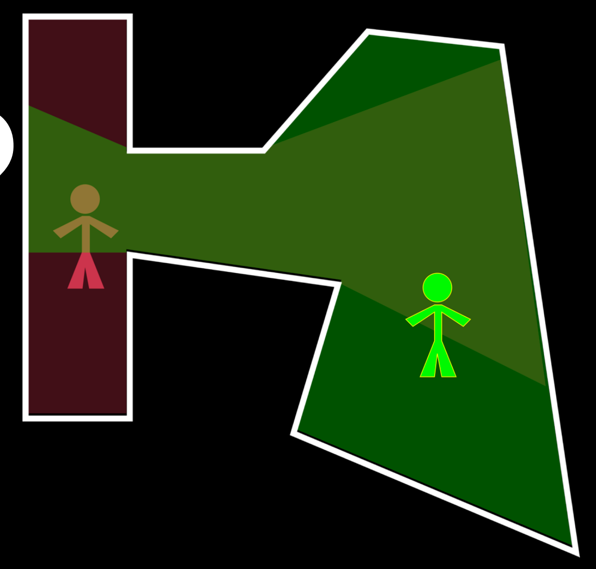

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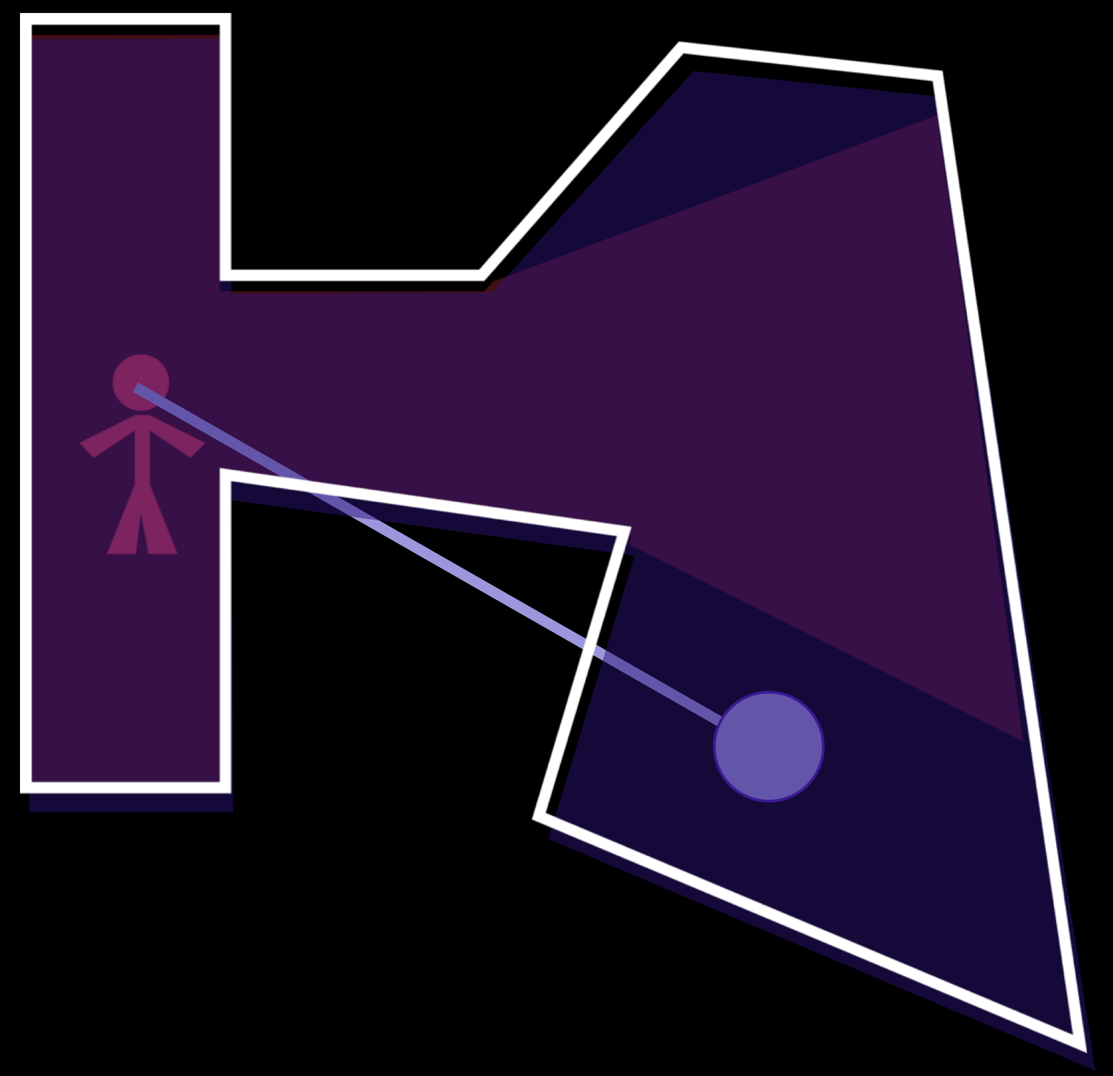

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Fading:

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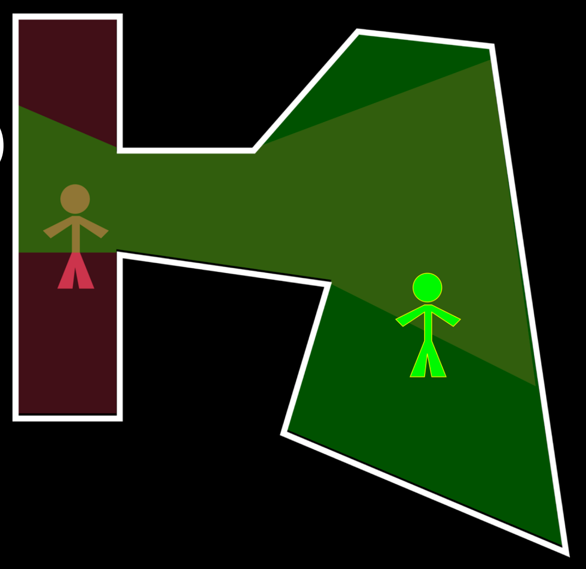

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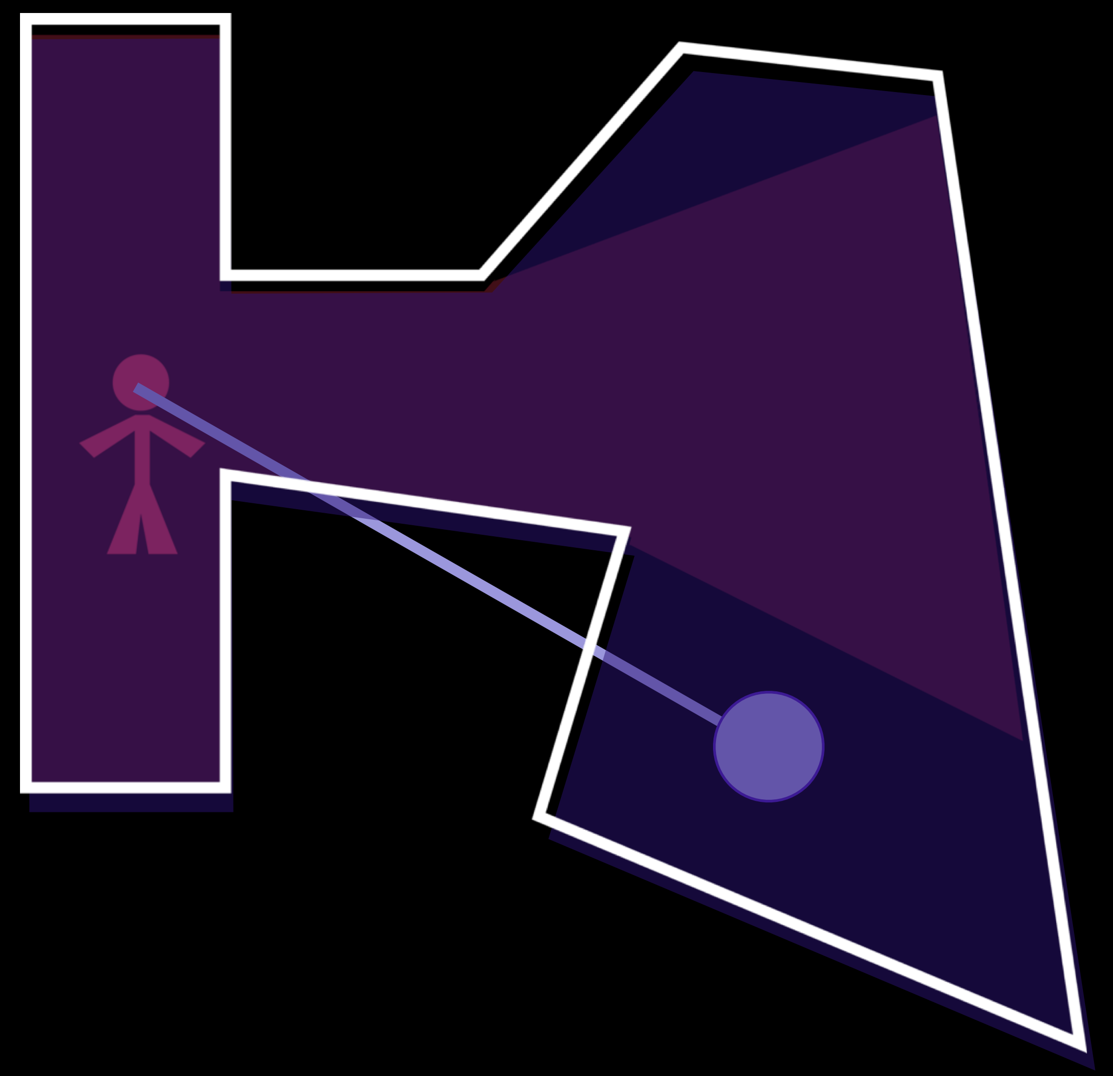

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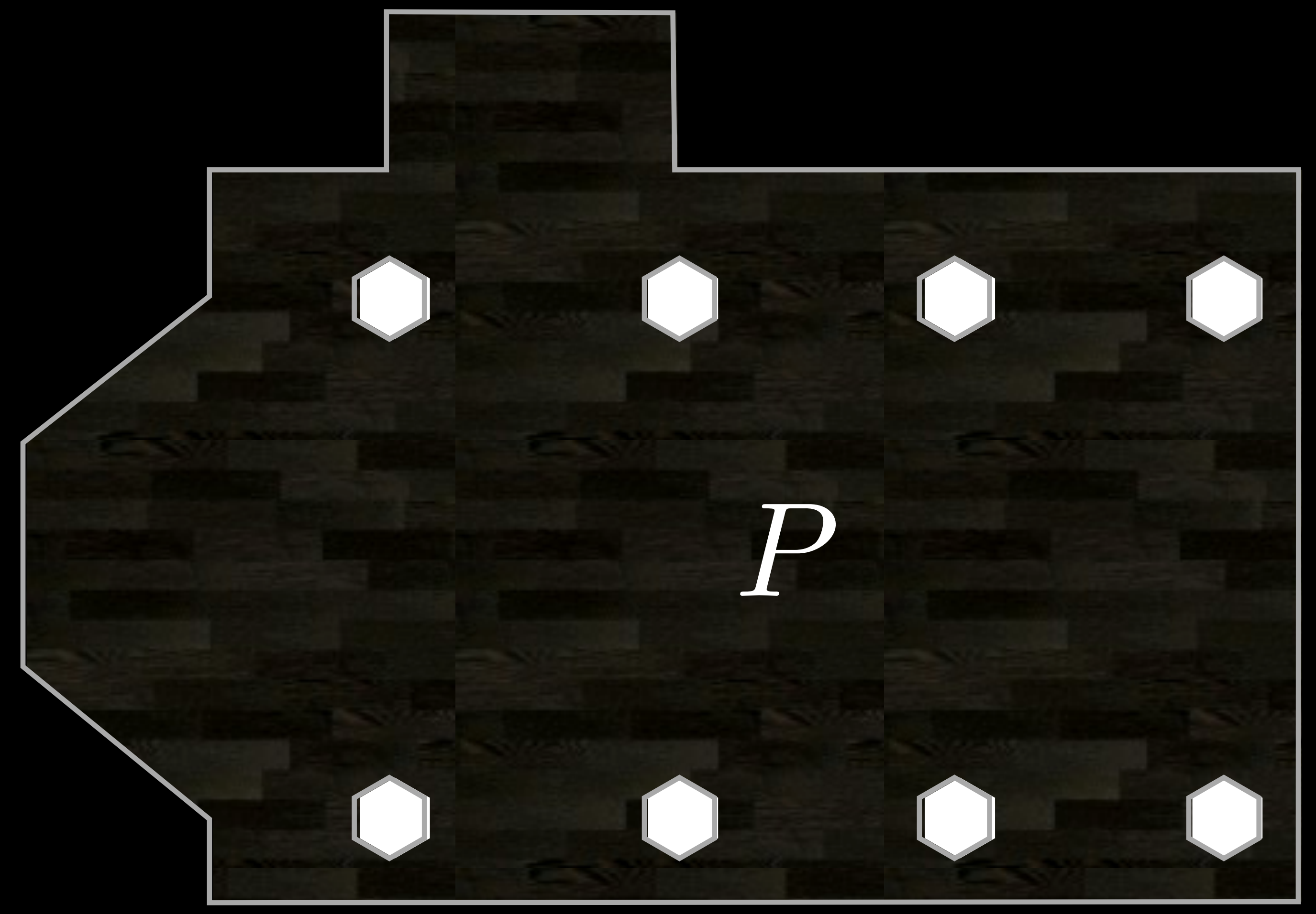

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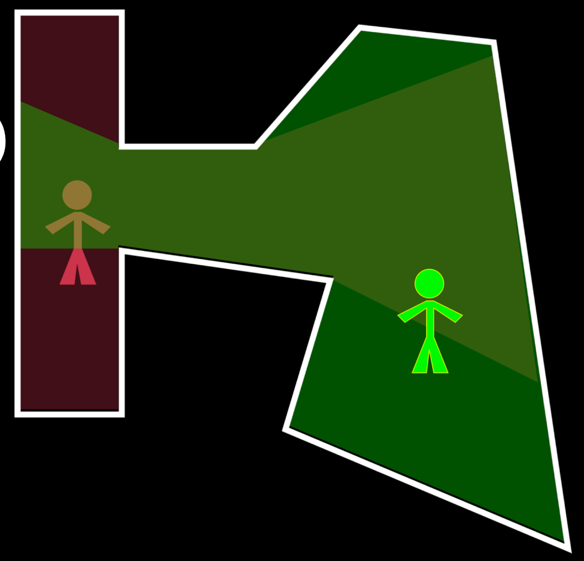

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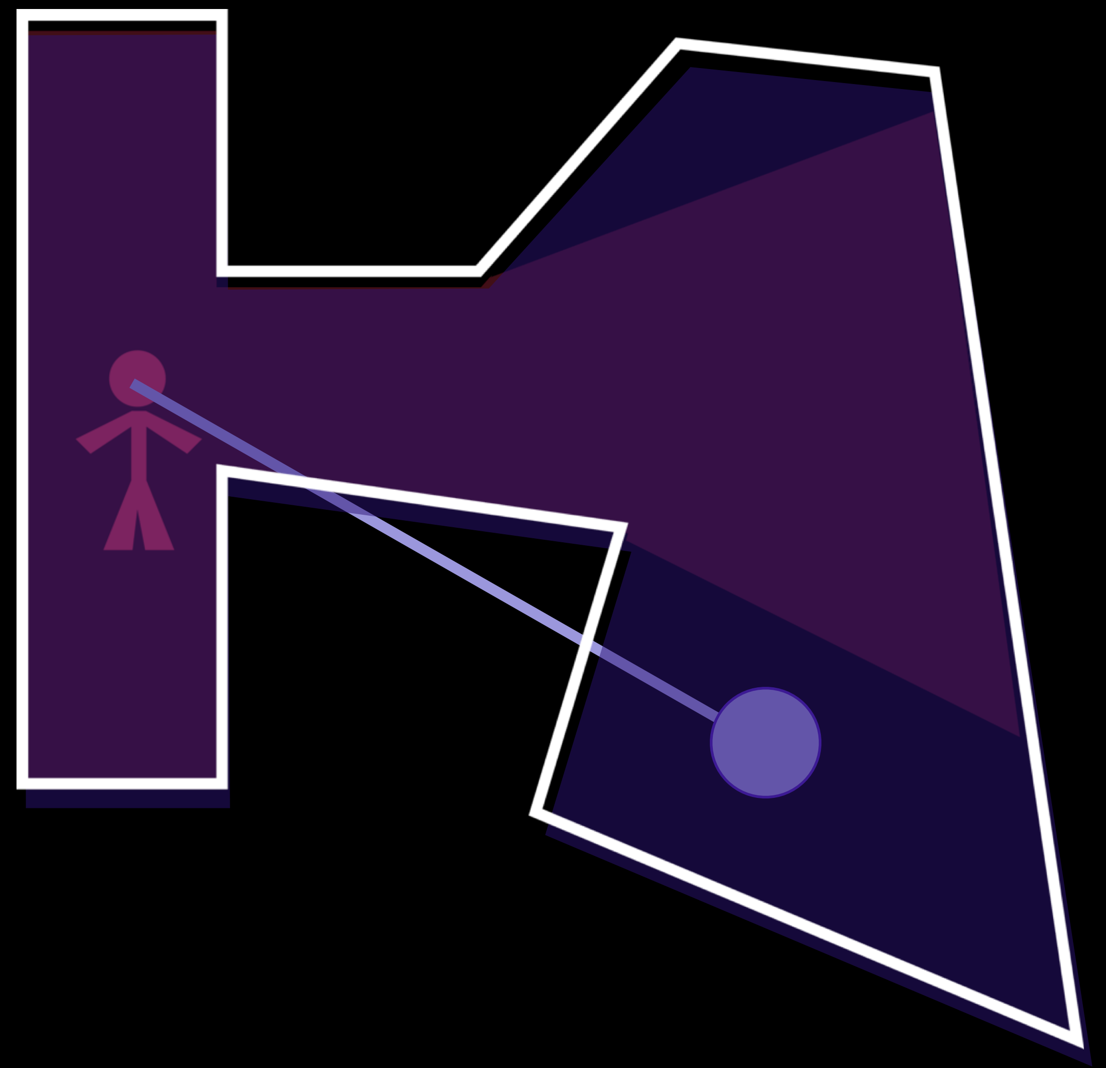

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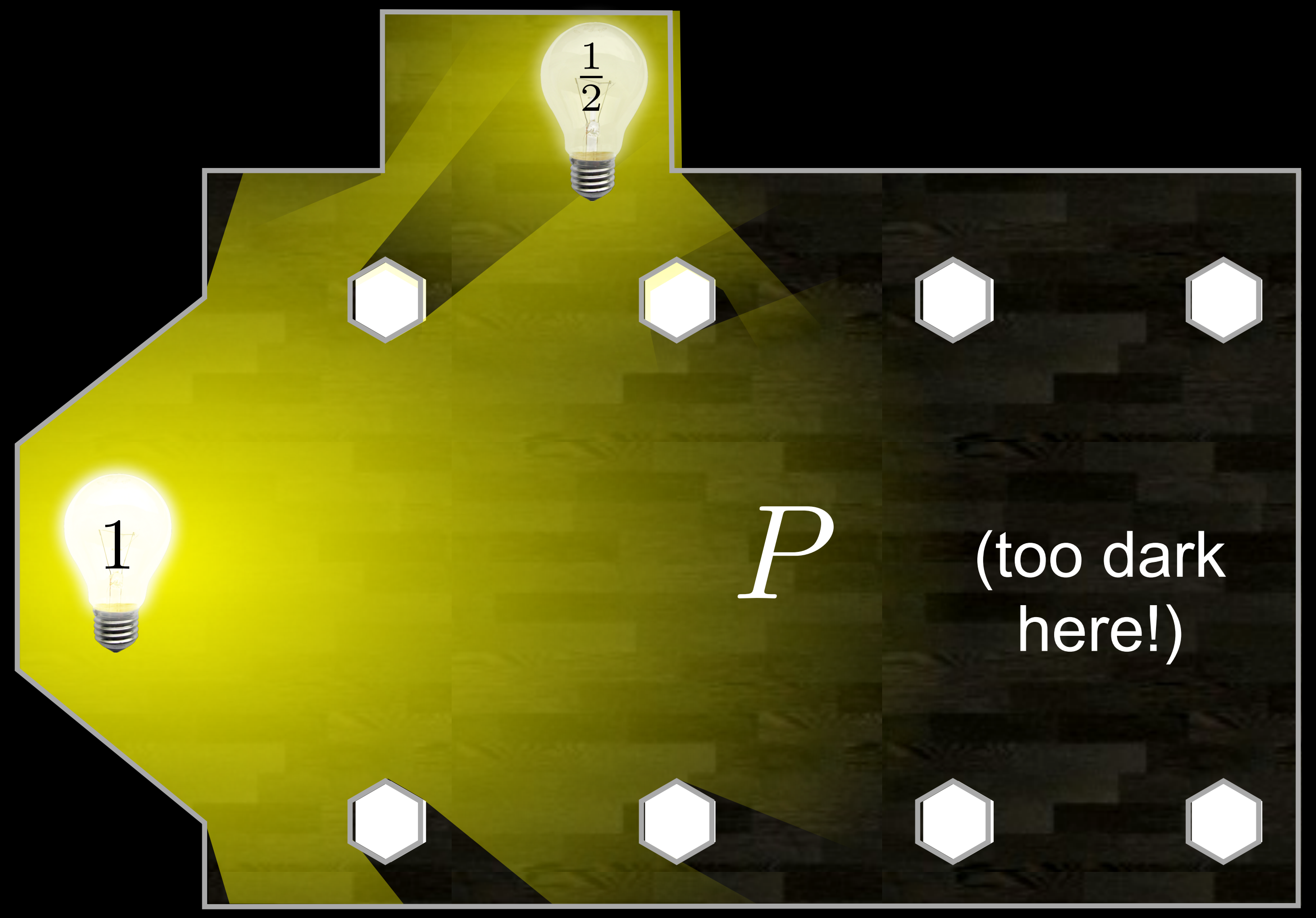

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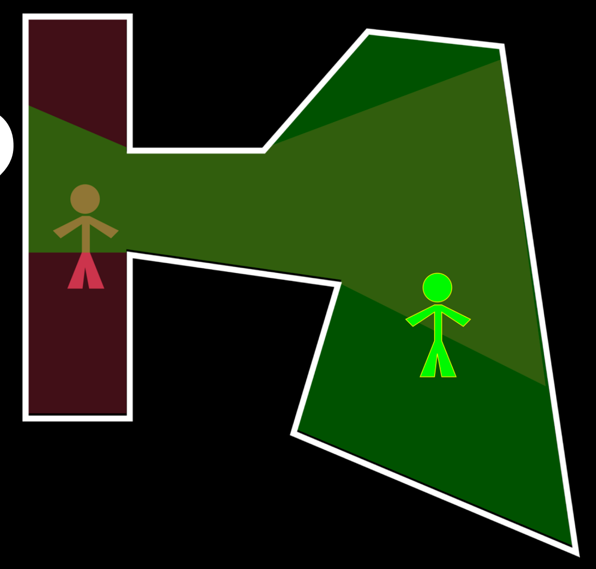

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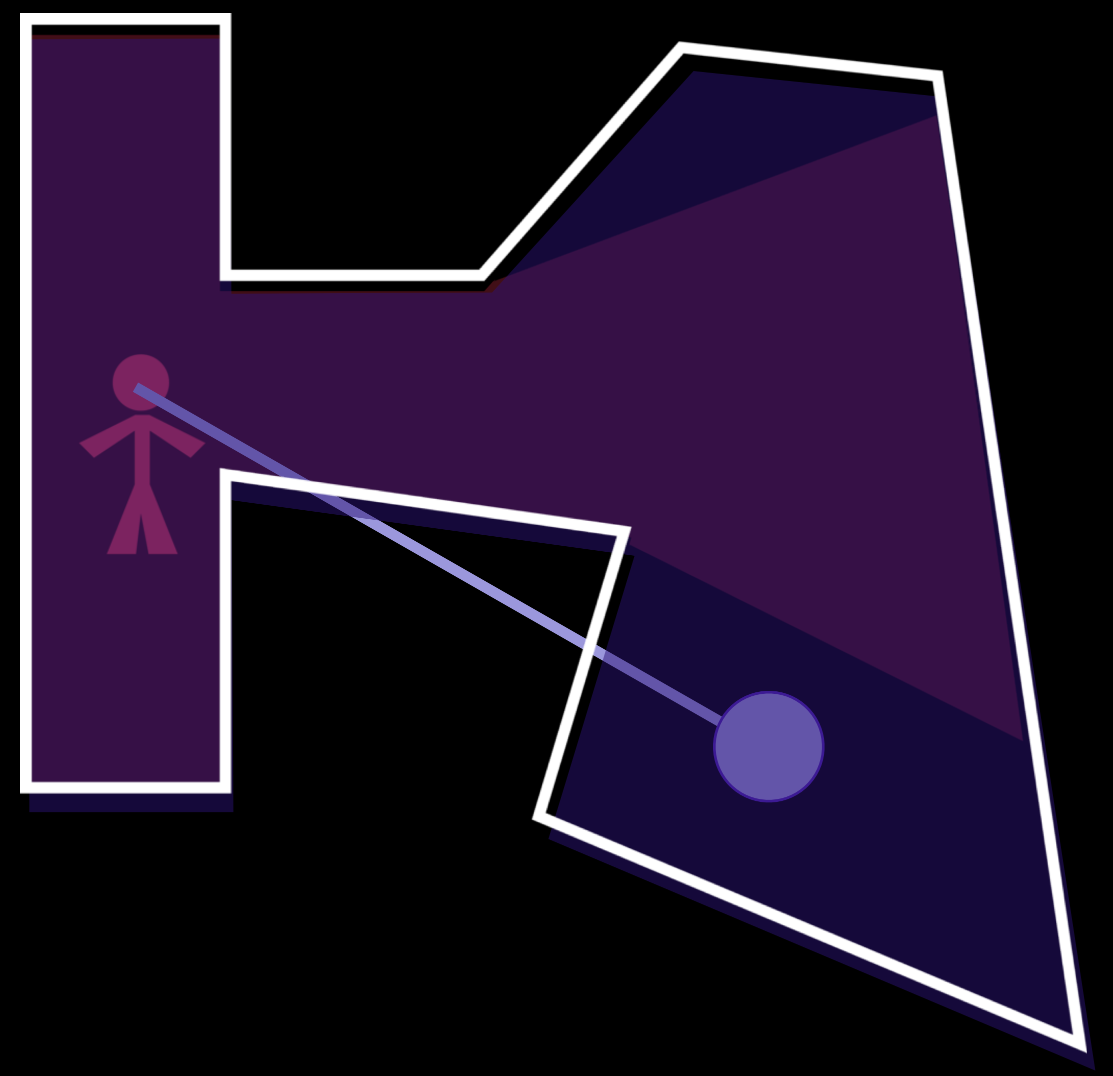

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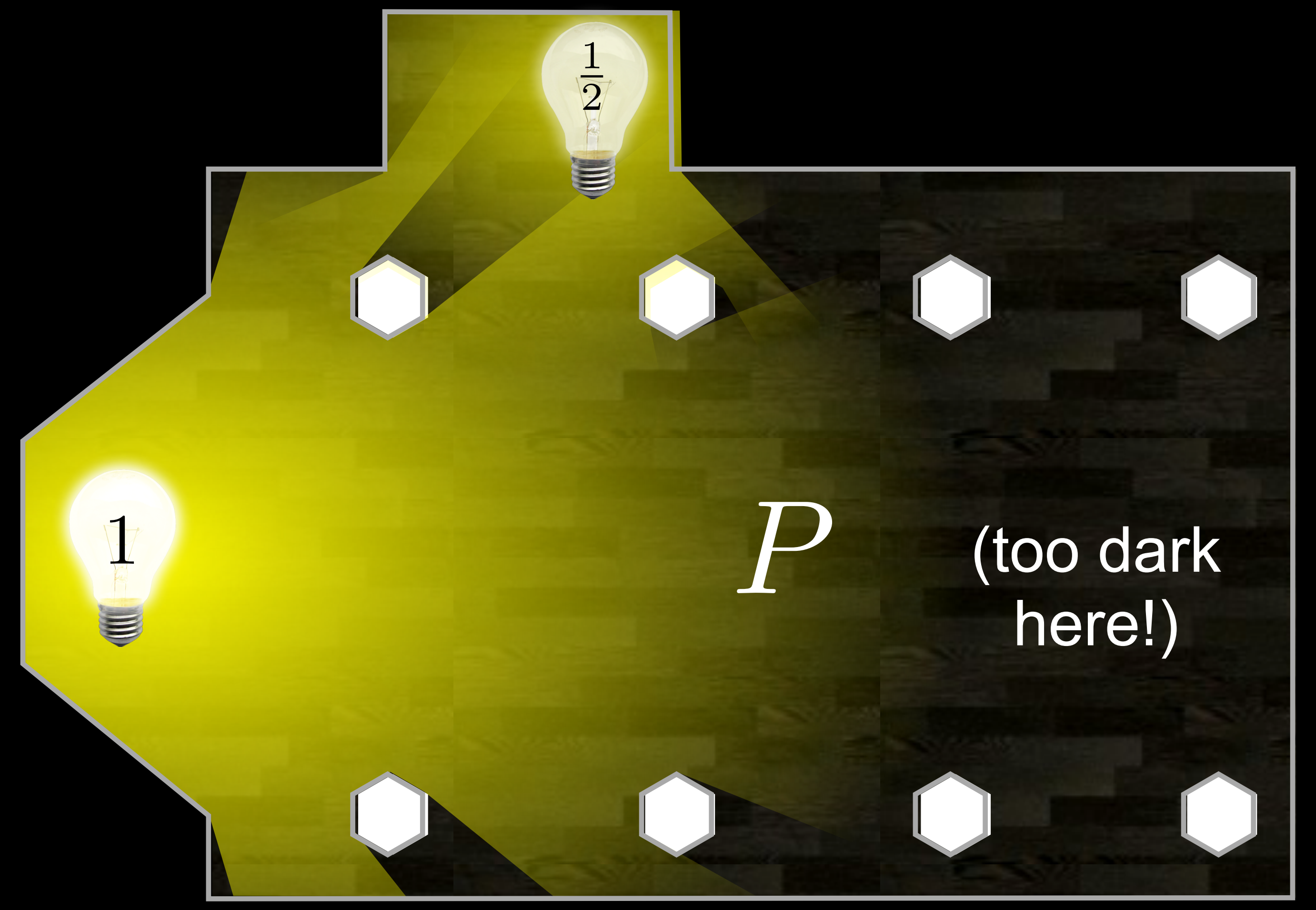

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“Sufficiently” (normalize to 1) light everything — with fading!

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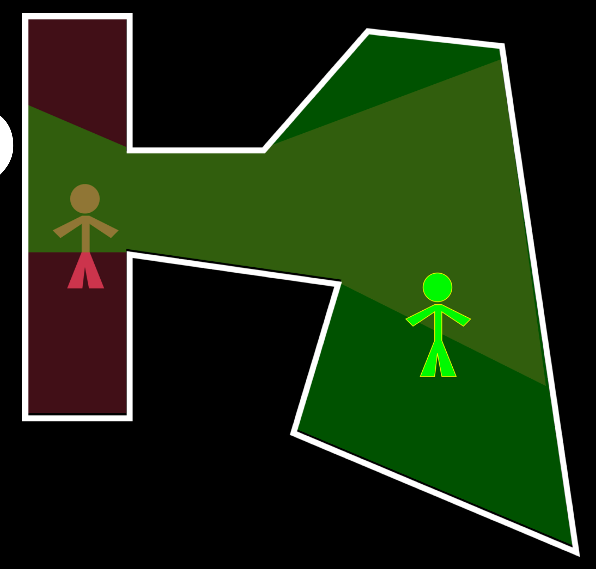

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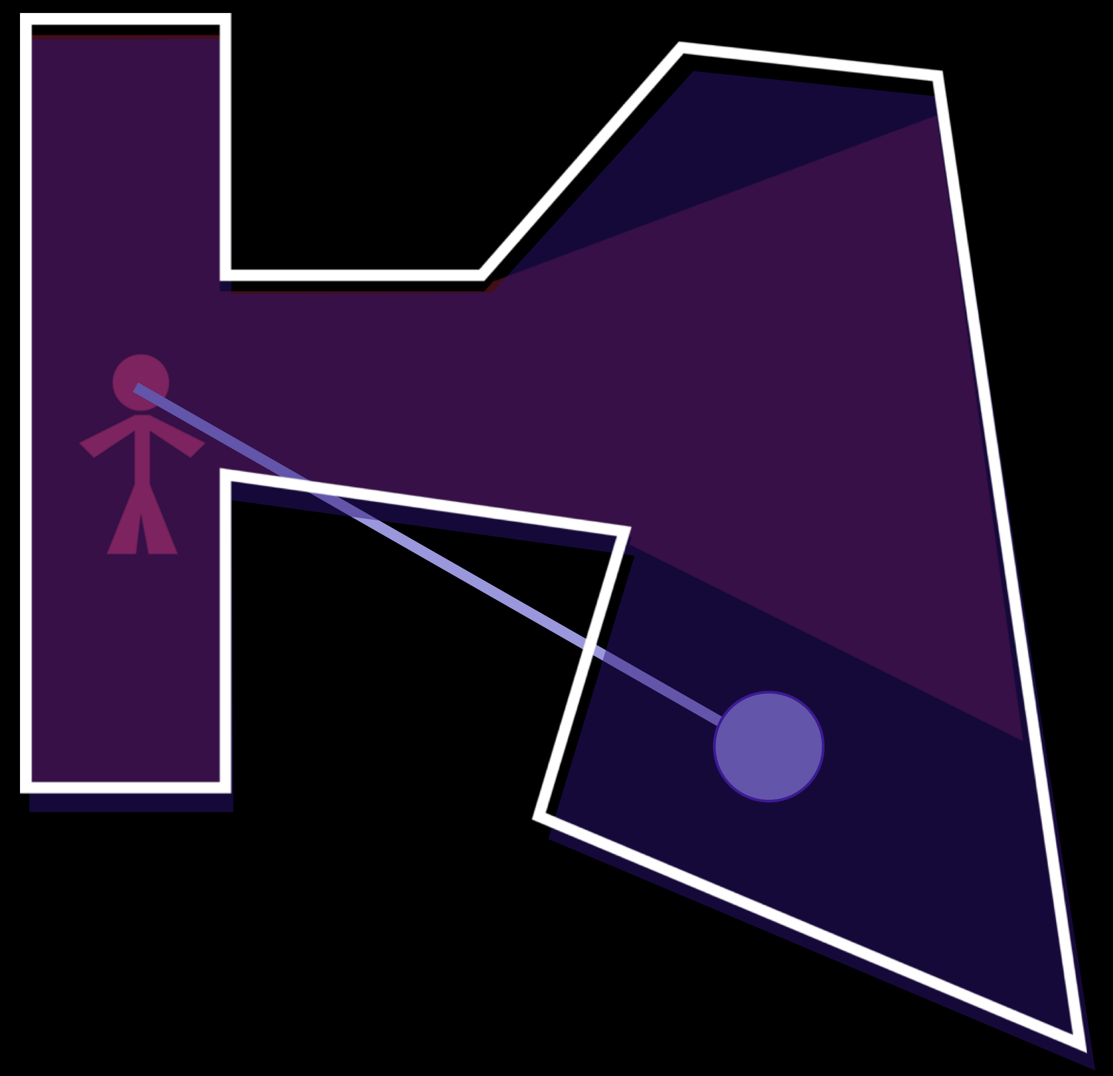
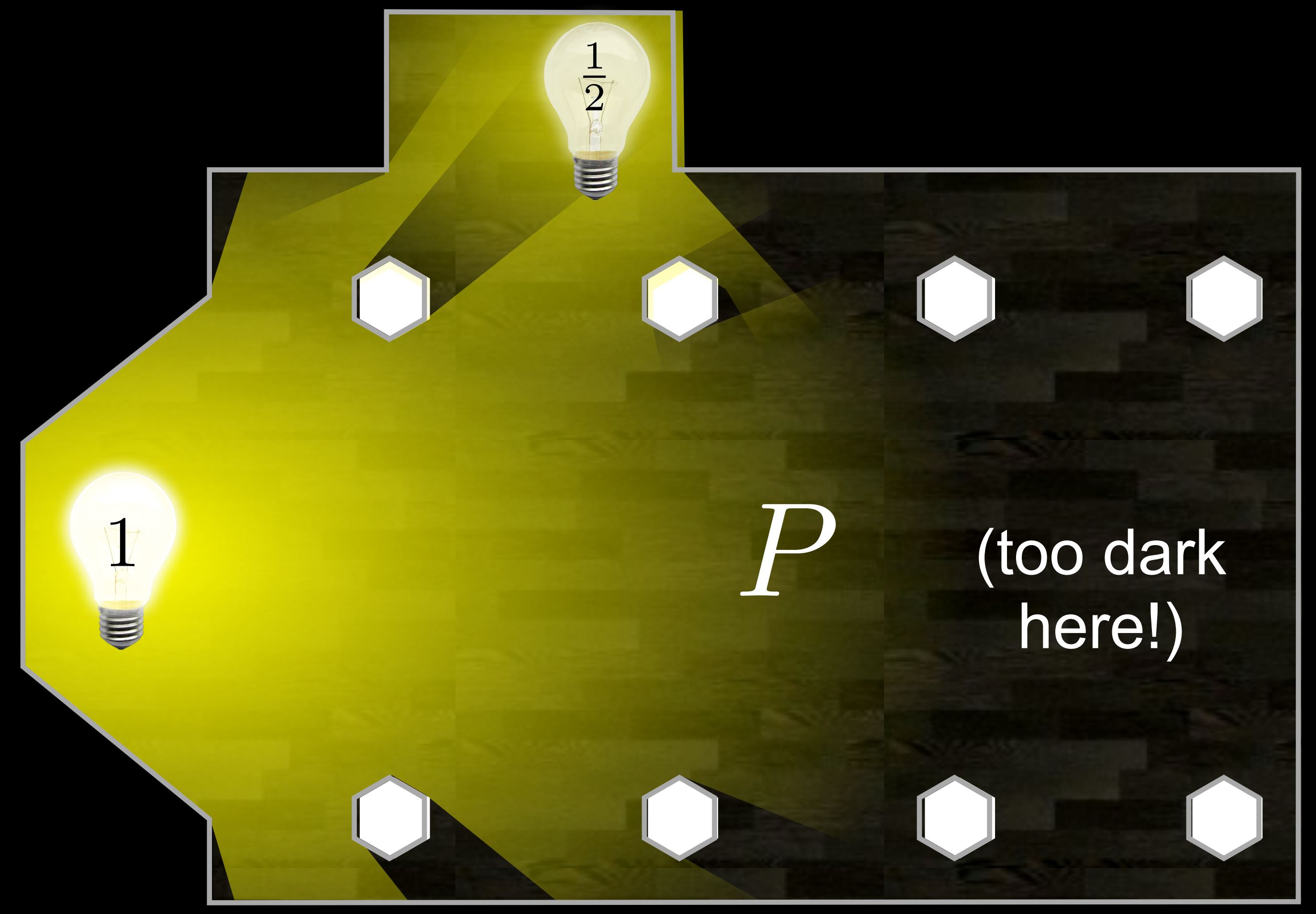
- Environment to be guarded

k-transmitter:



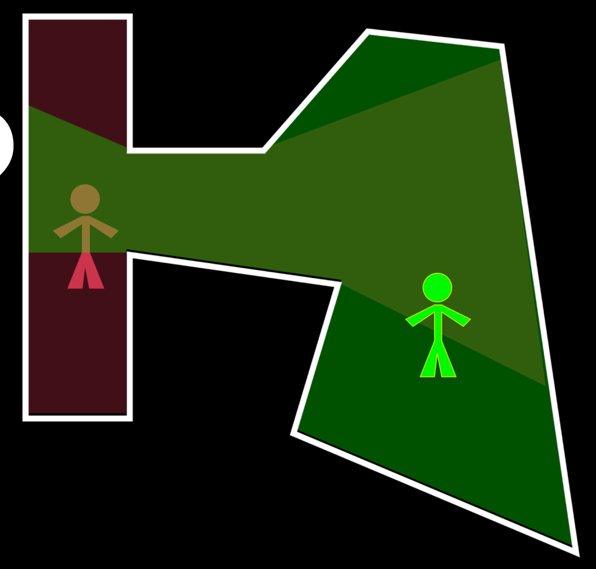
Line crosses at most 2 walls
⇒visible from the 2-transmitter

Fading:



Place lights,
assign energy (= brightness).
“Sufficiently” (normalize to 1) light everything — with fading!
Minimize total energy.

The Art Gallery Problem (AGP)—and Its Variants?



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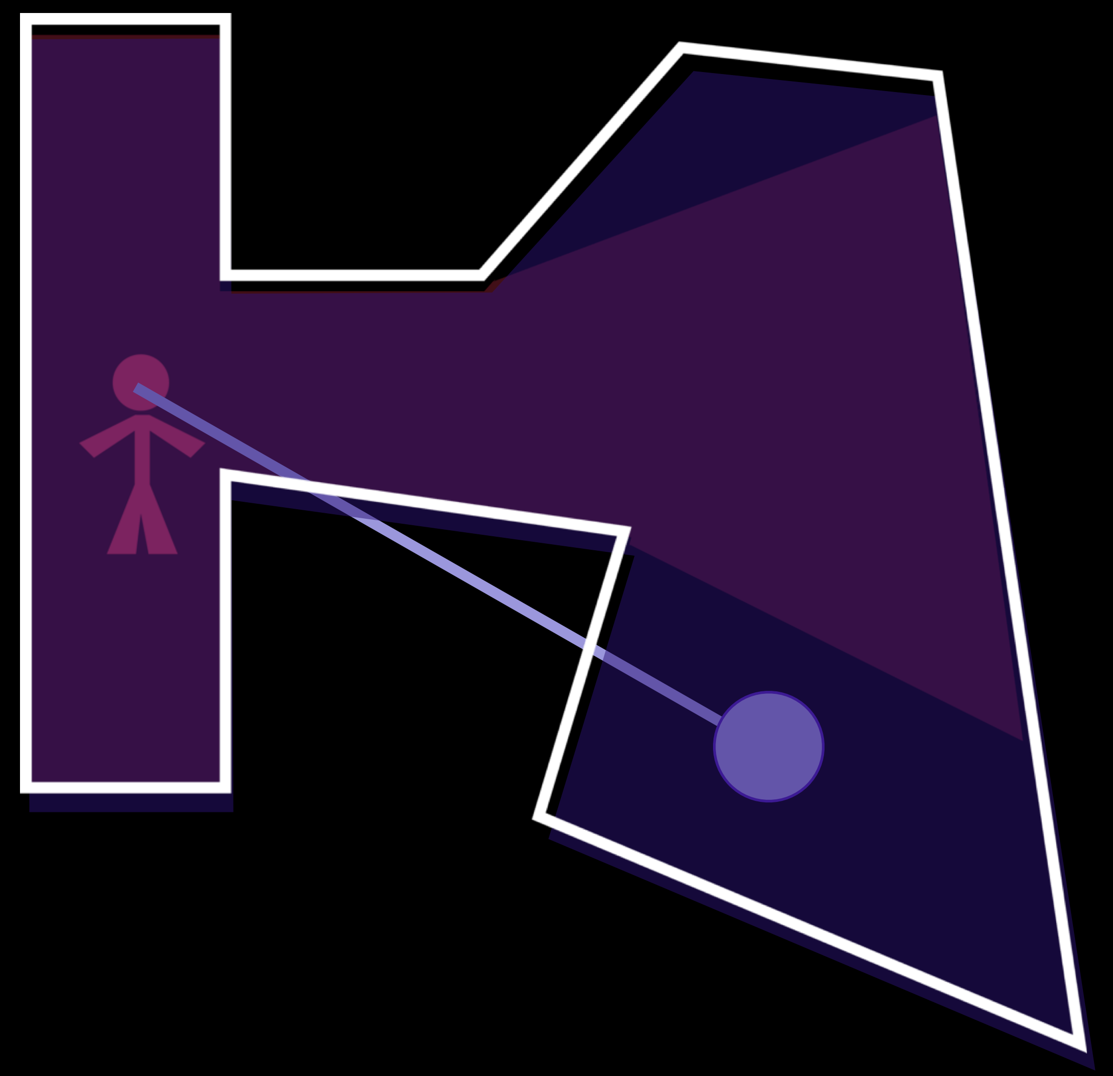

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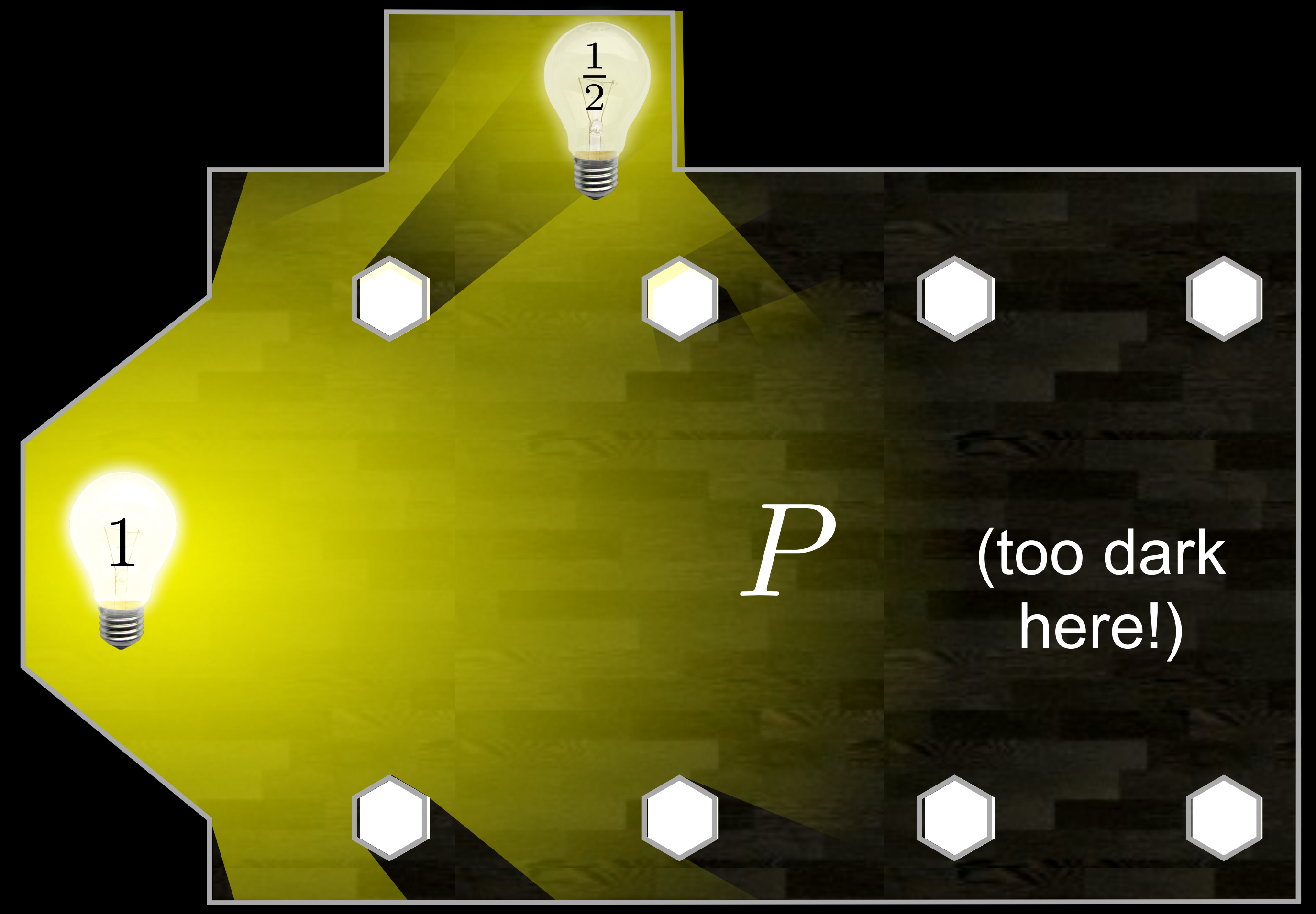

Place lights,
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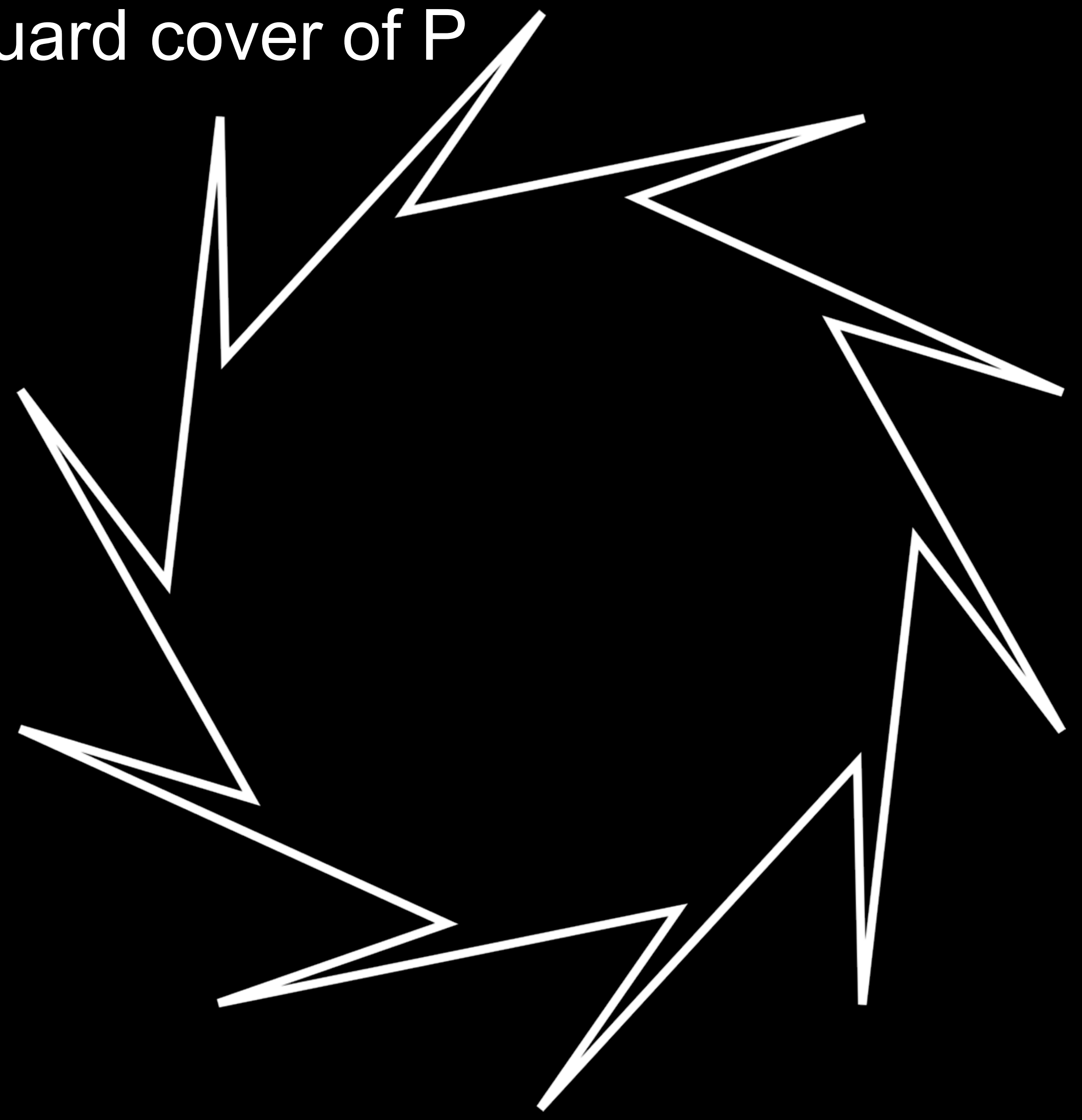
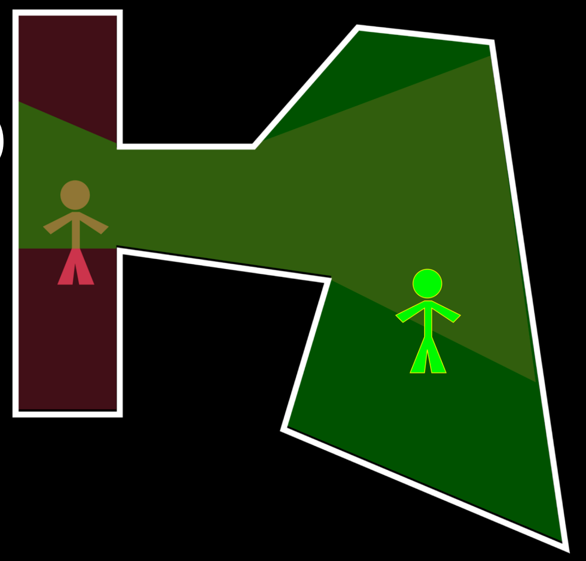
Chromatic AGP:

Given: a polygon P

Task: find a min guard cover of P



The Art Gallery Problem (AGP)—and Its Variants?



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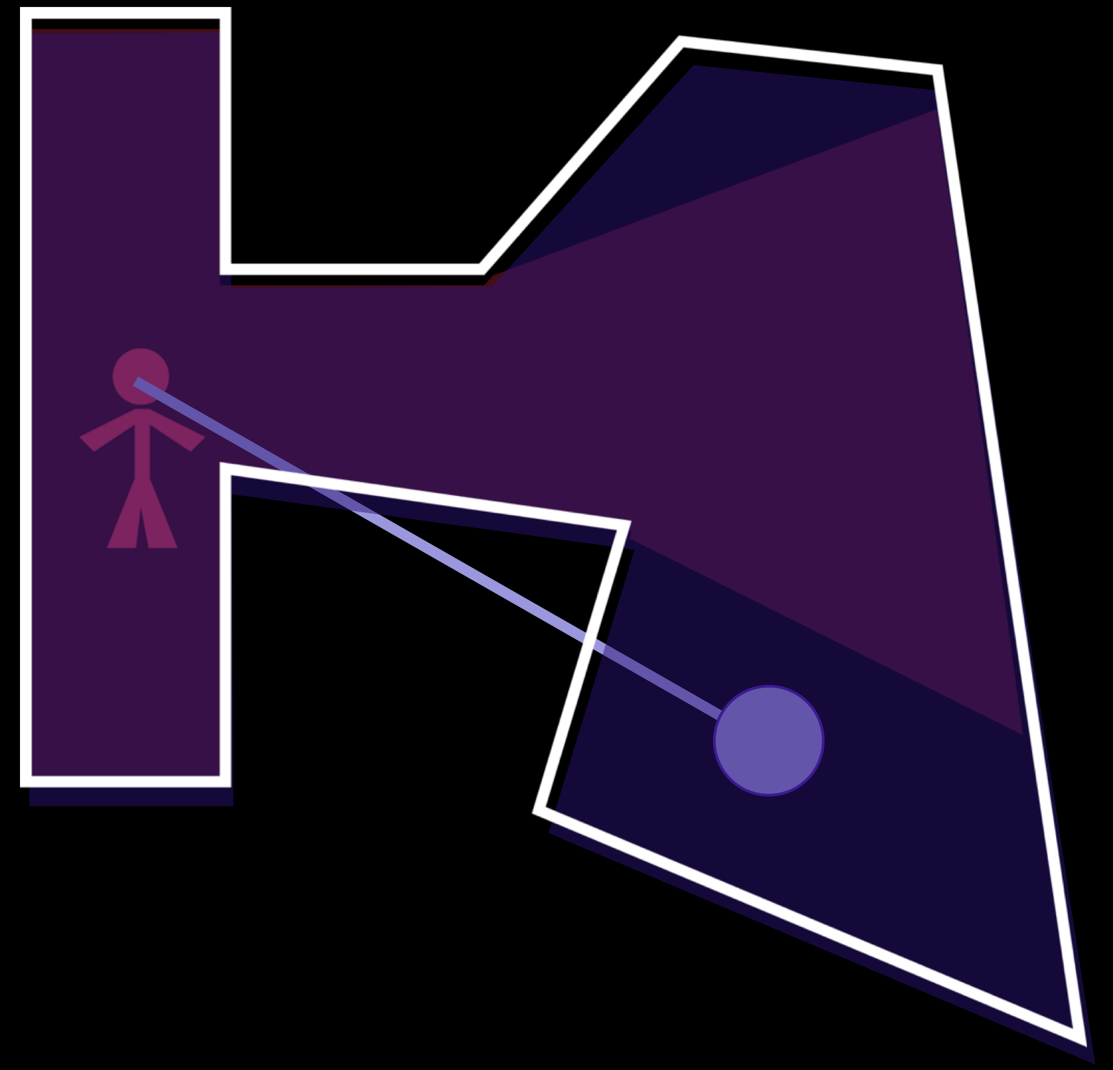

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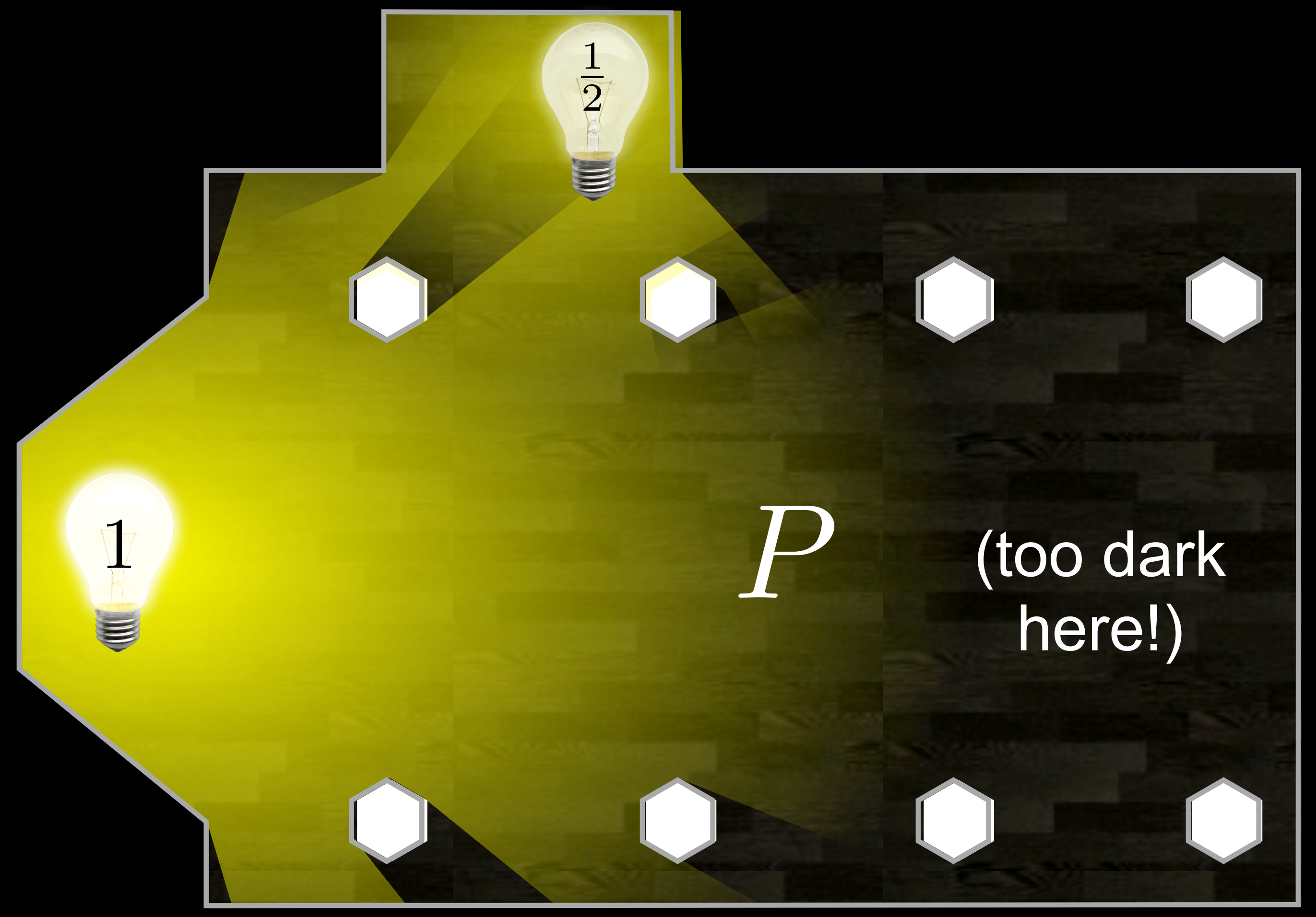

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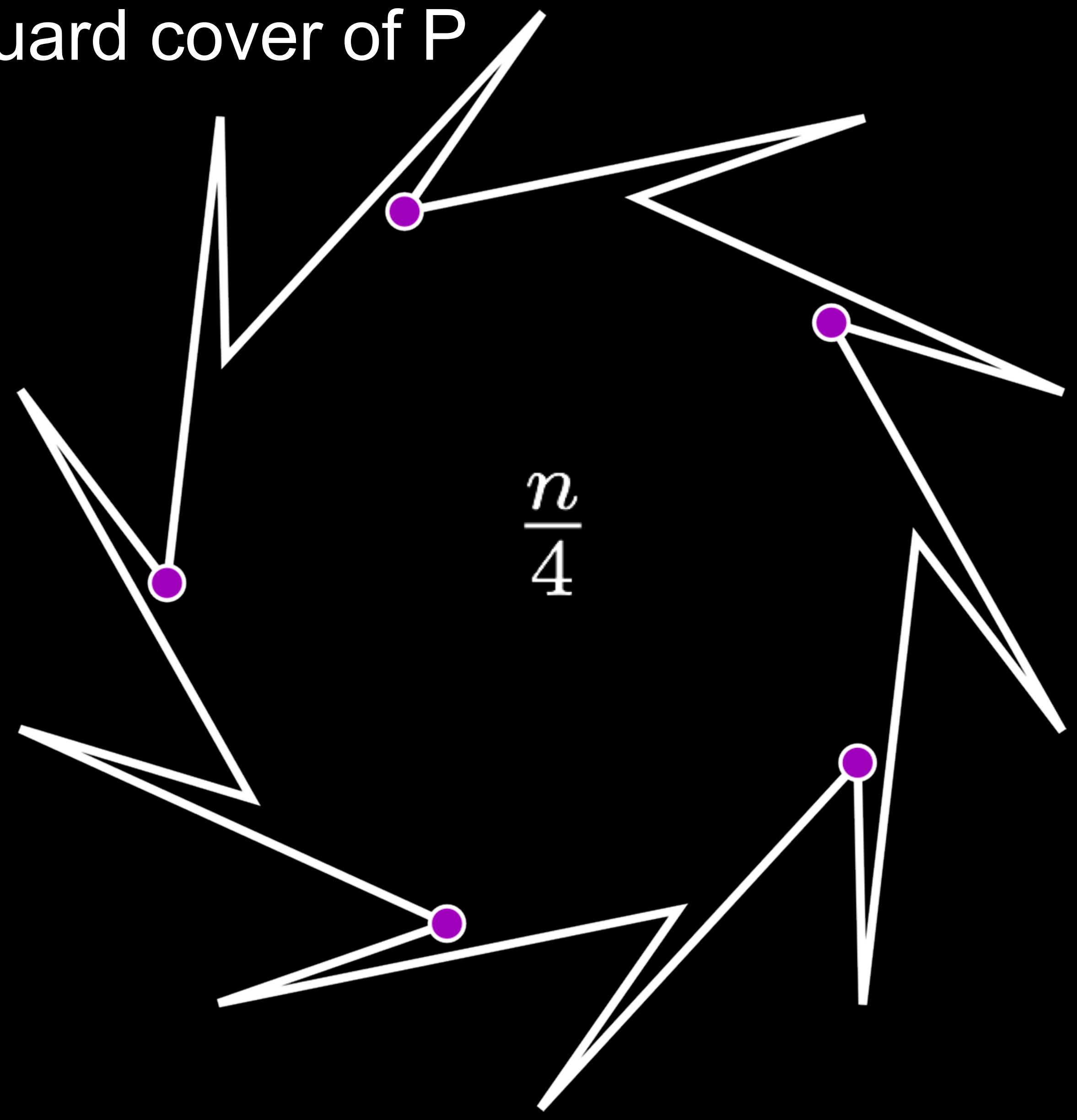
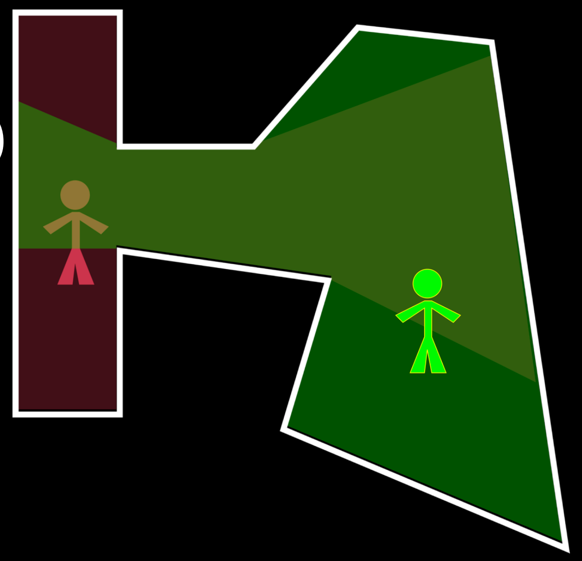


The Art Gallery Problem (AGP)—and Its Variants?



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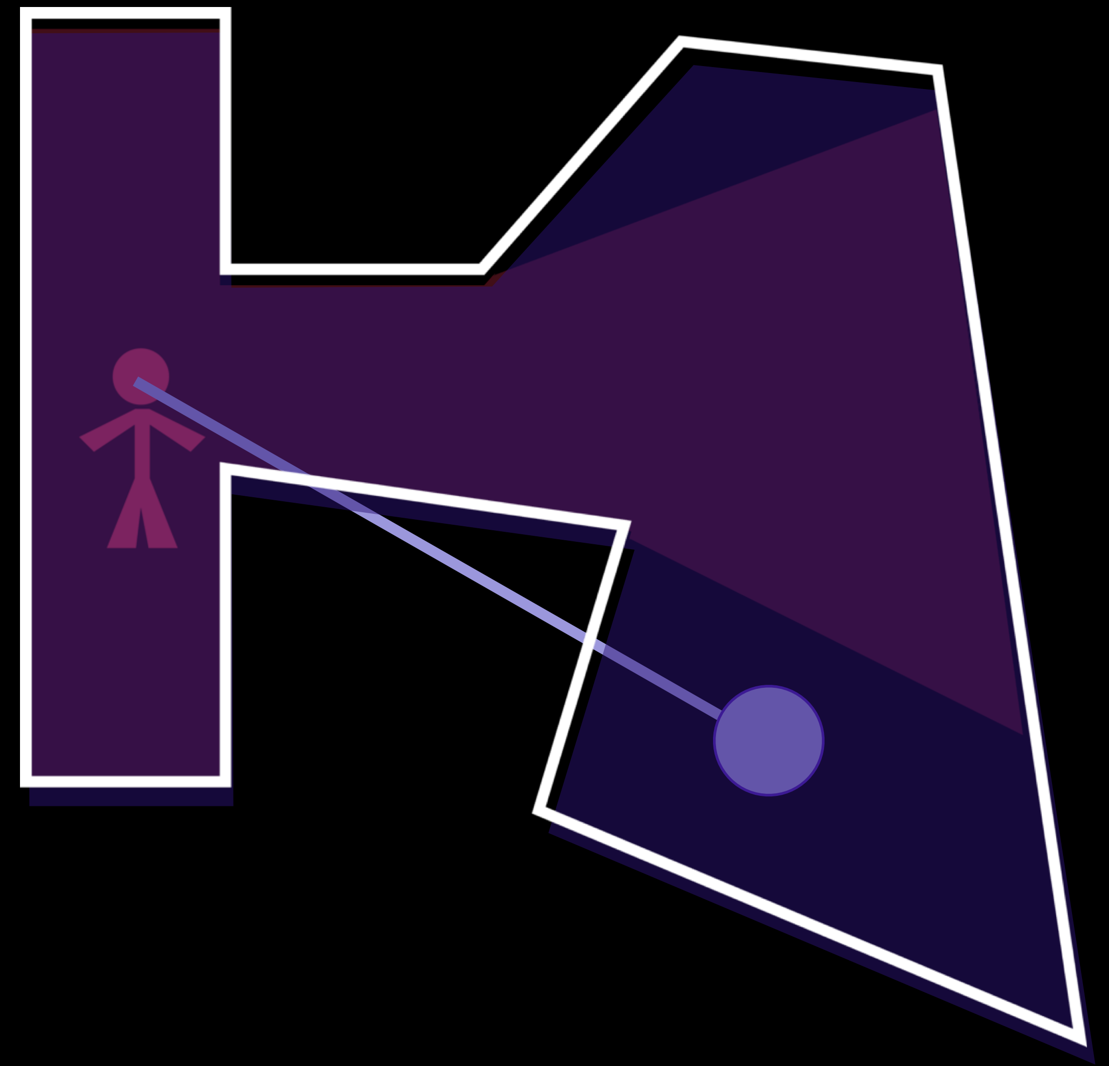

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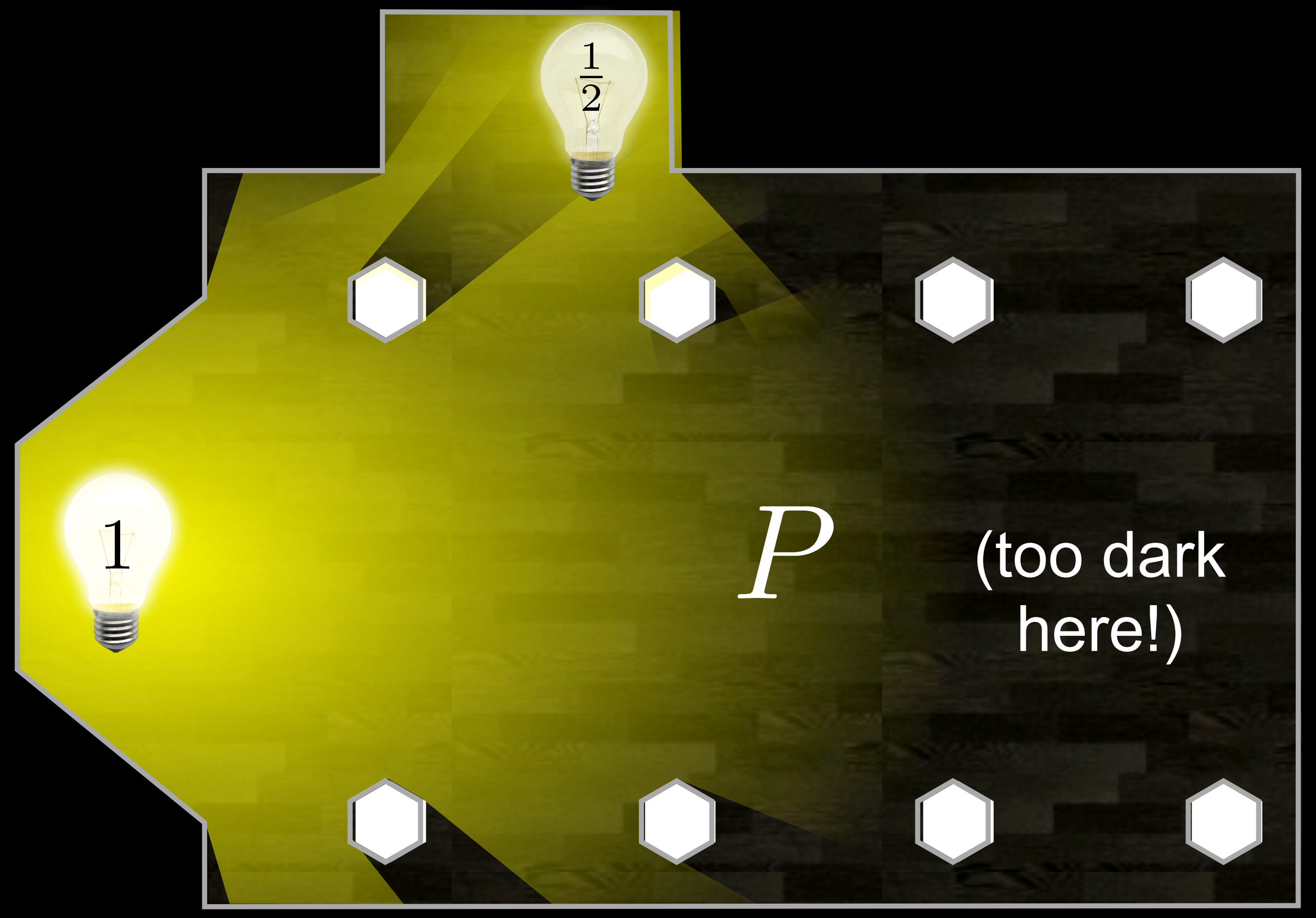

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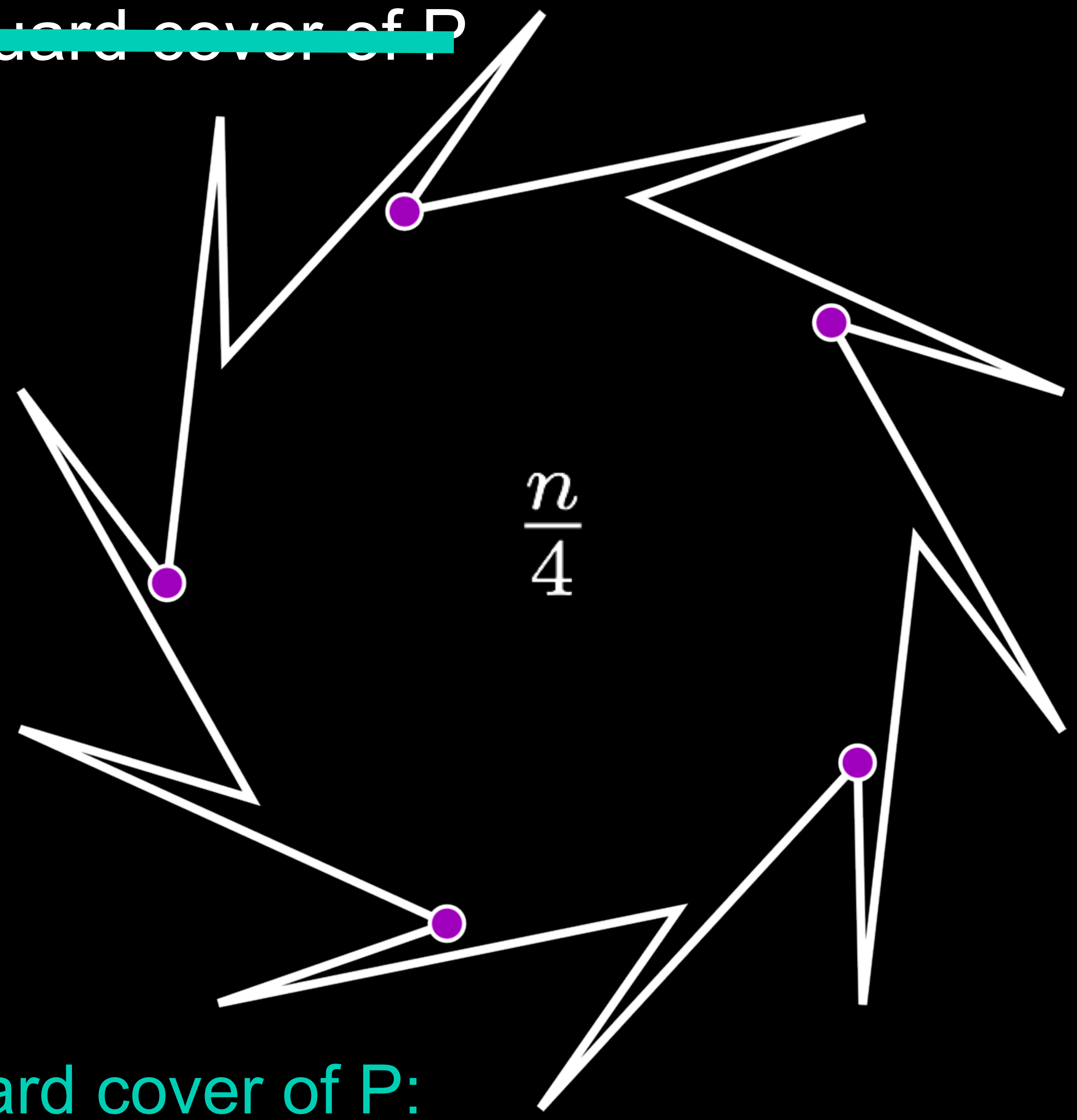
Chromatic AGP:

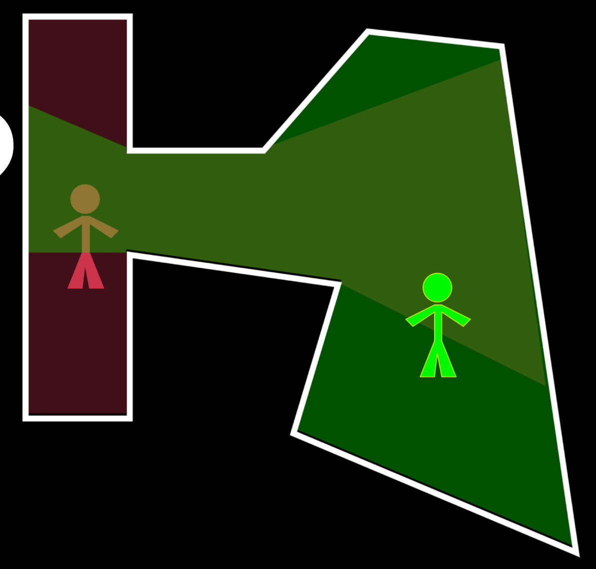
Given: a polygon P

Task: ~~find a min guard cover of P~~



Find a **colored** guard cover of P:
No point in P is seen by two guards
of the same color.

The Art Gallery Problem (AGP)—and Its Variants?



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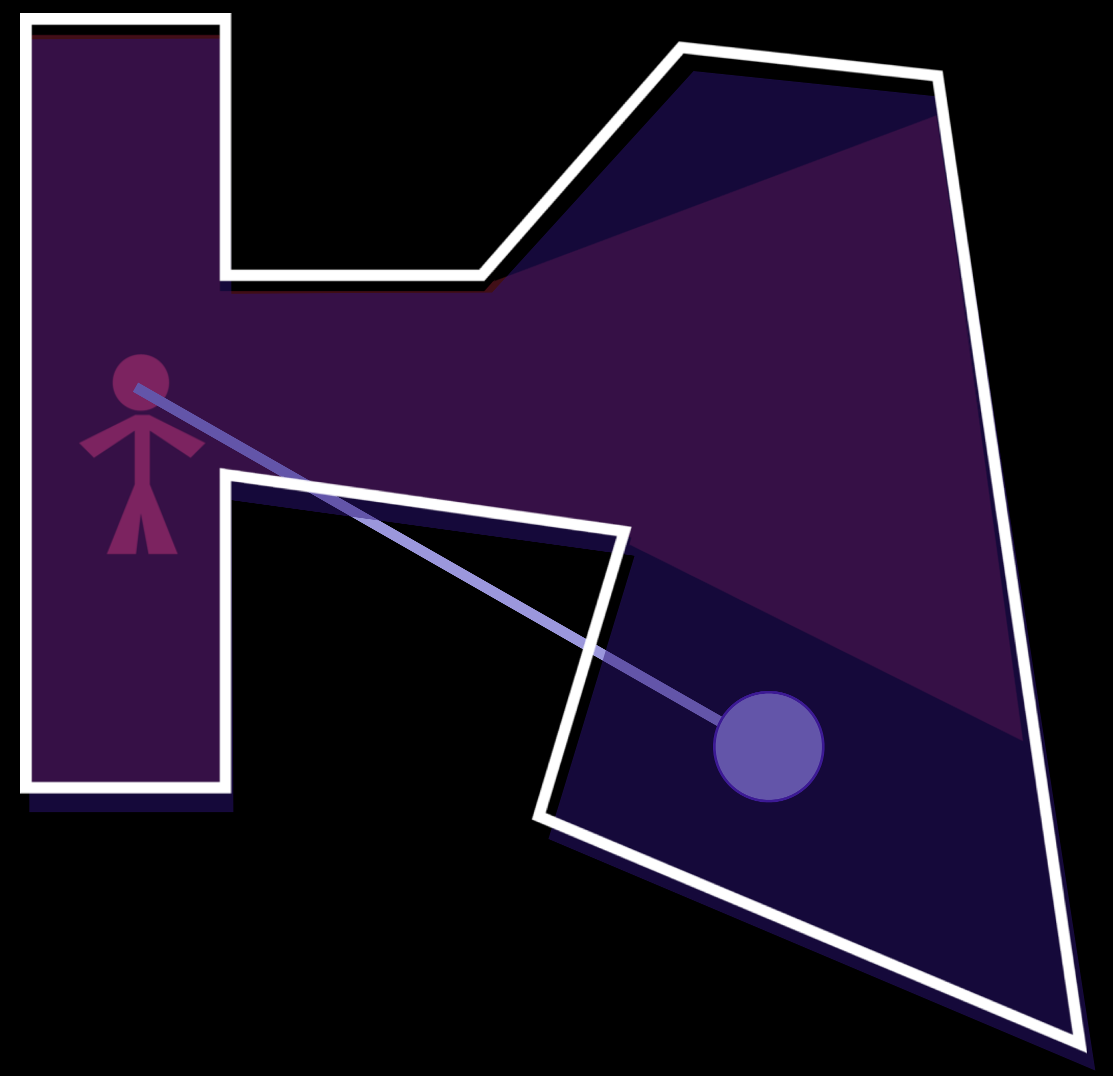

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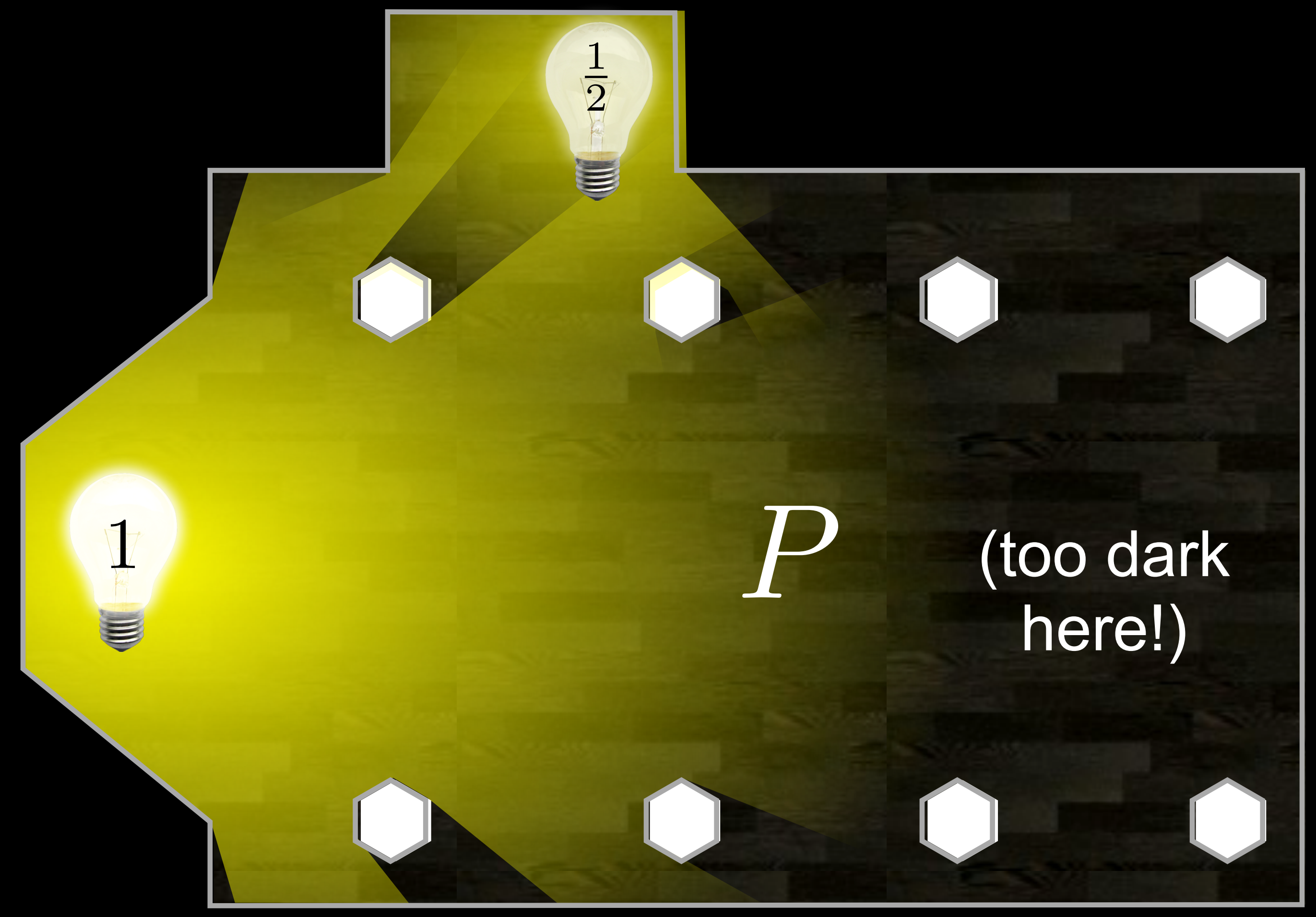

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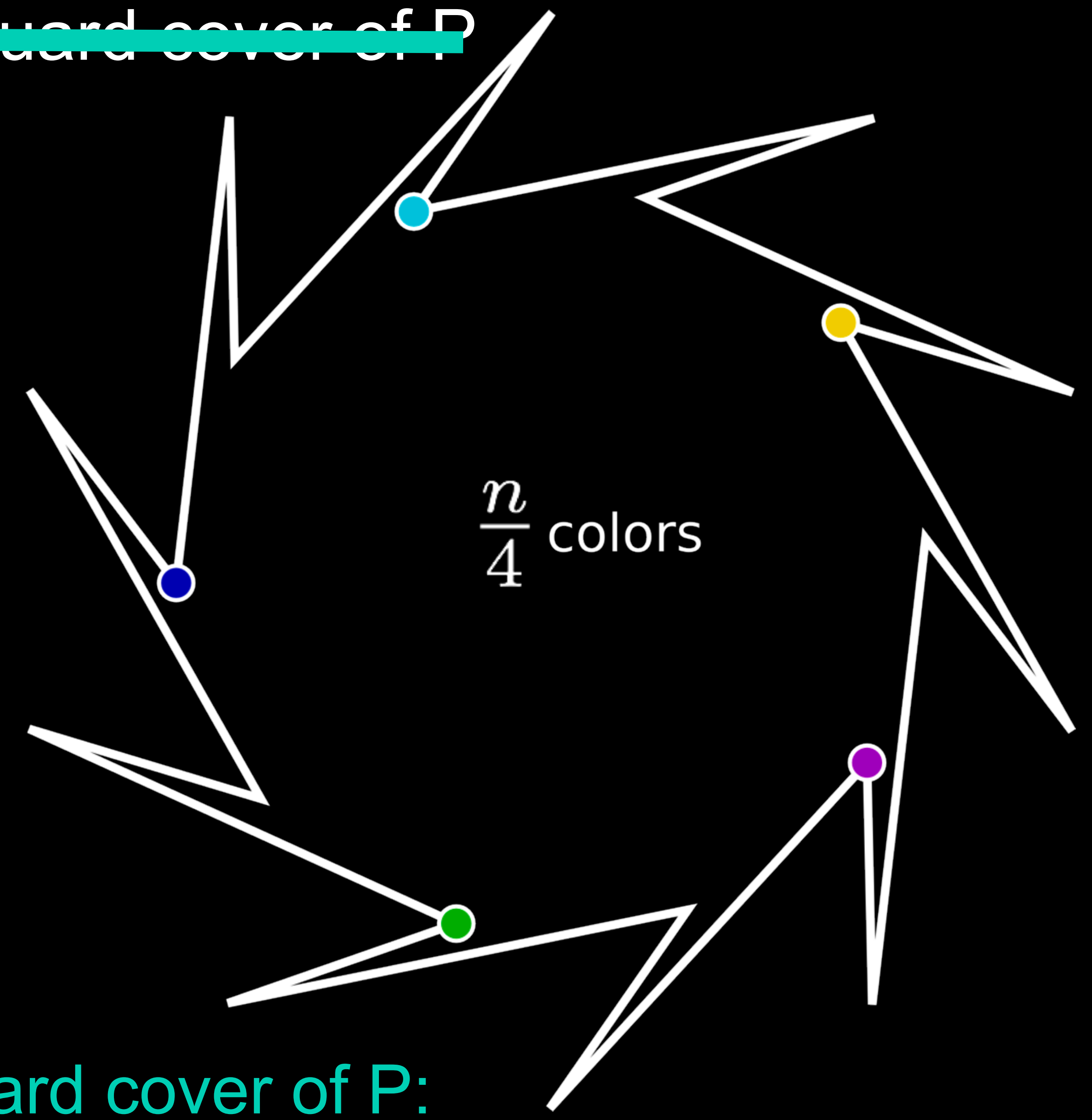
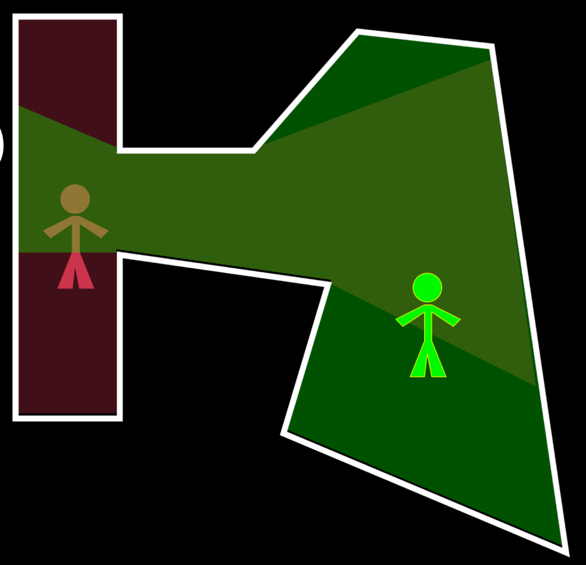


Find a **colored** guard cover of P:
No point in P is seen by two guards of the same color.

The Art Gallery Problem (AGP)—and Its Variants?



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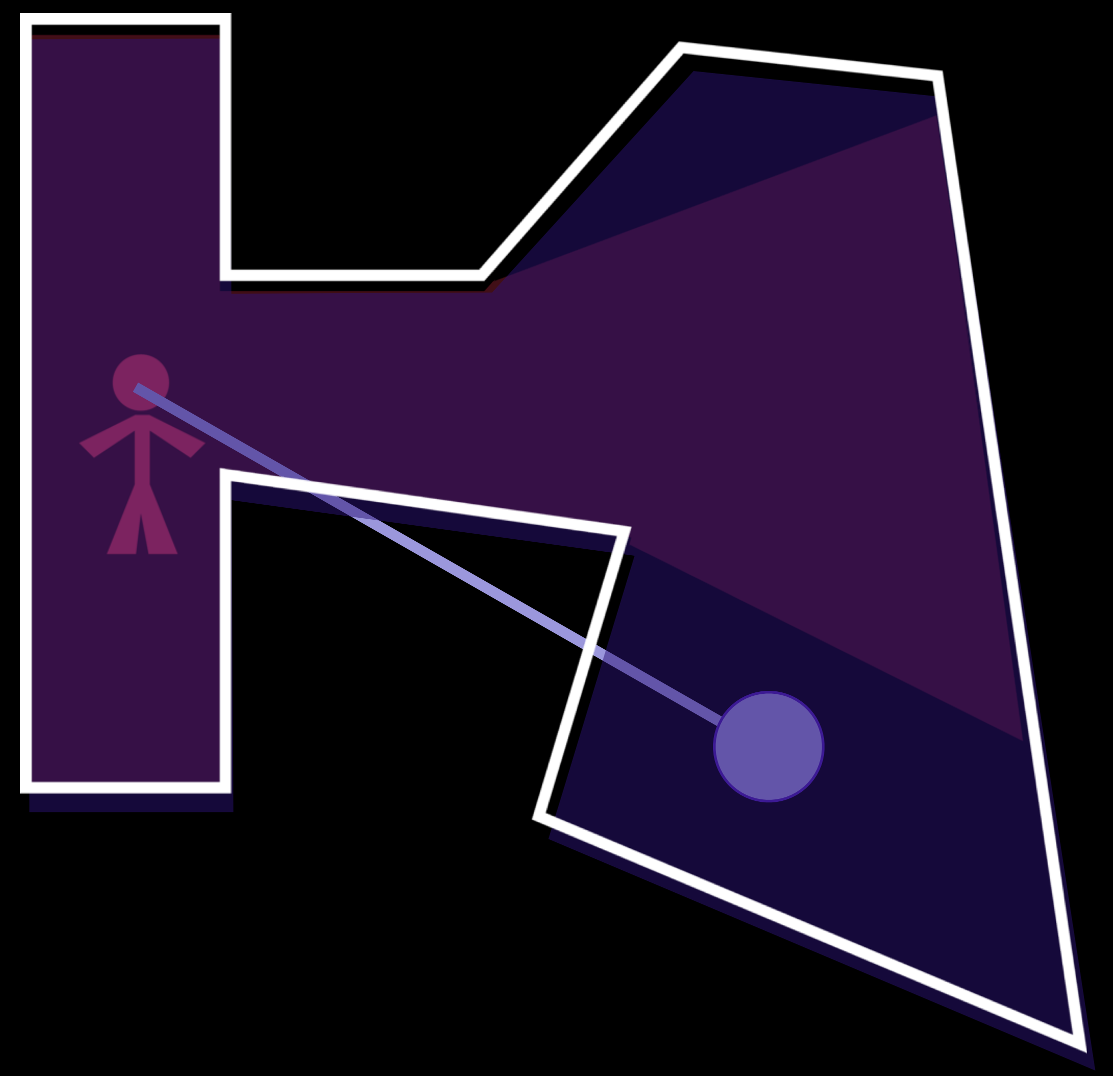

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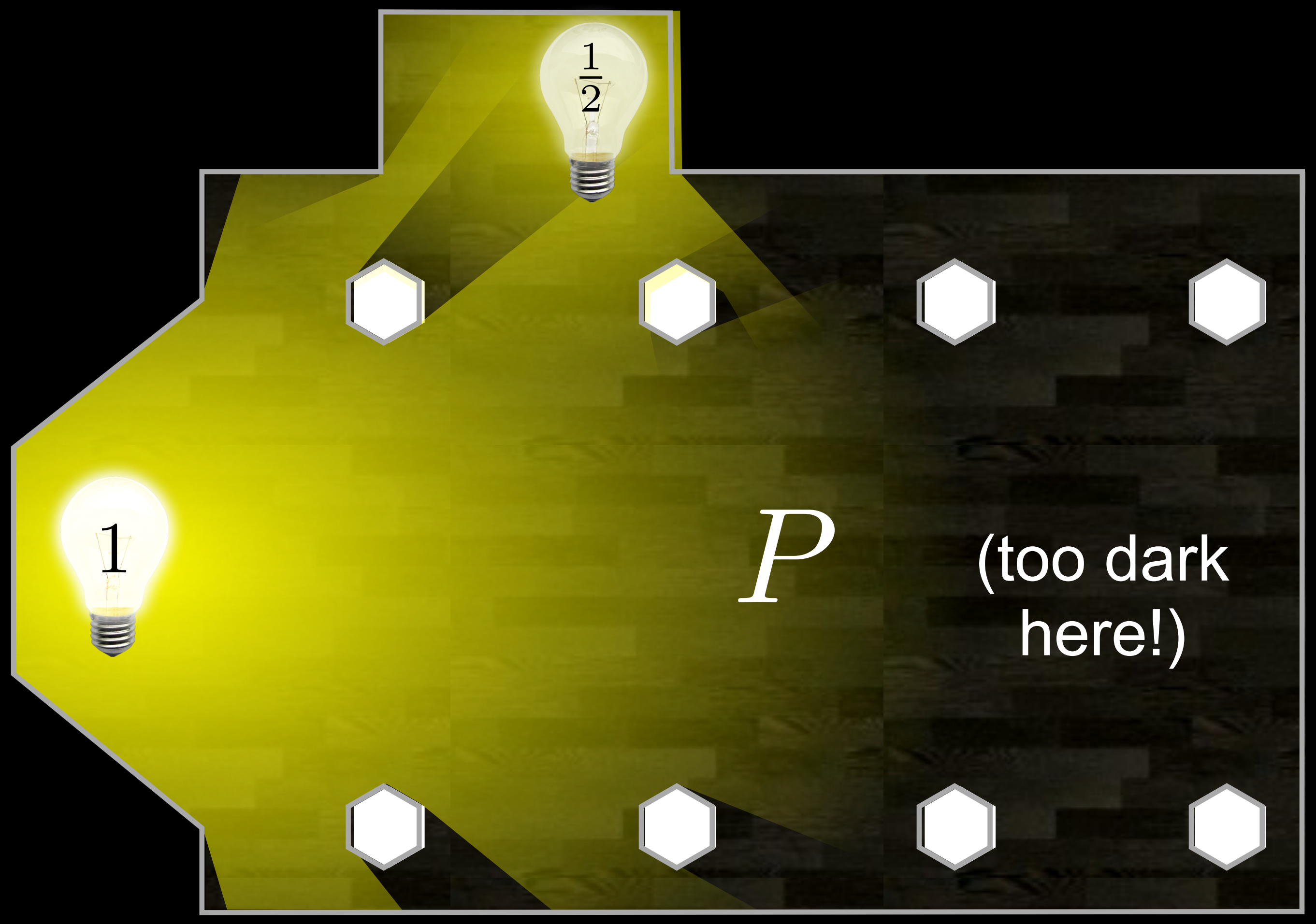

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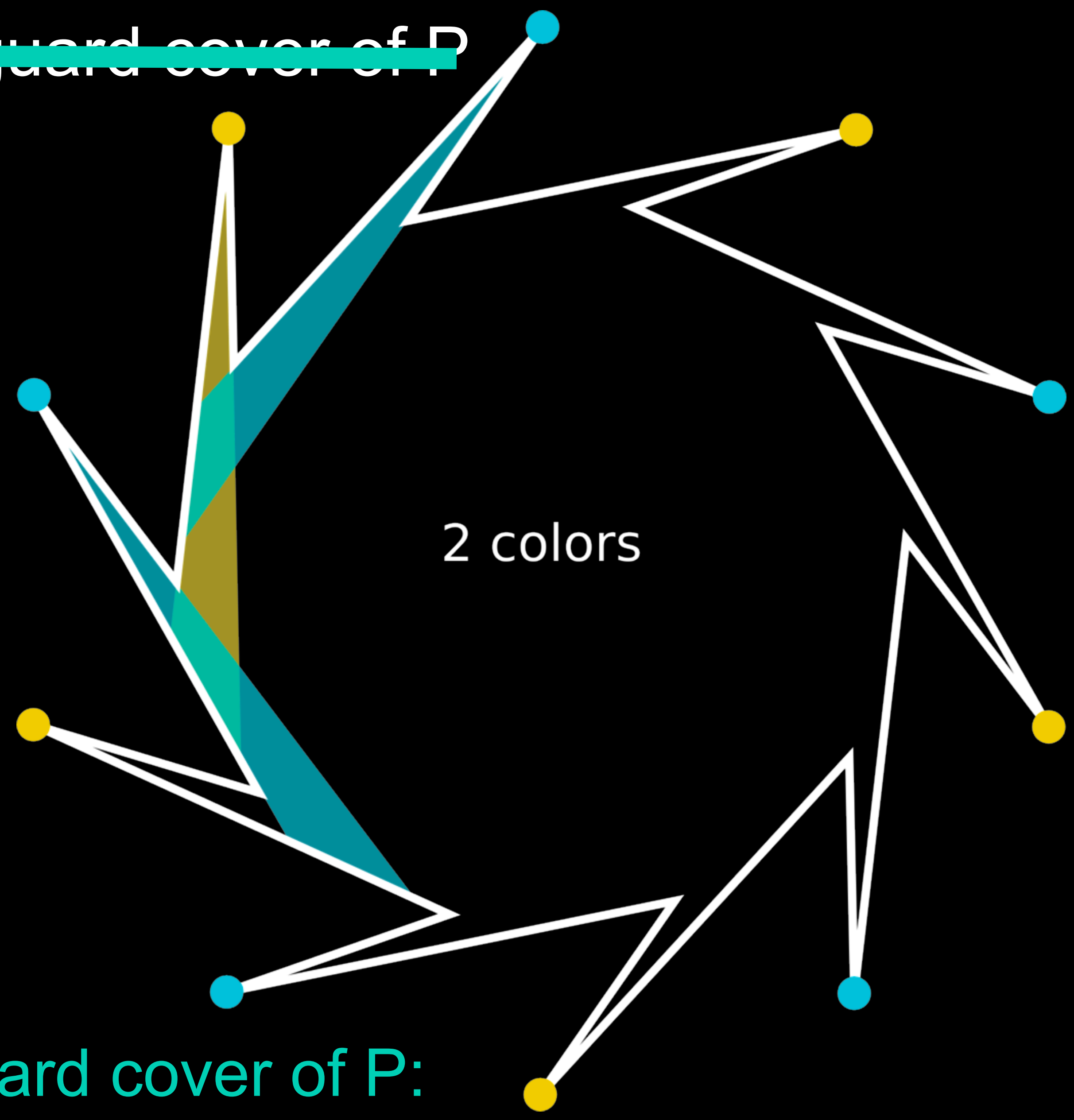

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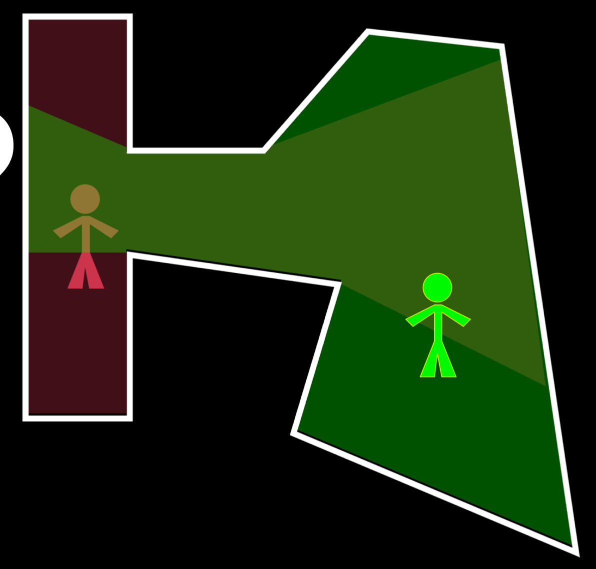
The Art Gallery Problem (AGP)—and Its Variants?



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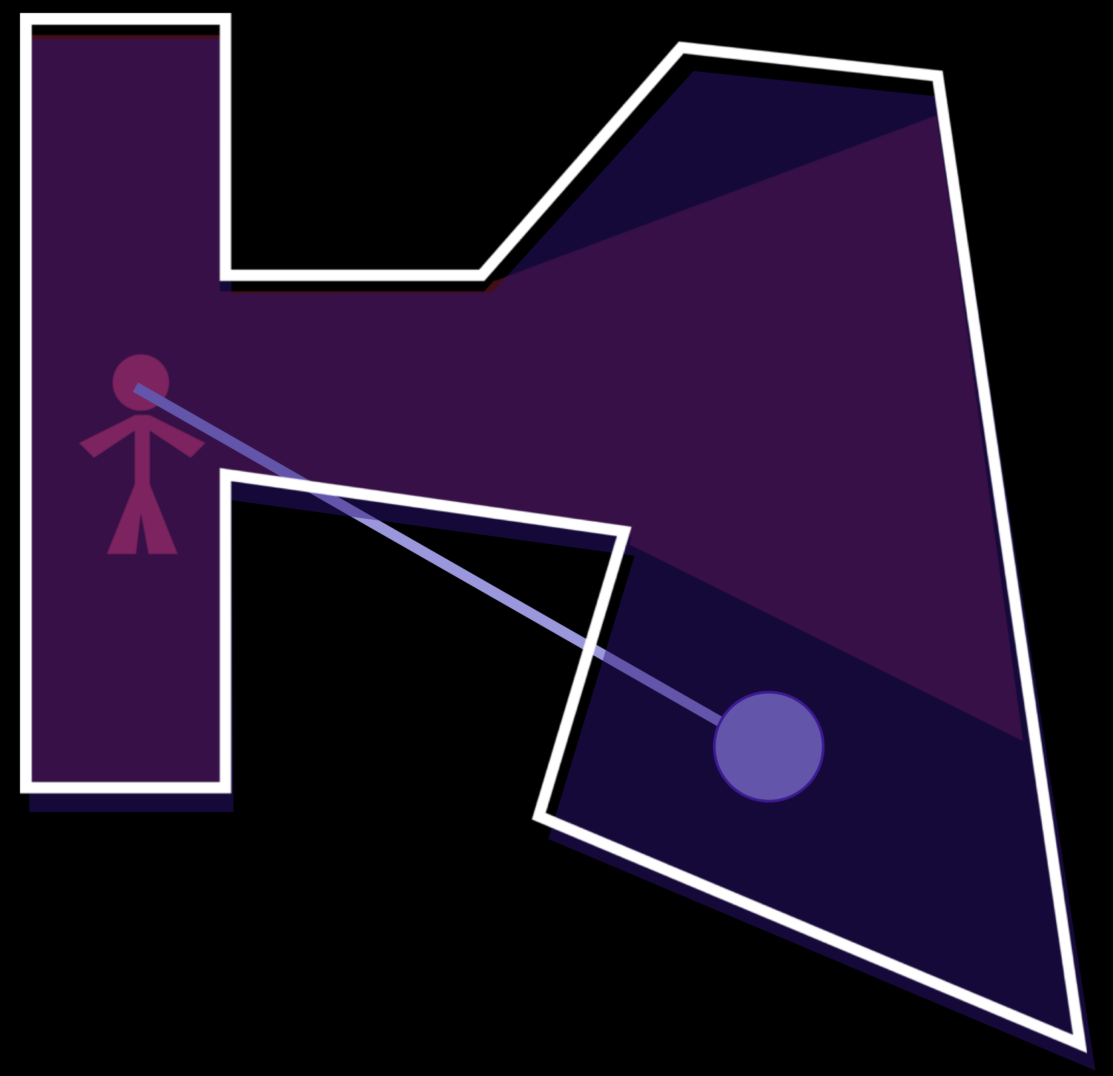

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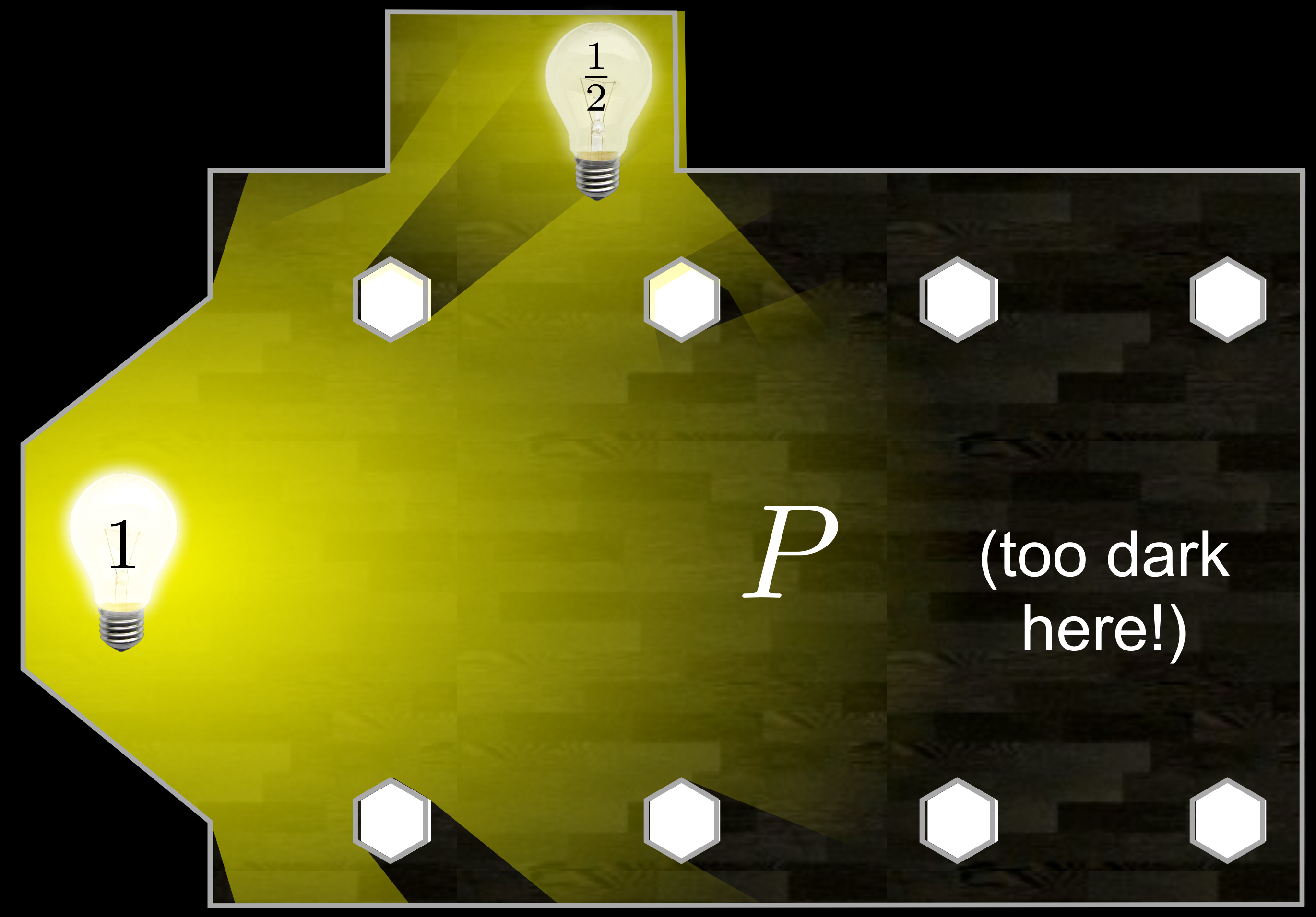

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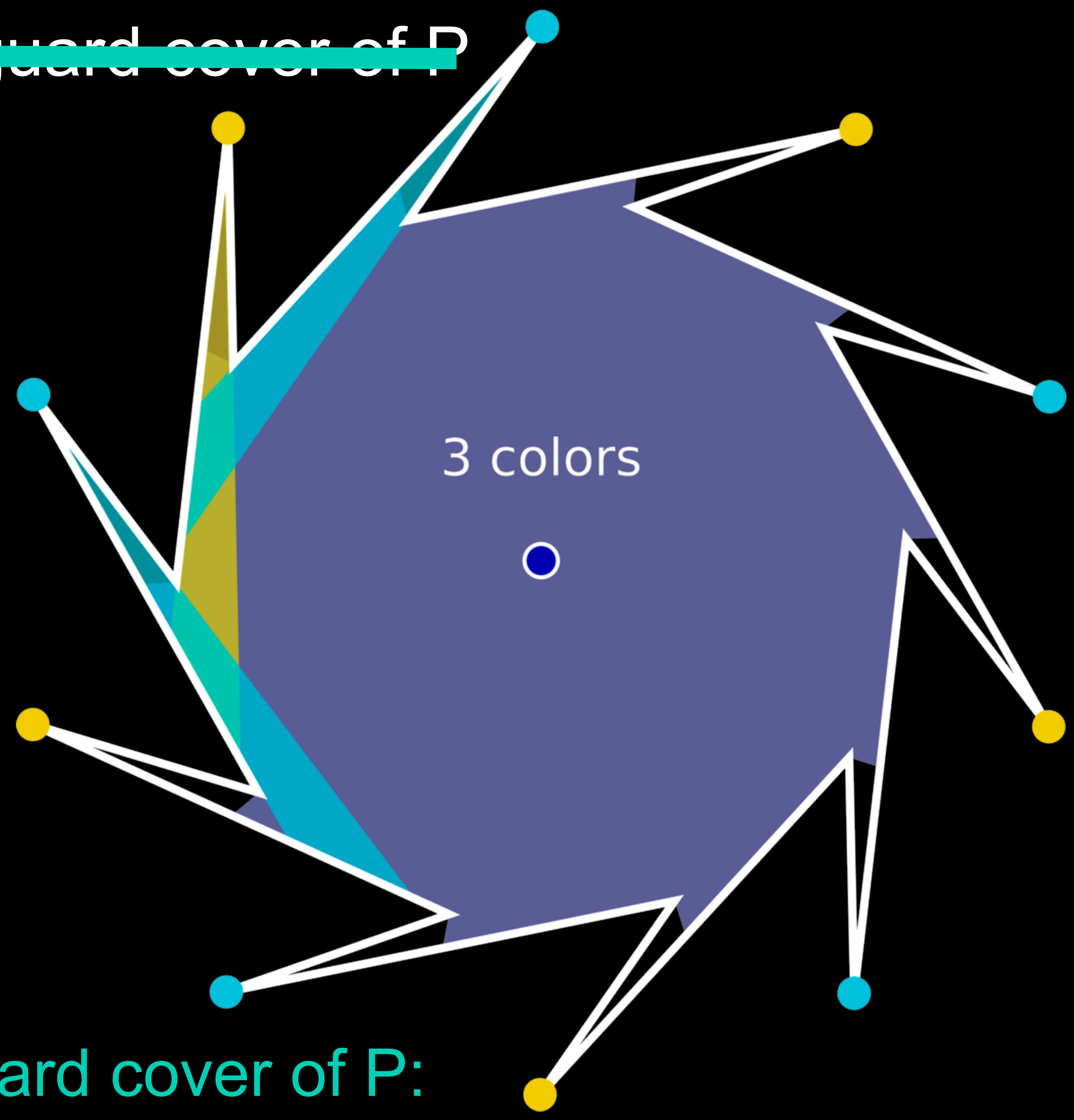

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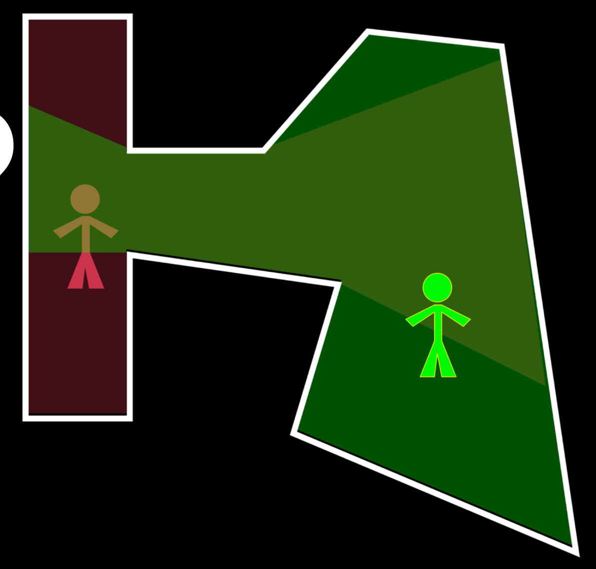

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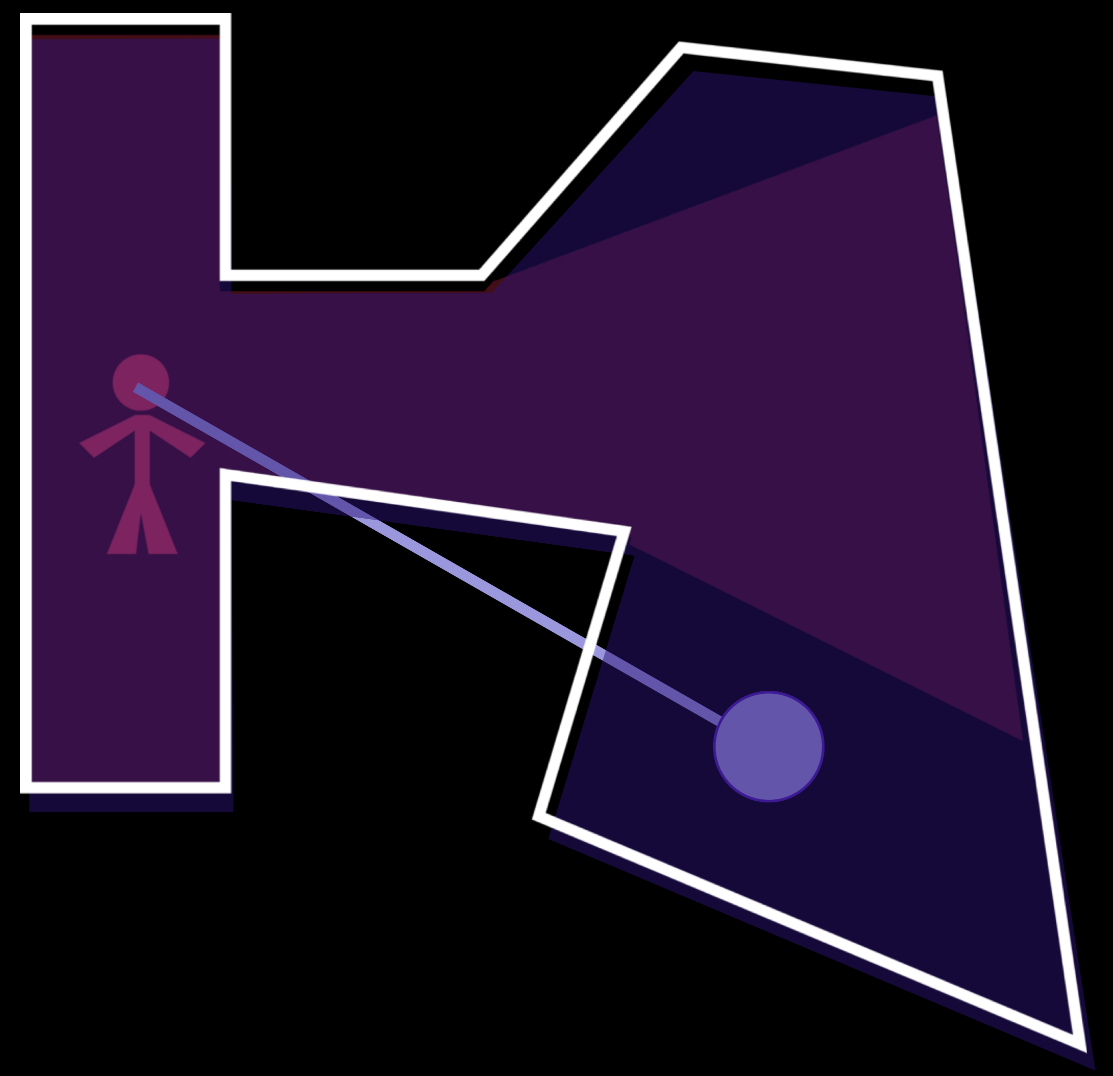

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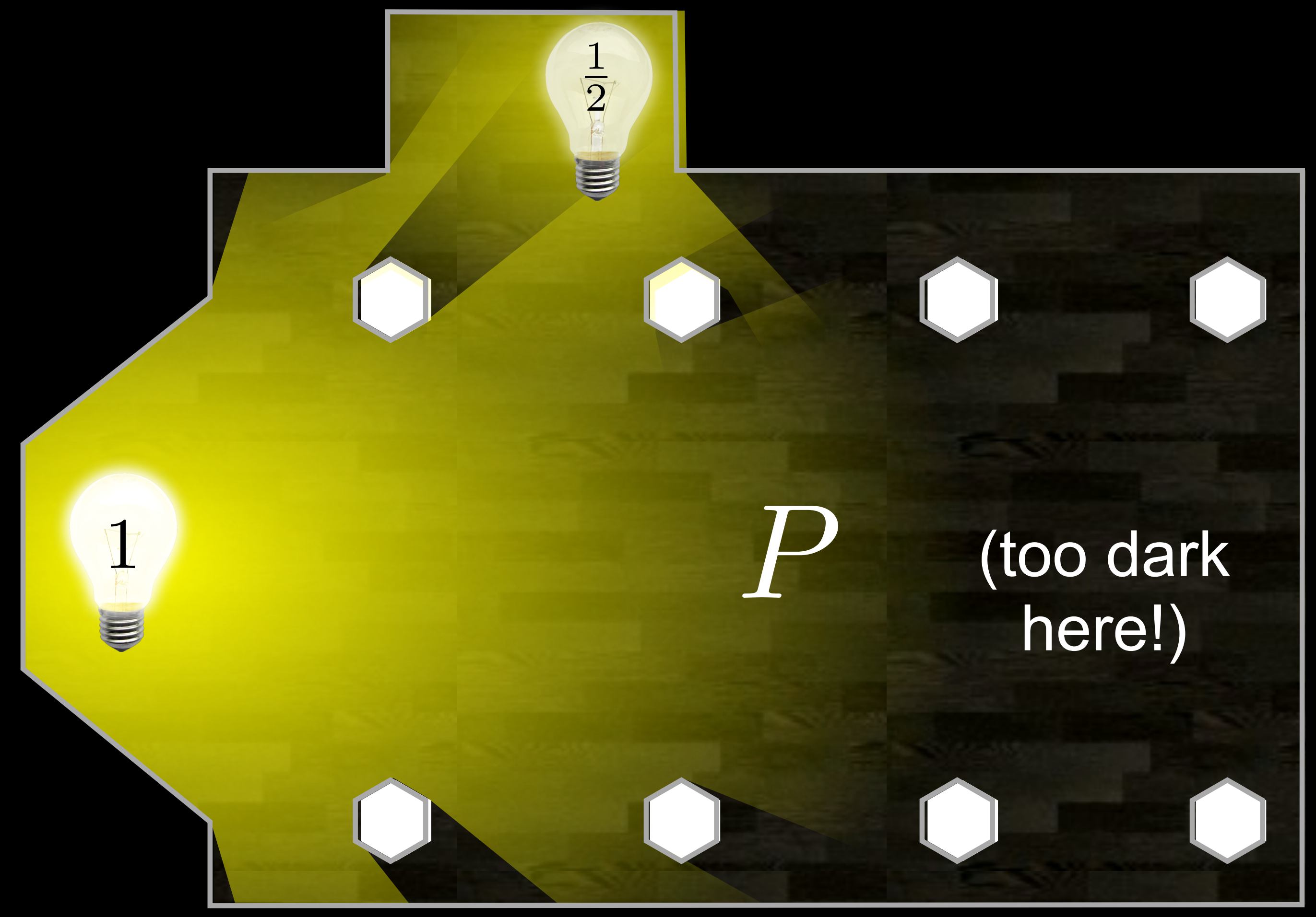

Place lights,
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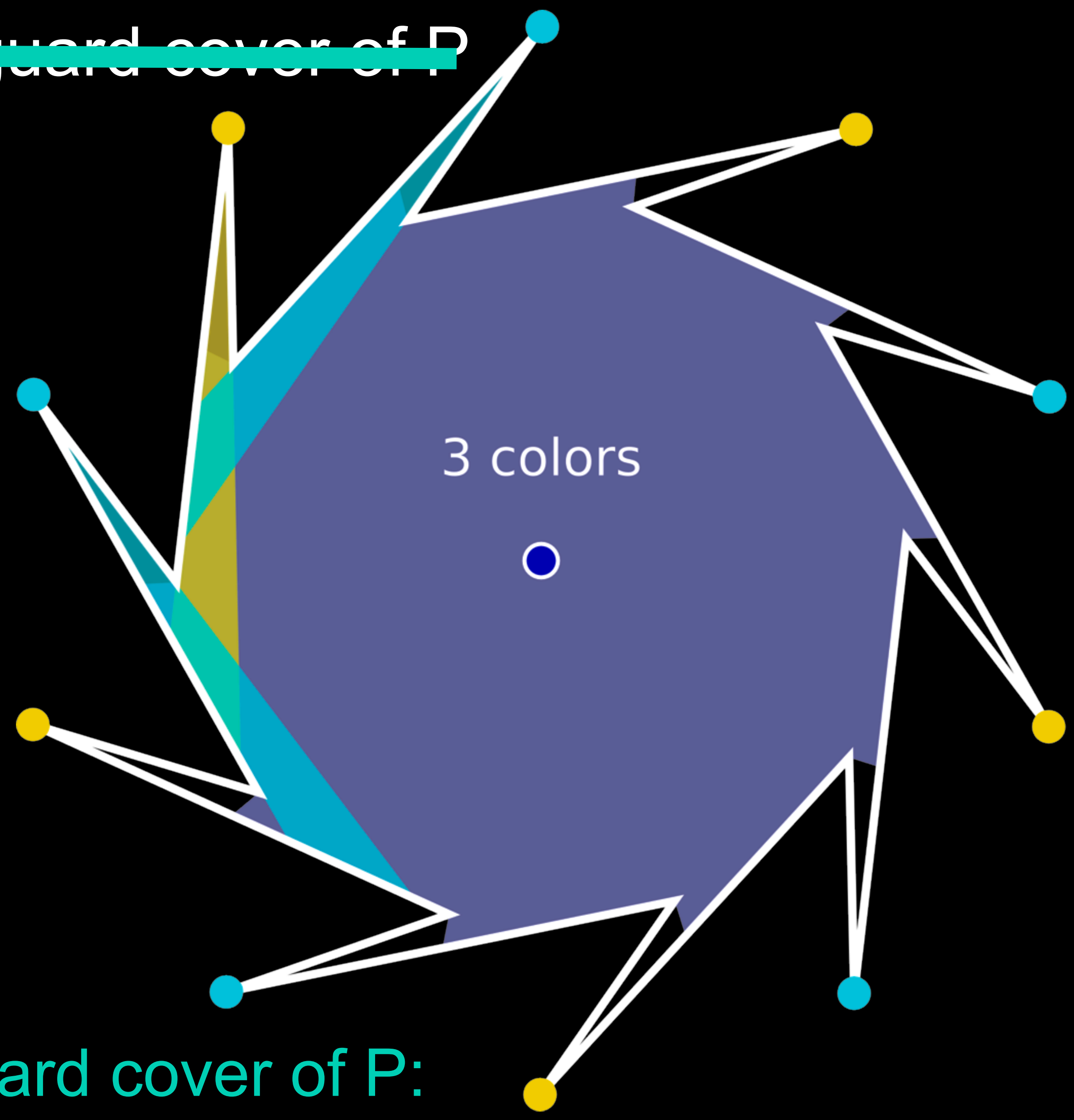
Minimize total energy.

Chromatic AGP:

We do not care about the number of guards, but about the number of colors!

Given: a polygon P

Task: ~~find a min guard cover of P~~



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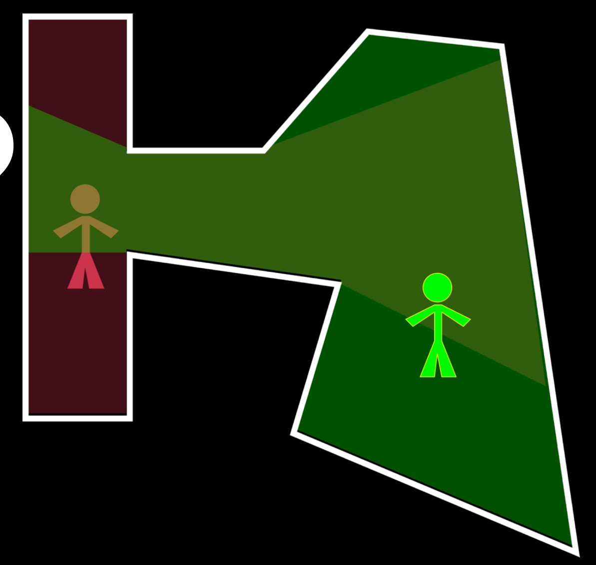

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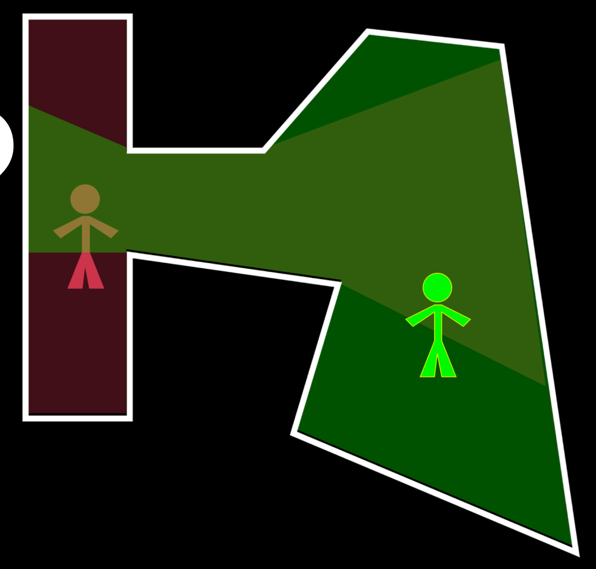

We can alter:

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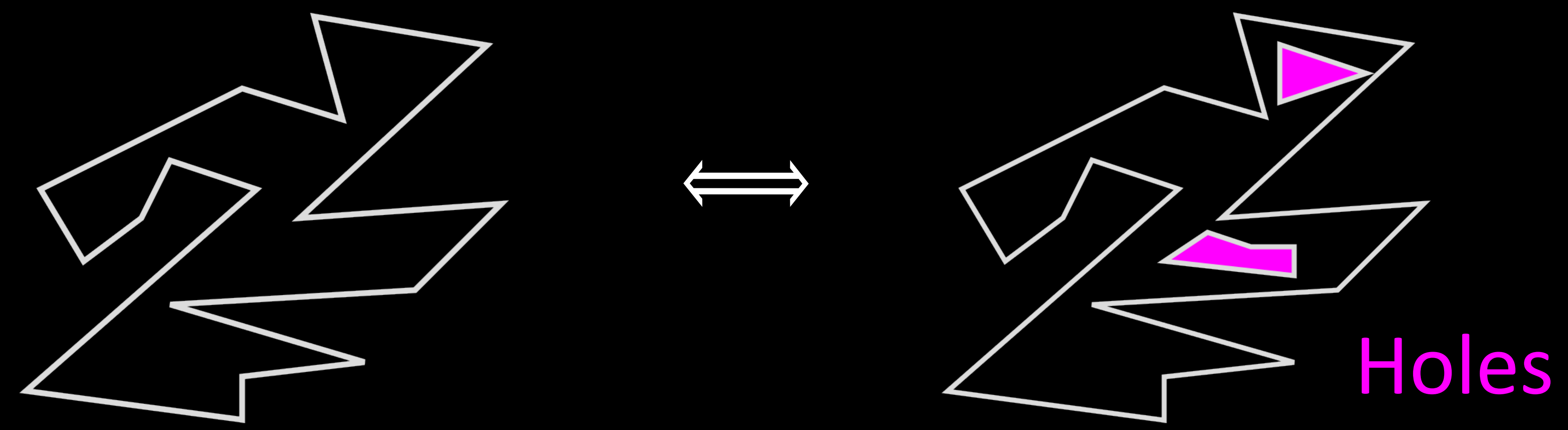
Alter the polygon class:

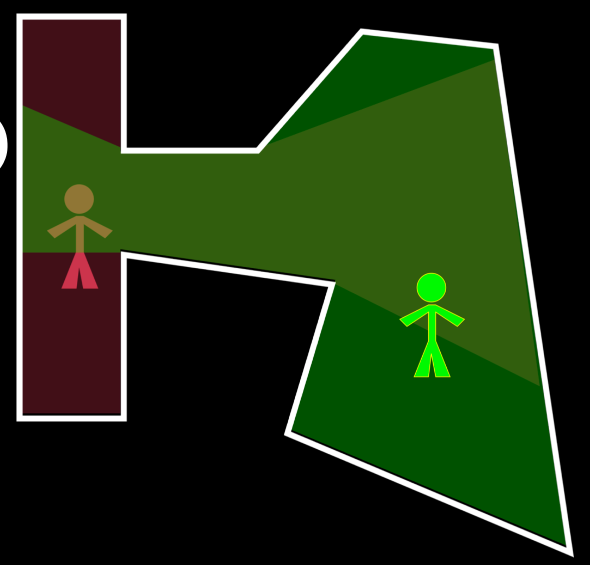
Traditionally:
Simple polygons or polygons with holes



Simple polygon:

- Does not intersect itself
- No holes

The Art Gallery Problem (AGP)—and Its Variants?



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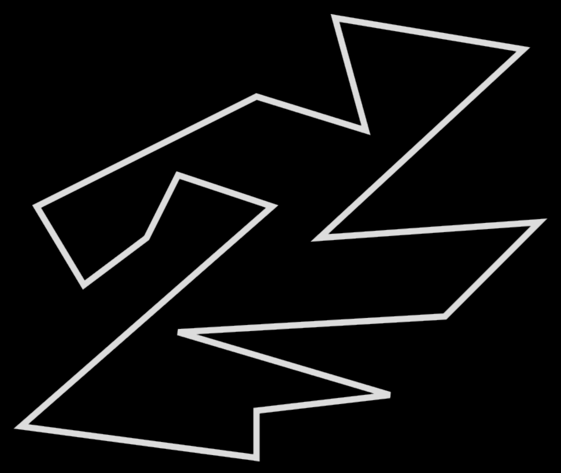
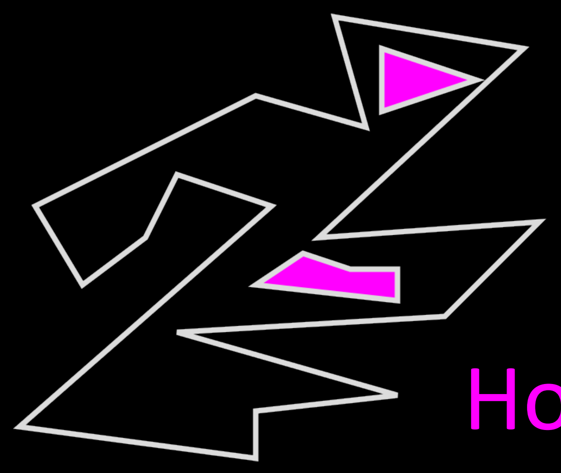
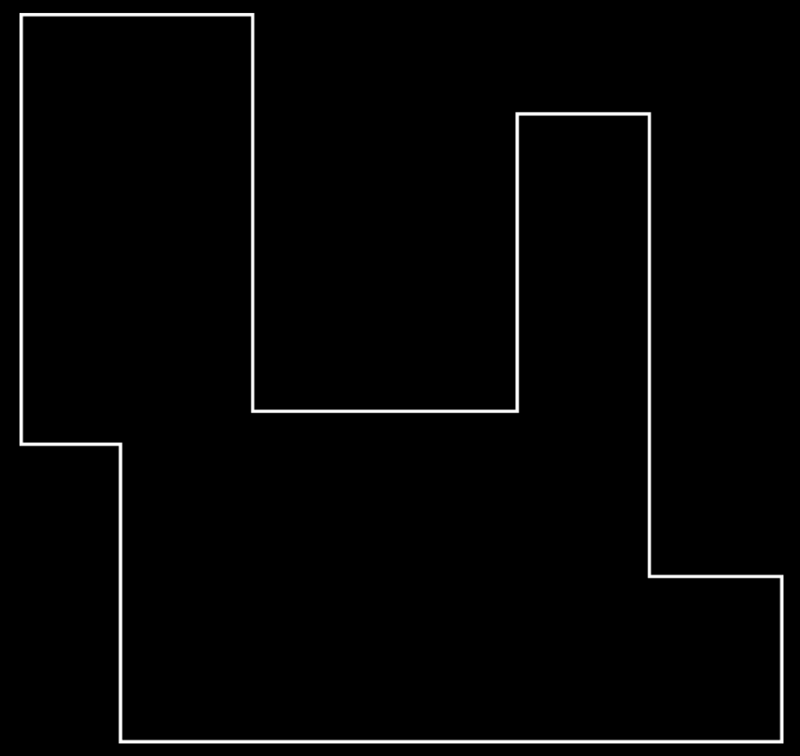


⟺



Rectilinear polygons



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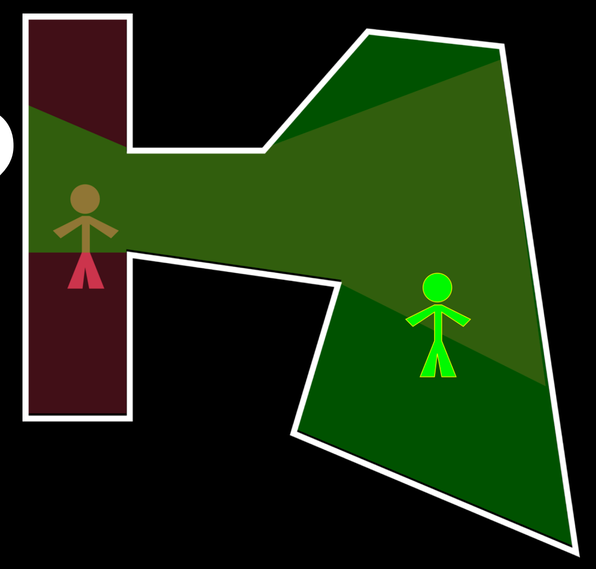
The Art Gallery Problem (AGP)—and Its Variants?



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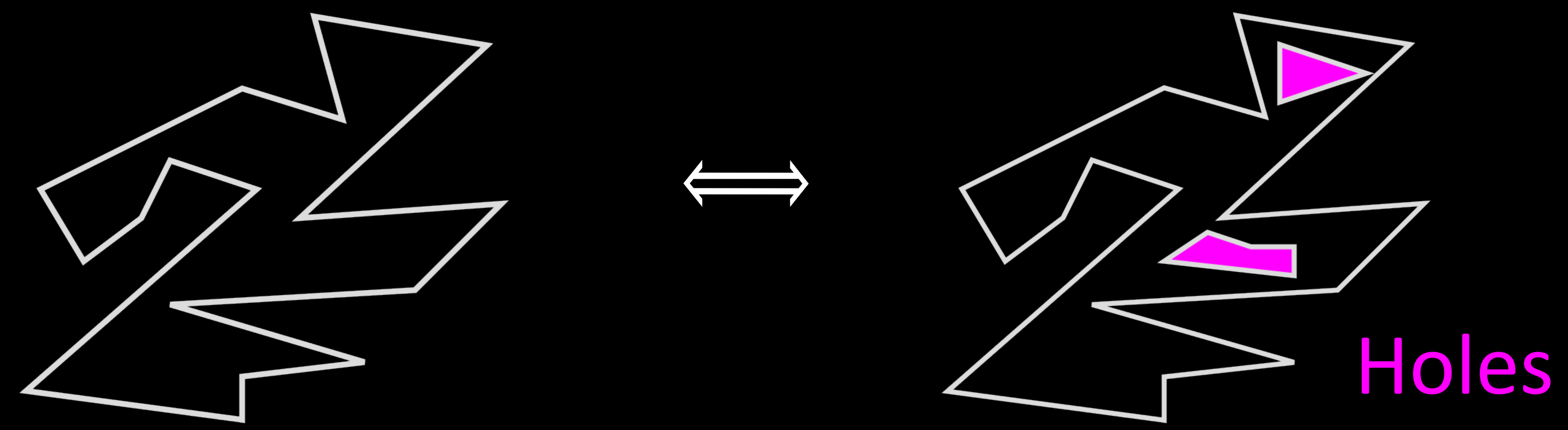

Simple polygon:

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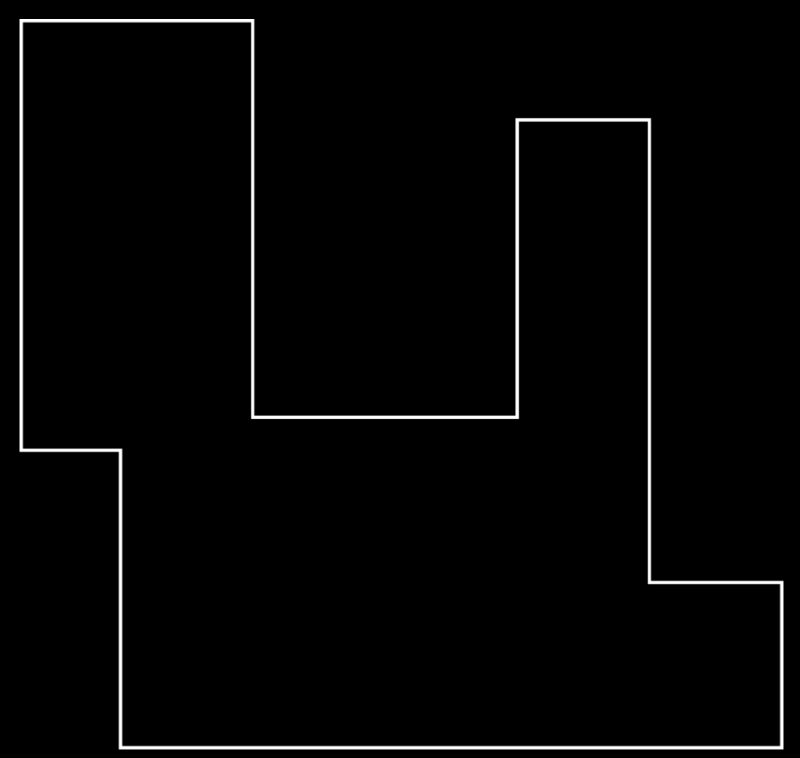
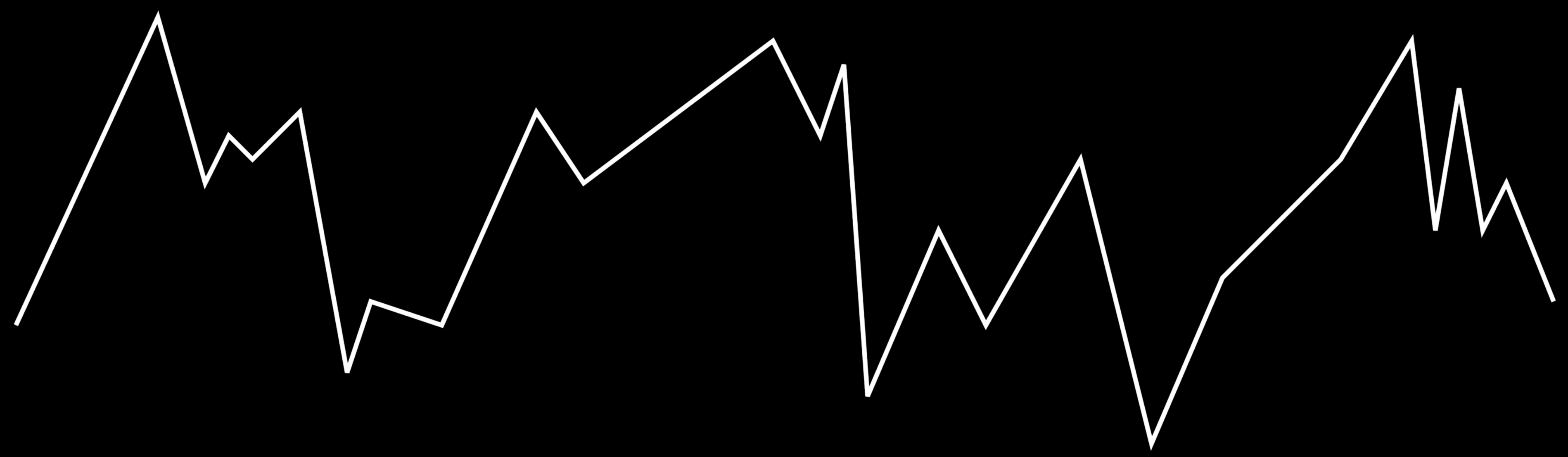
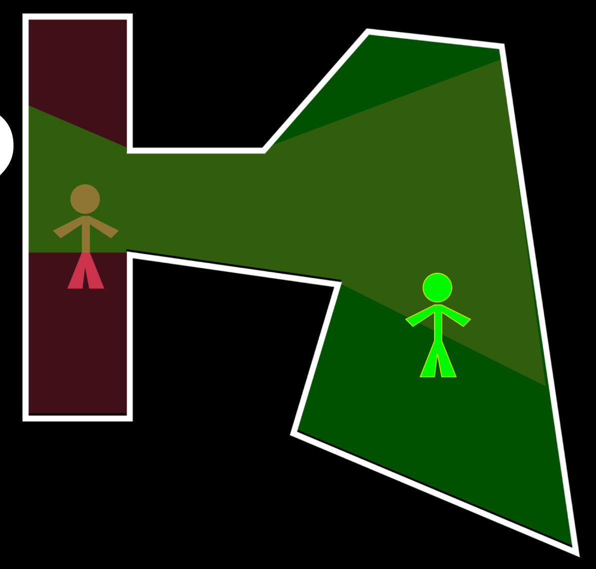
Rectilinear polygons



Guard a 1.5D-Terrain



The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

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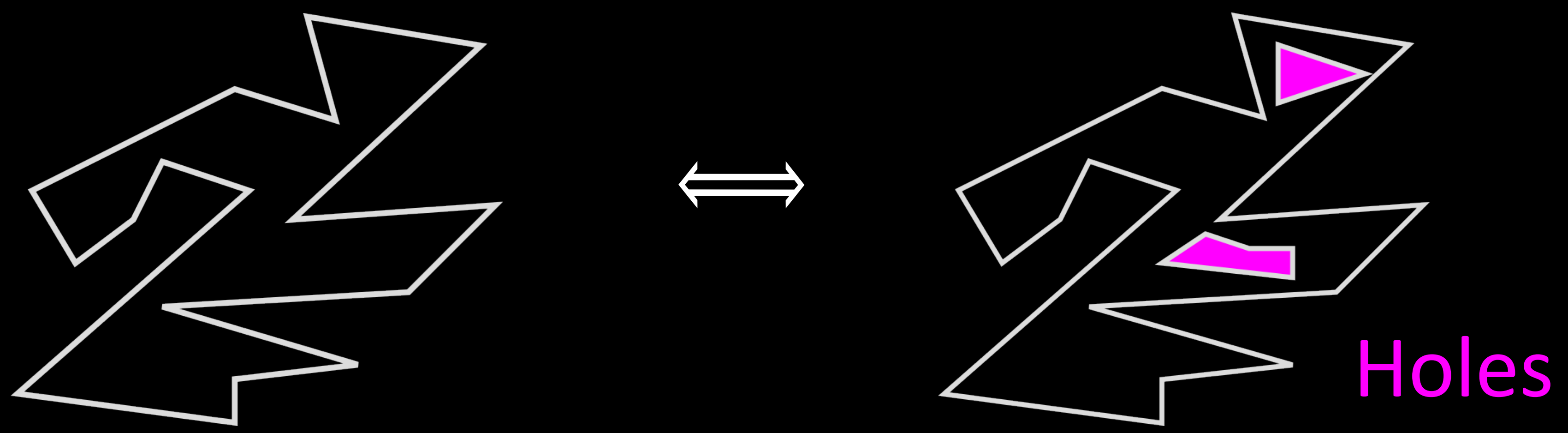
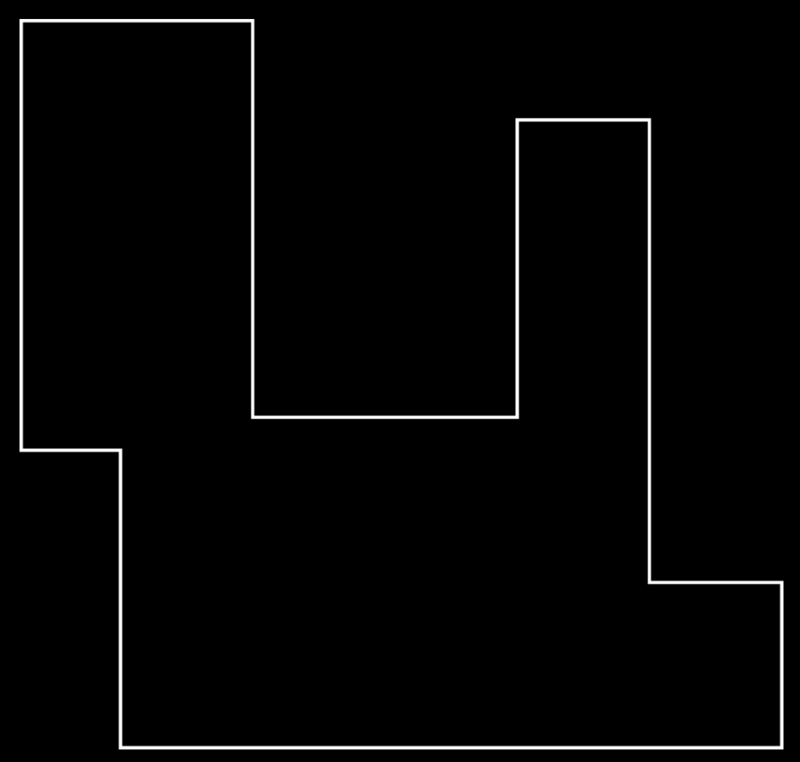


Simple polygon:

- Does not intersect itself
- No holes

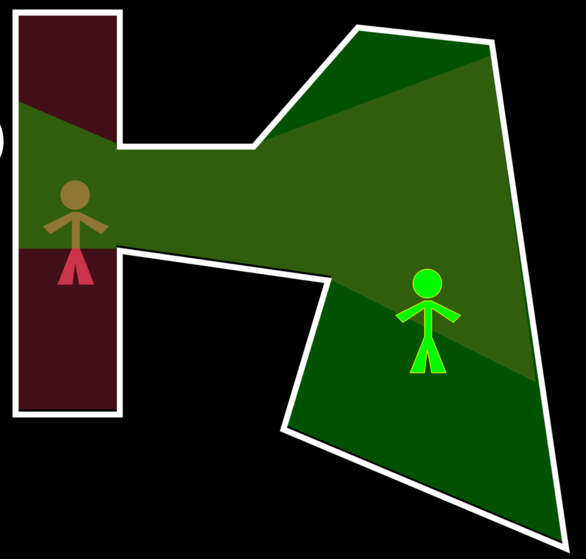
Rectilinear polygons



Guard a 1.5D-Terrain

- With guards on the terrain

The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

- Capabilities of the guards
- Environment to be guarded

Alter the polygon class:

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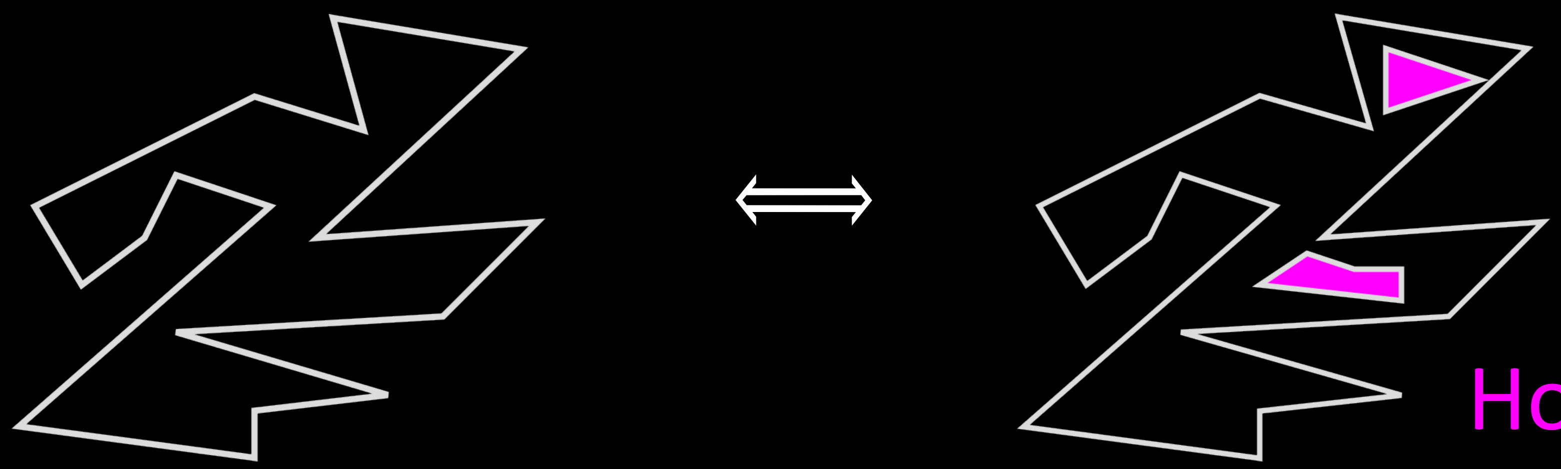
Simple polygons or polygons with holes

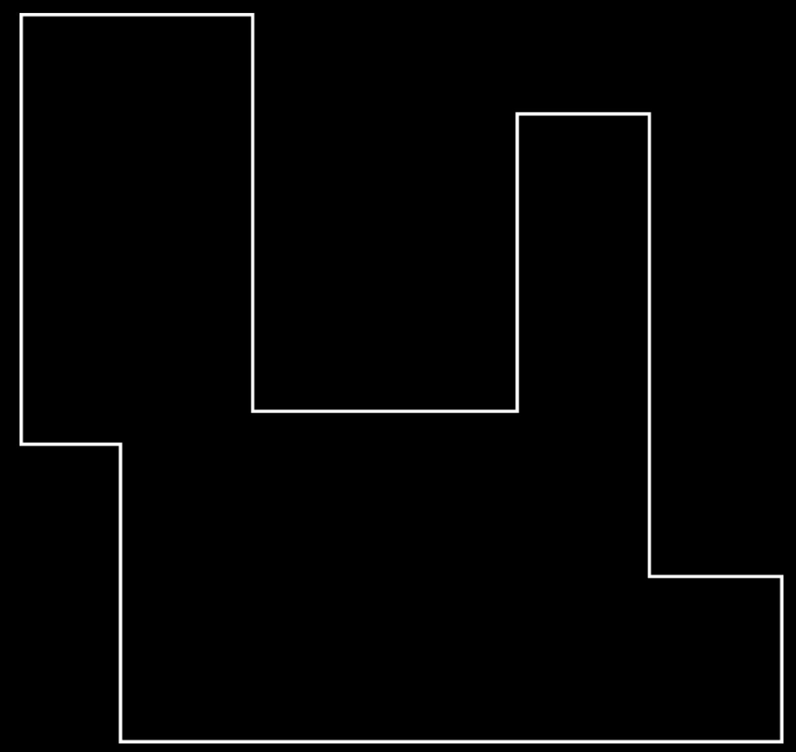


Holes

Simple polygon:

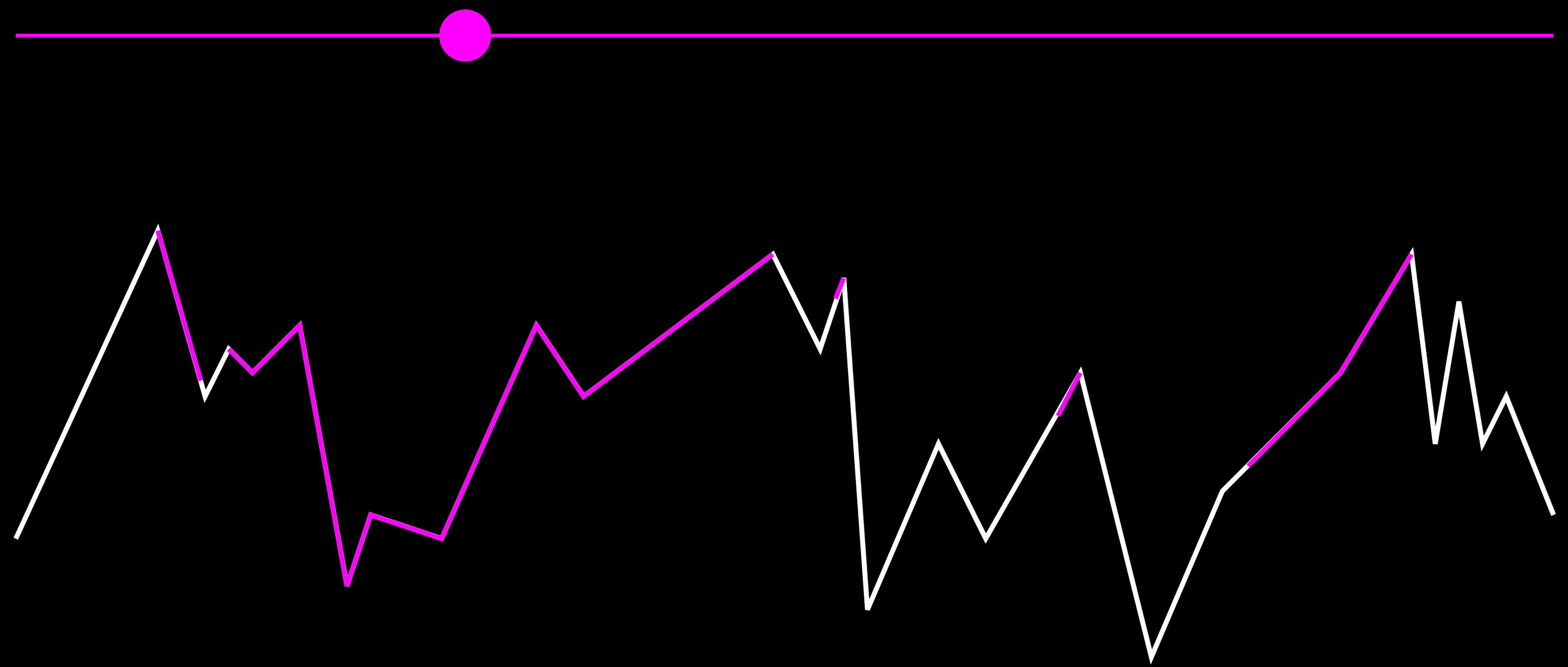
- Does not intersect itself
- No holes

Rectilinear polygons



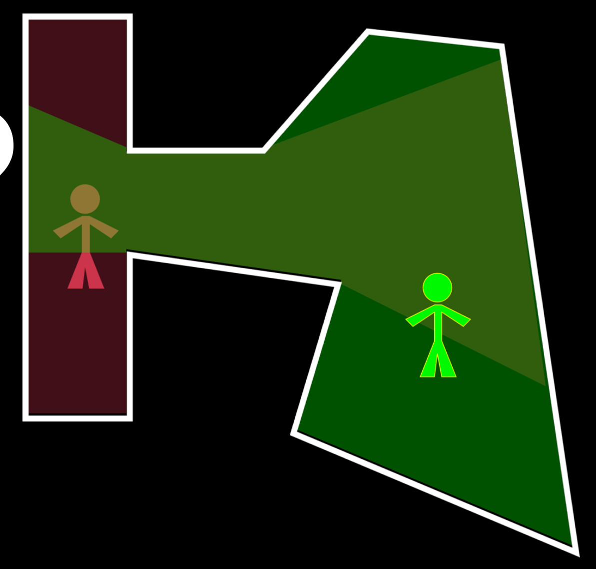
Guard a 1.5D-Terrain

- With guards on the terrain
- With guards on an altitude line above the terrain



k-Transmitters

The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

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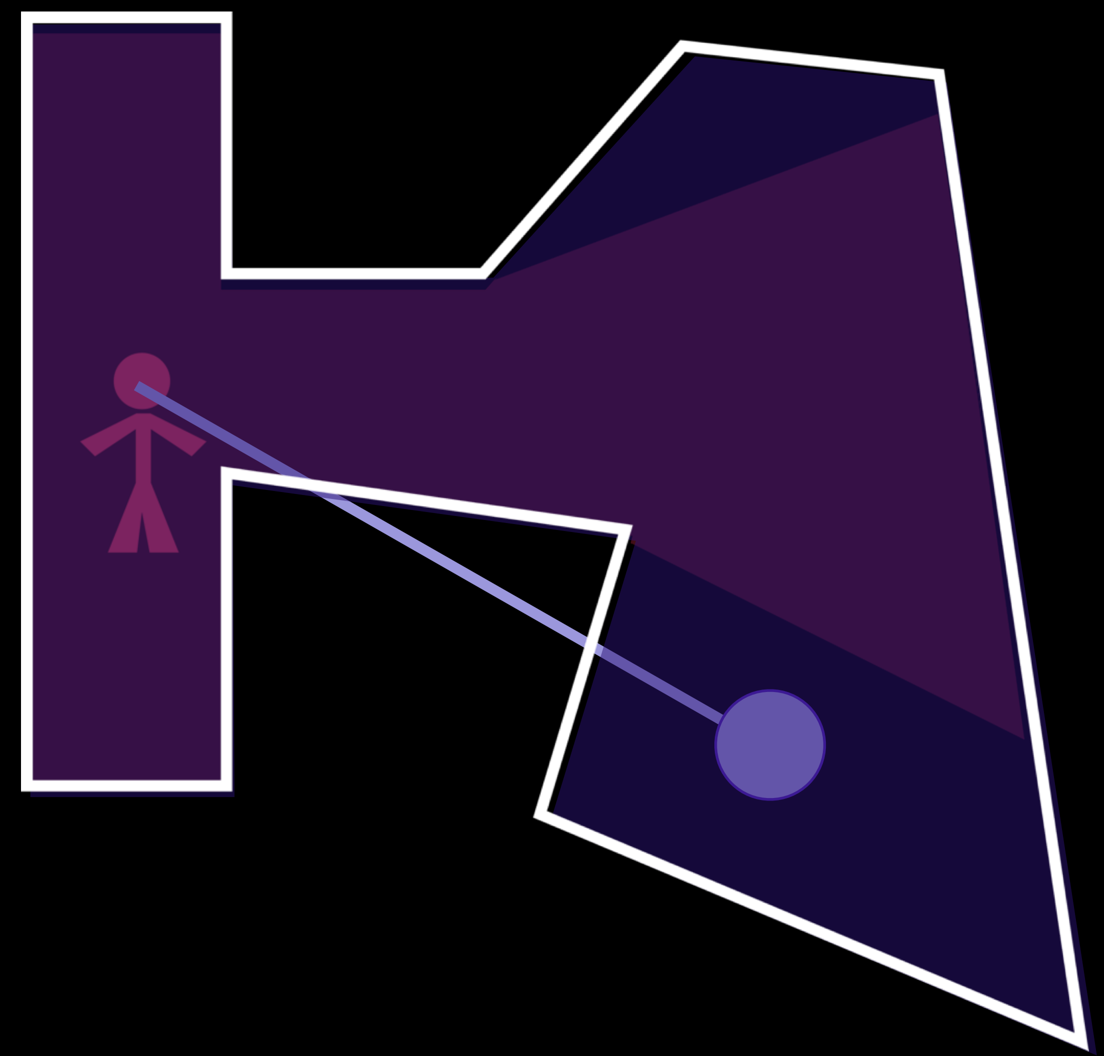
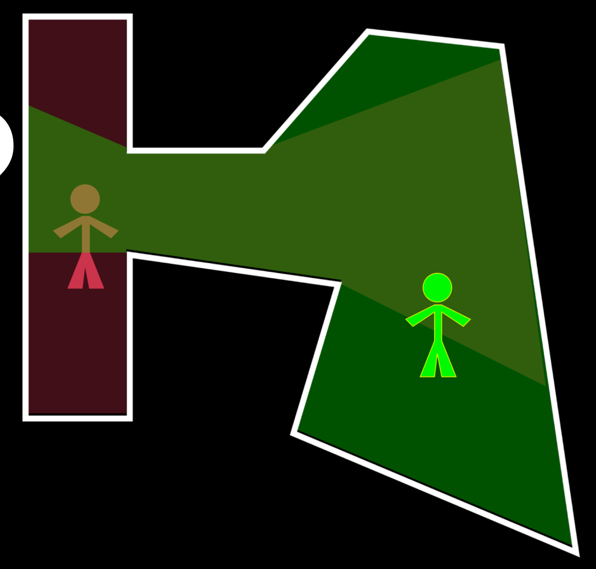


Line crosses at most 2 walls
⇒visible from the 2-transmitter

The Art Gallery Problem (AGP)—and Its Variants?



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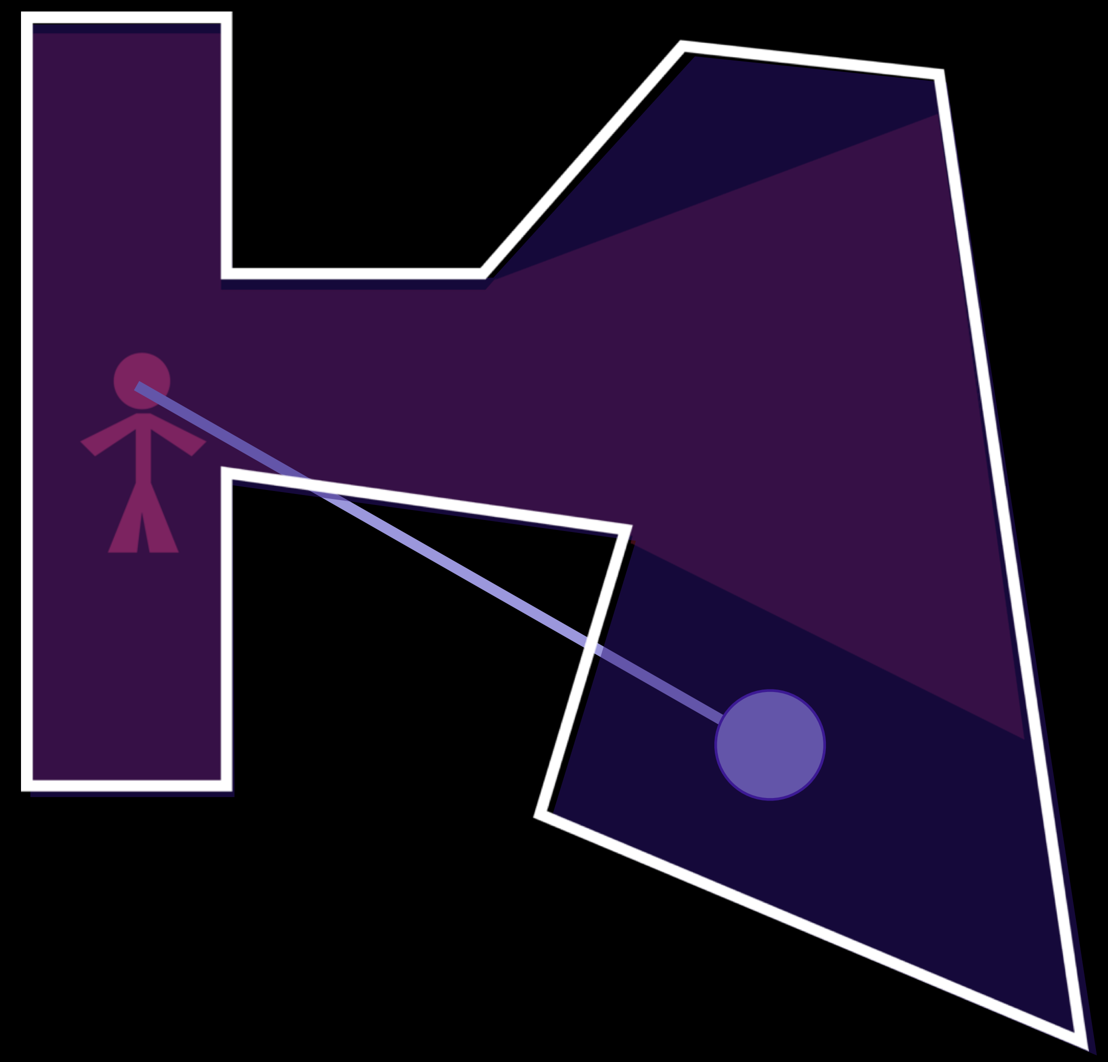

Line crosses at most 2 walls
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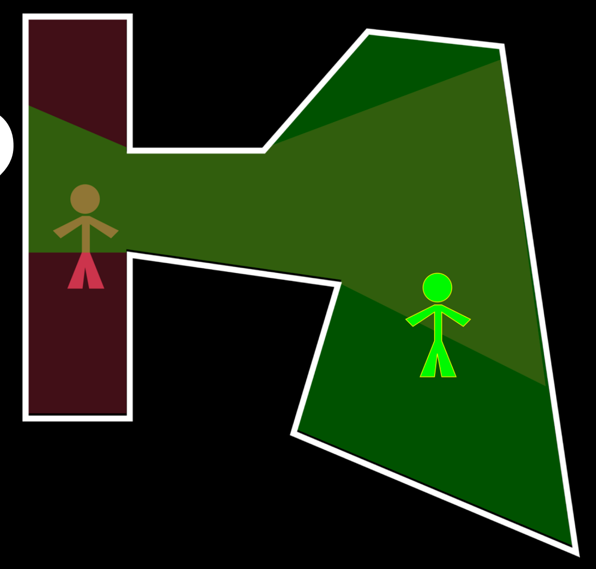
Formally: a point *p* is **2(k)-visible** from a point *q*, if the straight line connection *pq* intersects *P* in at most two **(k)** connected components.

The Art Gallery Problem (AGP)—and Its Variants?



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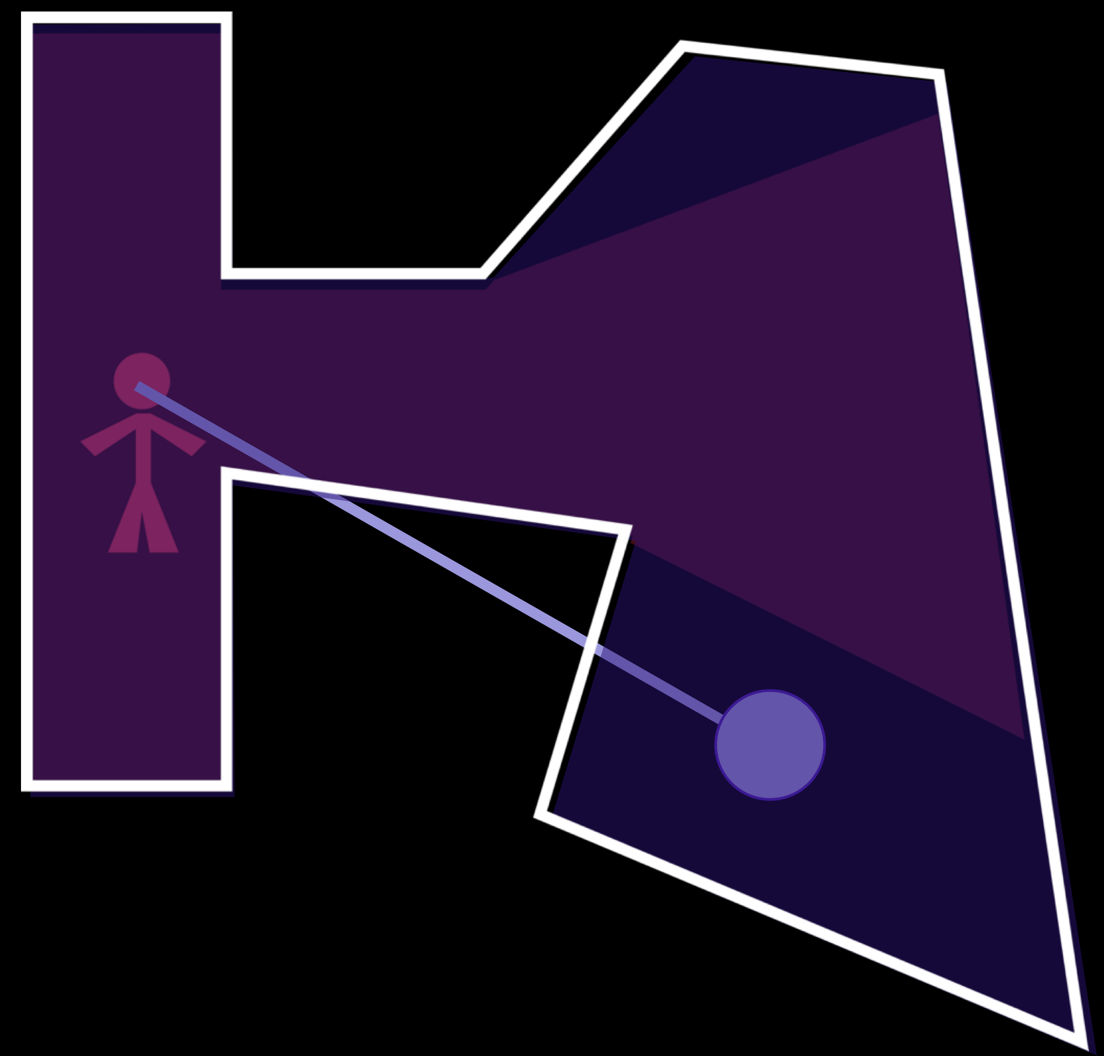

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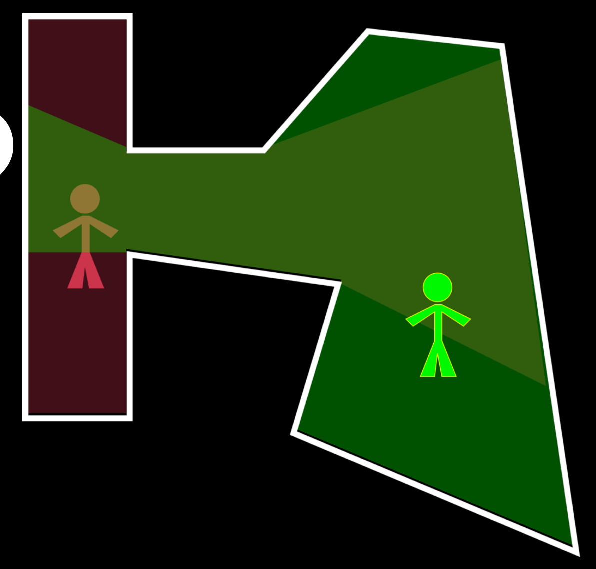
Formally: a point *p* is **2(k)-visible** from a point *q*, if the straight line connection *pq* intersects *P* in at most two **(k)** connected components.

2VR(p) = set of points in P, 2-visible from p
*k*VR(*p*) = set of points in P, *k*-visible from *p*

The Art Gallery Problem (AGP)—and Its Variants?



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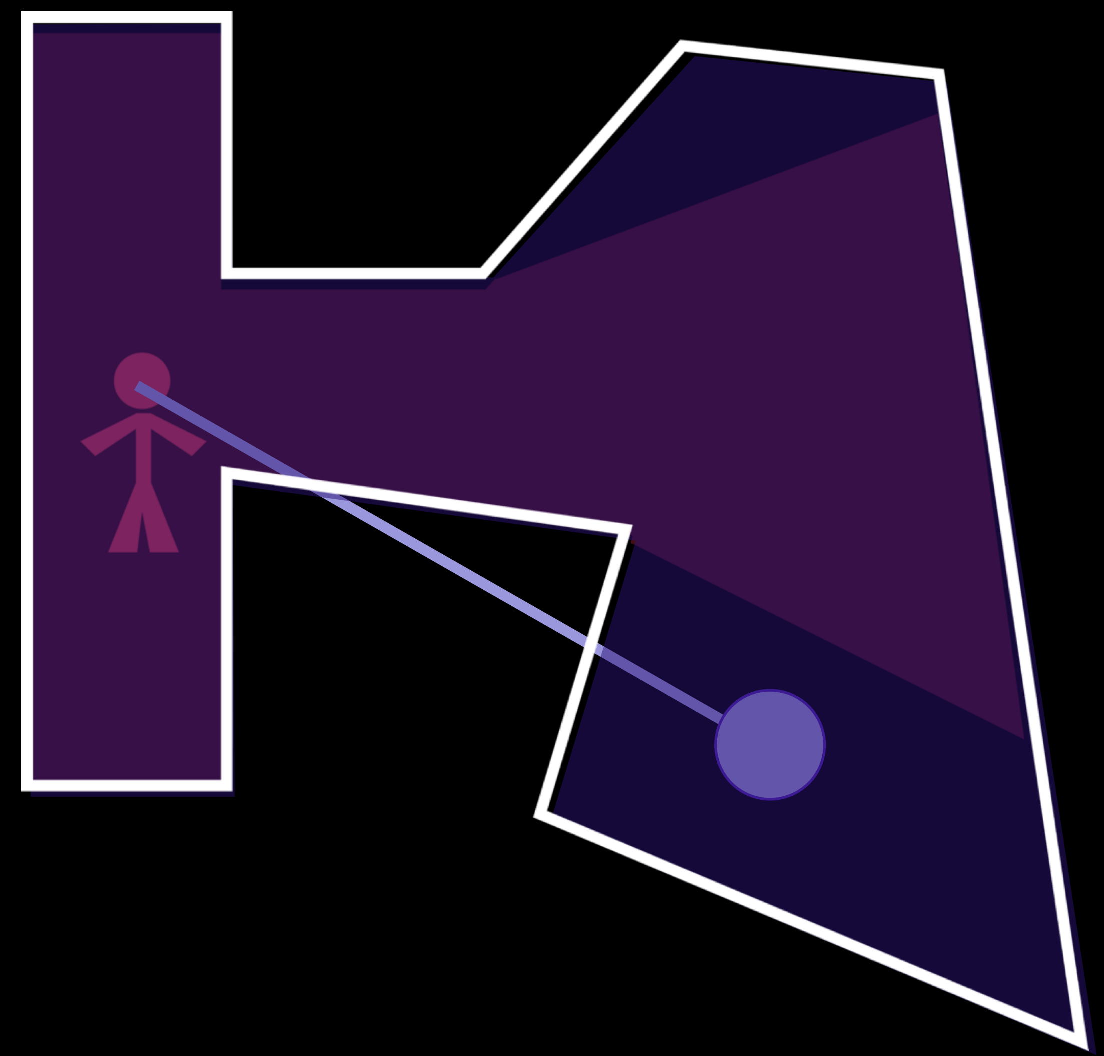

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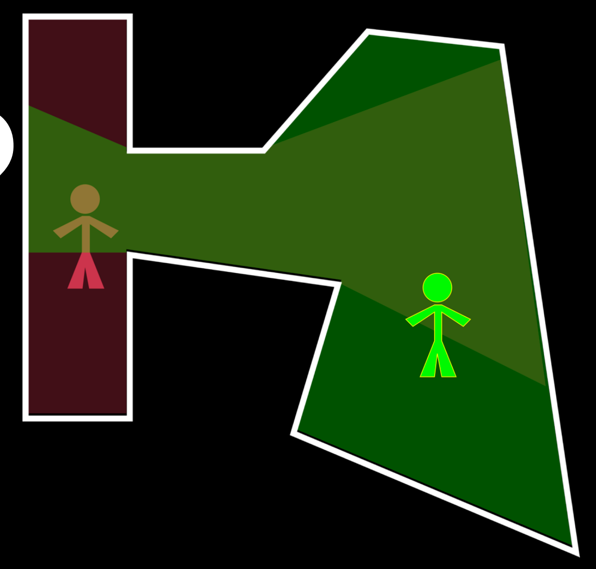
analogue of the visibility polygon

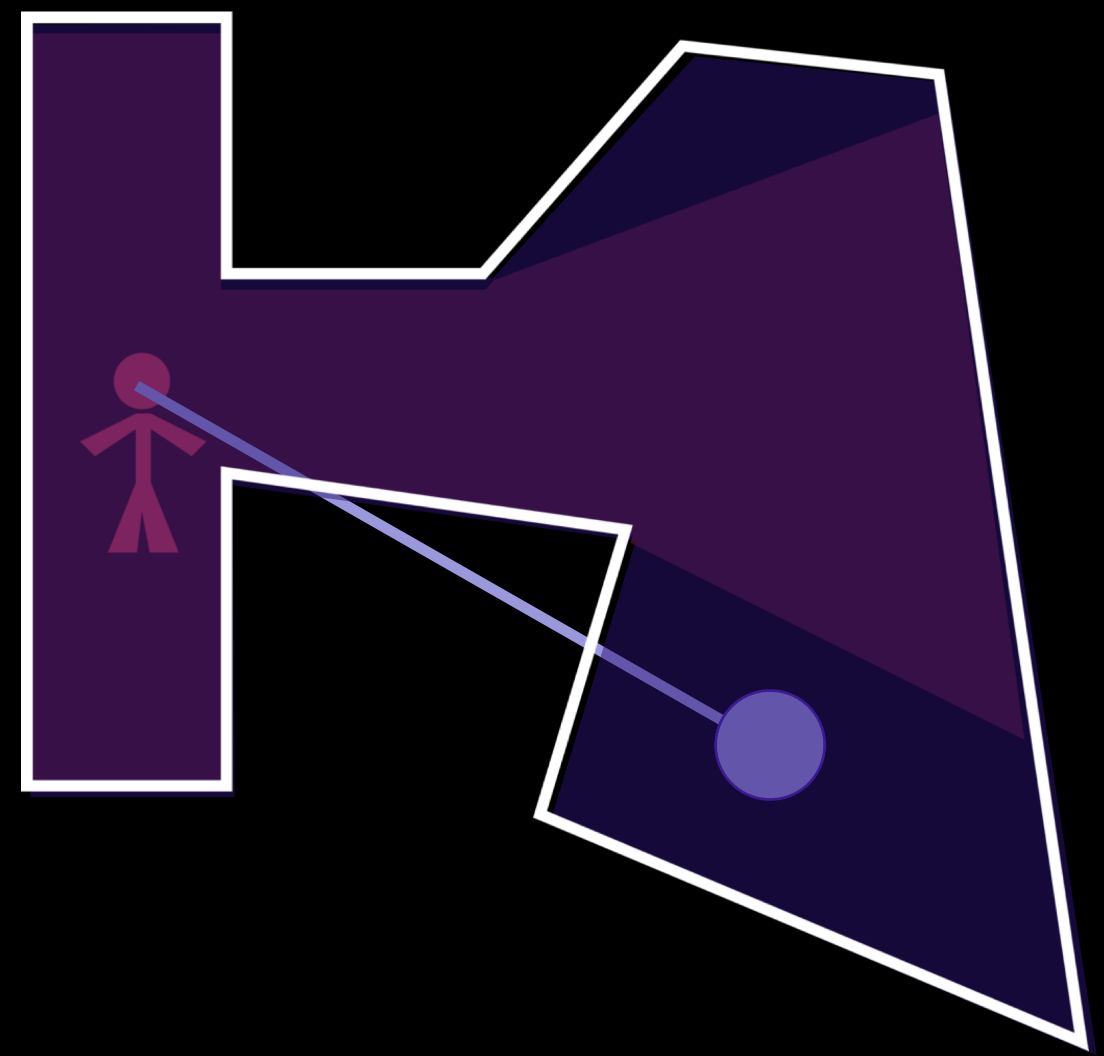
The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

- Capabilities of the guards
- Environment to be guarded

k-transmitter:



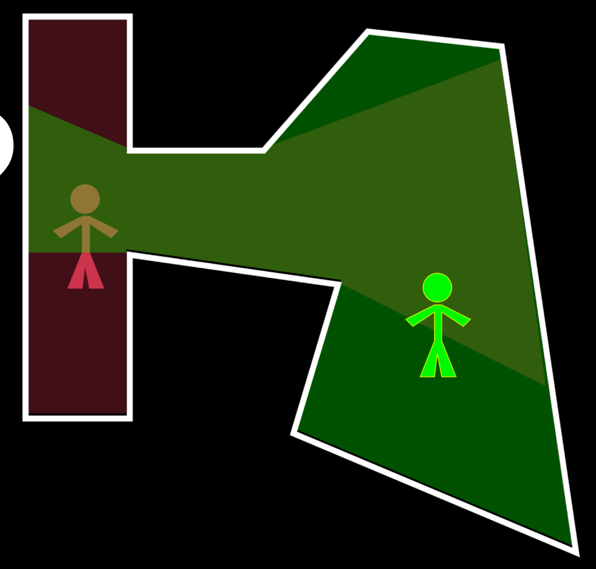
Line crosses at most 2 walls
⇒visible from the 2-transmitter

Formally: a point *p* is **2(k)-visible** from a point *q*, if the straight line connection *pq* intersects *P* in at most two **(k)** connected components.

2VR(p) = set of points in P, 2-visible from p
*k*VR(*p*) = set of points in P, *k*-visible from *p*

Stationary: analogue of the visibility polygon

The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

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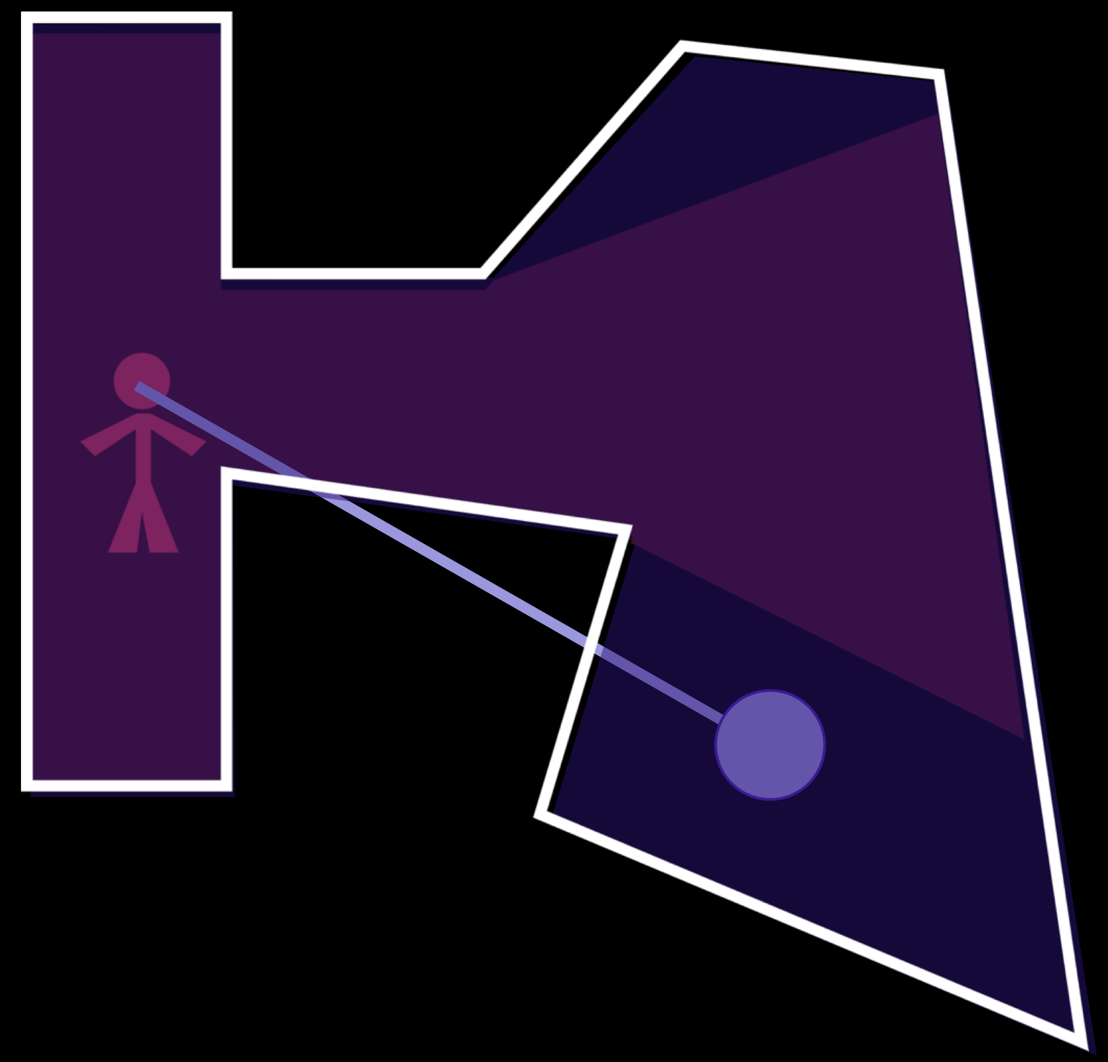

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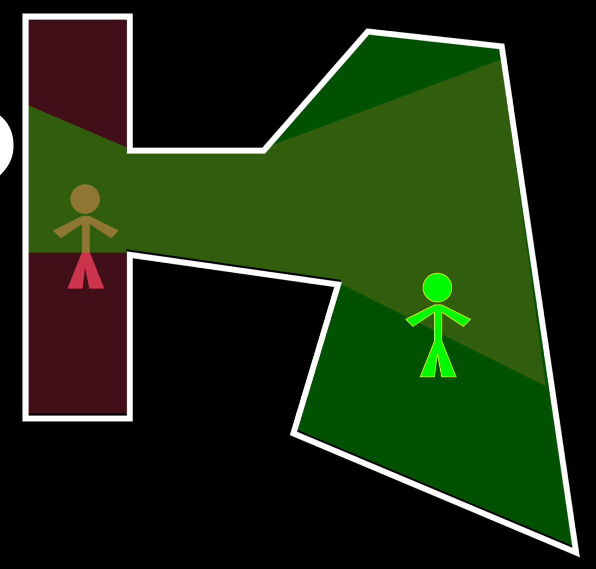

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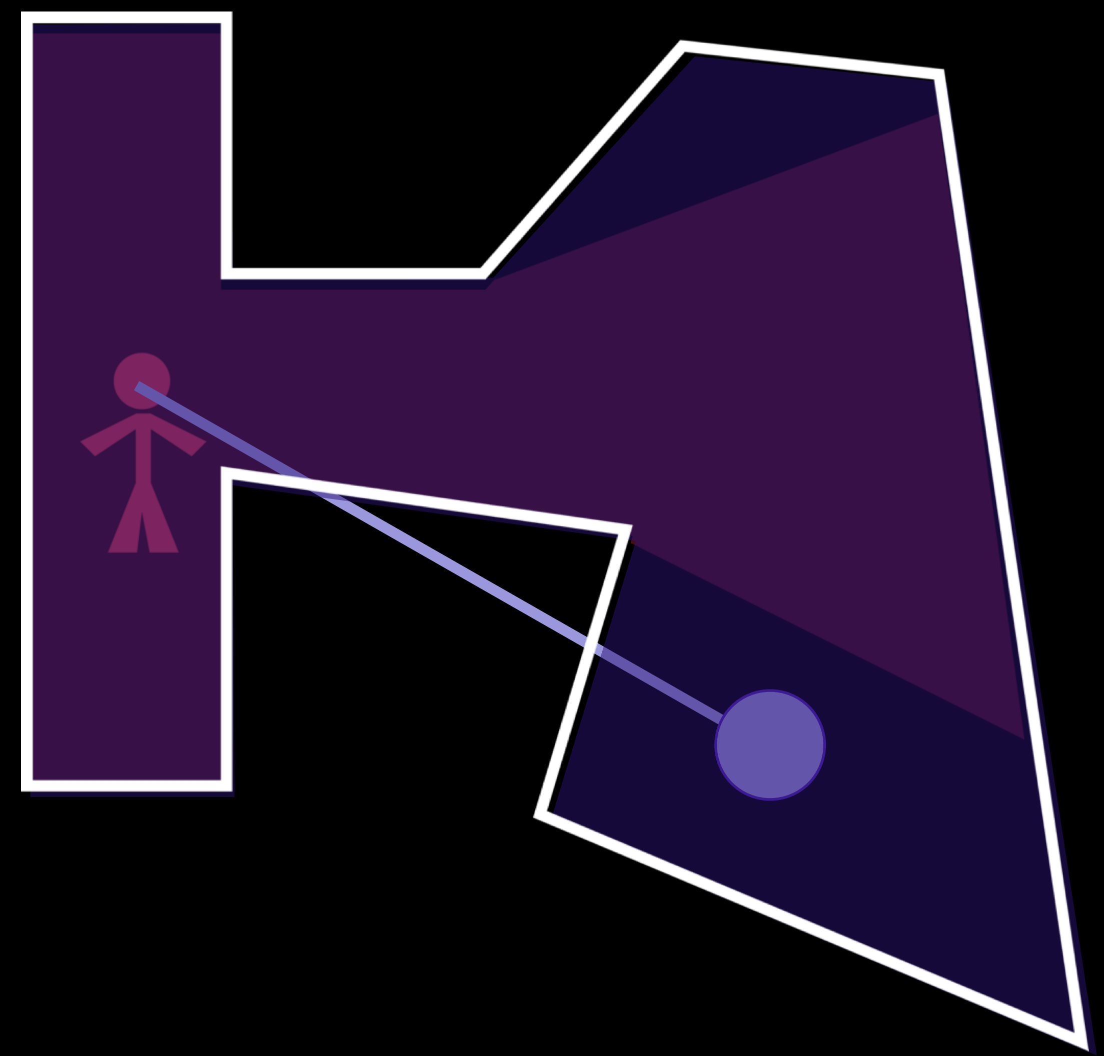

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Stationary: analogue of the visibility polygon

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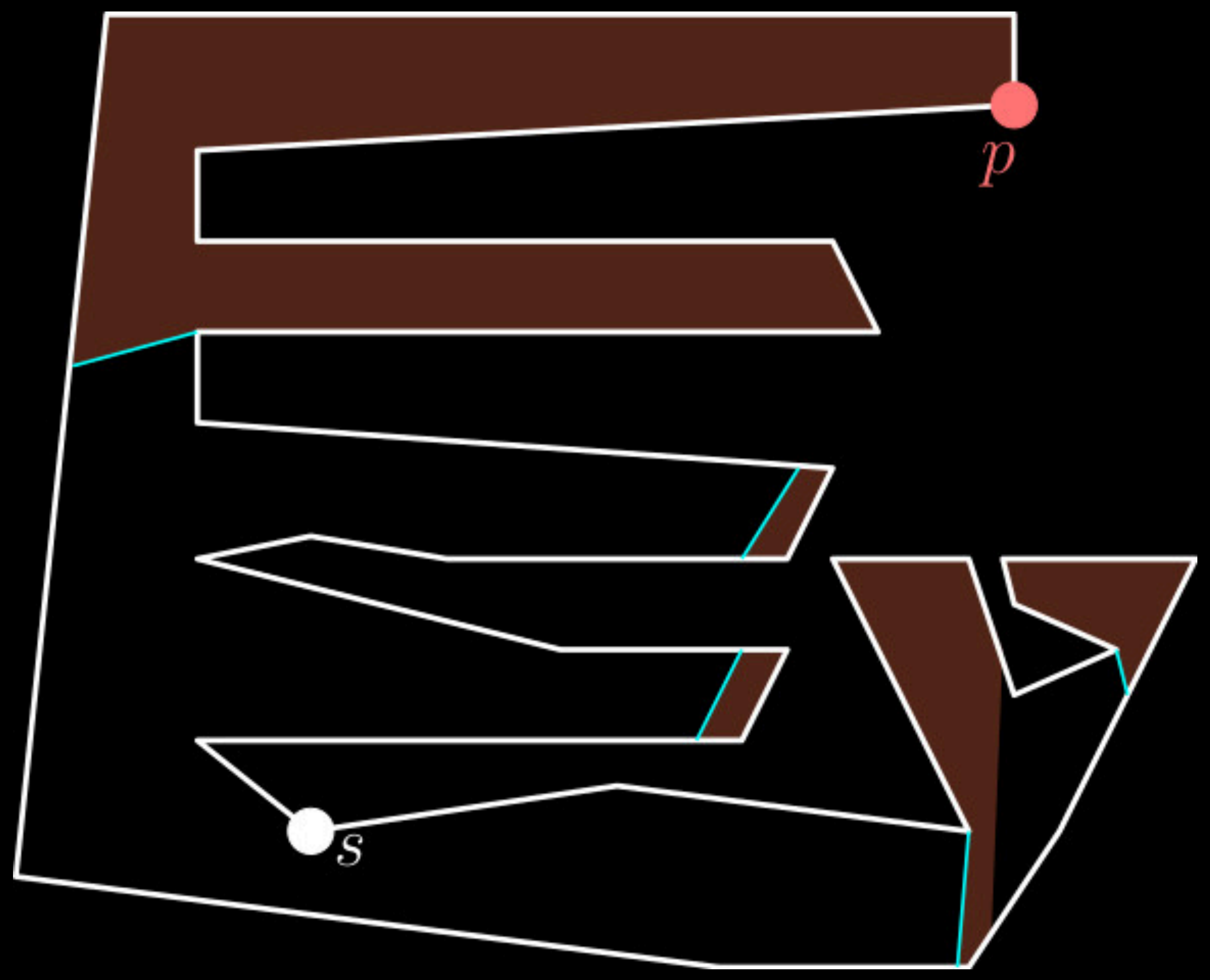
k-/2-Transmitter



2VR(p)/kVR(p) can have O(n) connected components.

k-Transmitters

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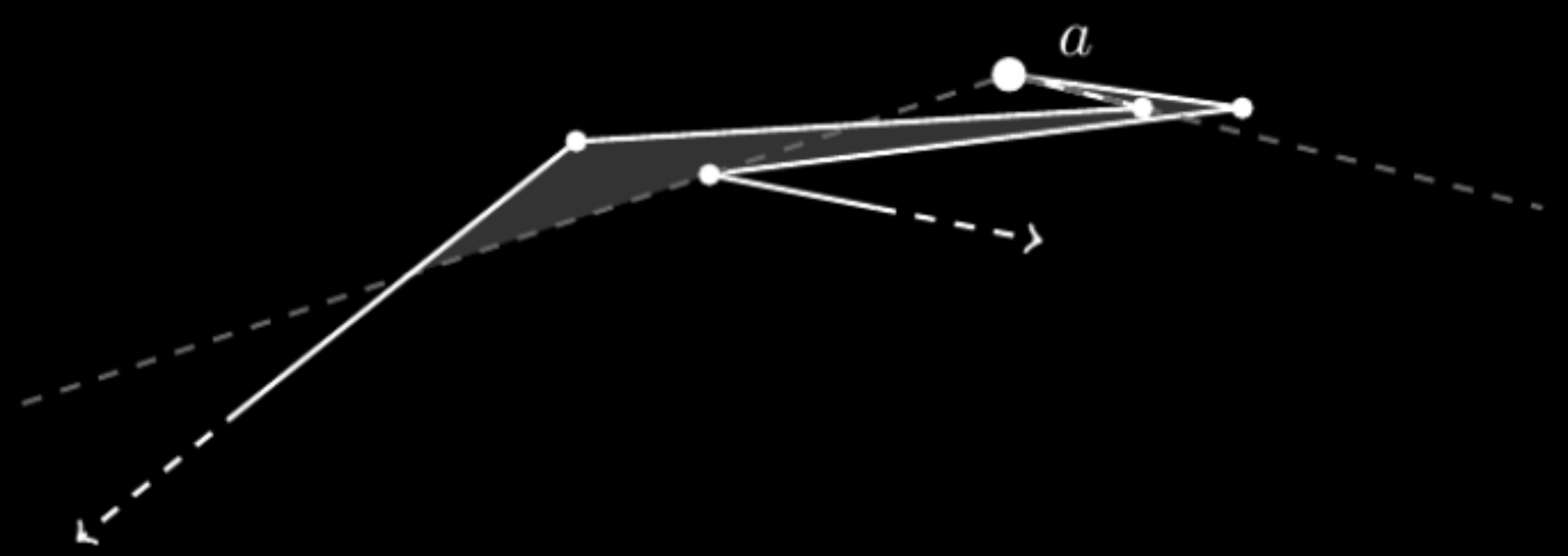

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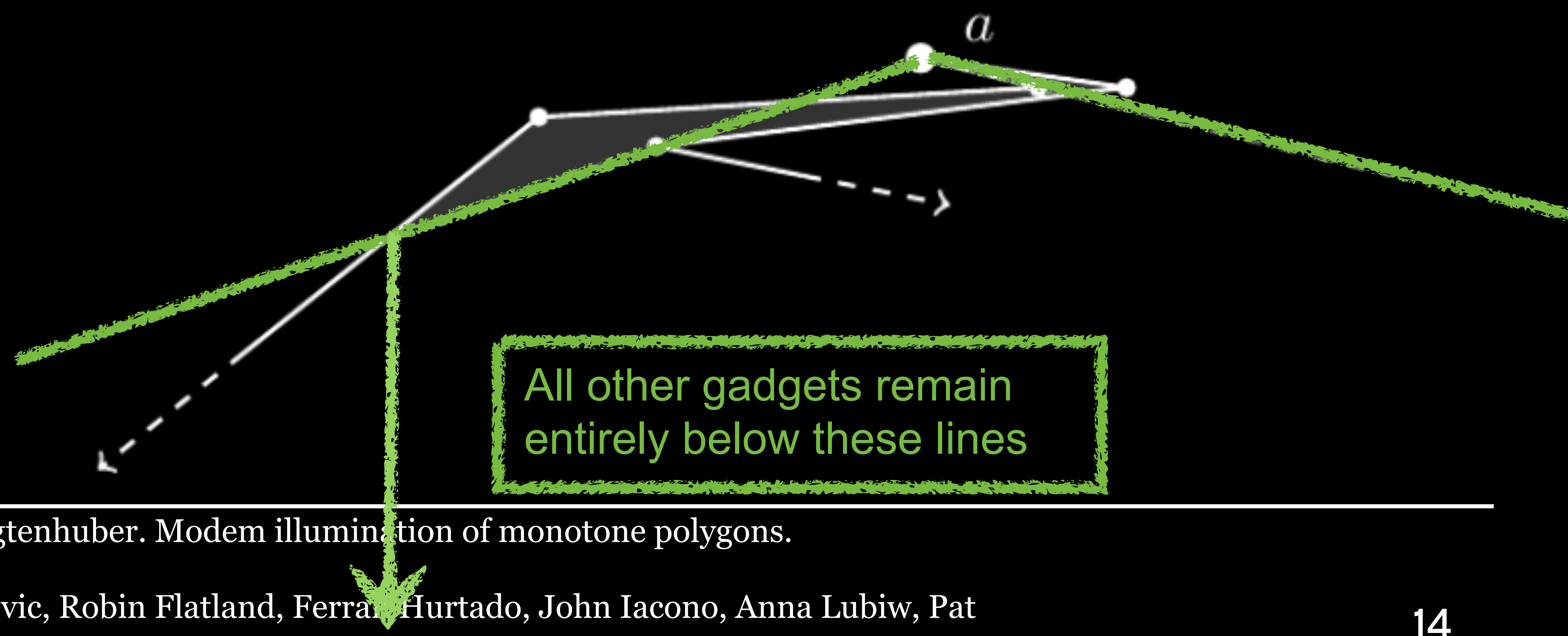

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  - CFIL**S**2018: NP-hard to compute point 2-transmitter/point k-transmitter/edge 2-transmitter cover in simple polygon, point 2-transmitter also for orthogonal polygons

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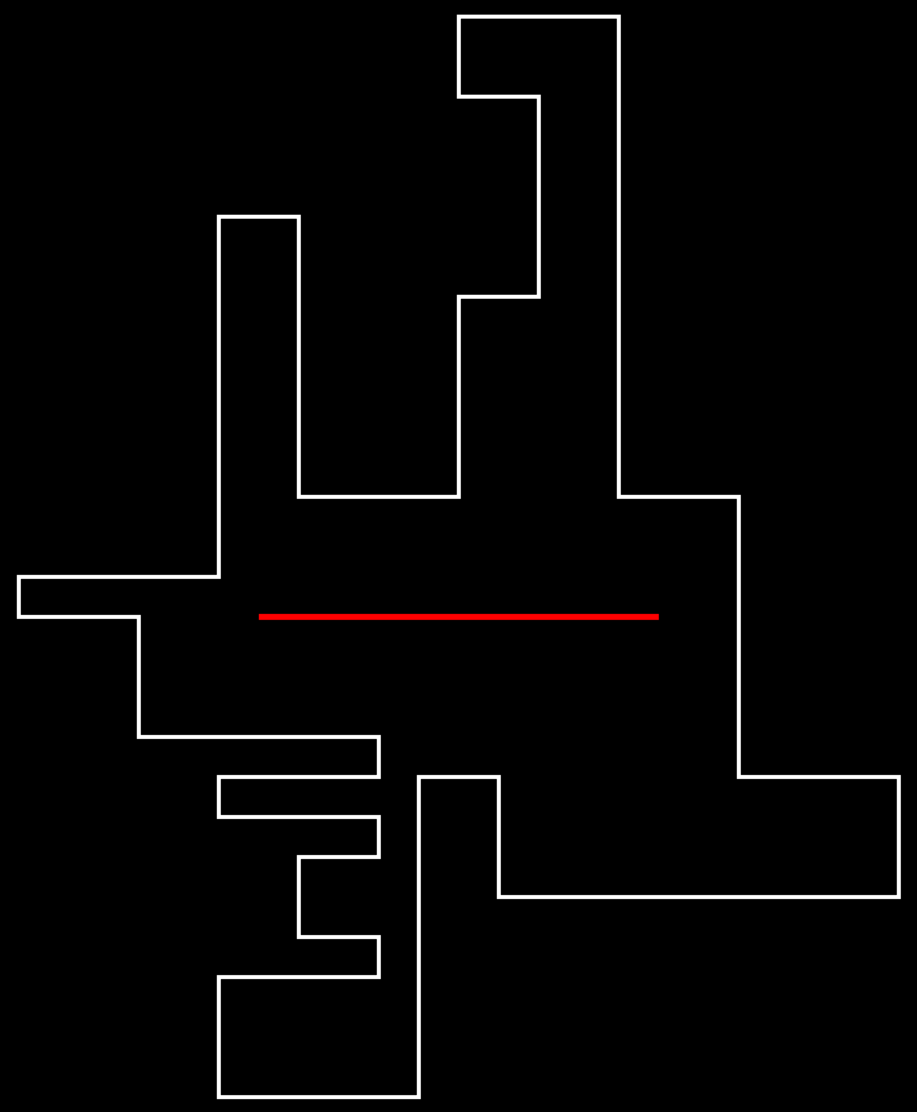

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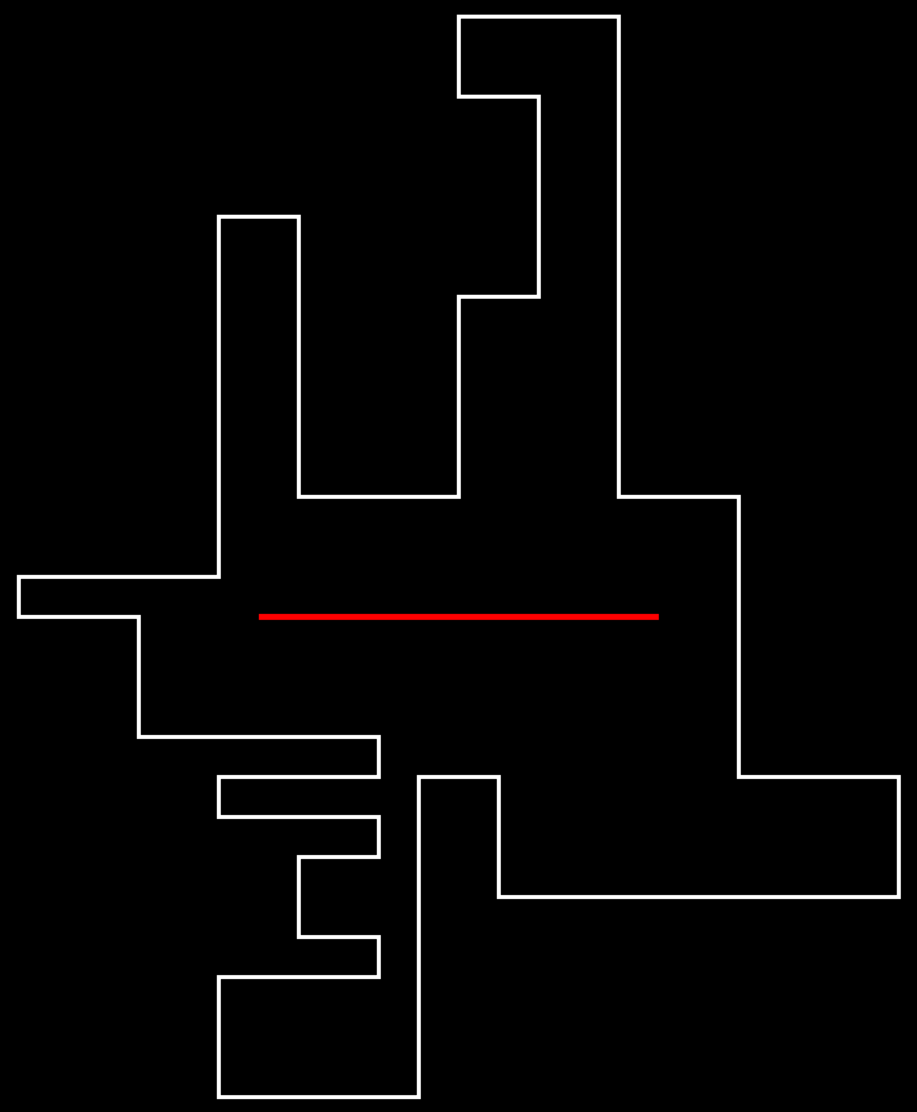

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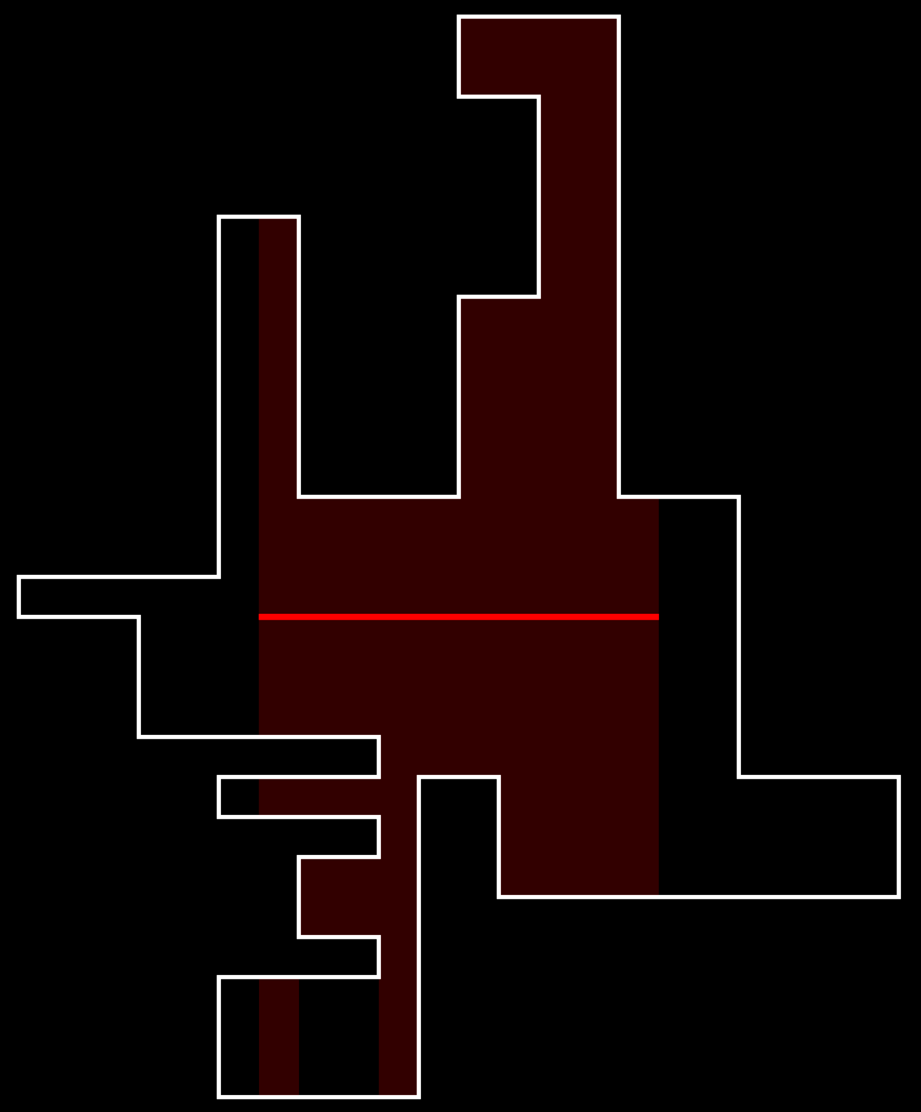

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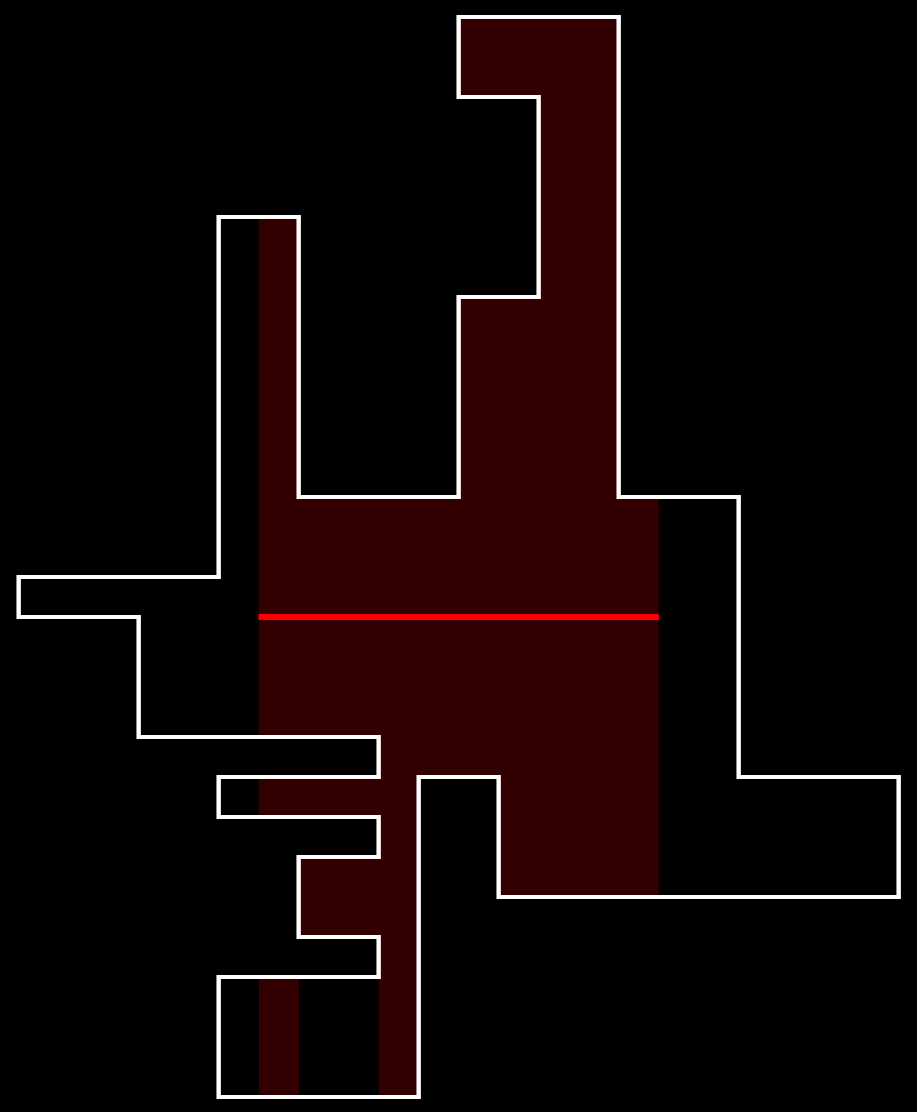
  - MSG2020(/2014): minimize total length of the k-transmitters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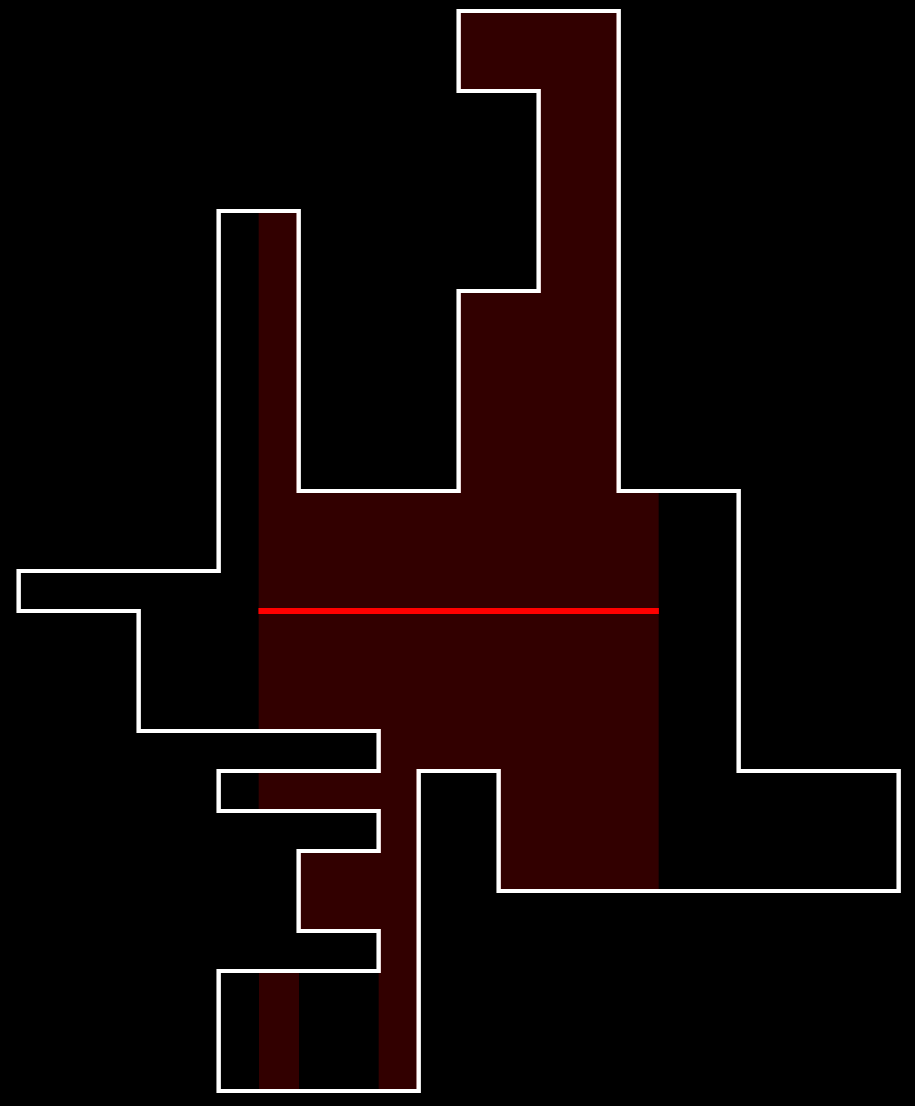
  - MSG2020(/2014): minimize total length of the k-transmitters
    - NP-hard for k=2



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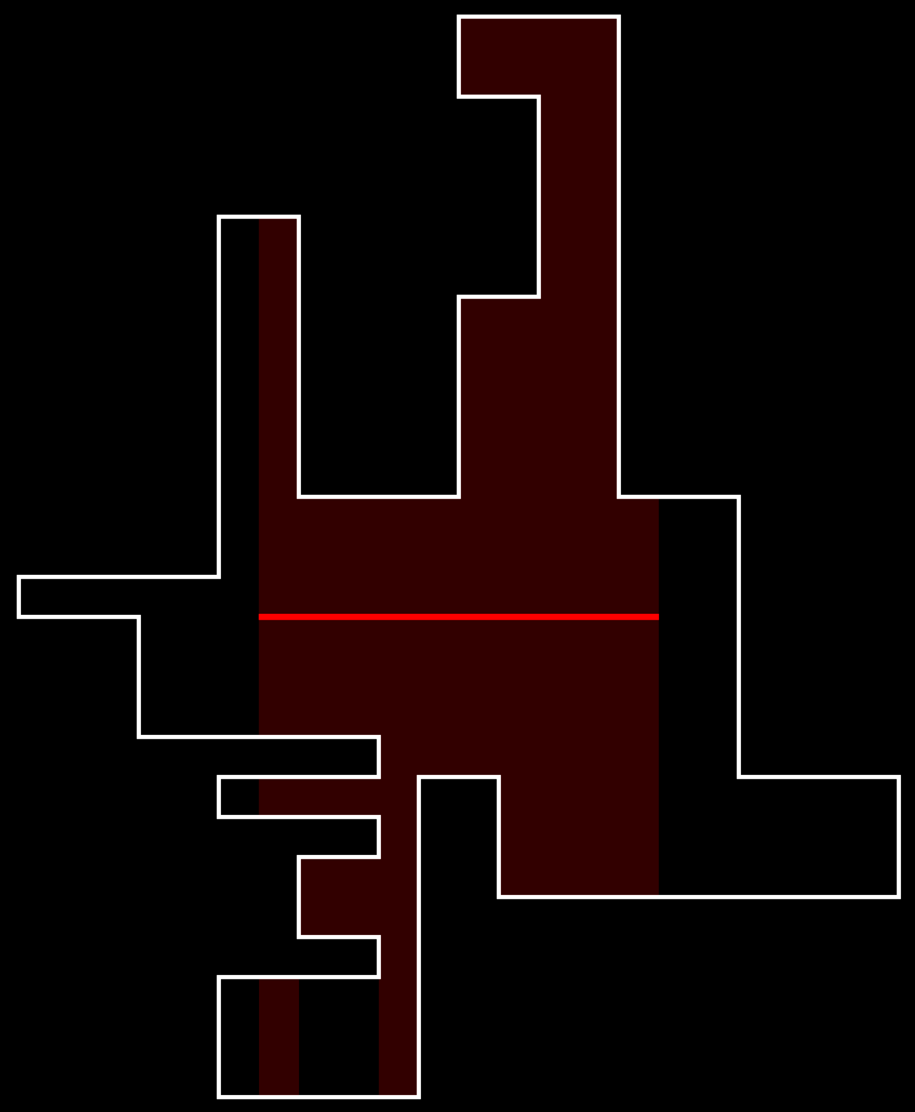
    - 2-approximation



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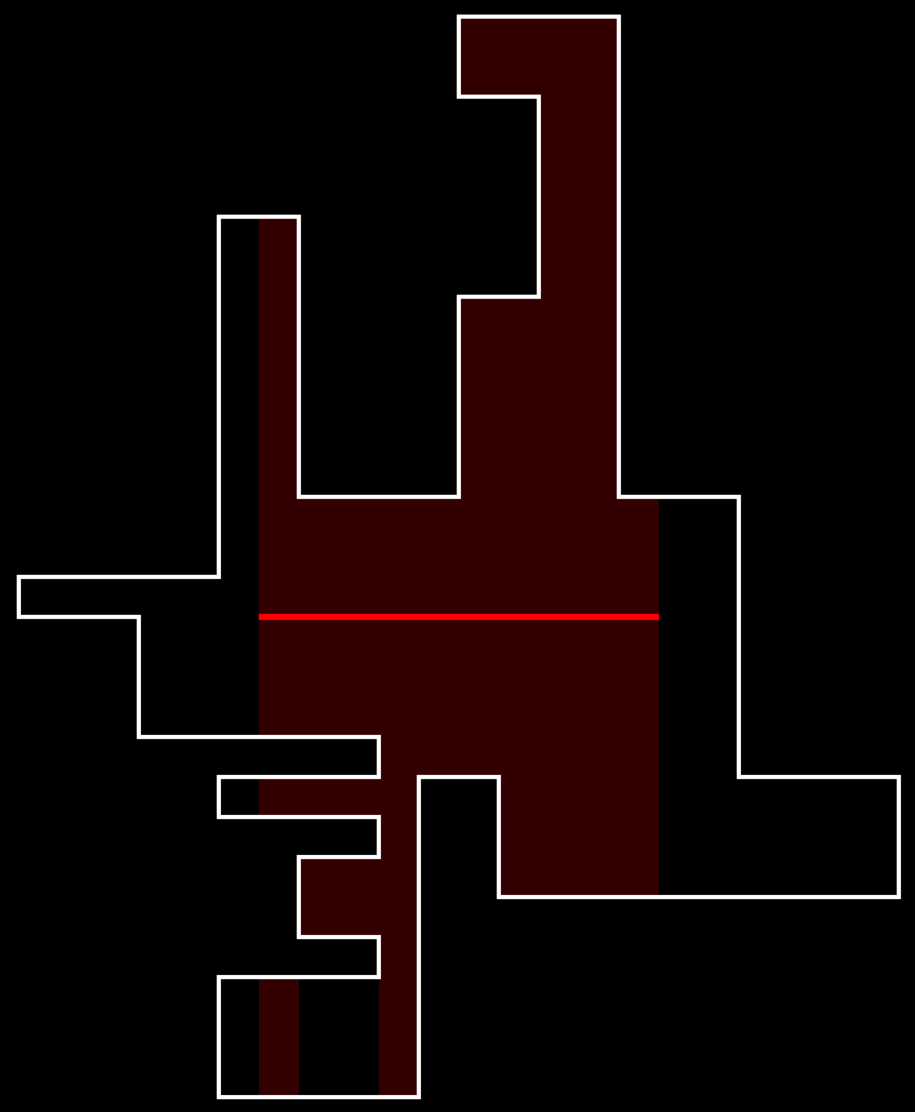

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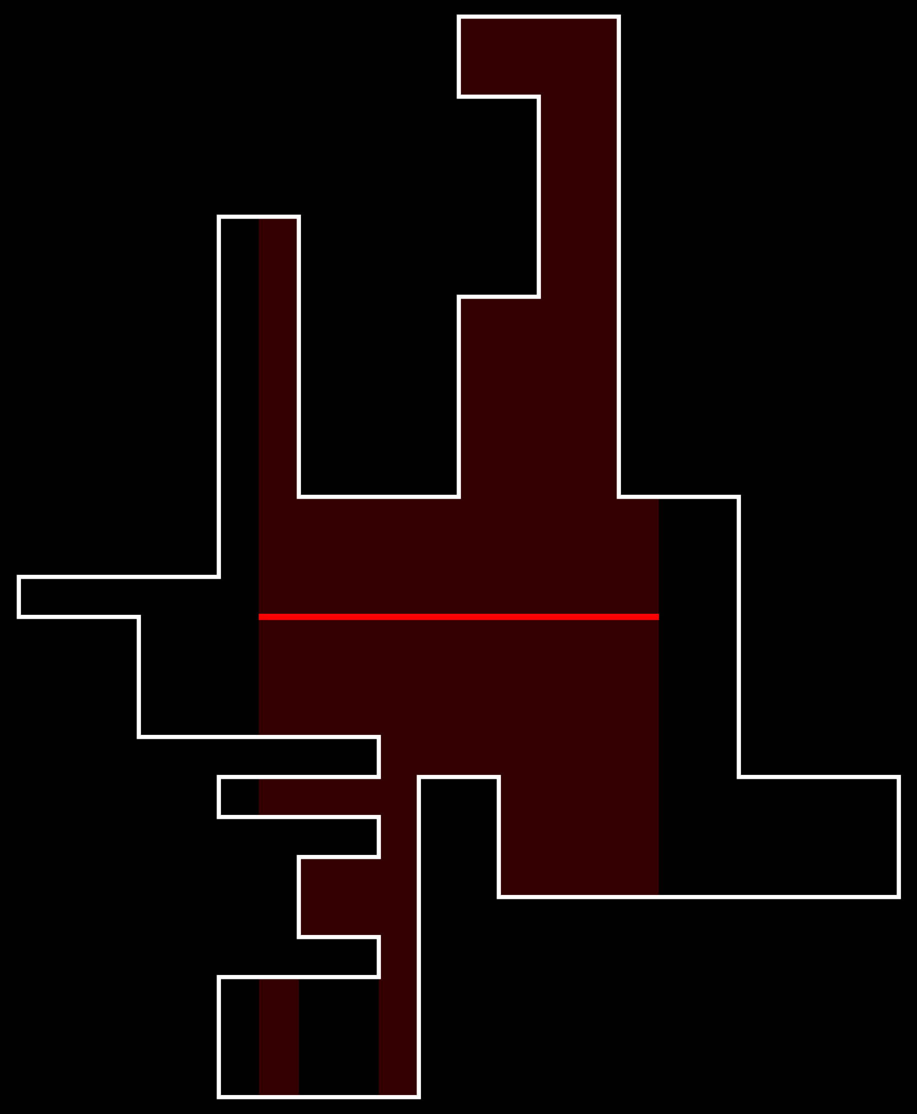
k-Transmitters



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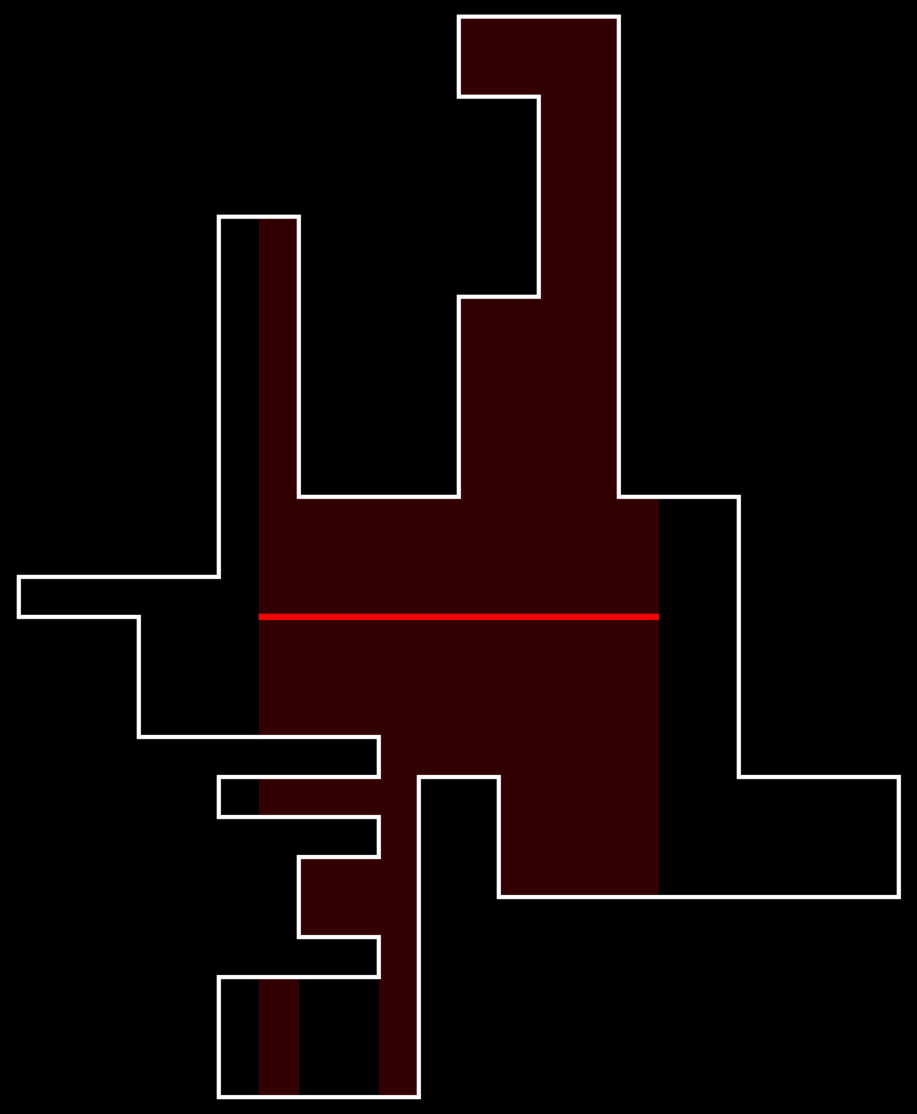

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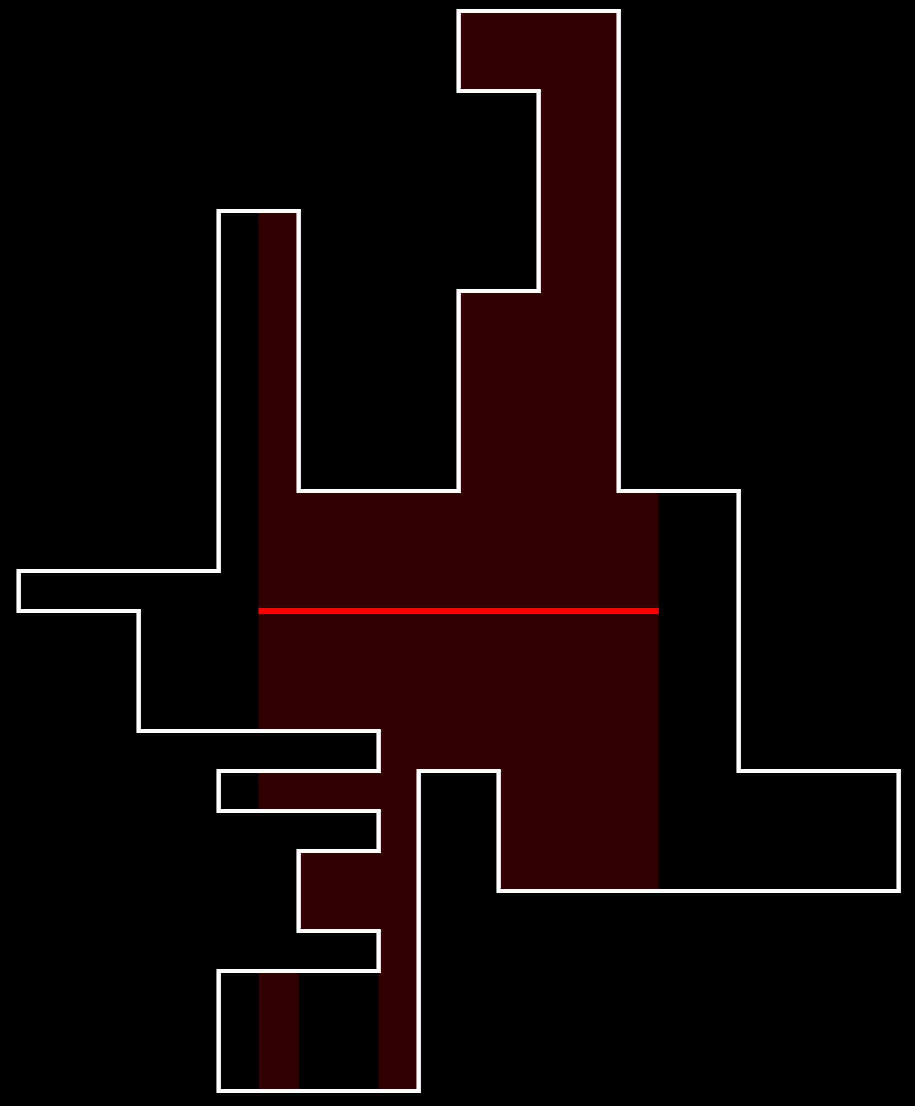

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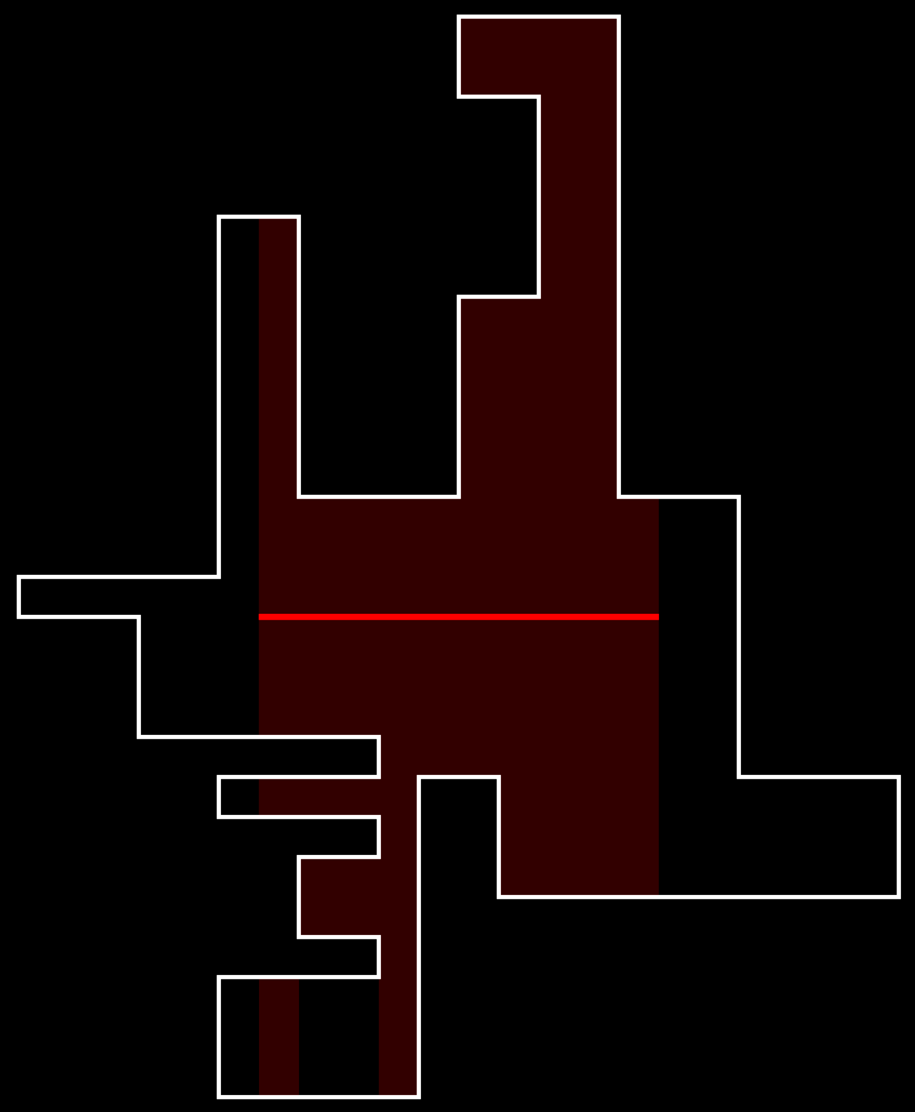

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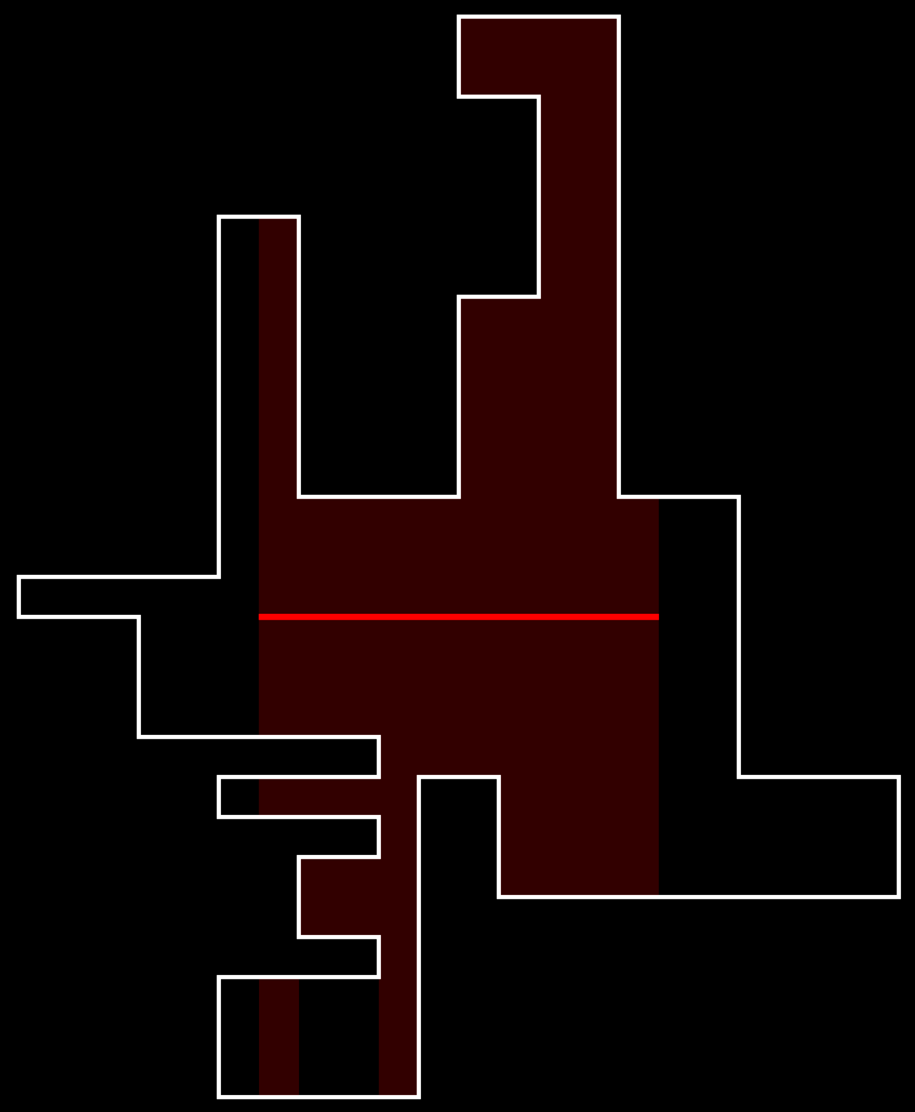

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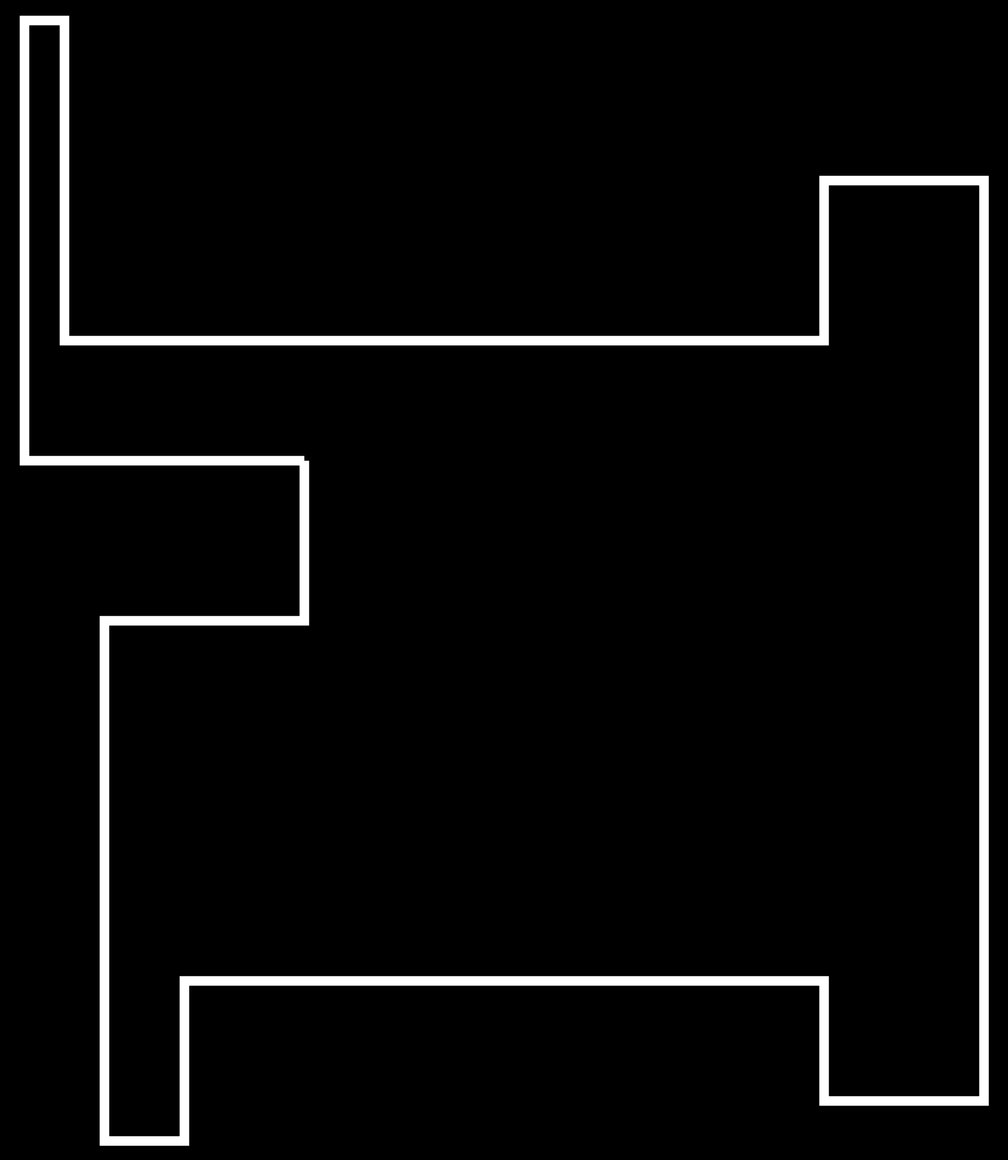

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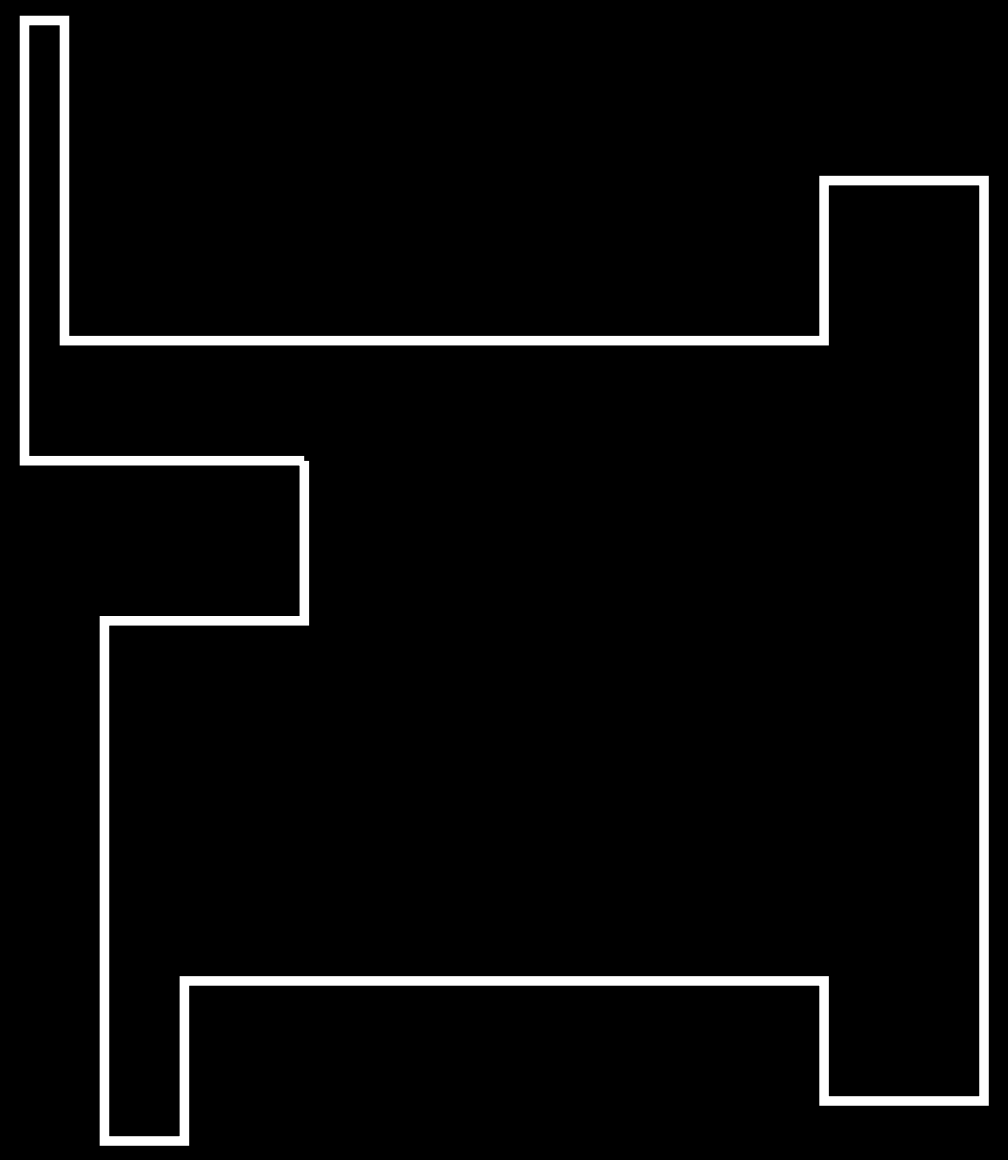

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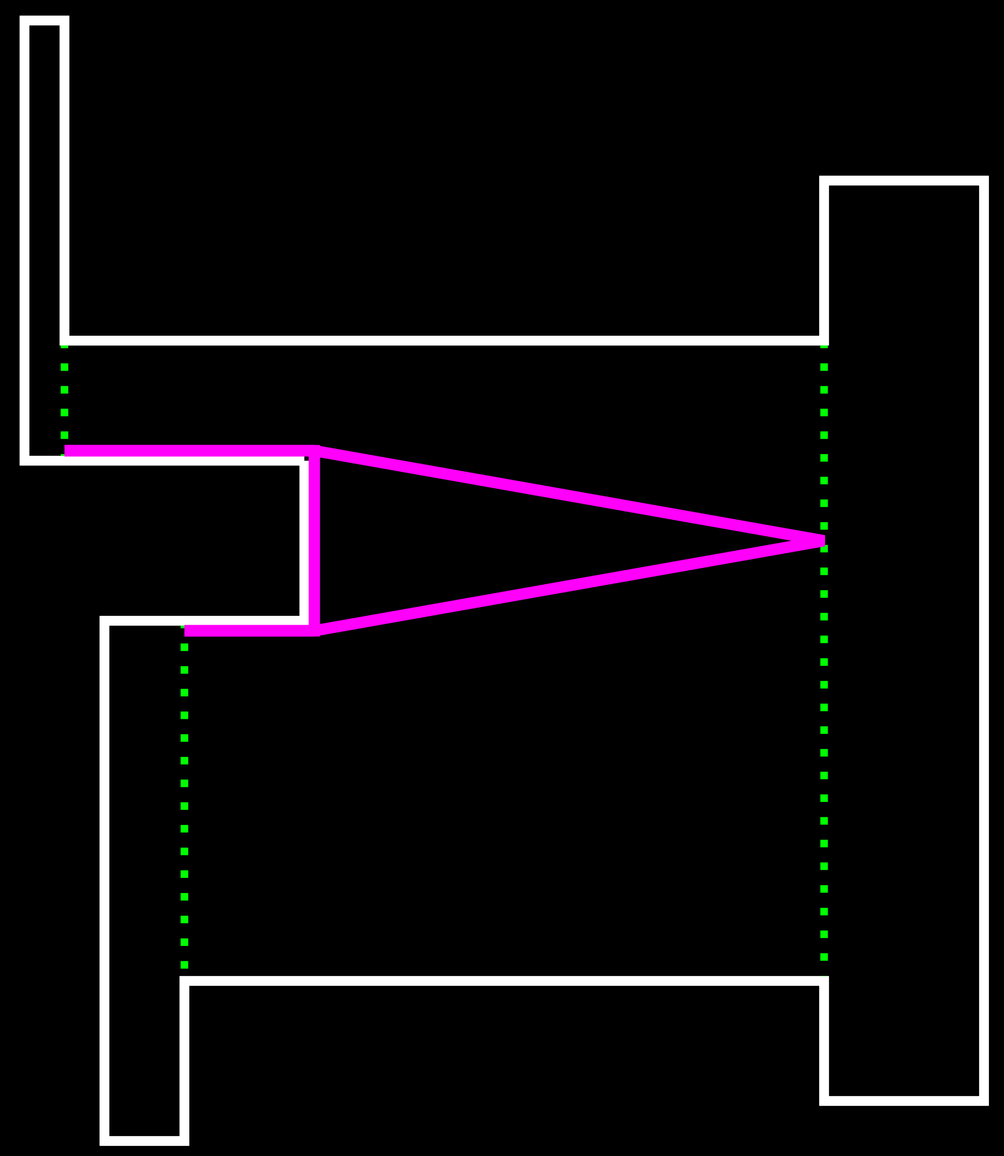

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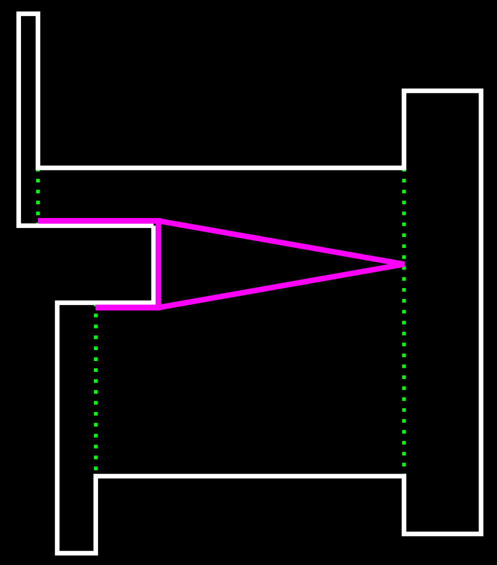

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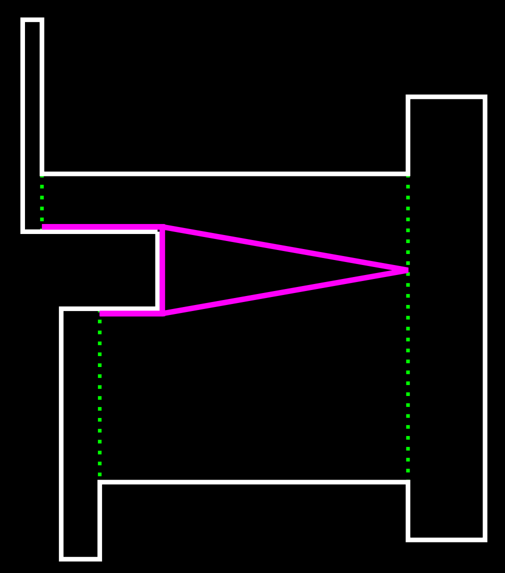

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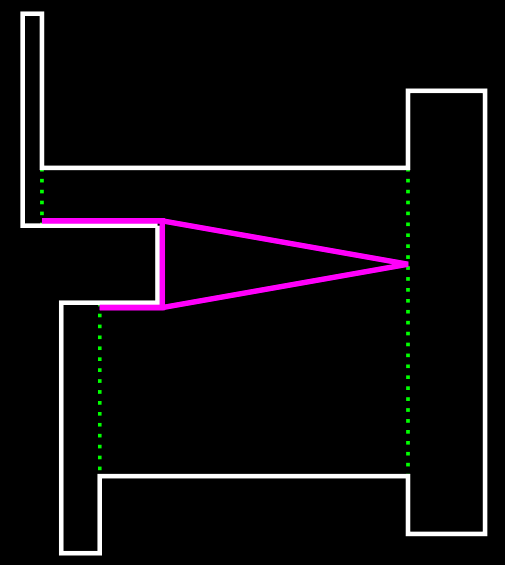

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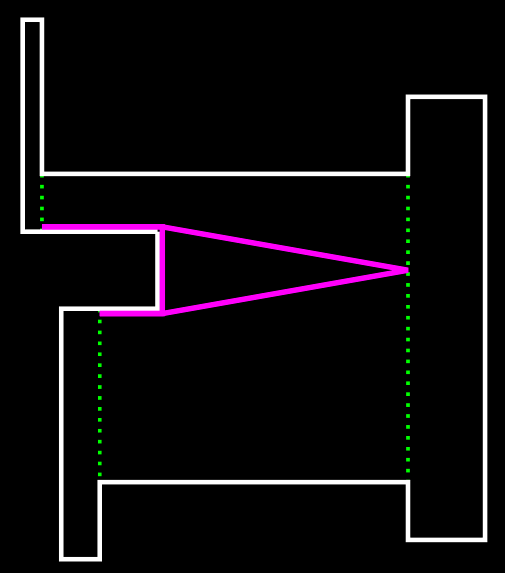

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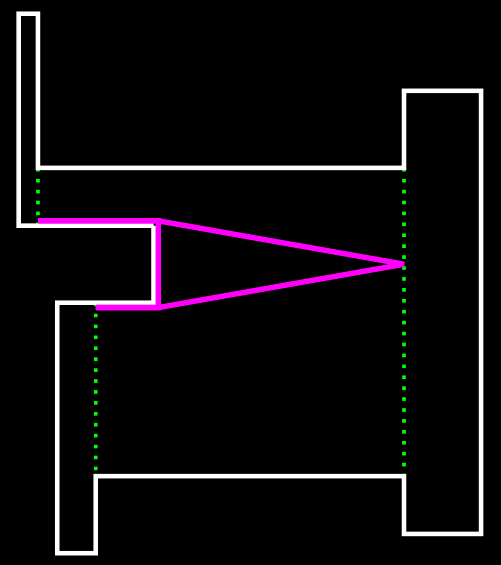

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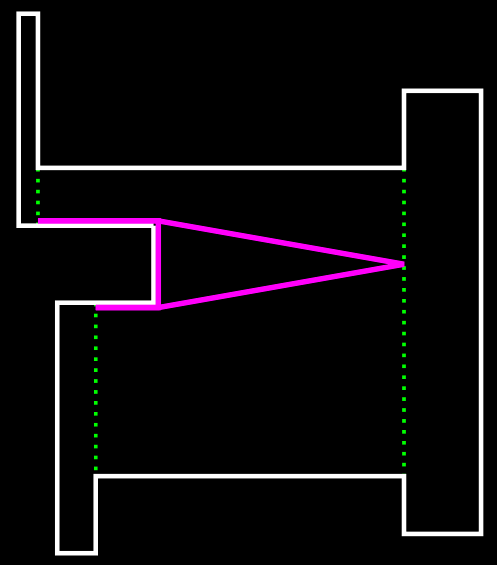

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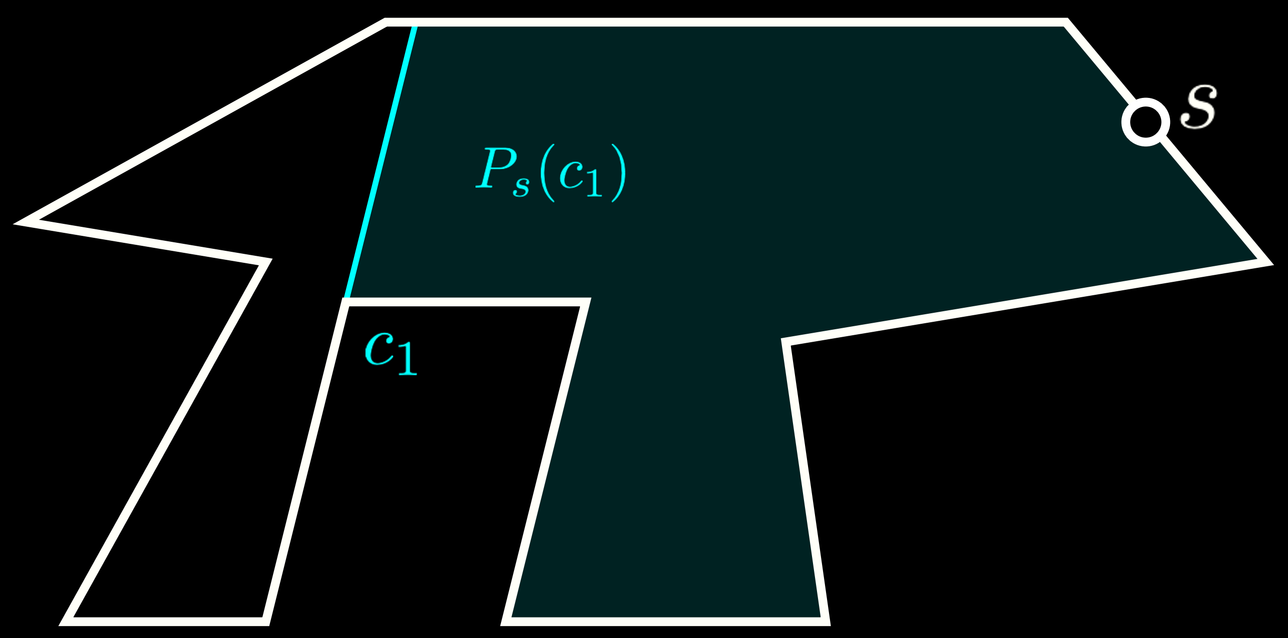

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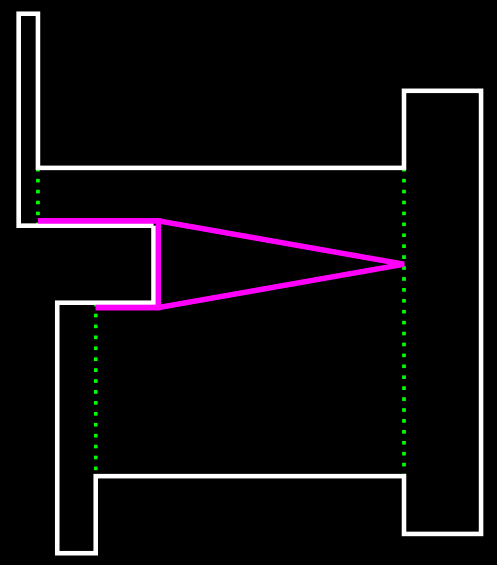

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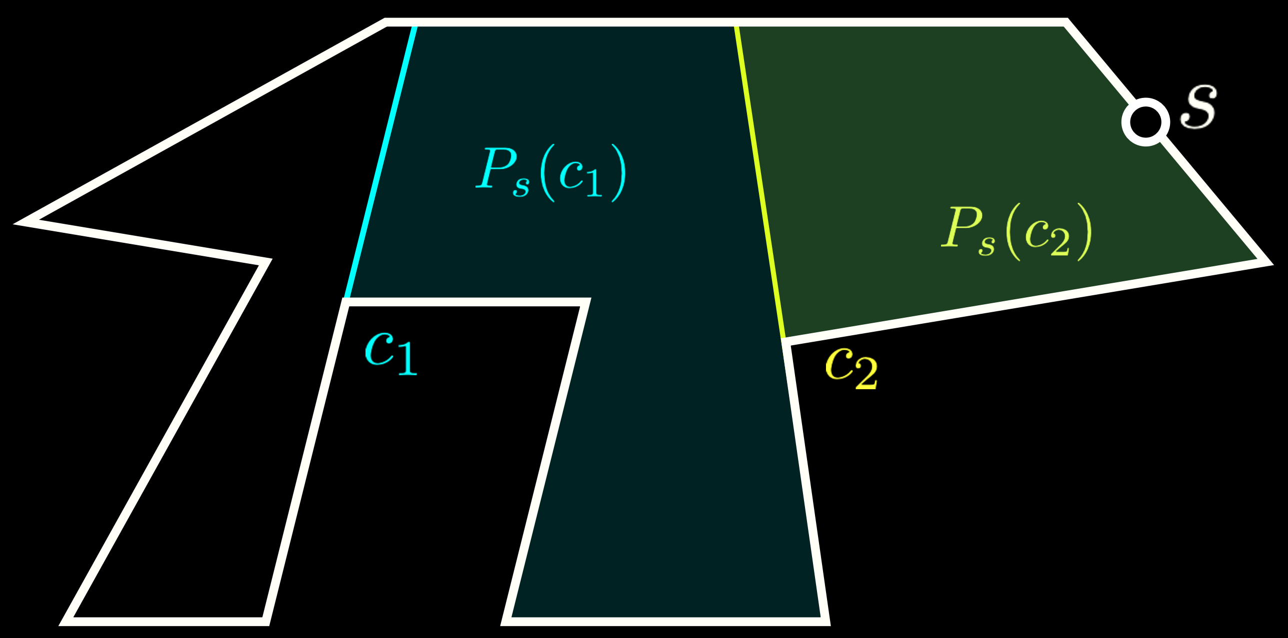

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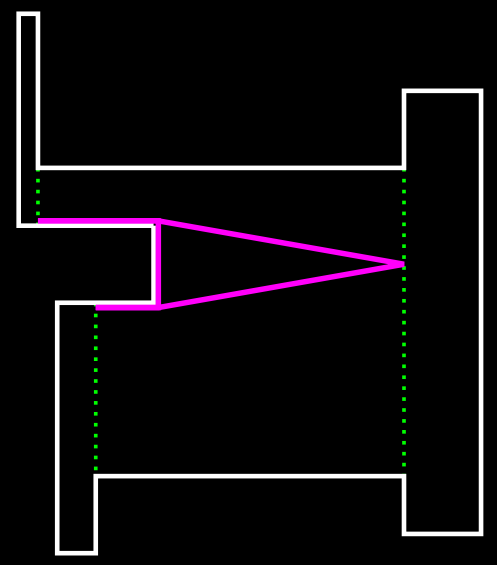

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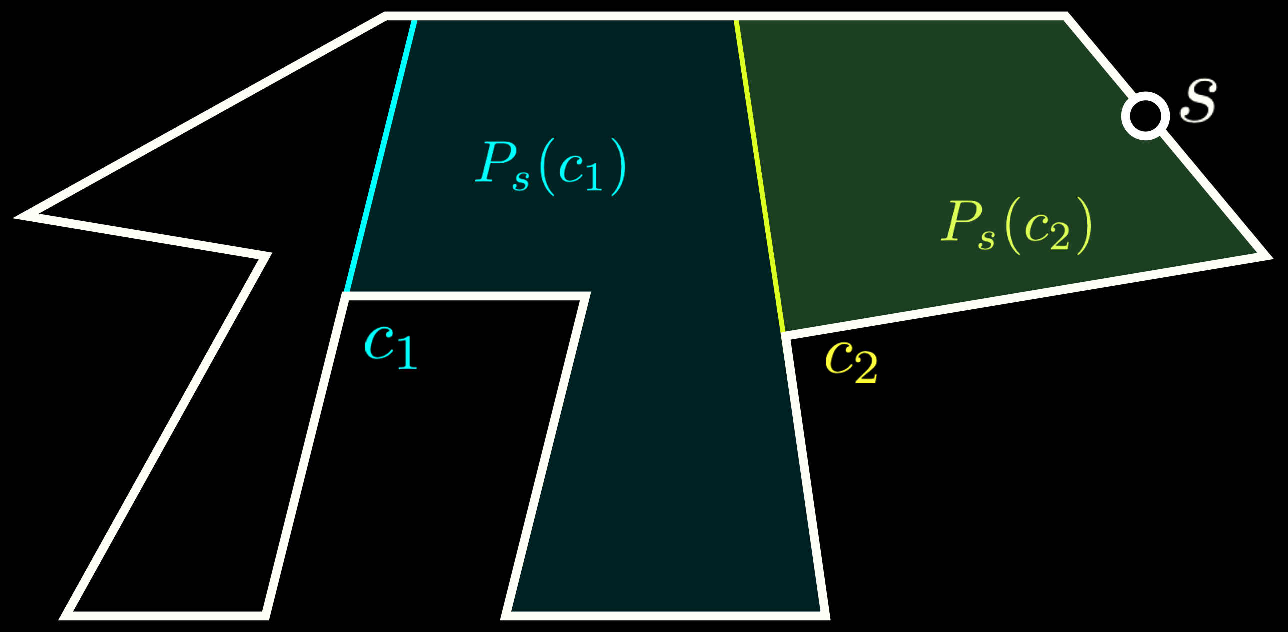

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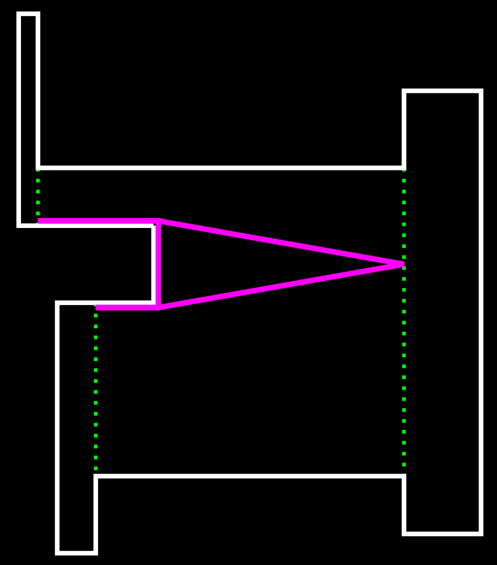

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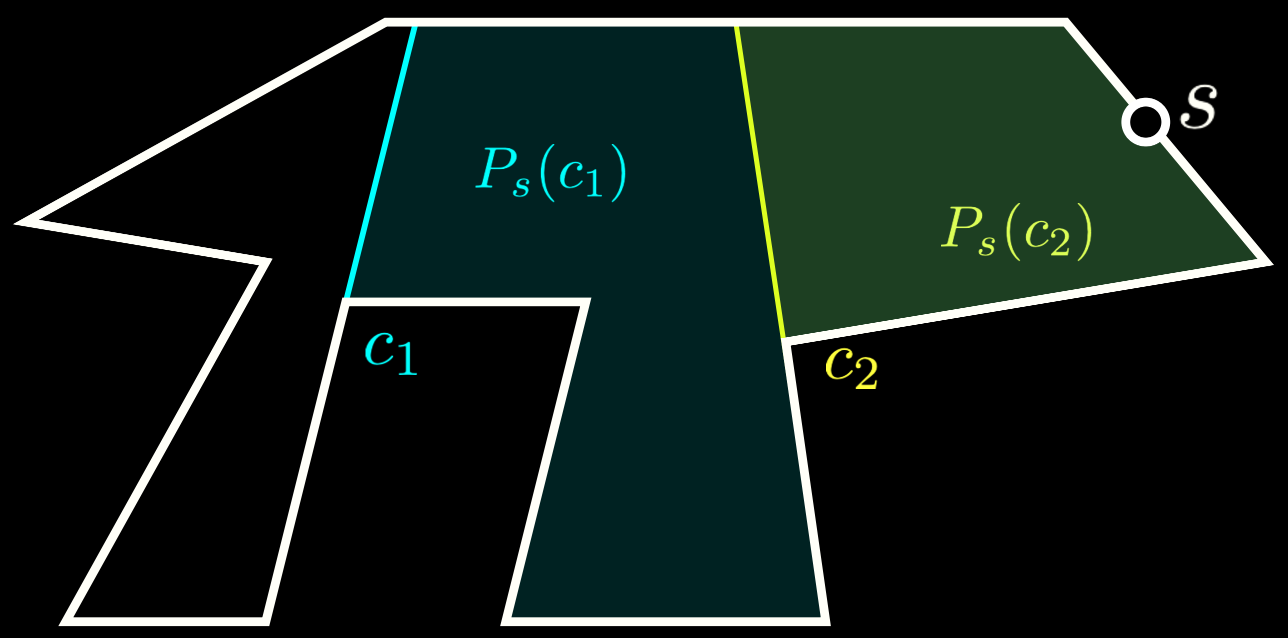

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k-Transmitter Watchman Routes

[Nilsson, S., 2022]

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To appear in WALCOM 2023
Preprint on arXiv: https://arxiv.org/abs/2202.01757

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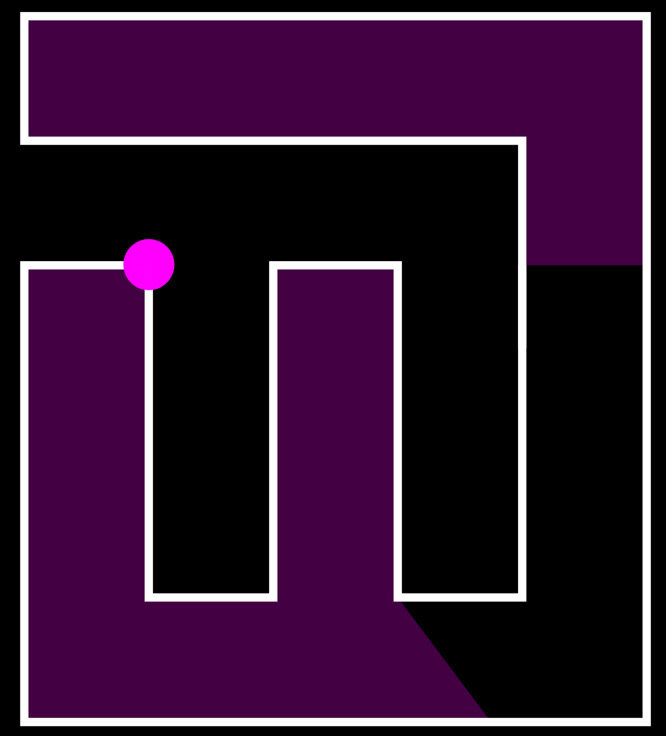

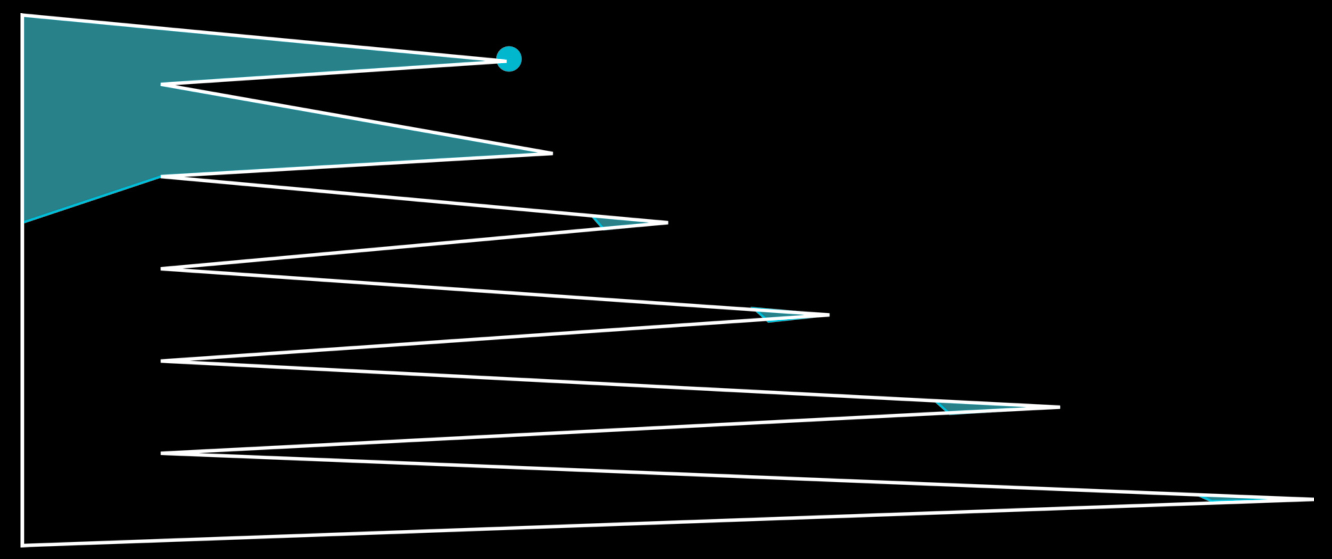

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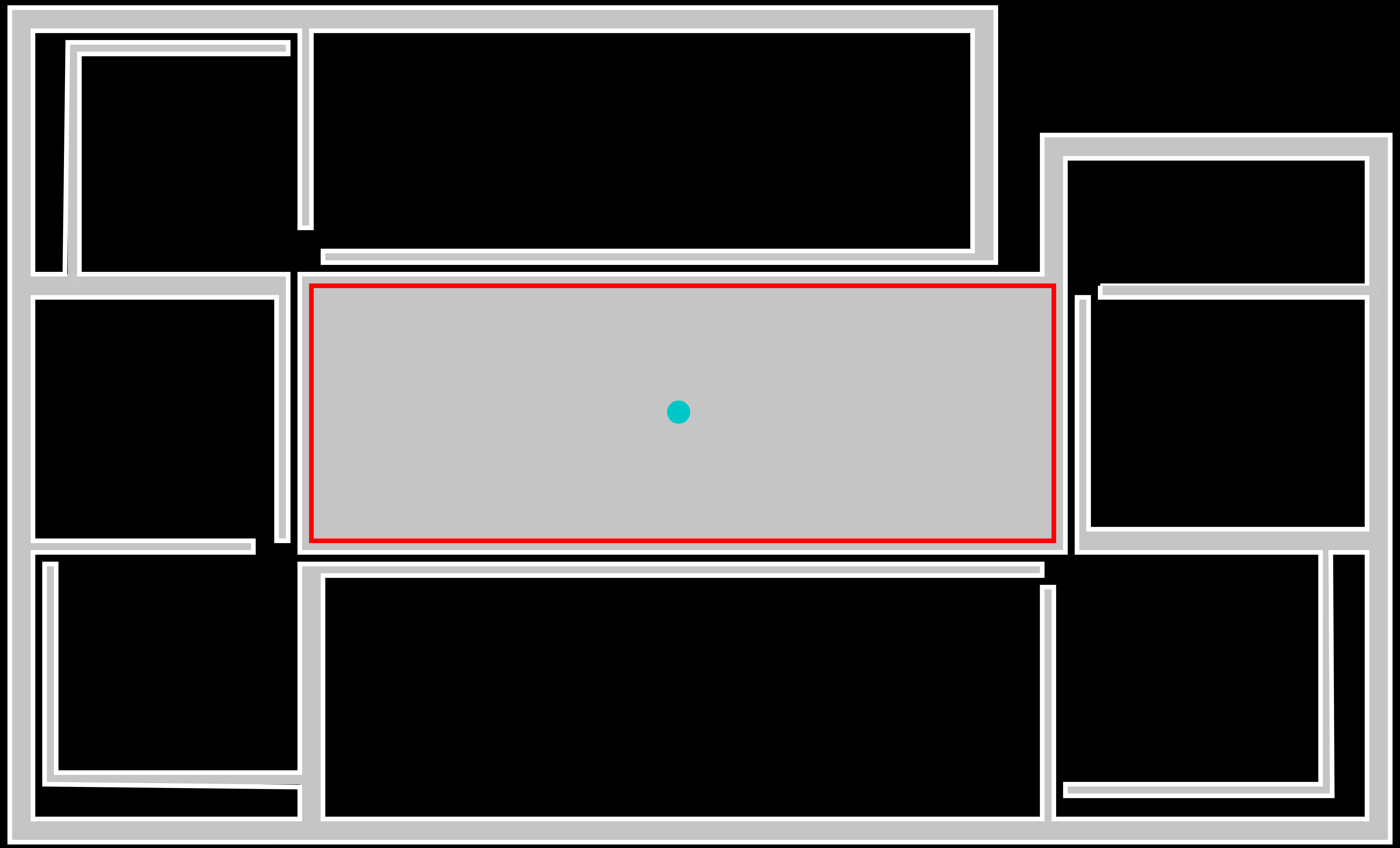

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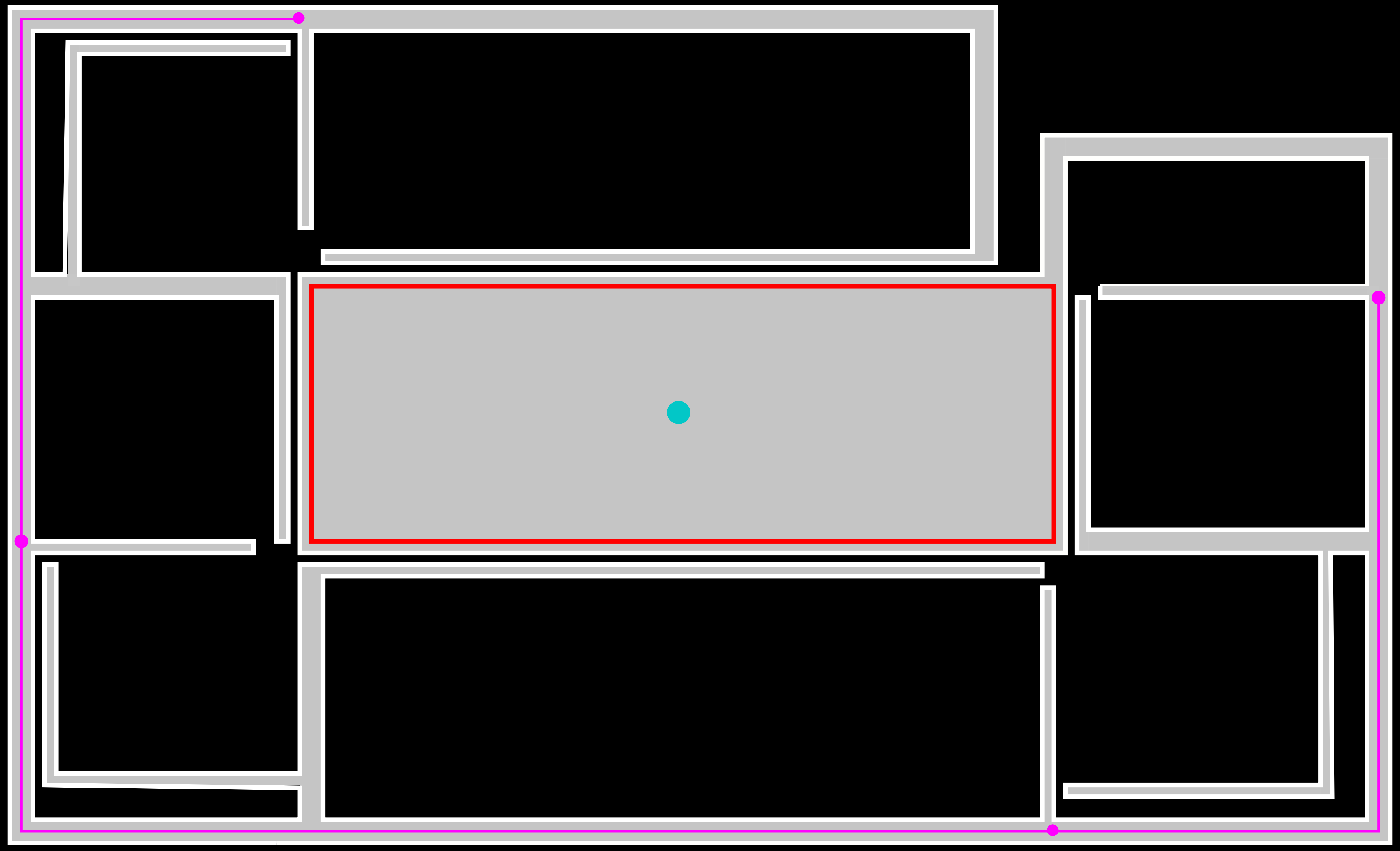

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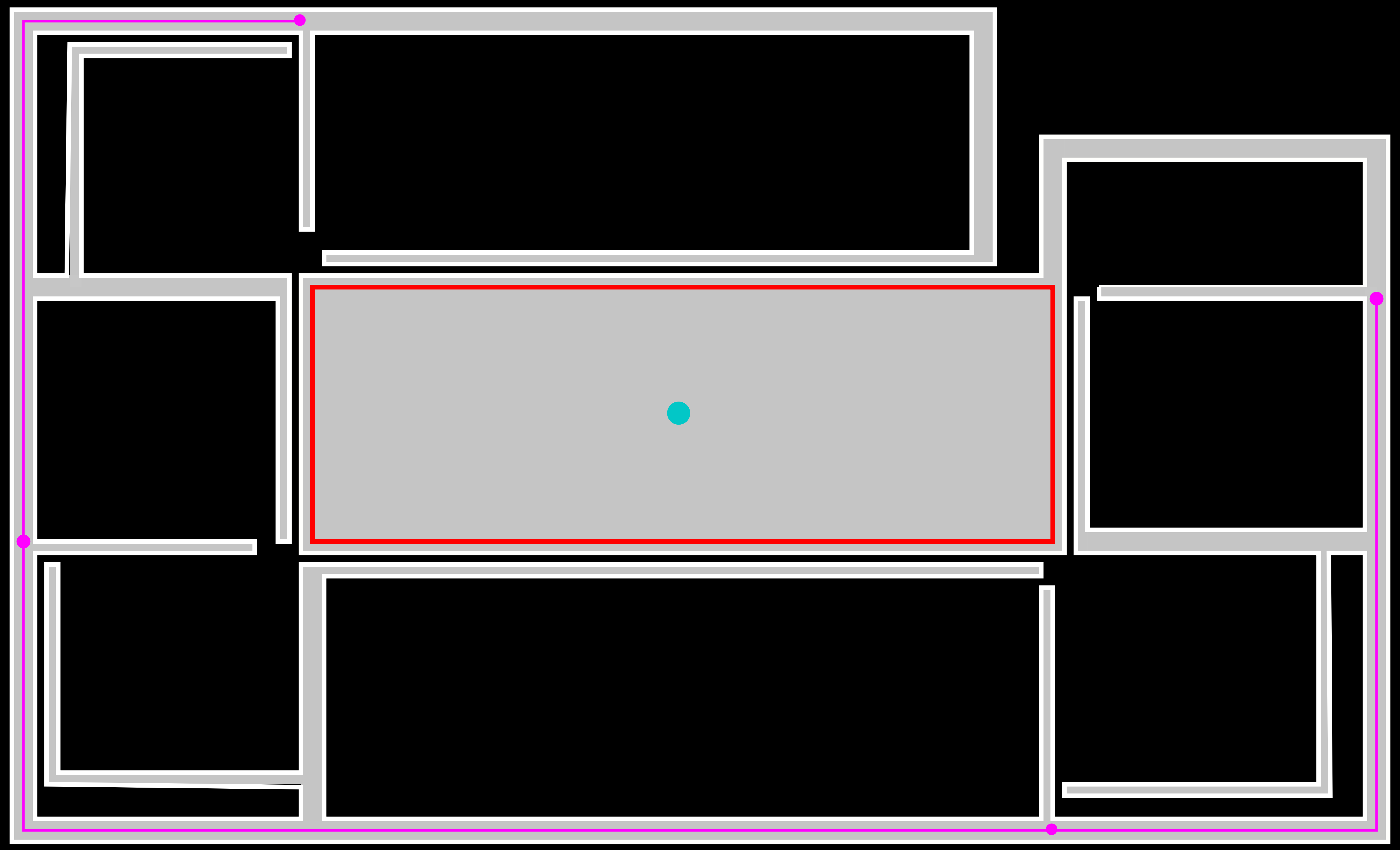

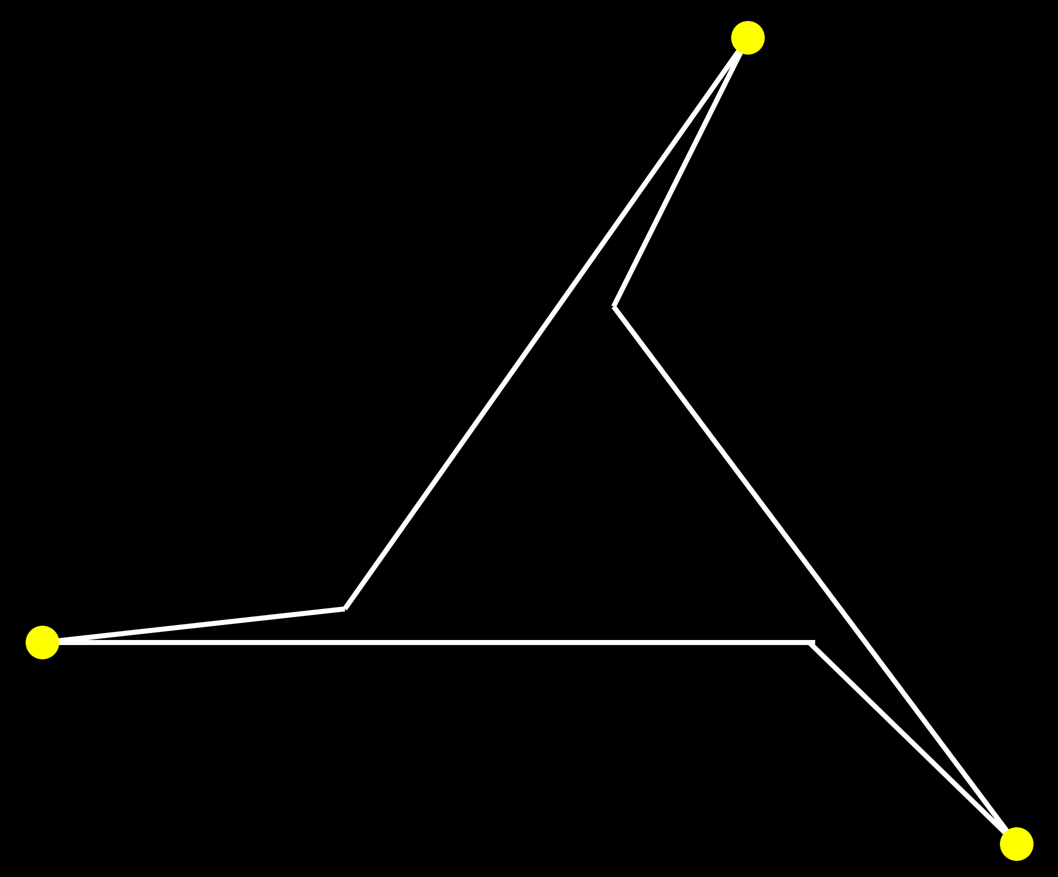

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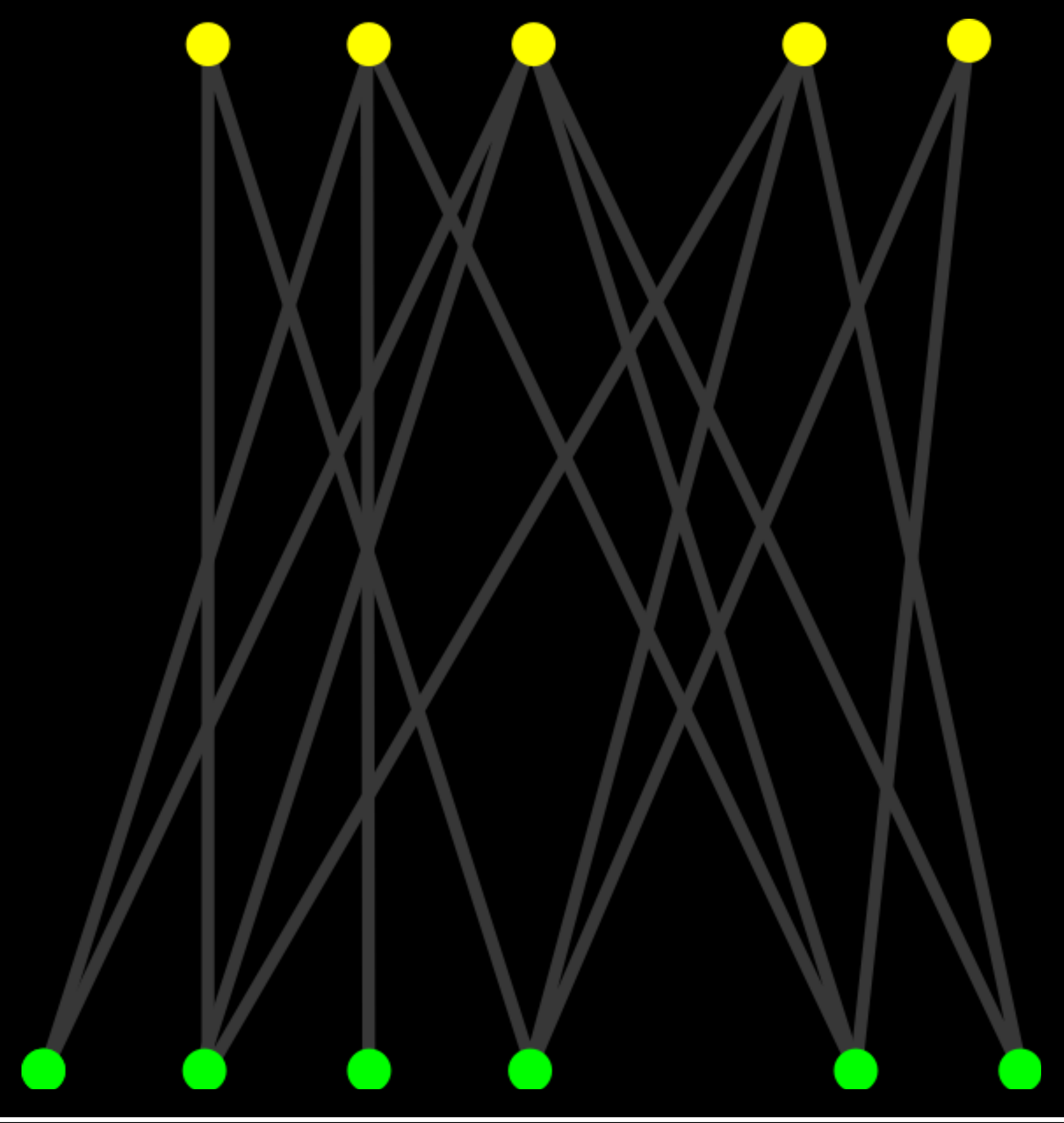

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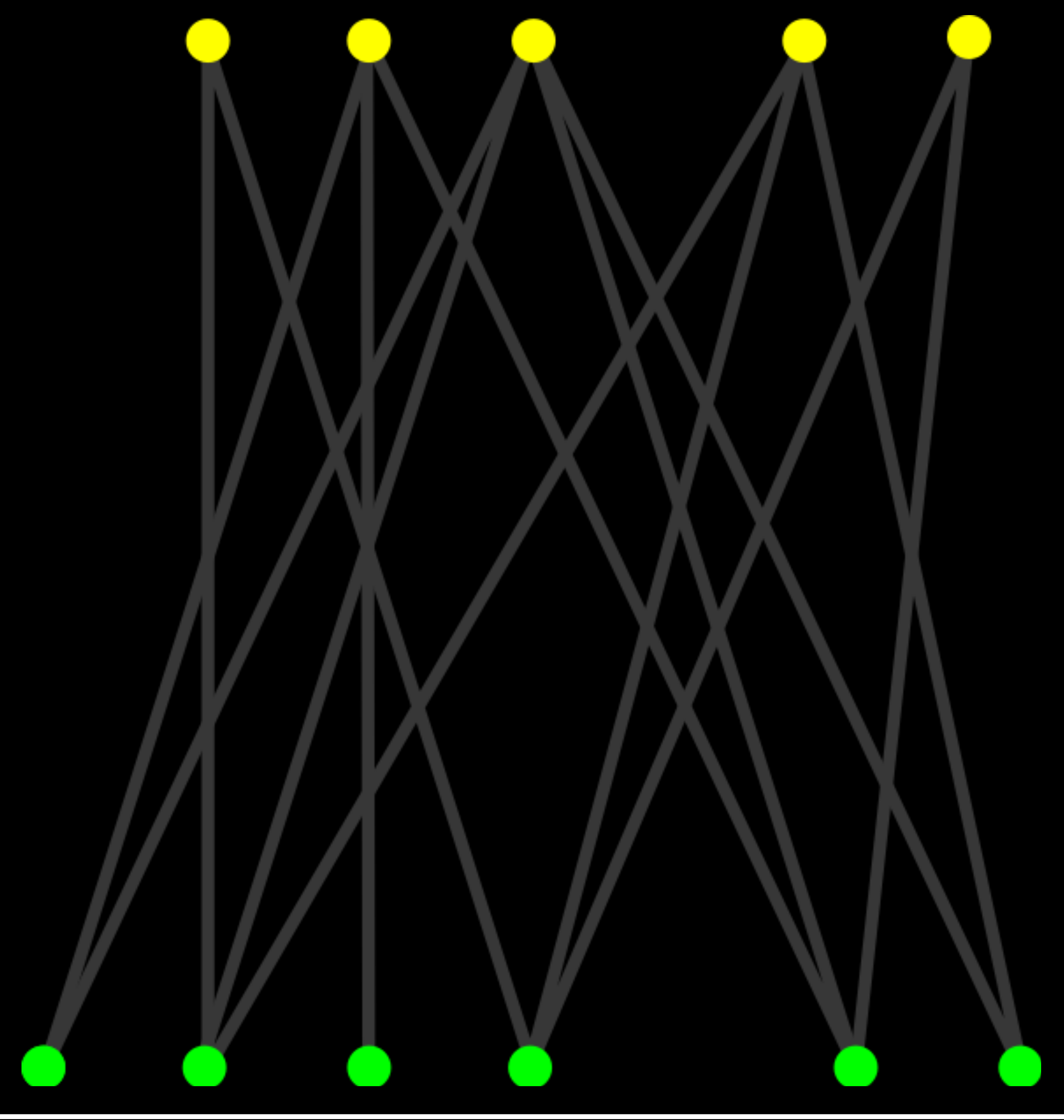

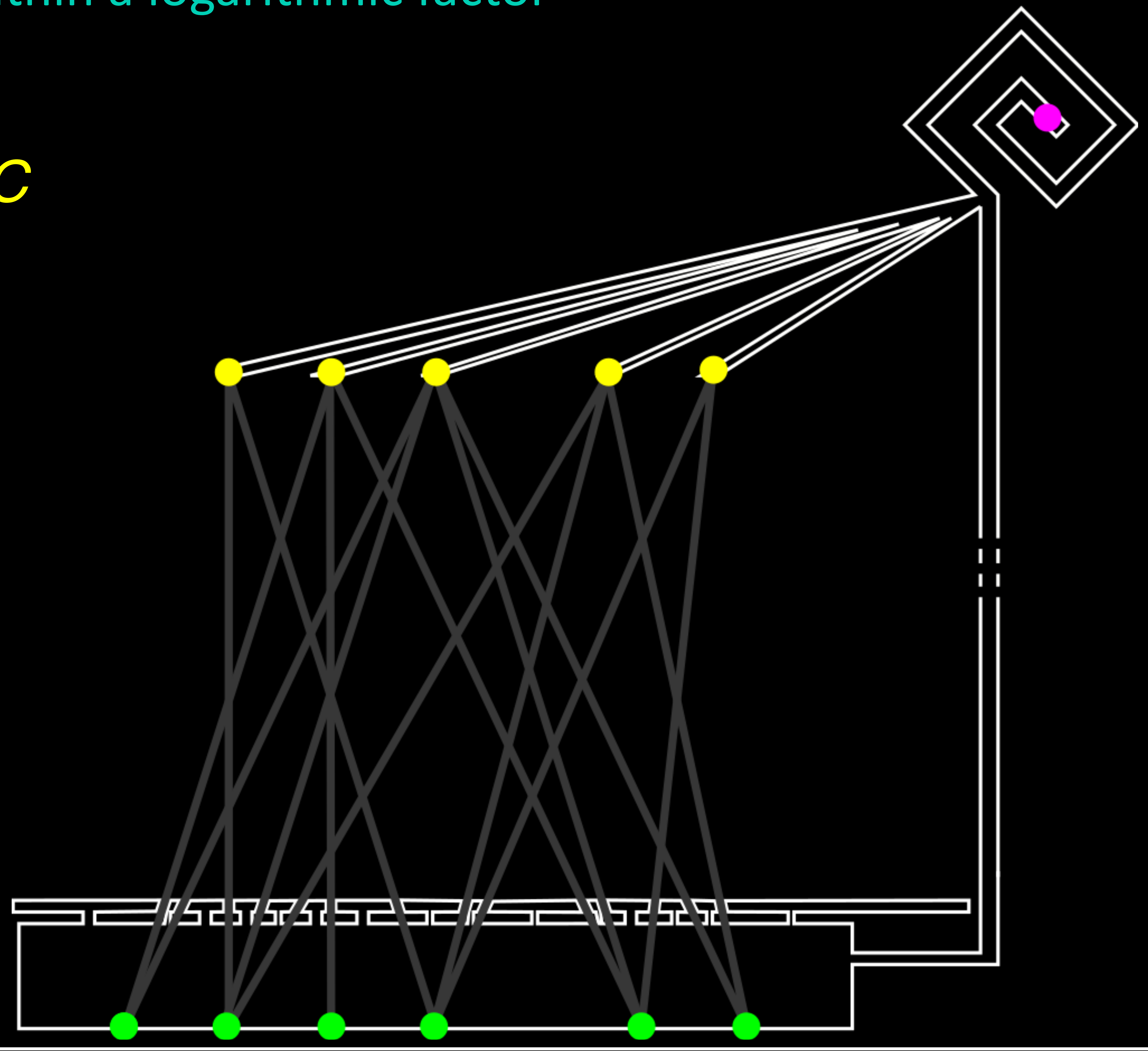

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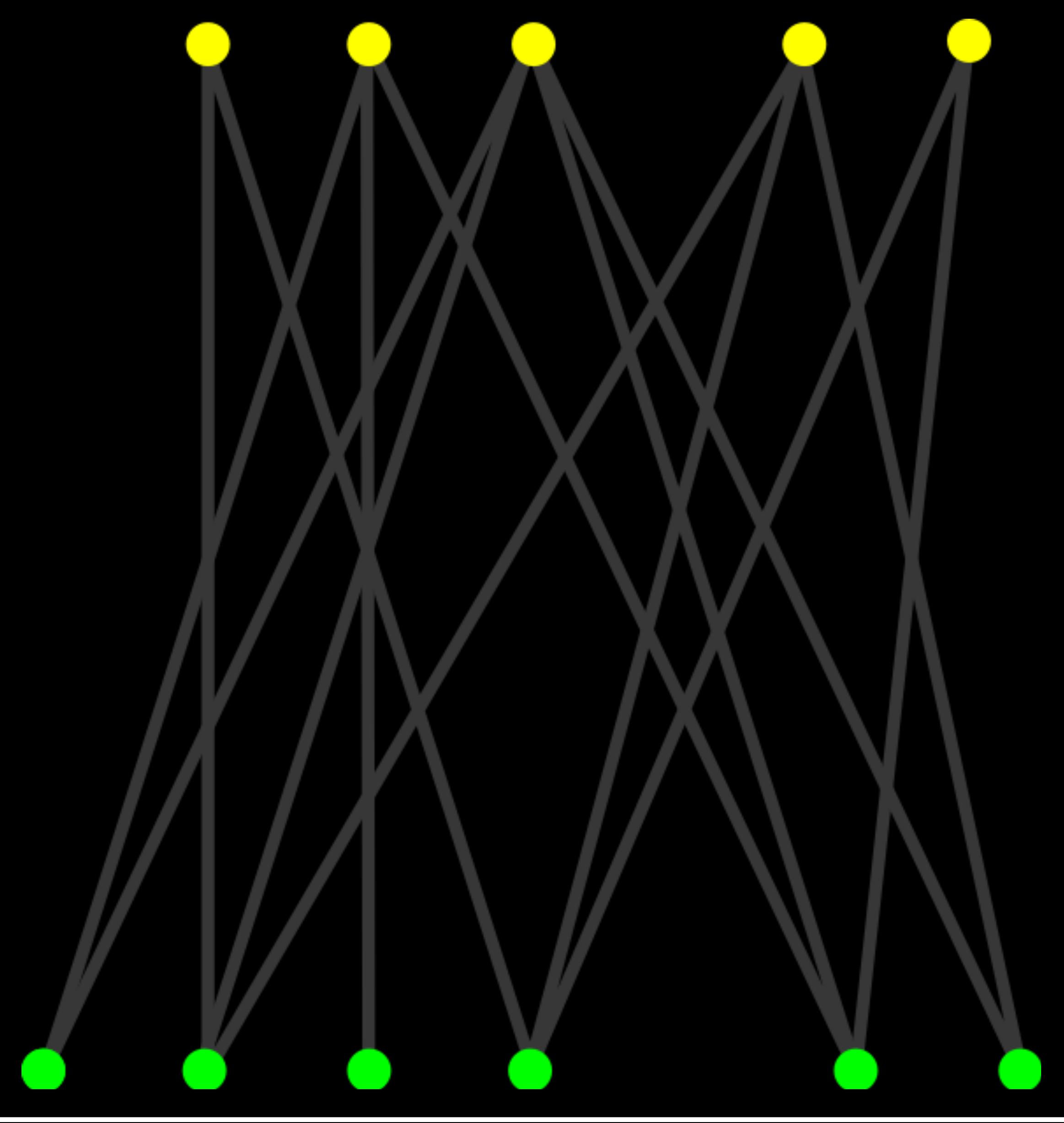

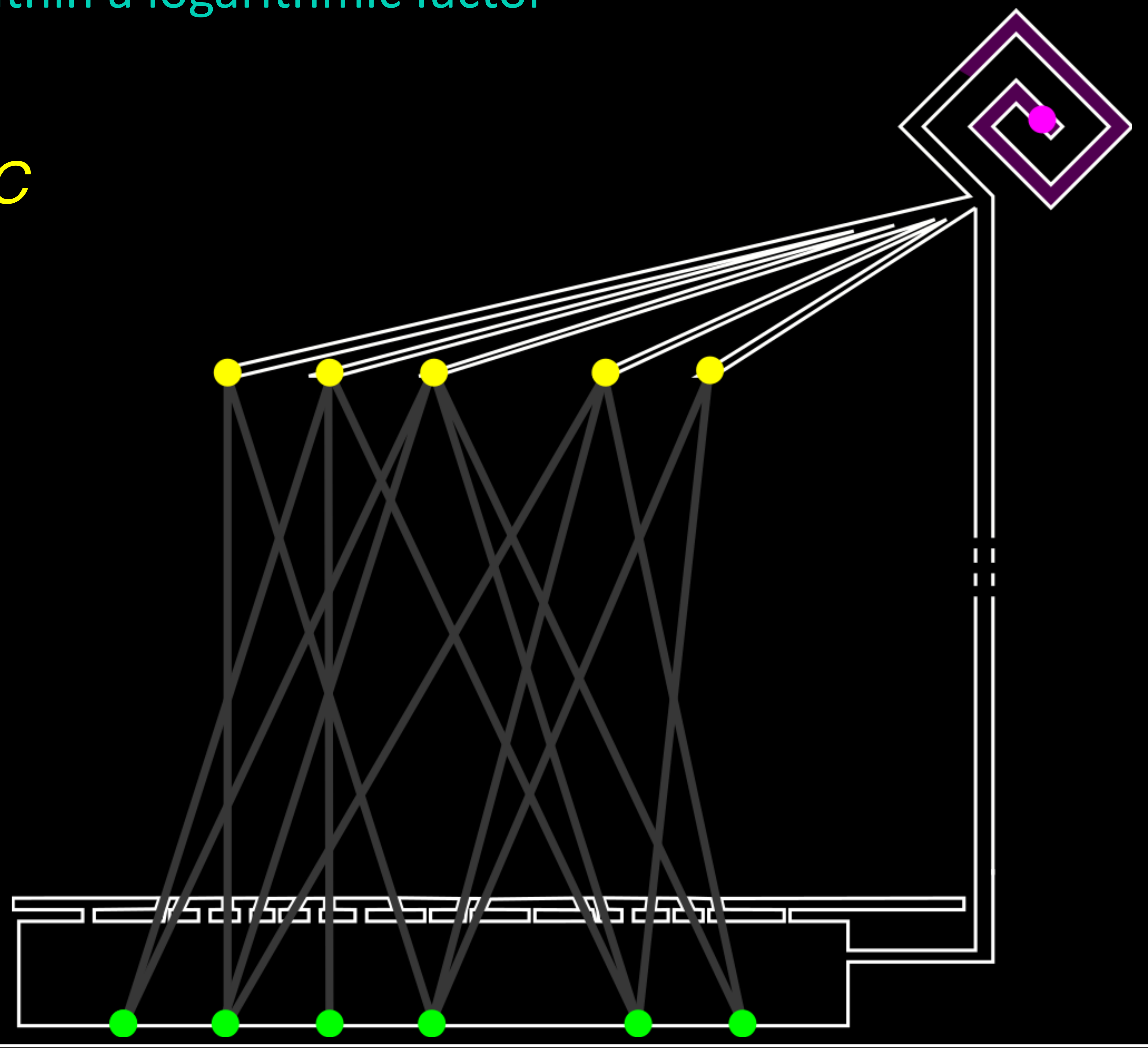

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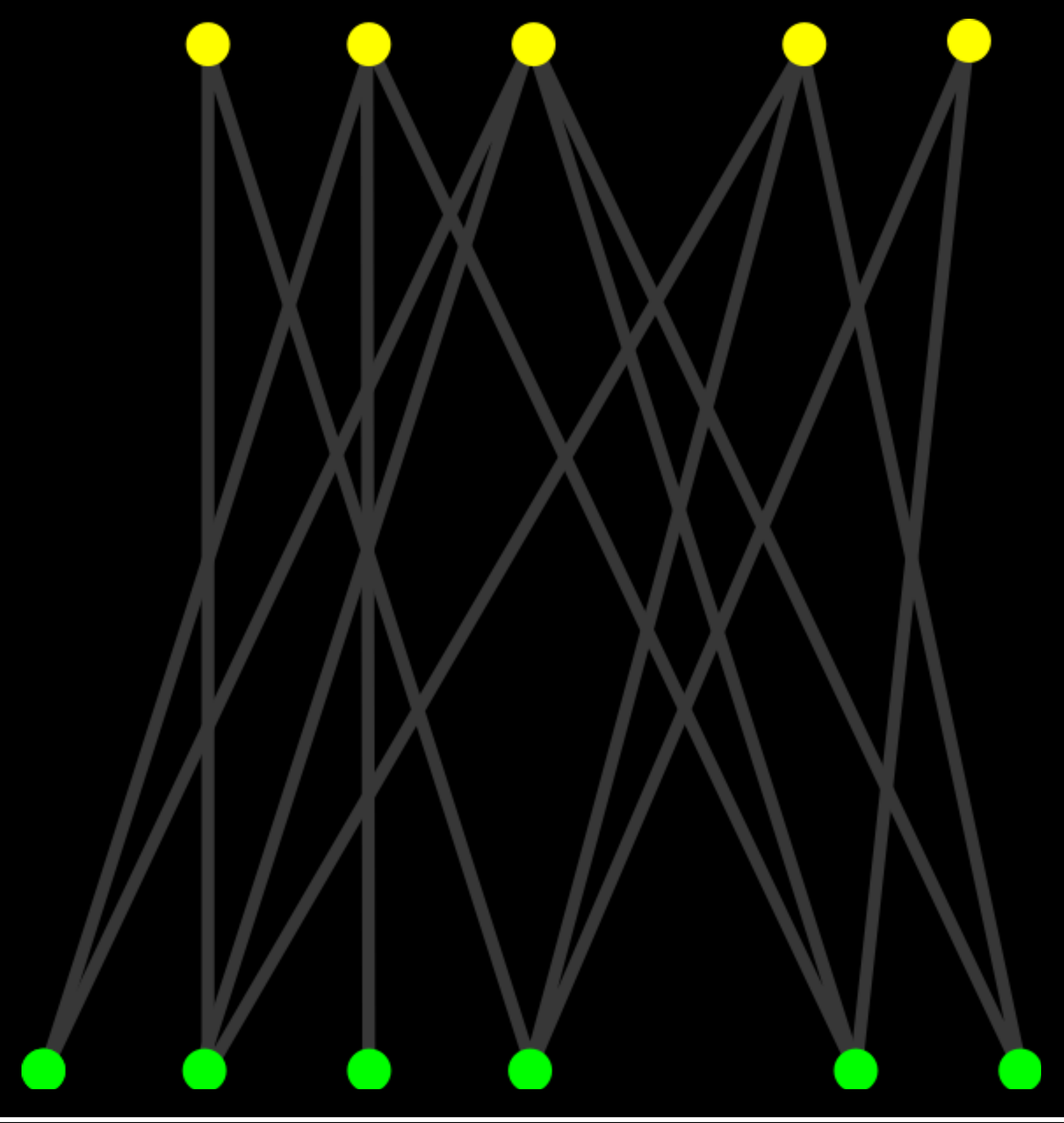
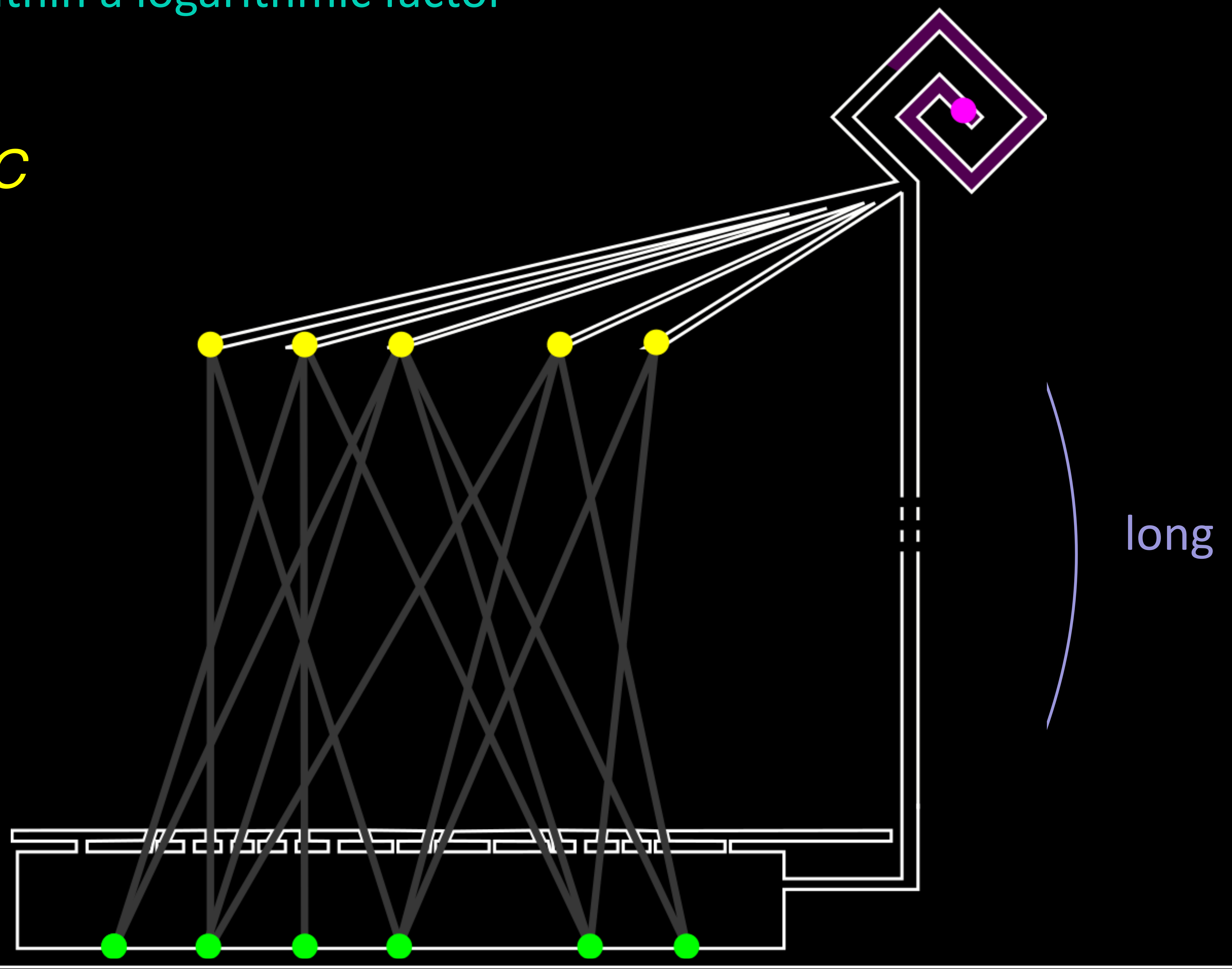
Theorem 1: For a discrete set of points *S* and a simple polygon *P*, the *k*-TrWRP(*S*,*P*) does not admit a polynomial-time approximation algorithm with approximation ratio *c* ln |*S*| unless P=NP, even for *k*=2. ➜ Inapproximability: Cannot be approximated to within a logarithmic factor

Proof: reduction from Set Cover

Set Cover instance: universe *U* and collection of sets *C*

We construct a polygon *P* with *S*=*U*∪{*v*}





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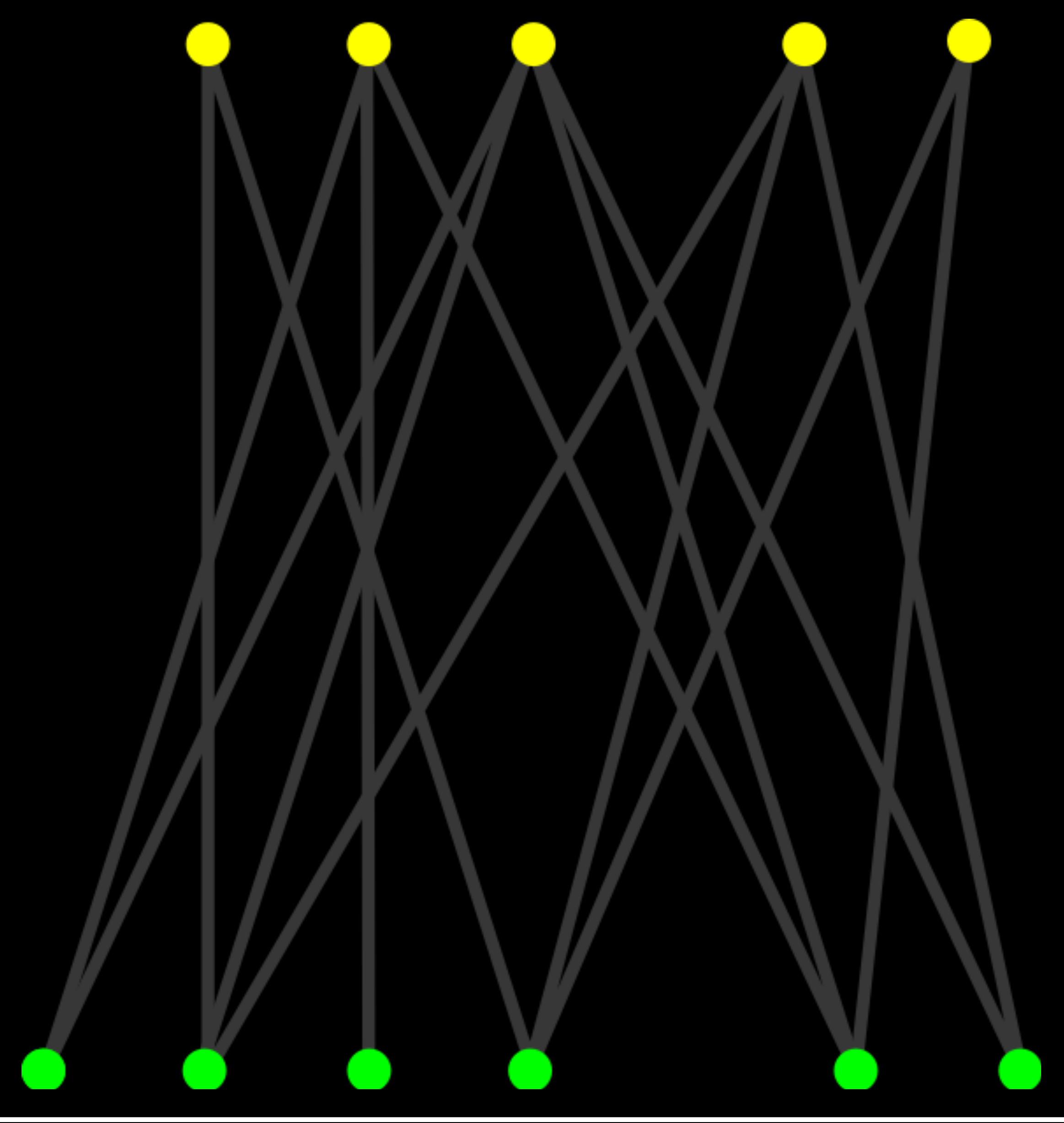

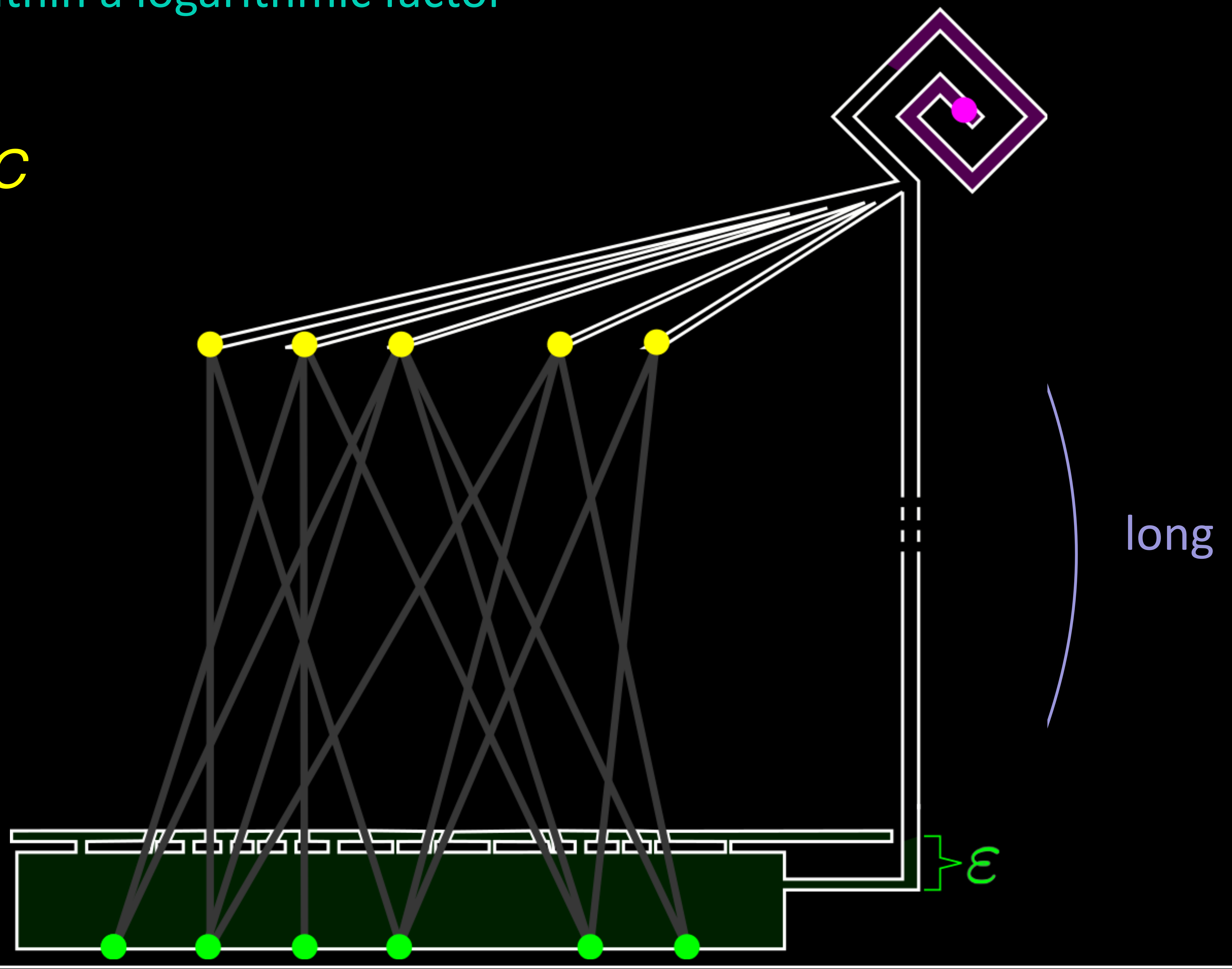

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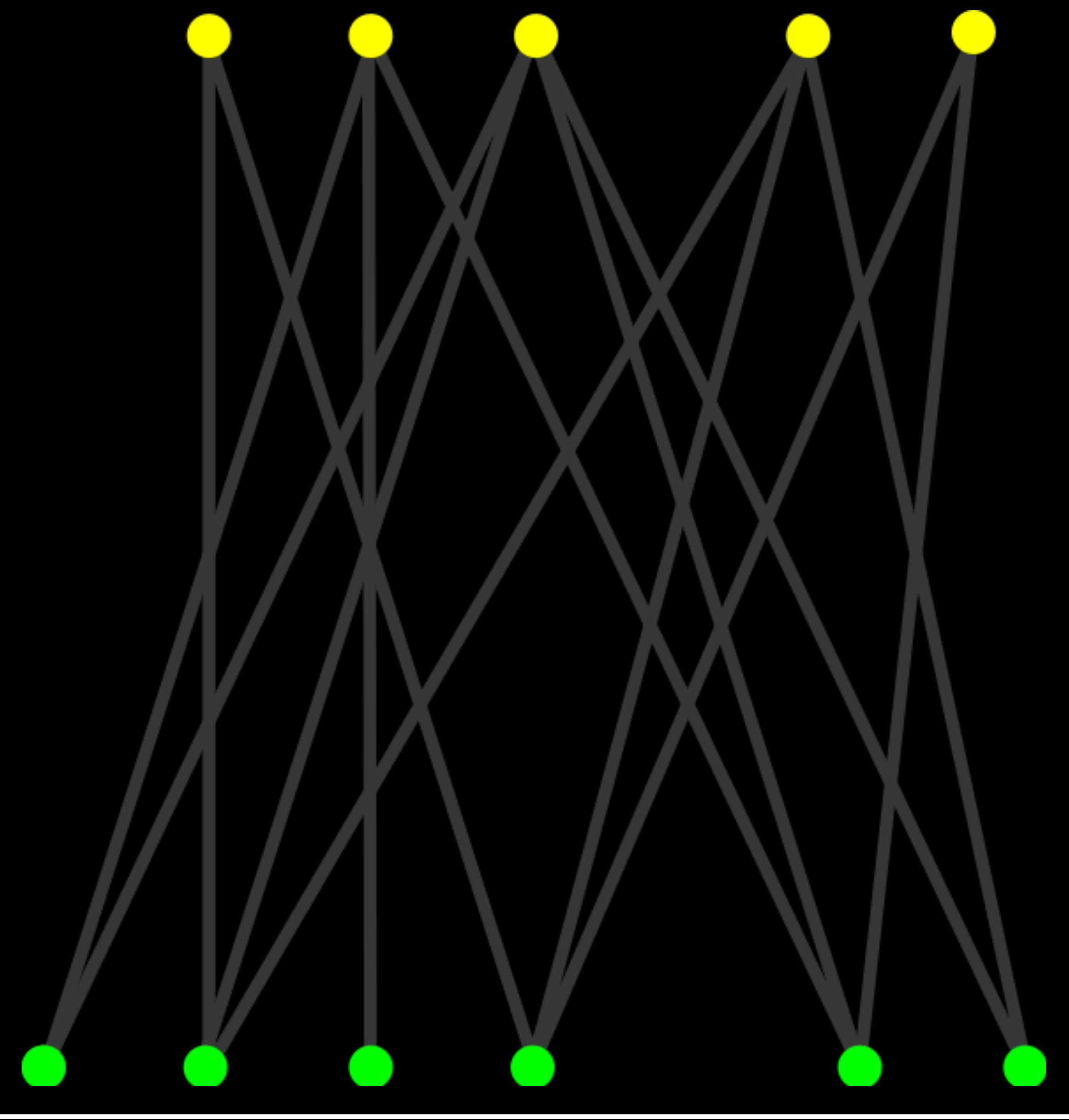

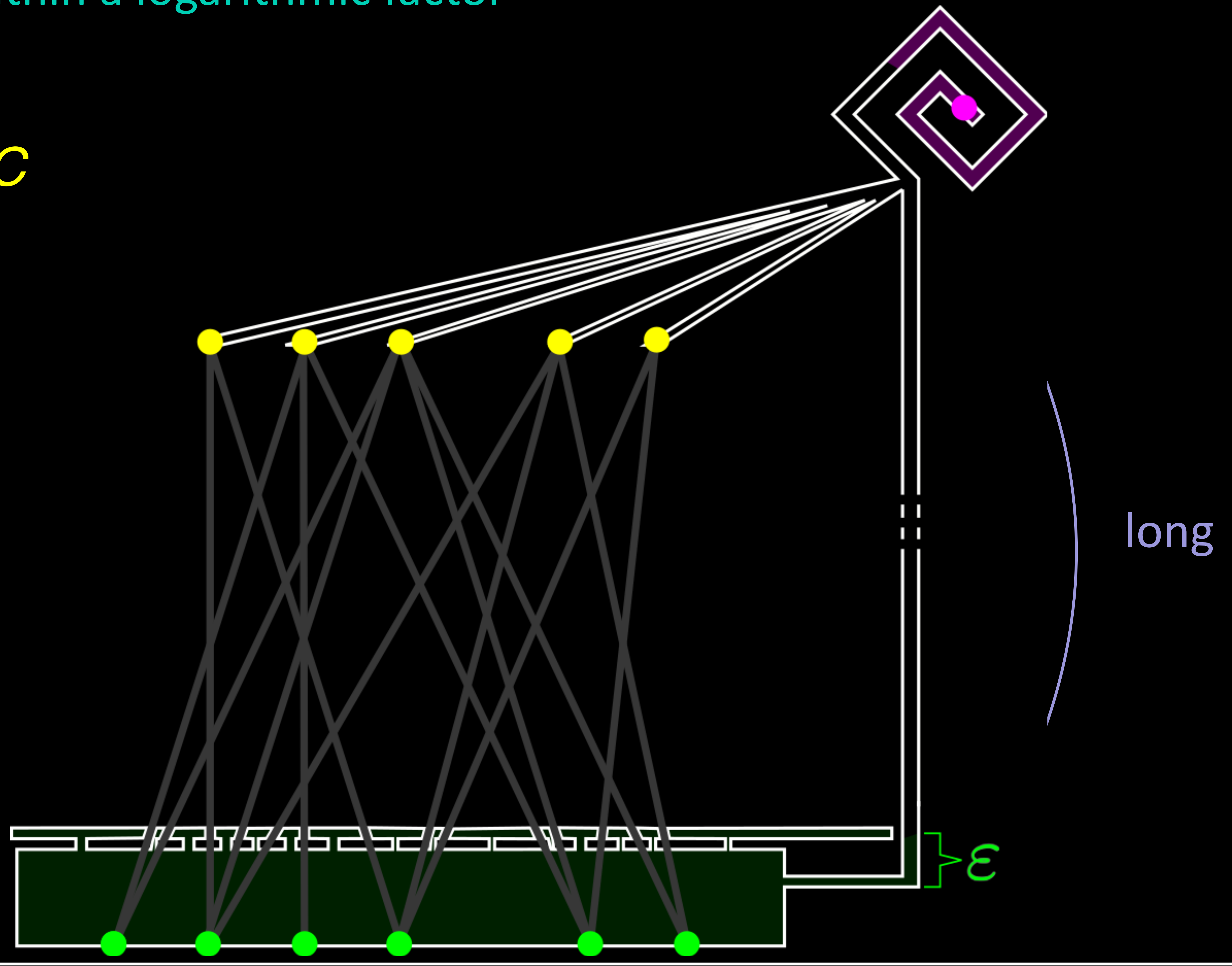

Corollary: The same holds for *k*-TrWRP(*S*,*P*,*s*).

Approximation Algorithm for *k*-TrWRP(*S*,*P*,*s*)

Approximation Algorithm for *k*-TrWRP(*S,P,s*)

Theorem 2: Let *P* be a simple polygon with $n=|P|$. Let OPT(*S,P,s*) be the optimal solution for the *k*-TrWRP(*S,P,s*) and let R be the solution by our algorithm ALG(*S,P,s*). Then R yields an approximation ratio of $O(\log^2 (|S|\, n) \log\log (|S|\, n) \log |S|)$.

Approximation Algorithm for *k*-TrWRP(S,P,s)

Approximation Algorithm for *k*-TrWRP(*S*,*P*,*s*)

- Create a candidate point for each connected component of the *k*-visibility region of each point in *S*.

Approximation Algorithm for *k*-TrWRP(*S*,*P*,*s*)

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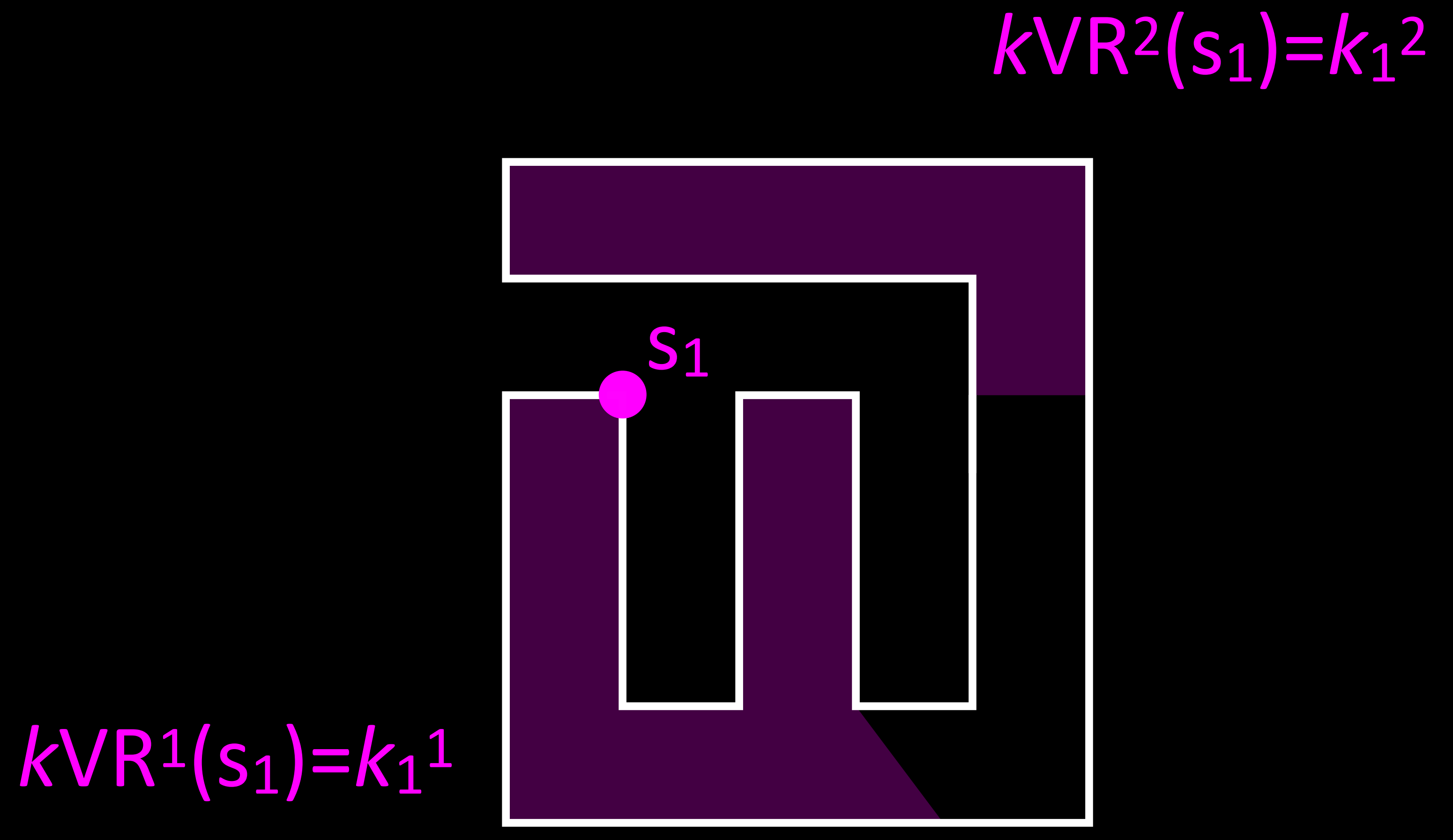

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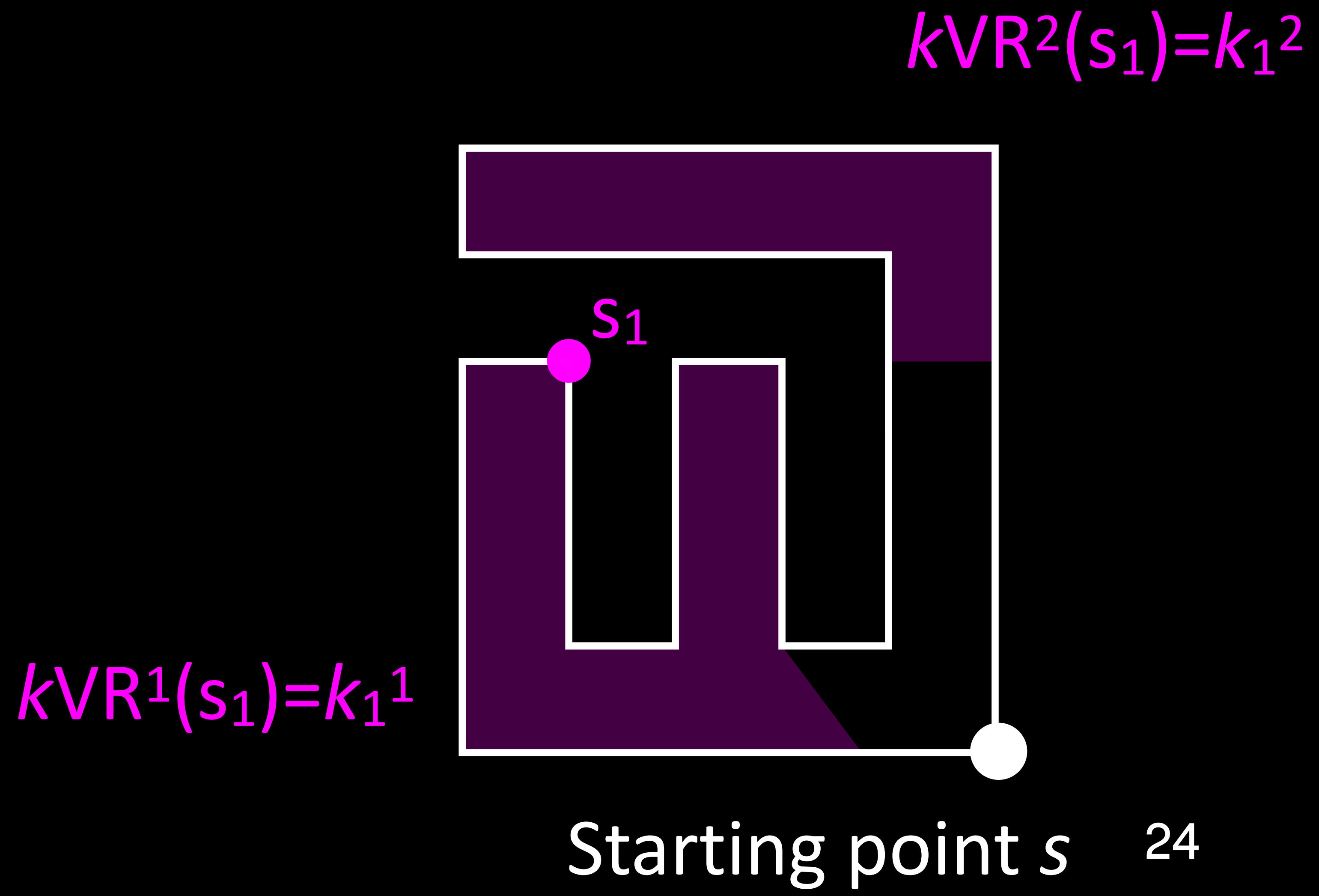

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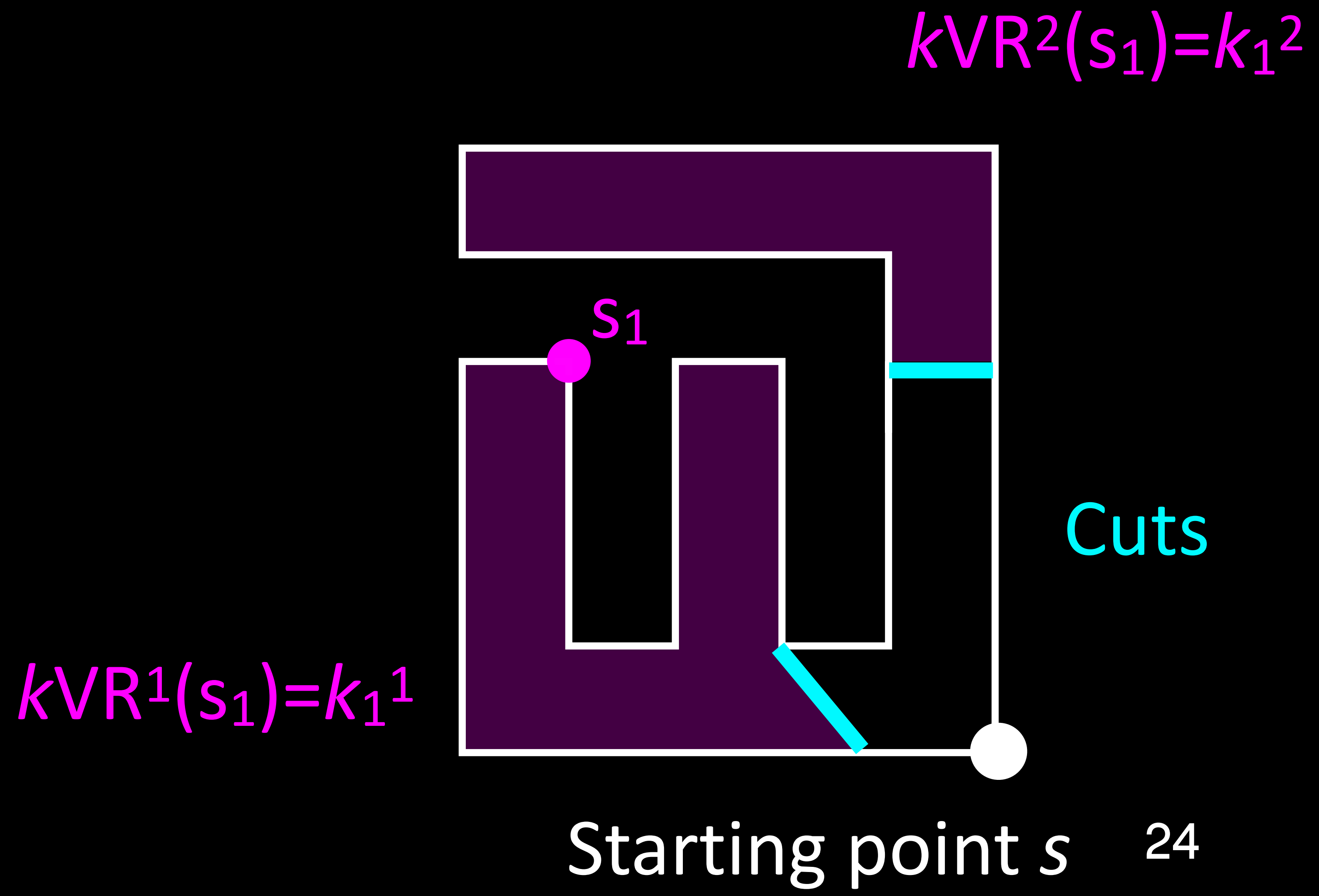

Approximation Algorithm for k-TrWRP(S,P,s)

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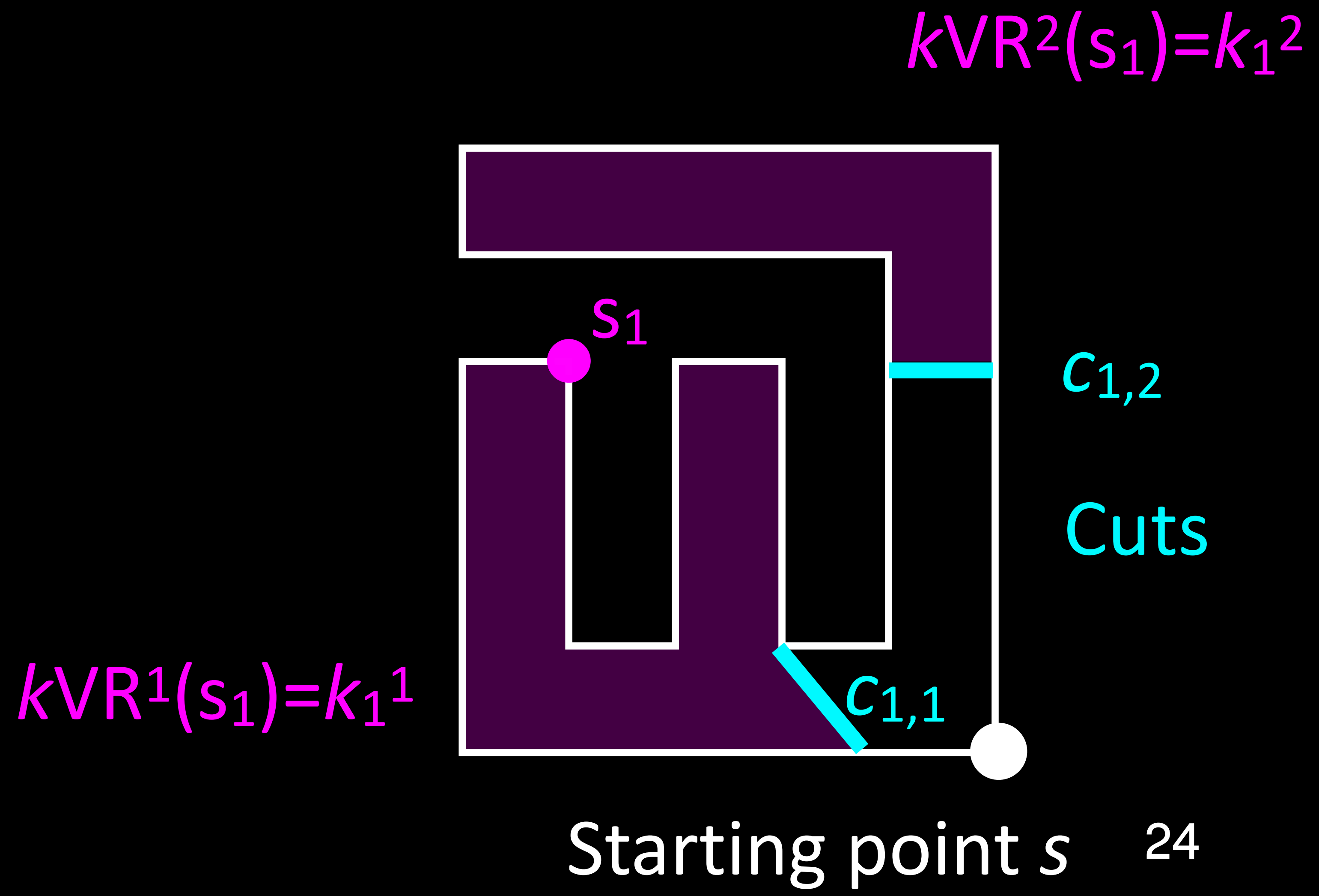

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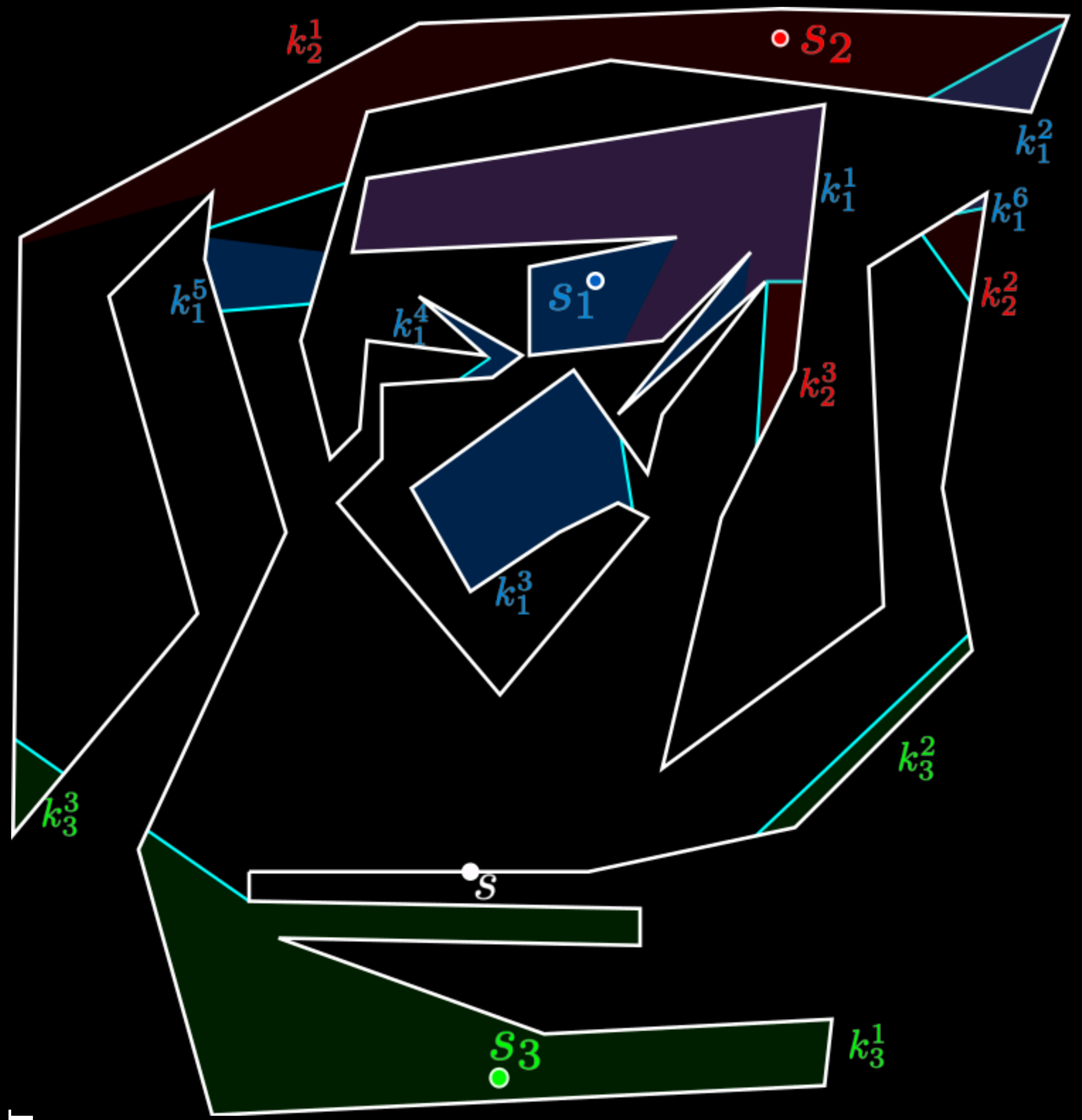

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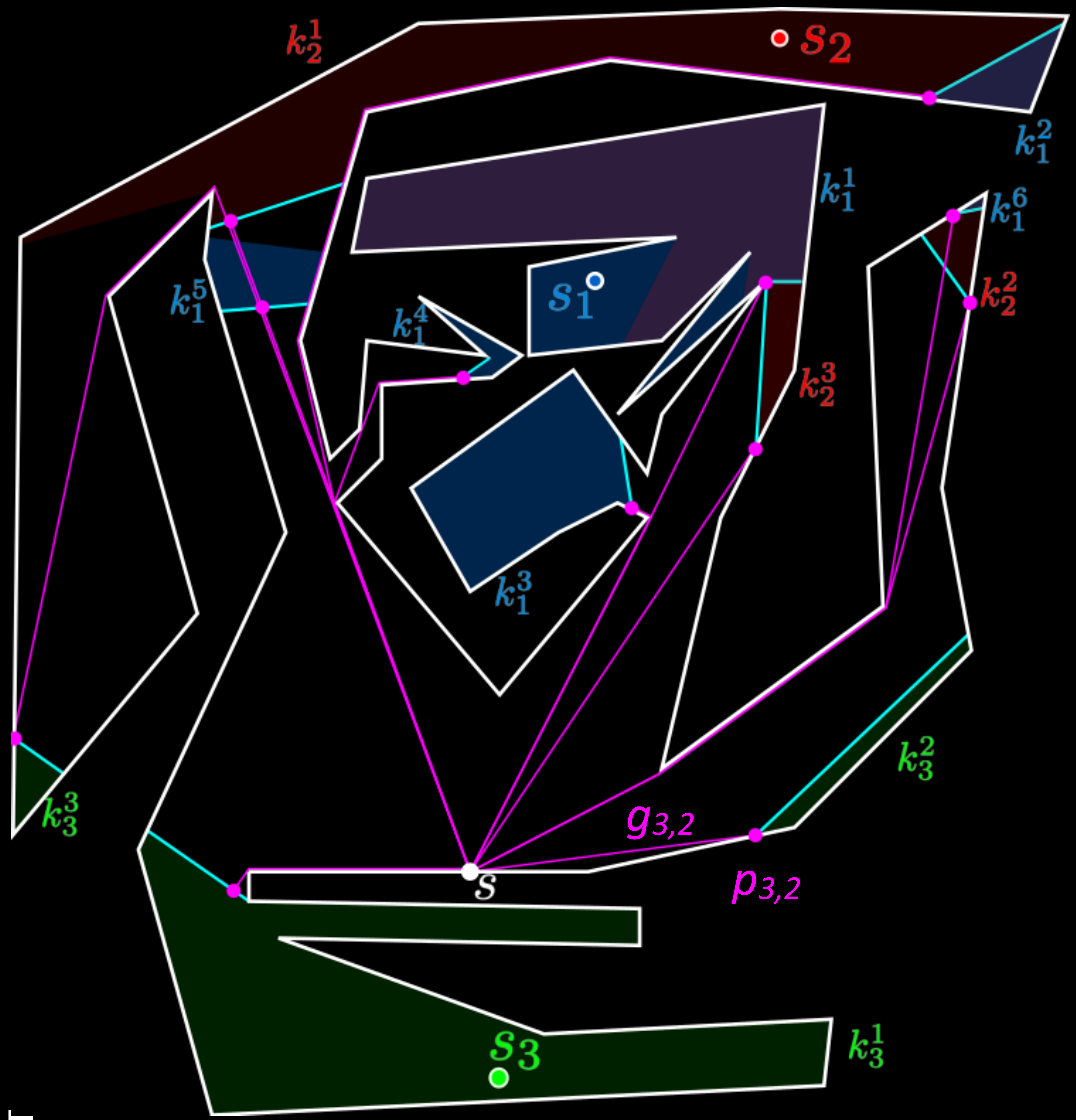

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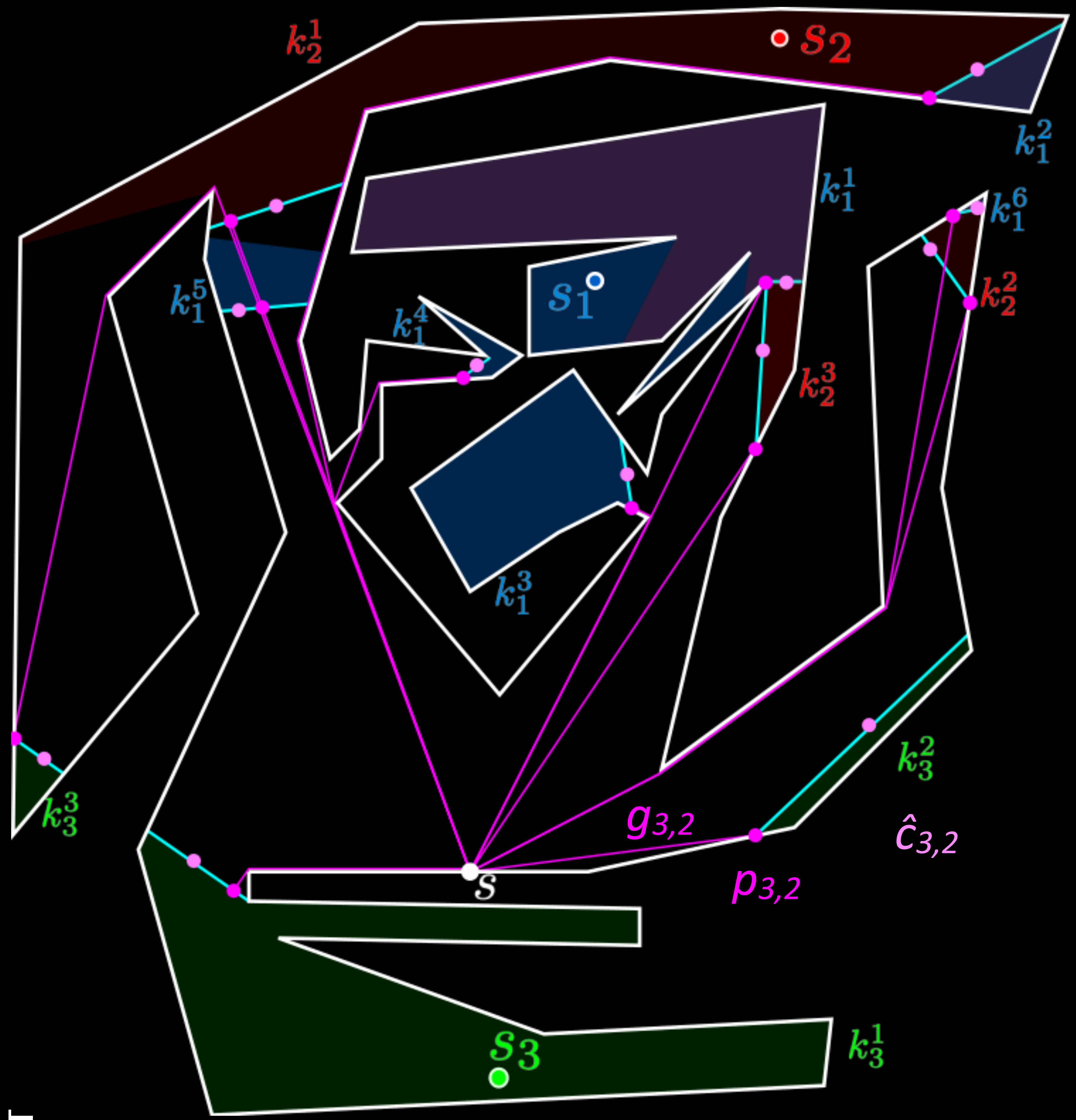

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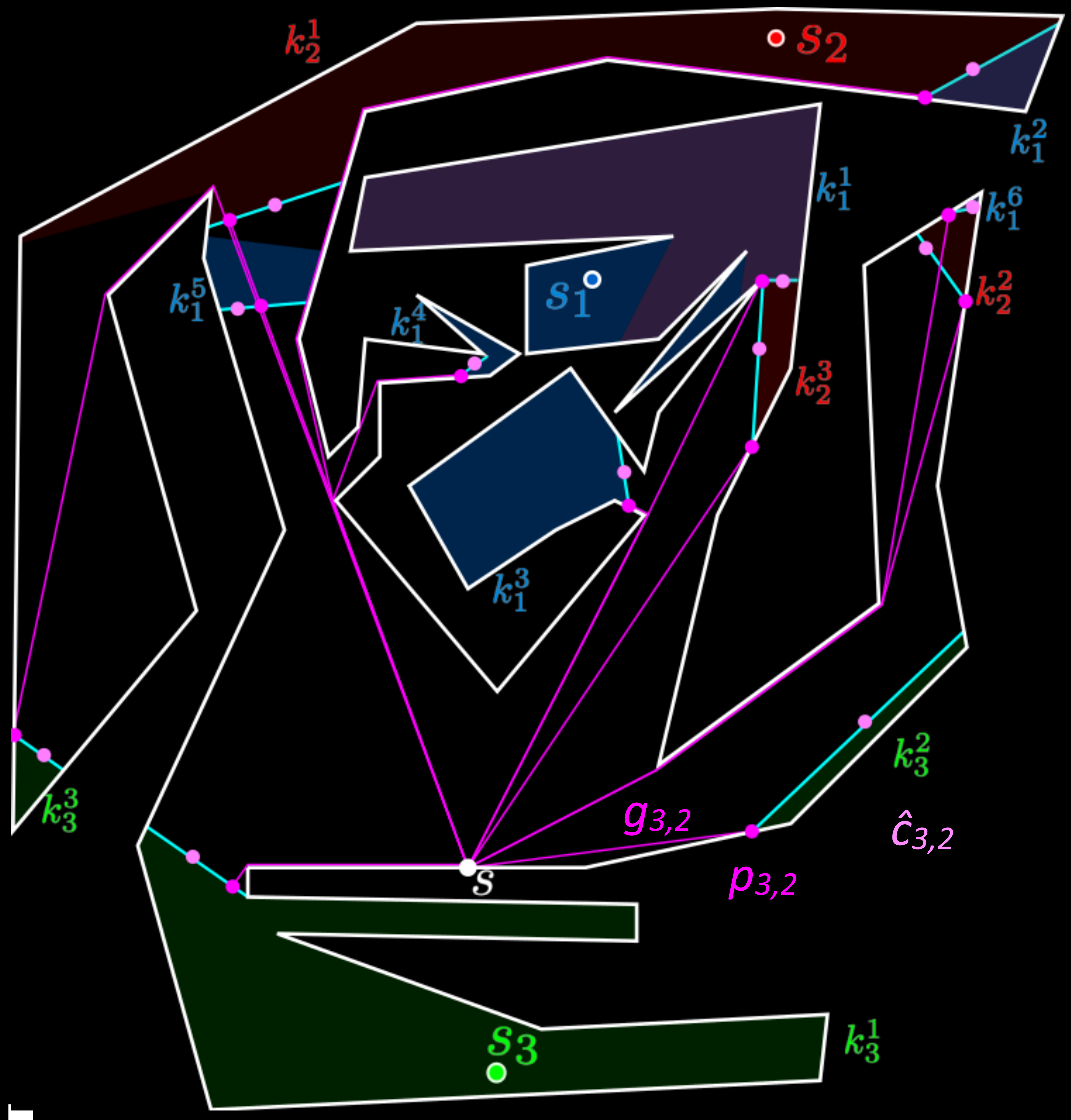

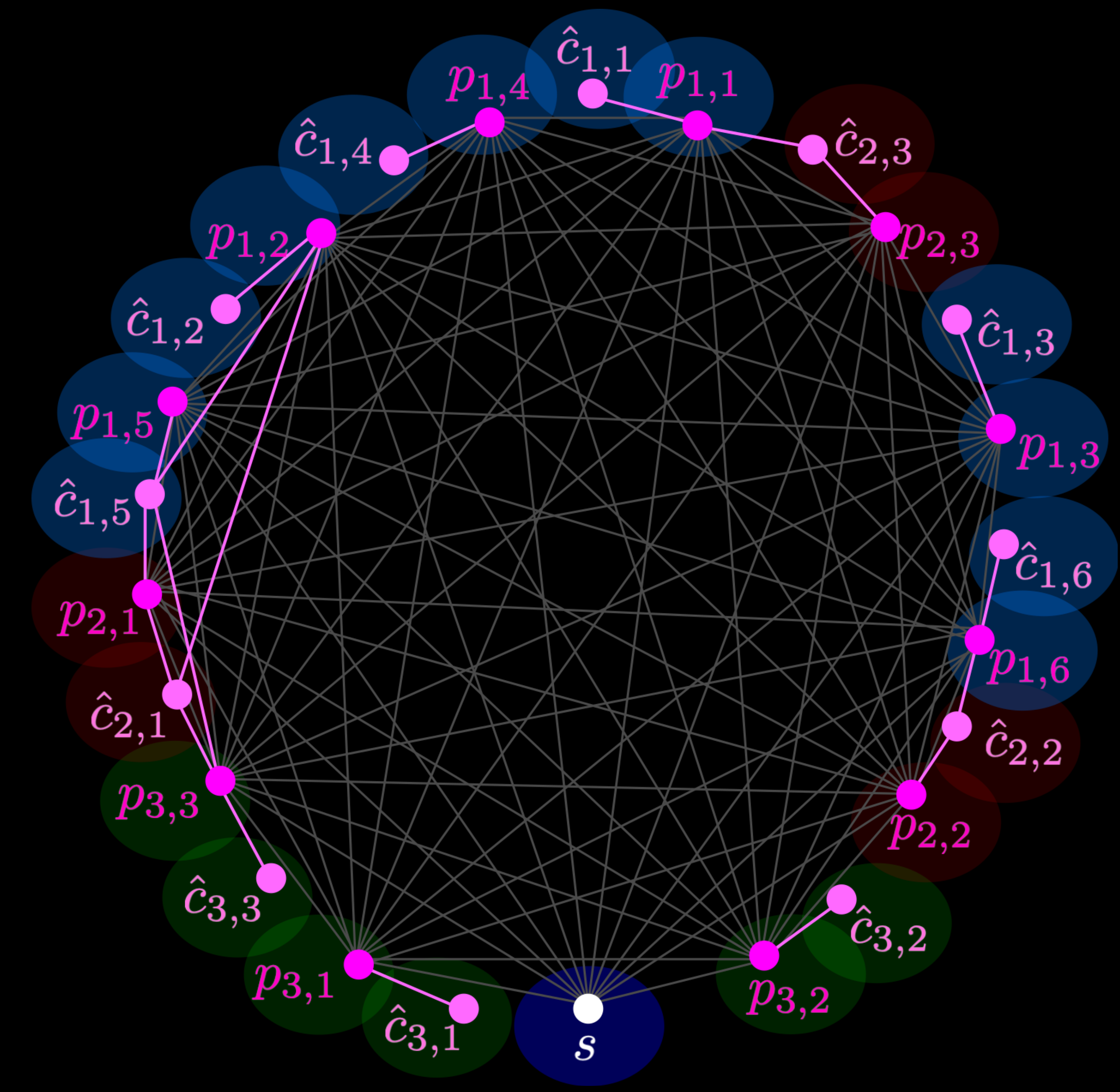

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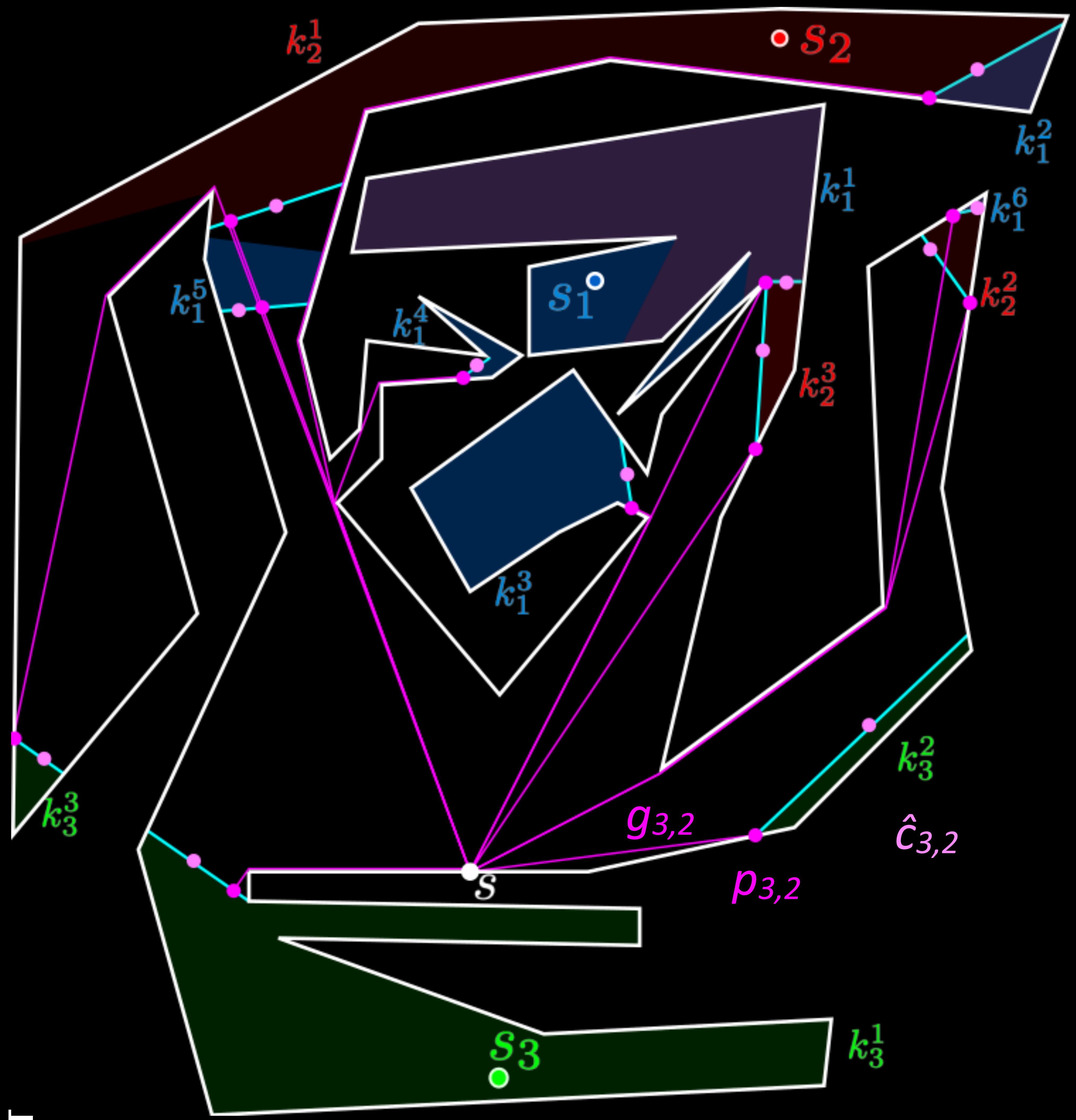

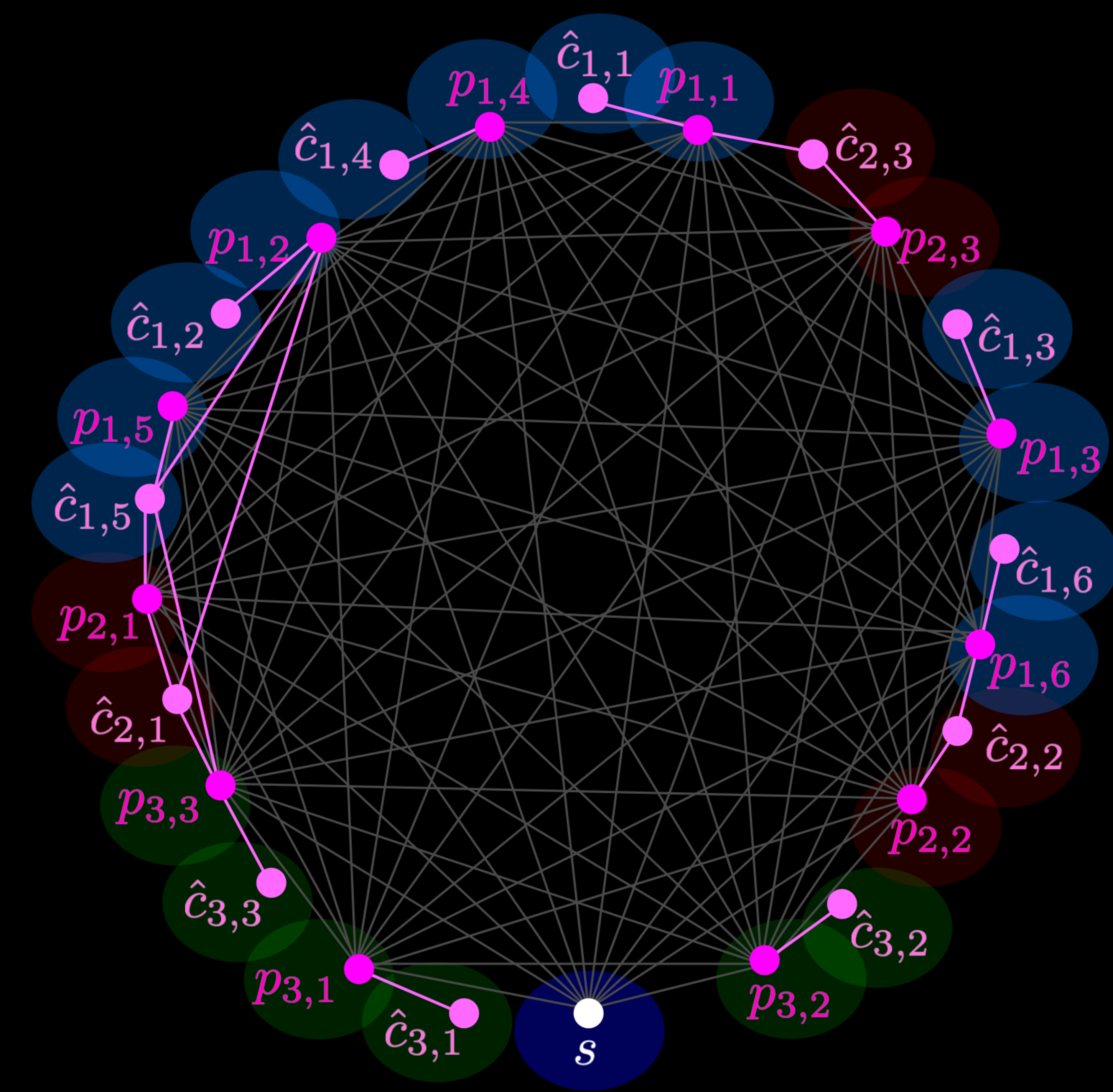

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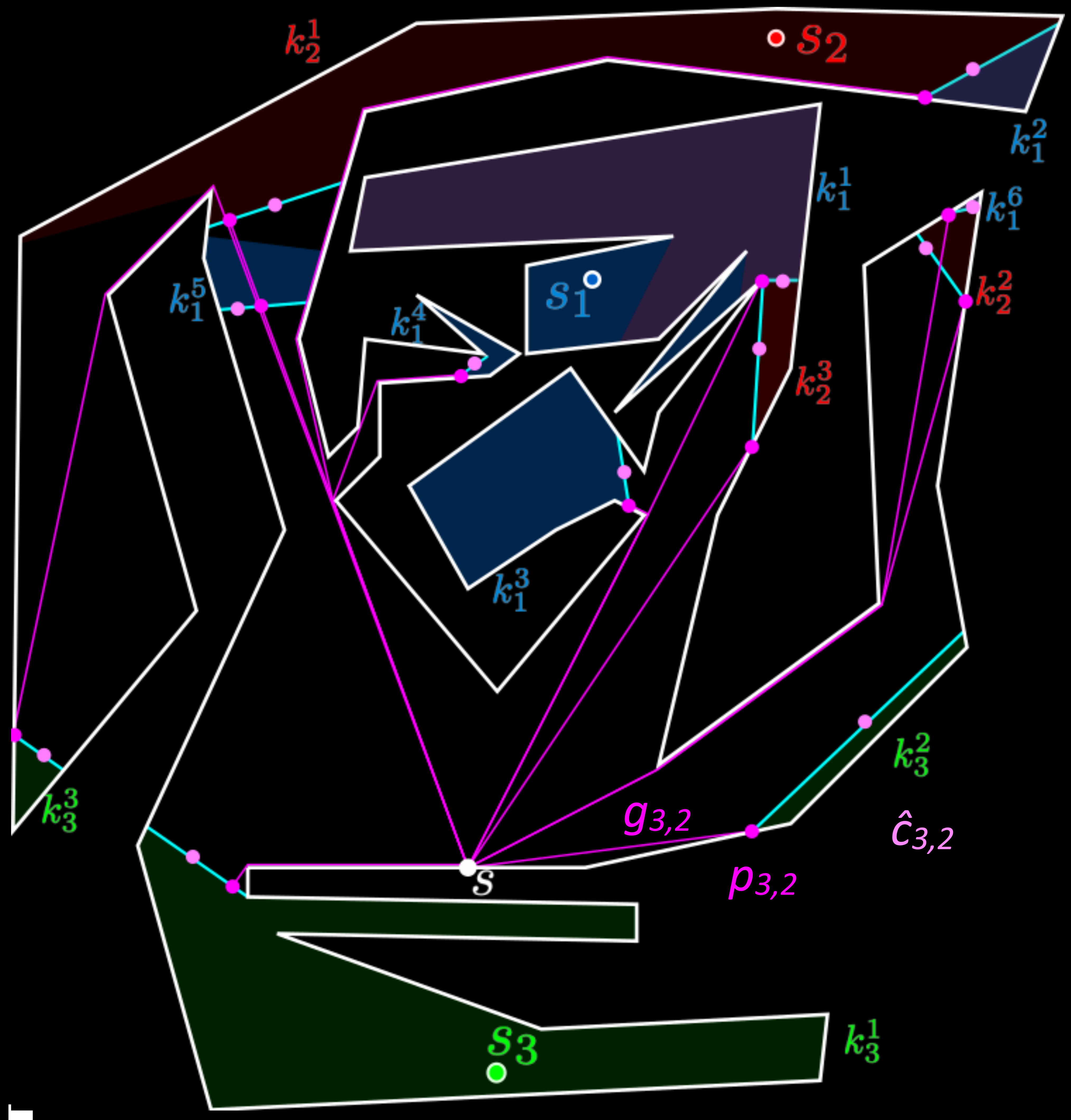

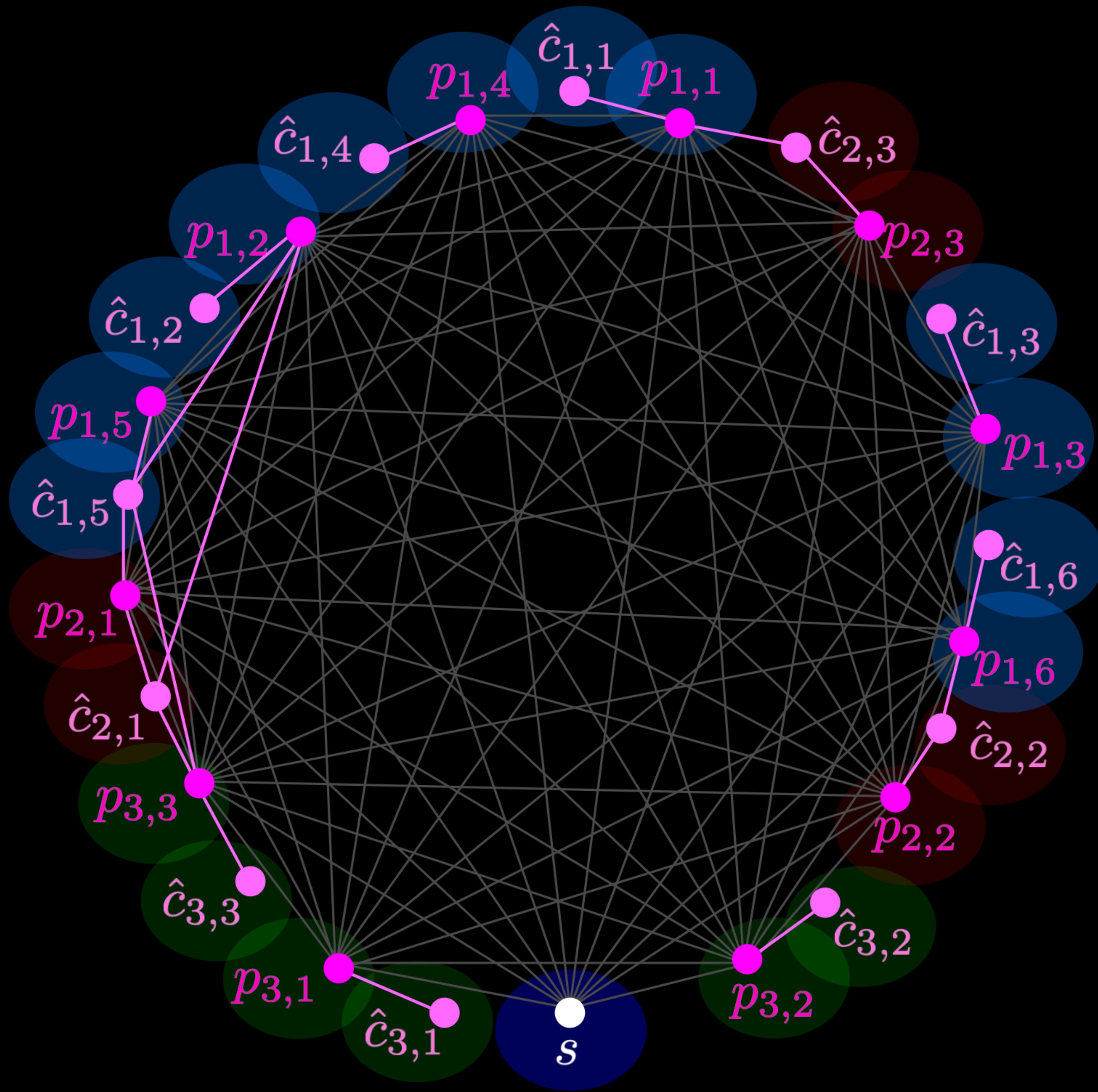

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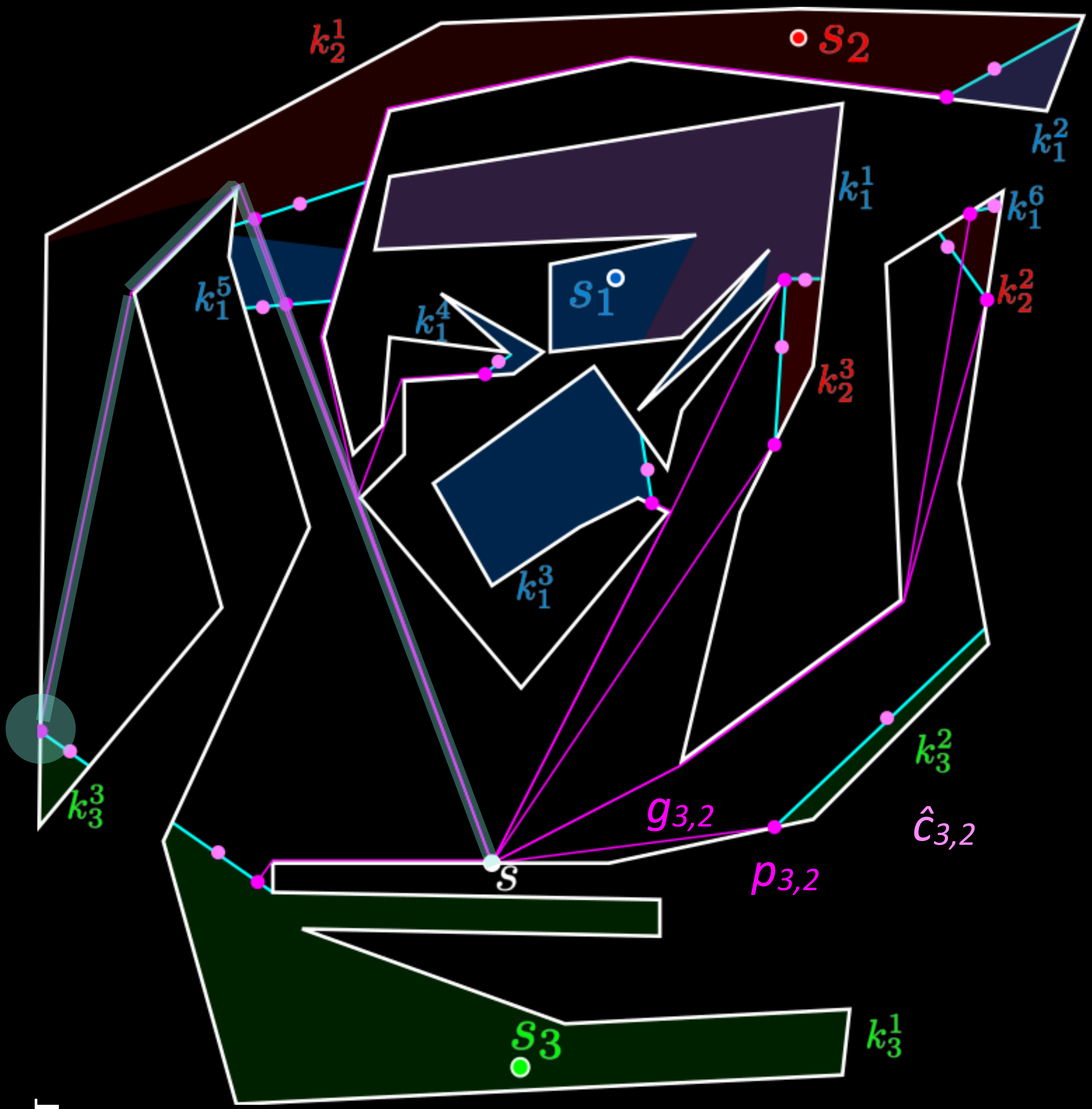
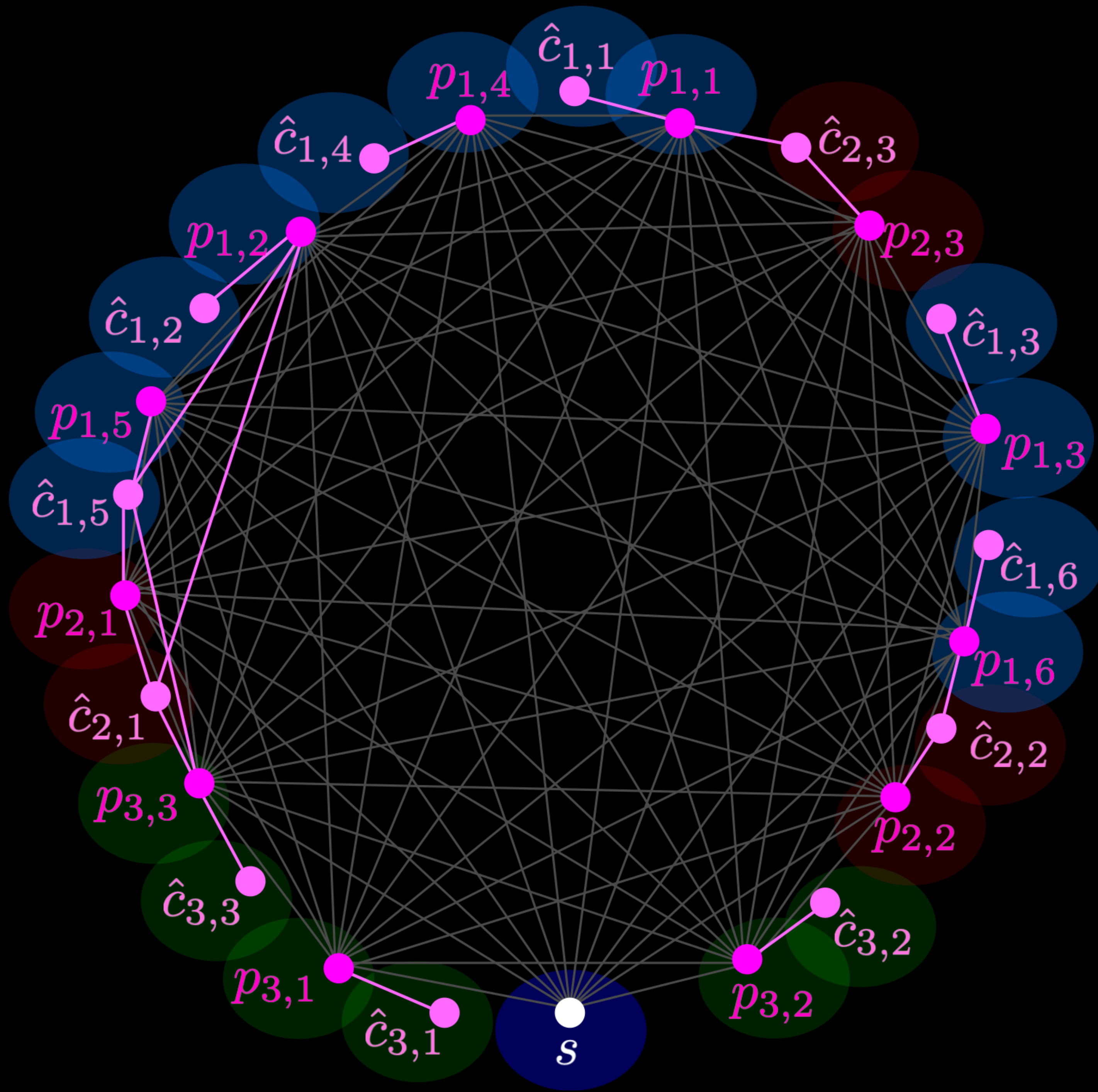
Thus, we have edges from p_3^3 to $\hat{c}_3^3$, $\hat{c}_2^1$, and $\hat{c}_1^5$.





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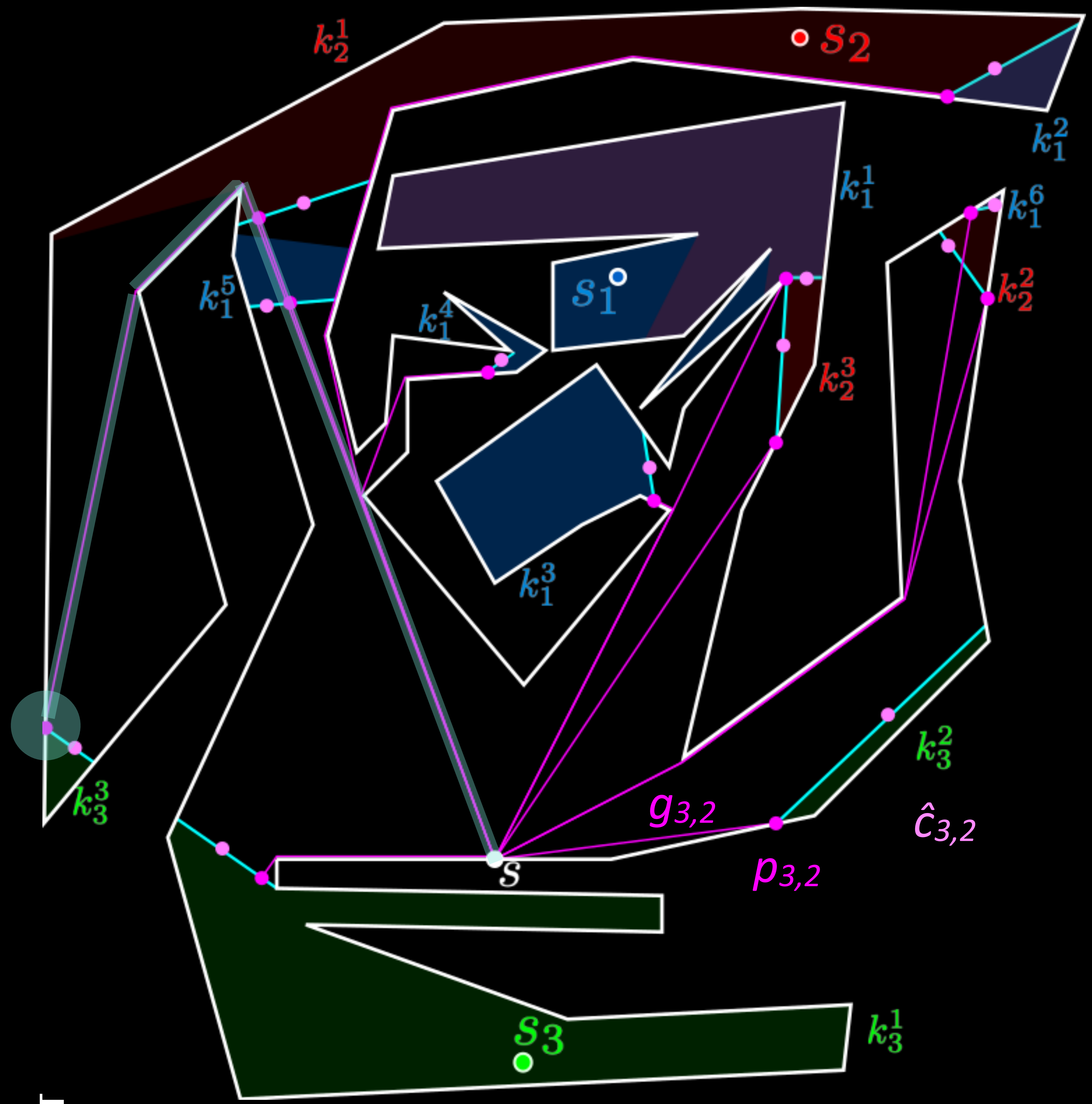

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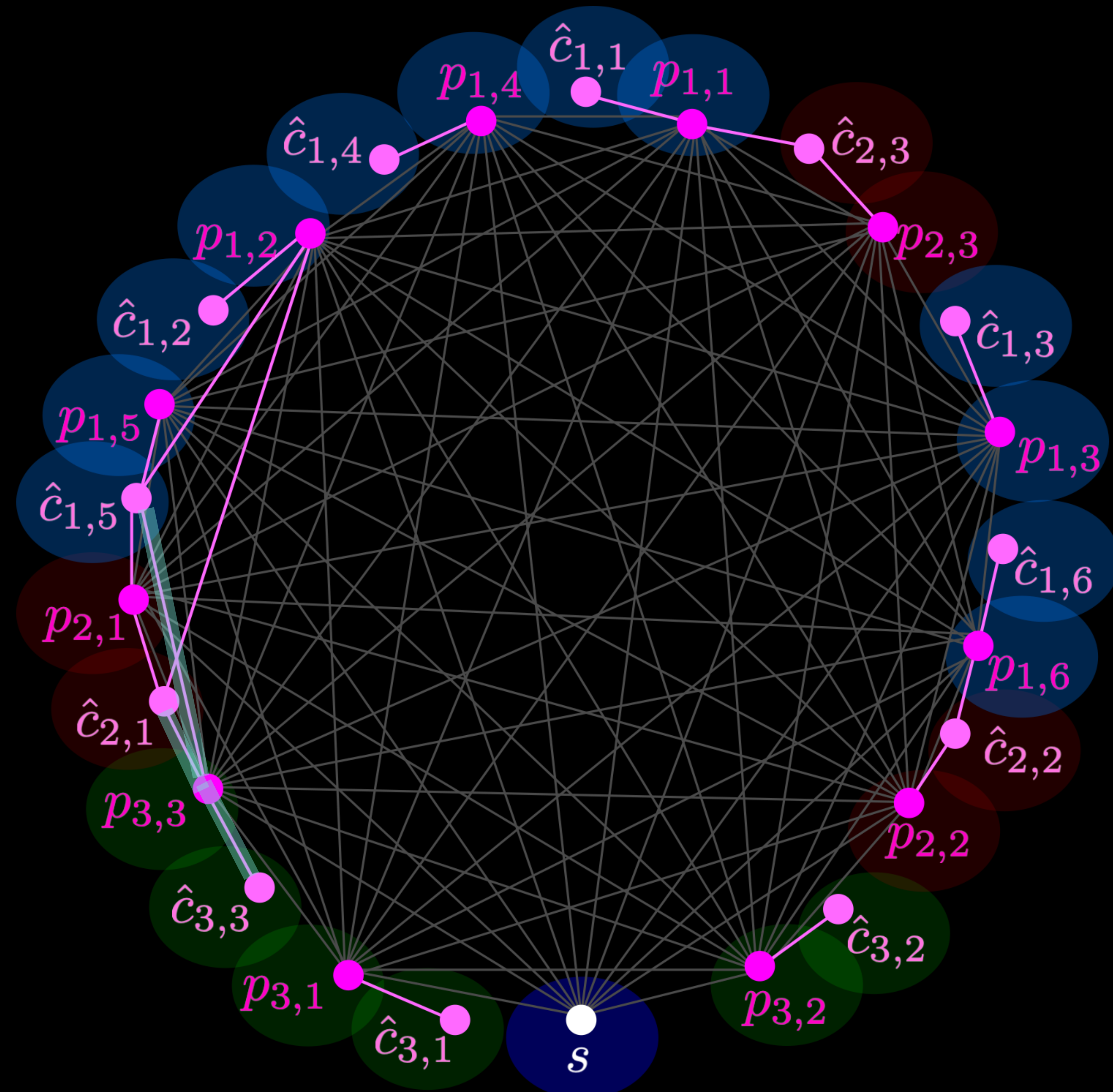

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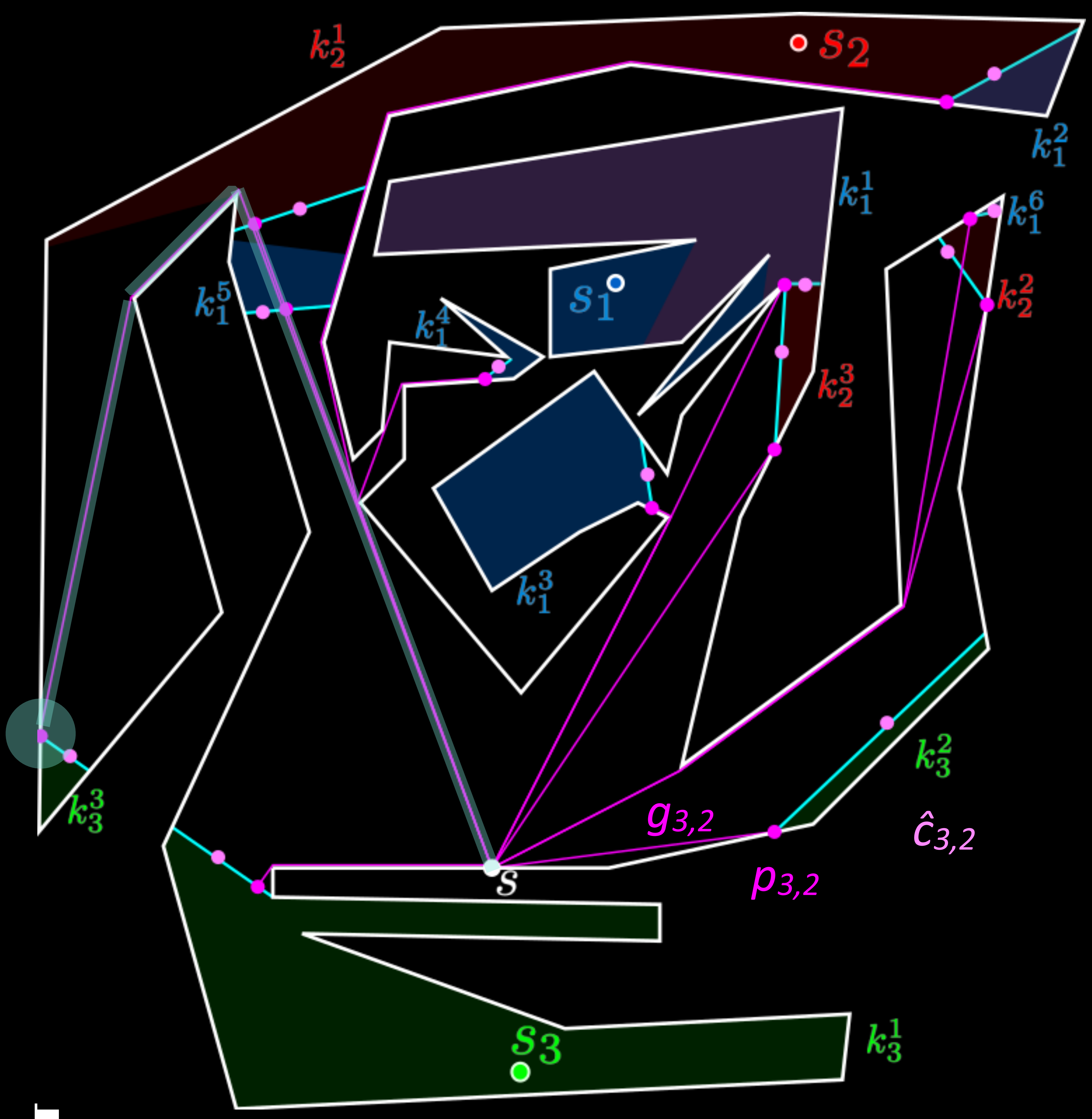

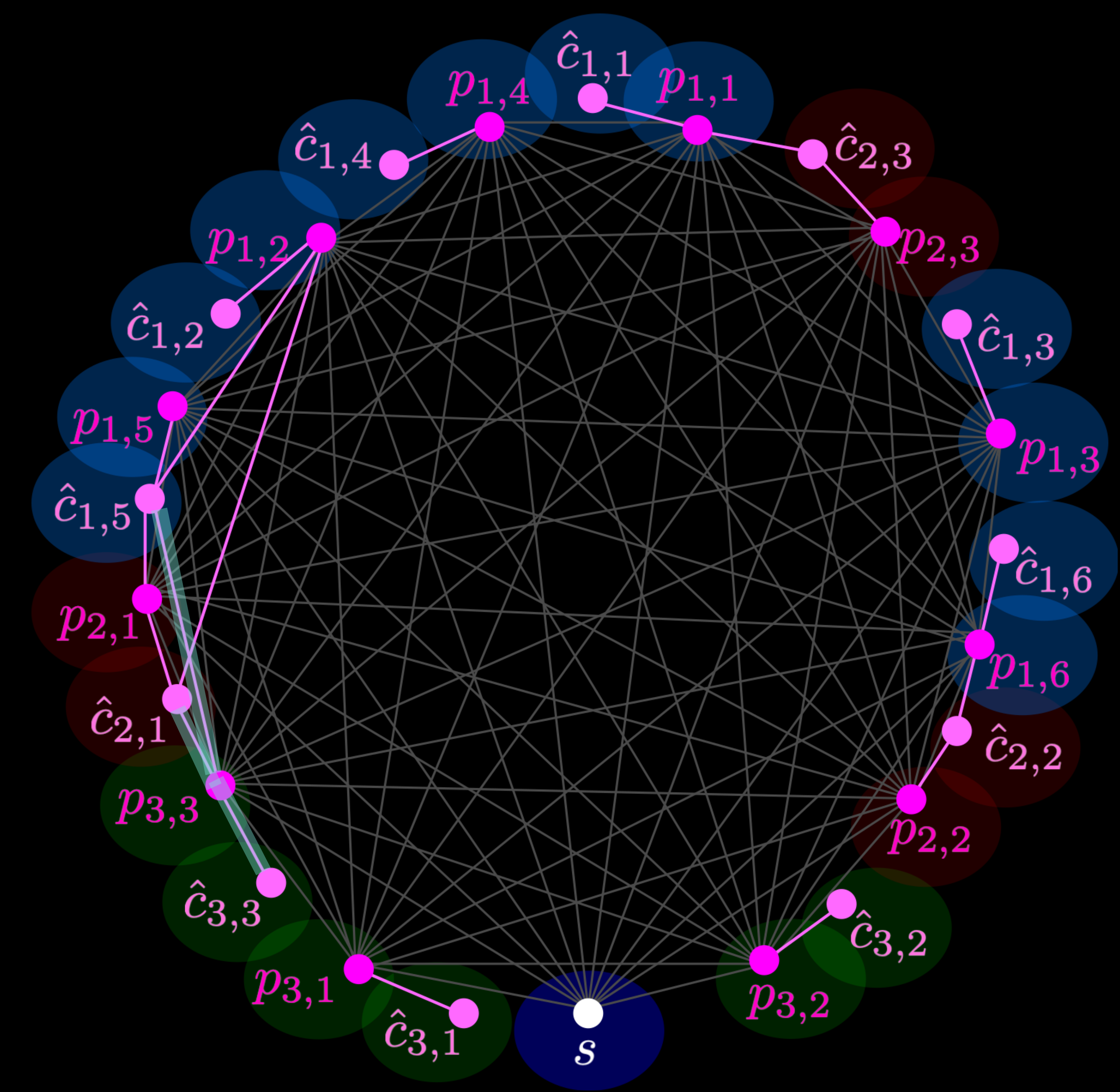

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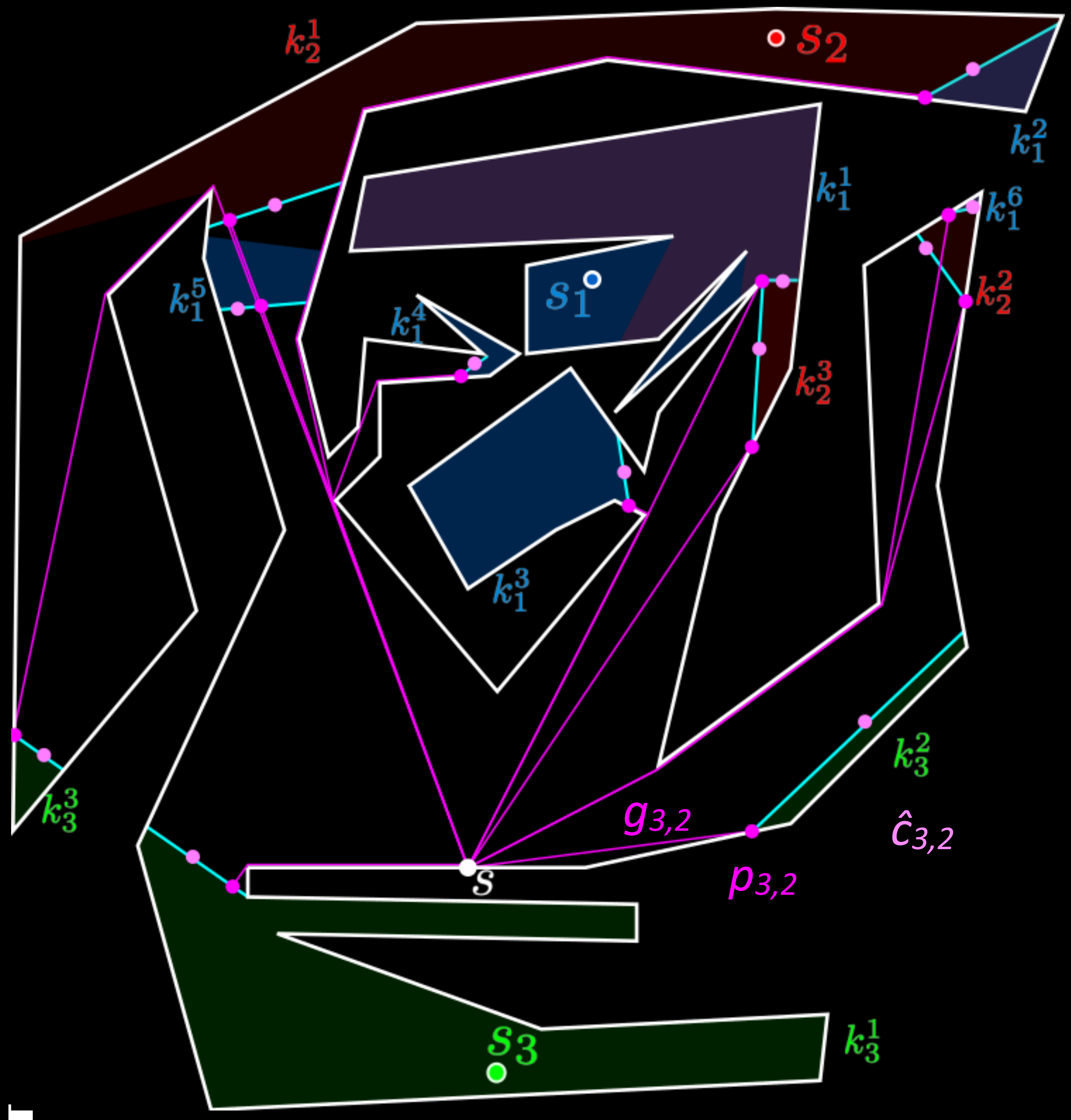

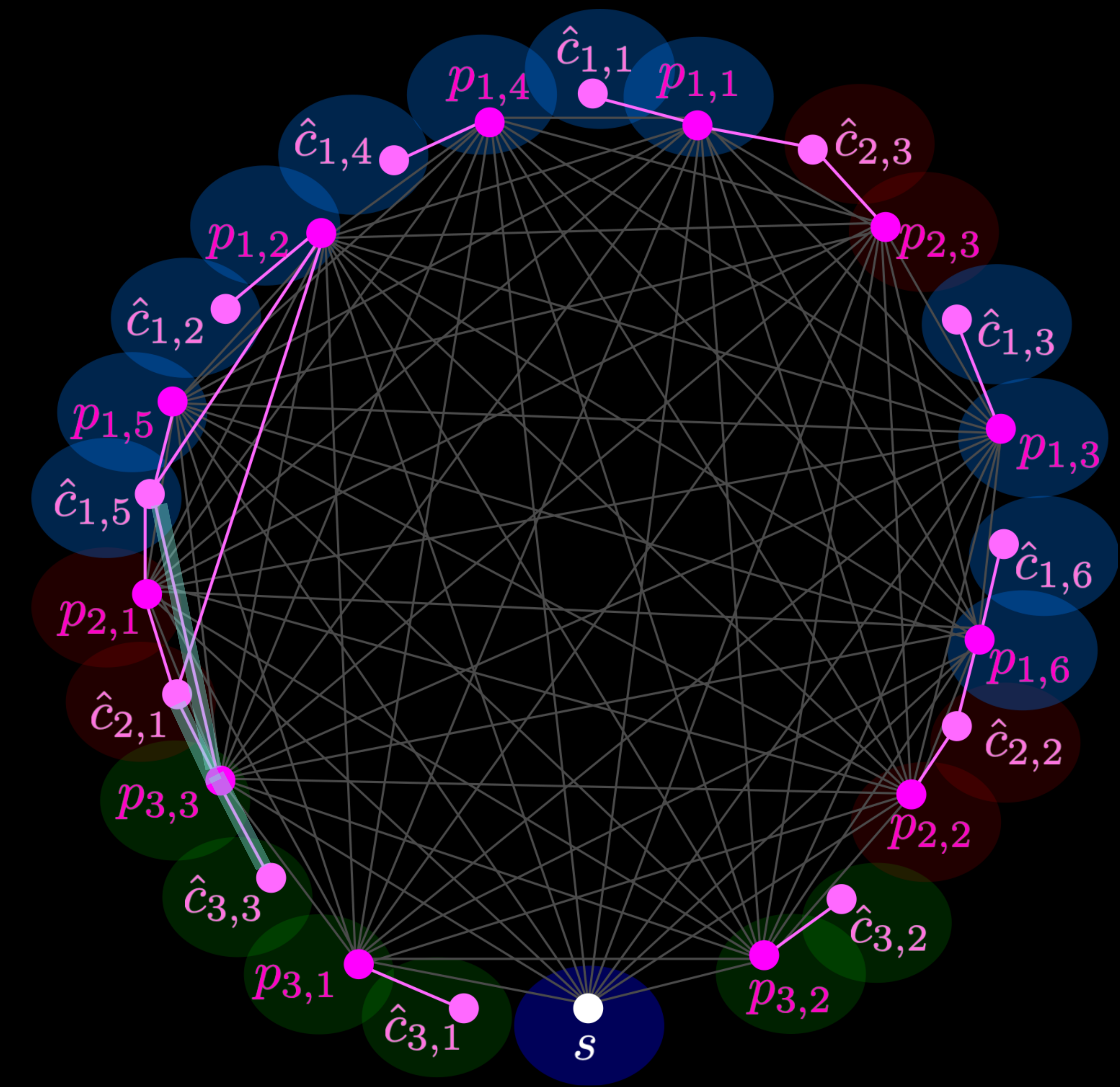

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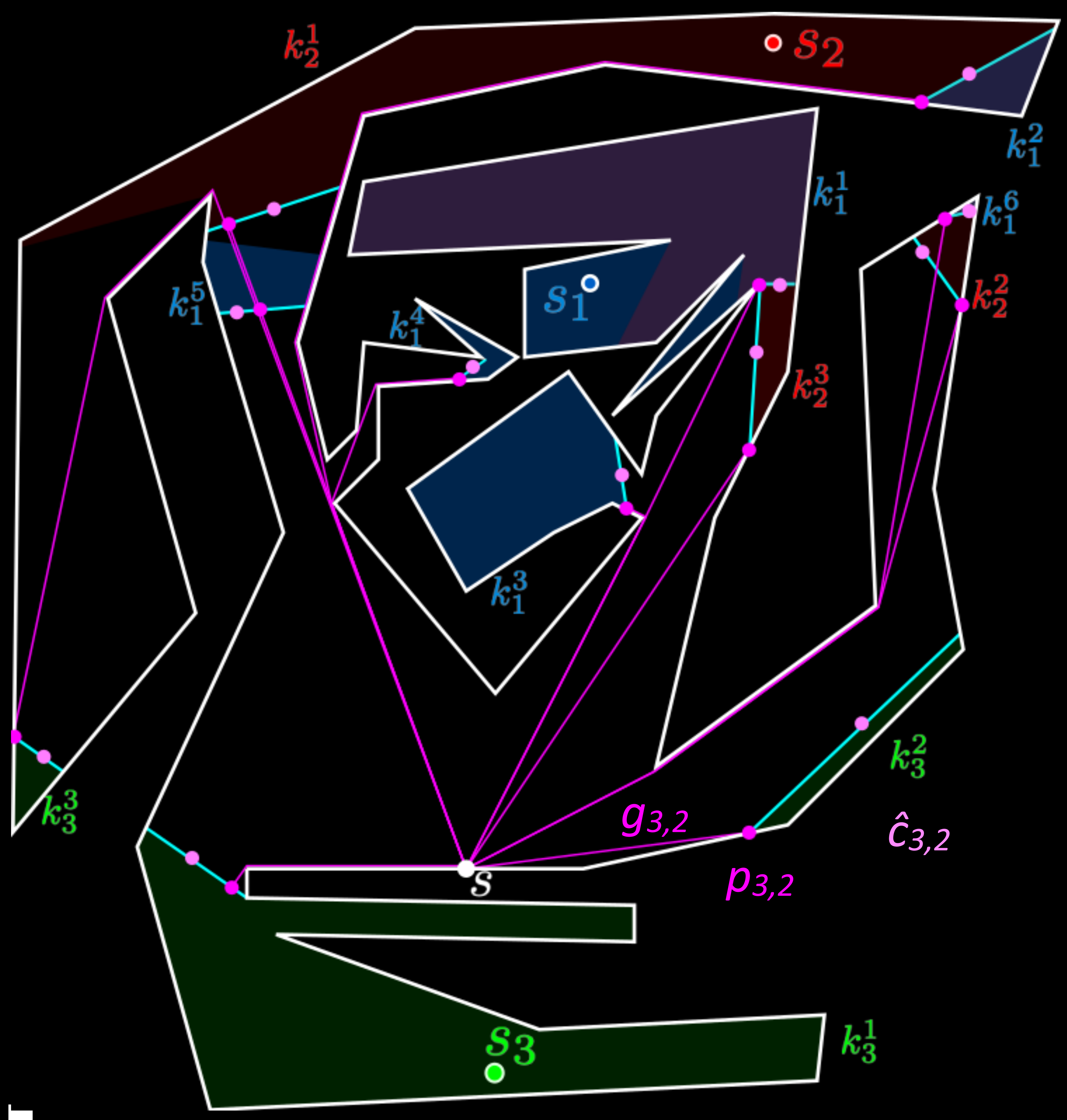

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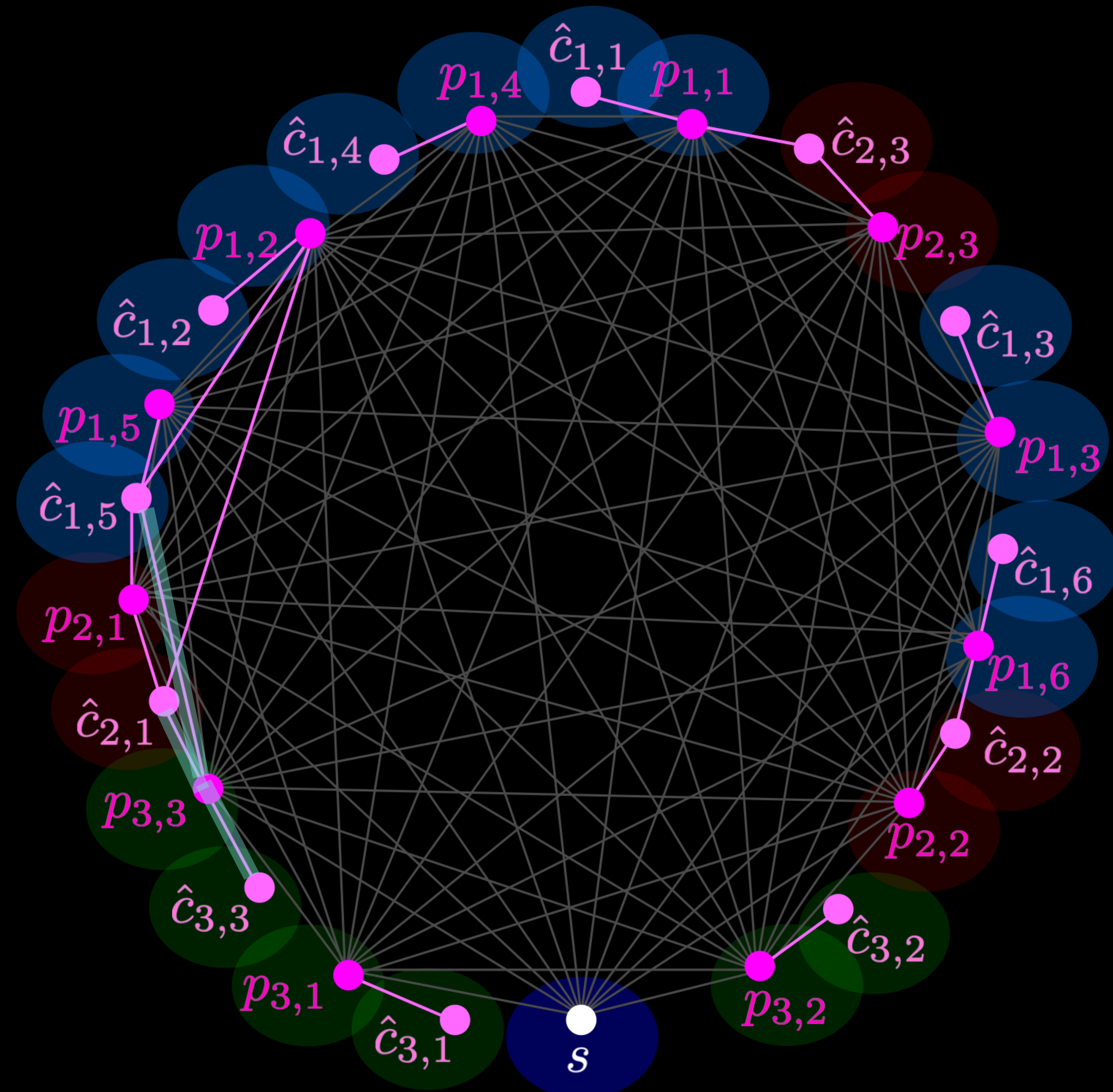

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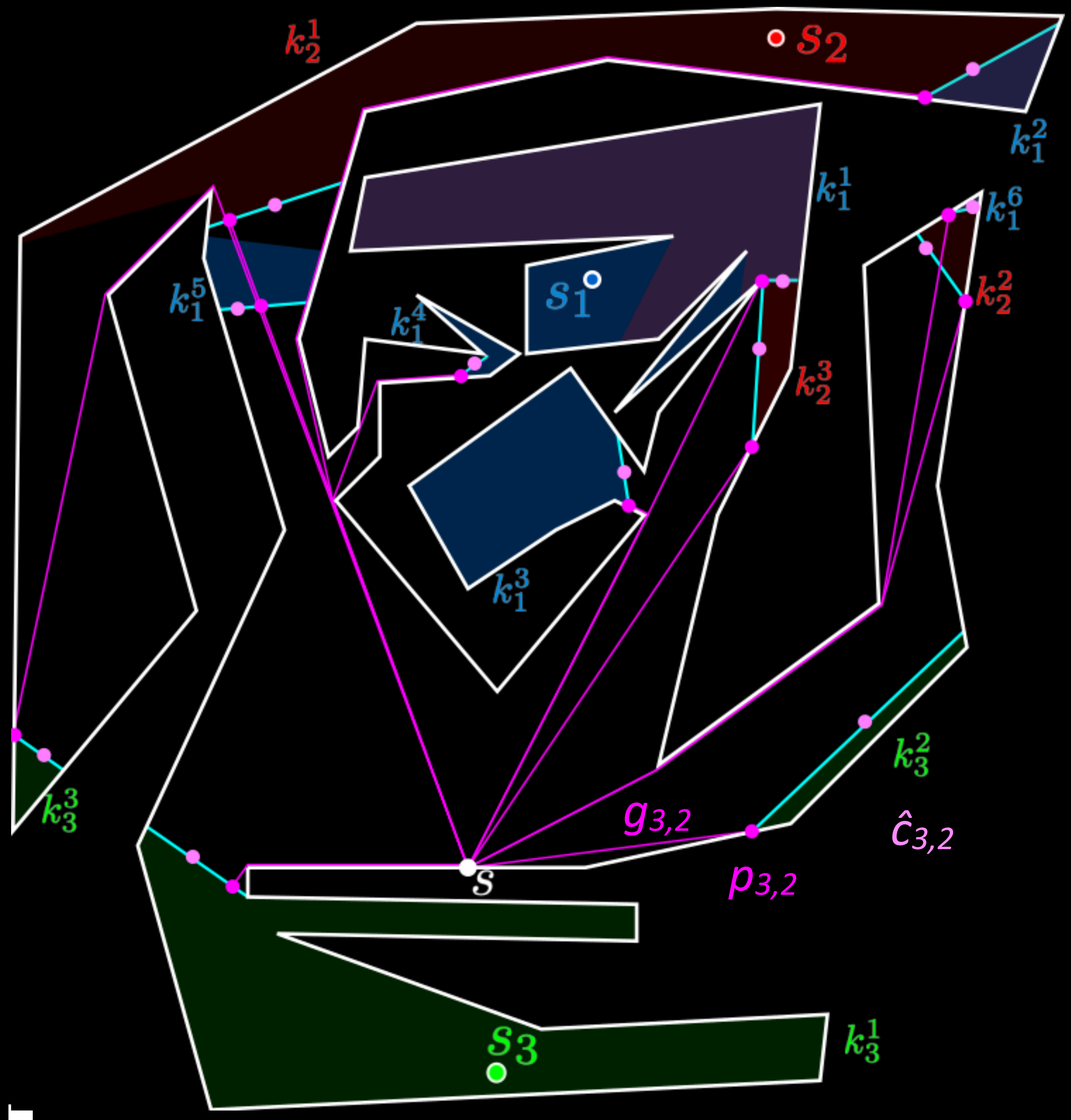

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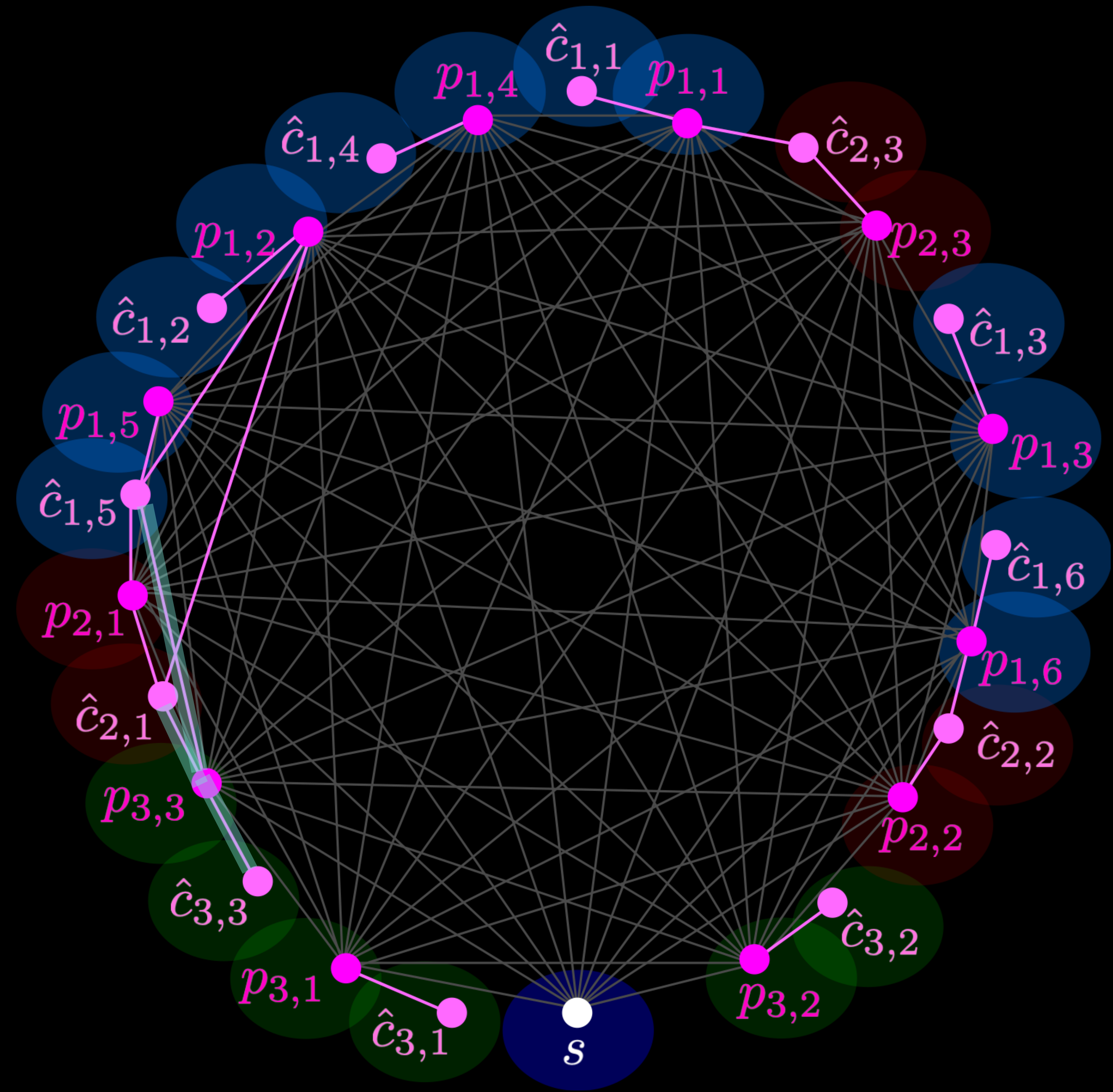

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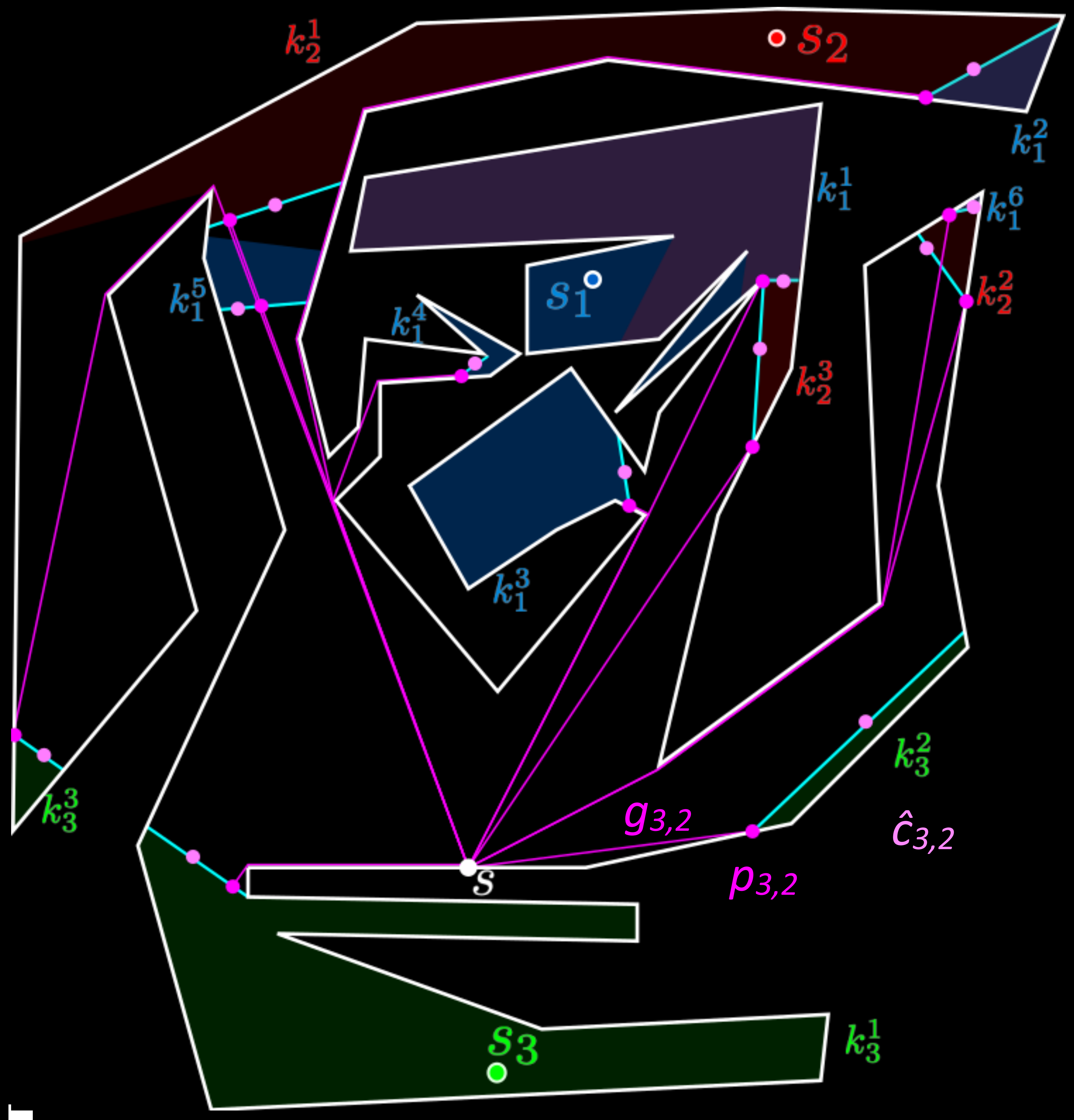

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Here:

γ_1 candidate points that belong to s_1,

γ_2 candidate points that belong to s_2,

γ_3 candidate points that belong to s_3,

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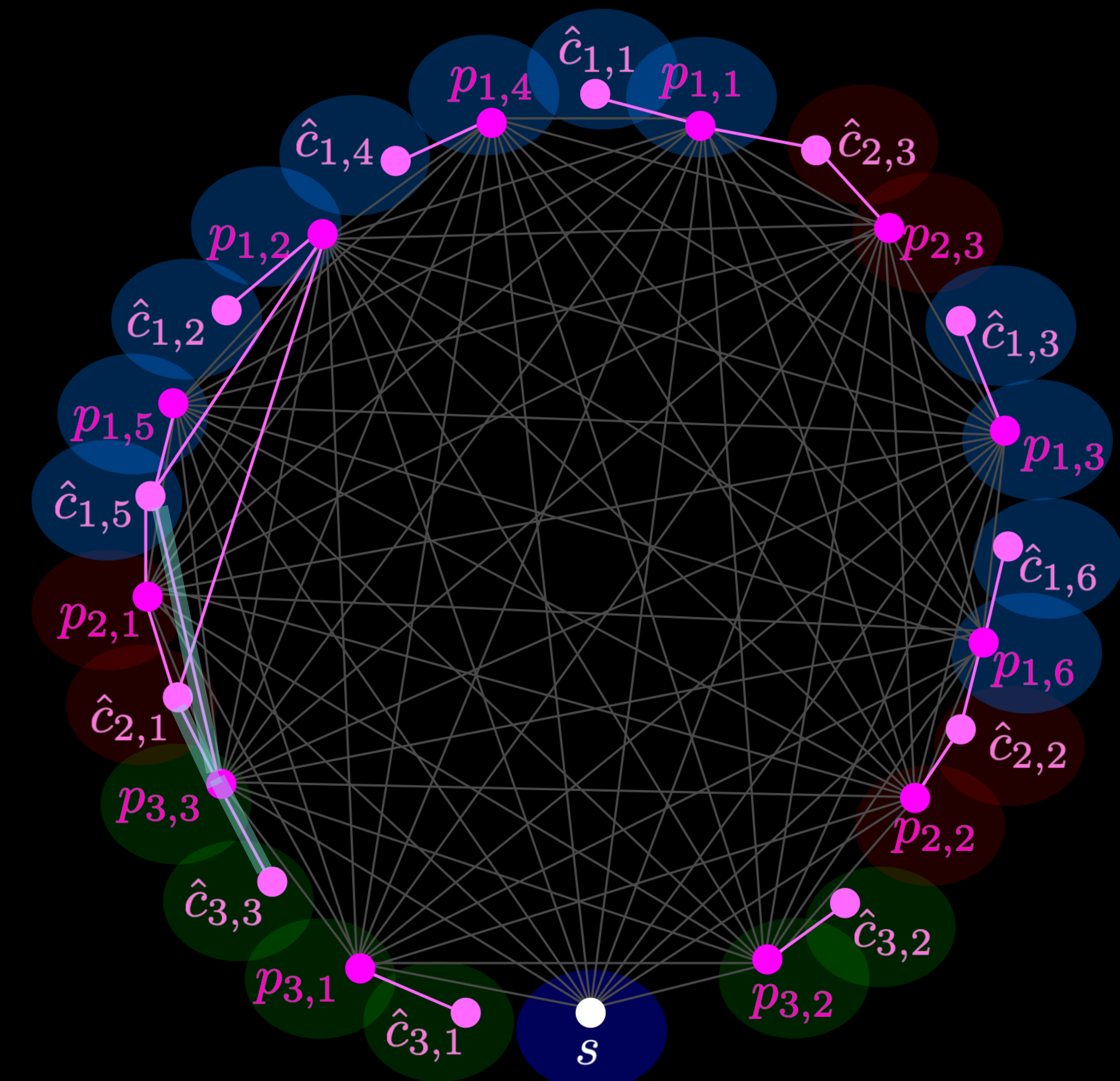

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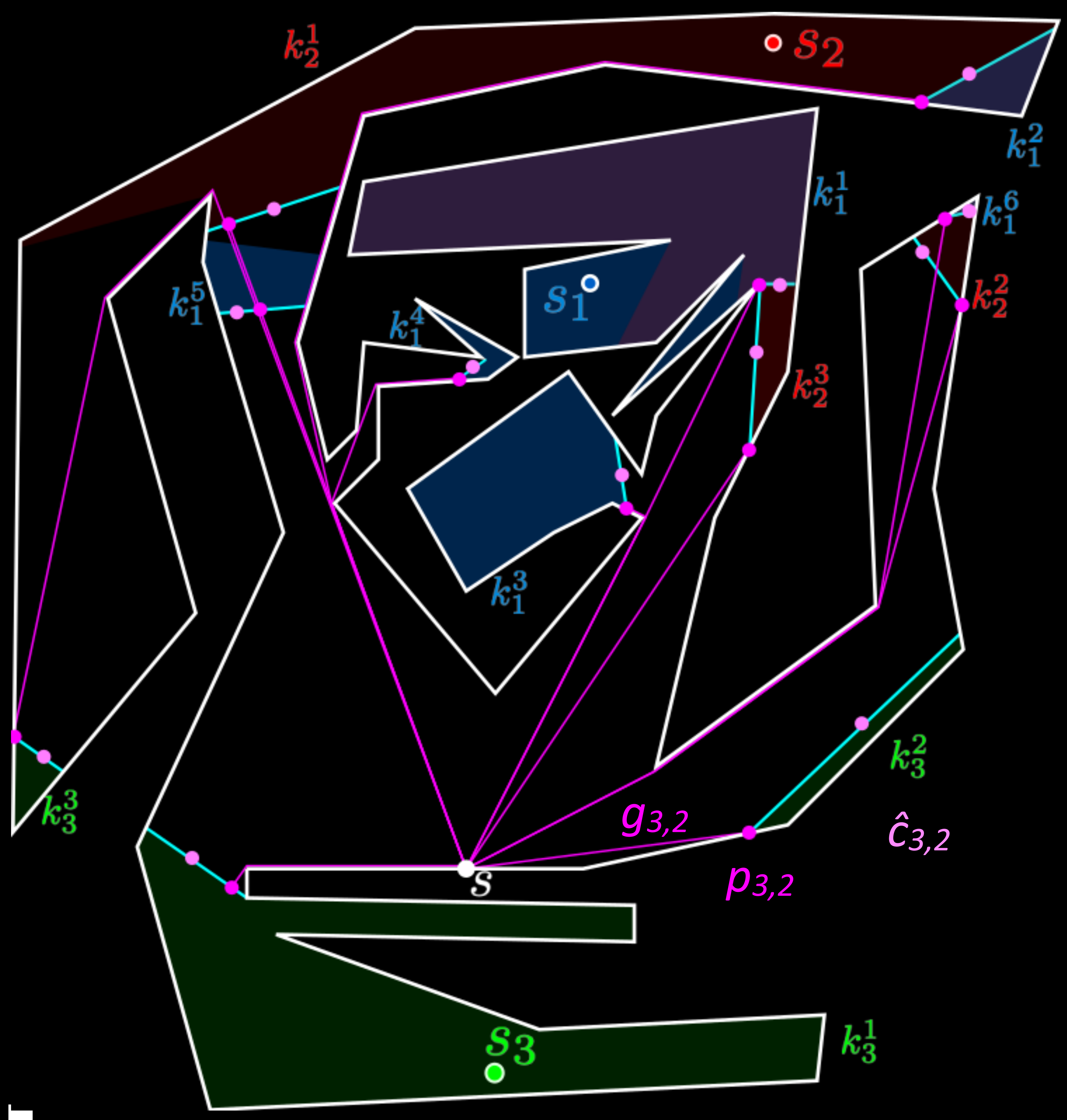

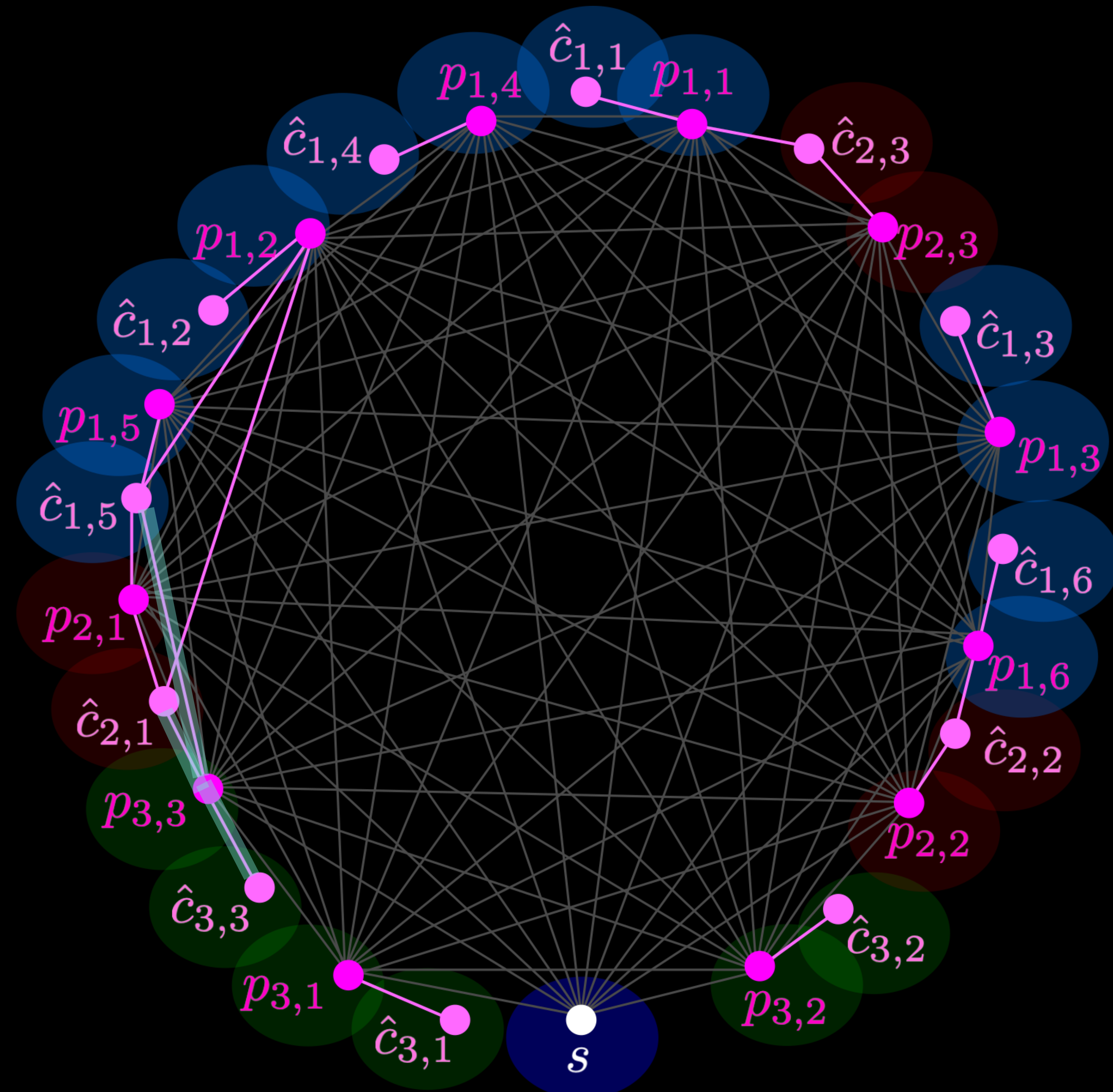

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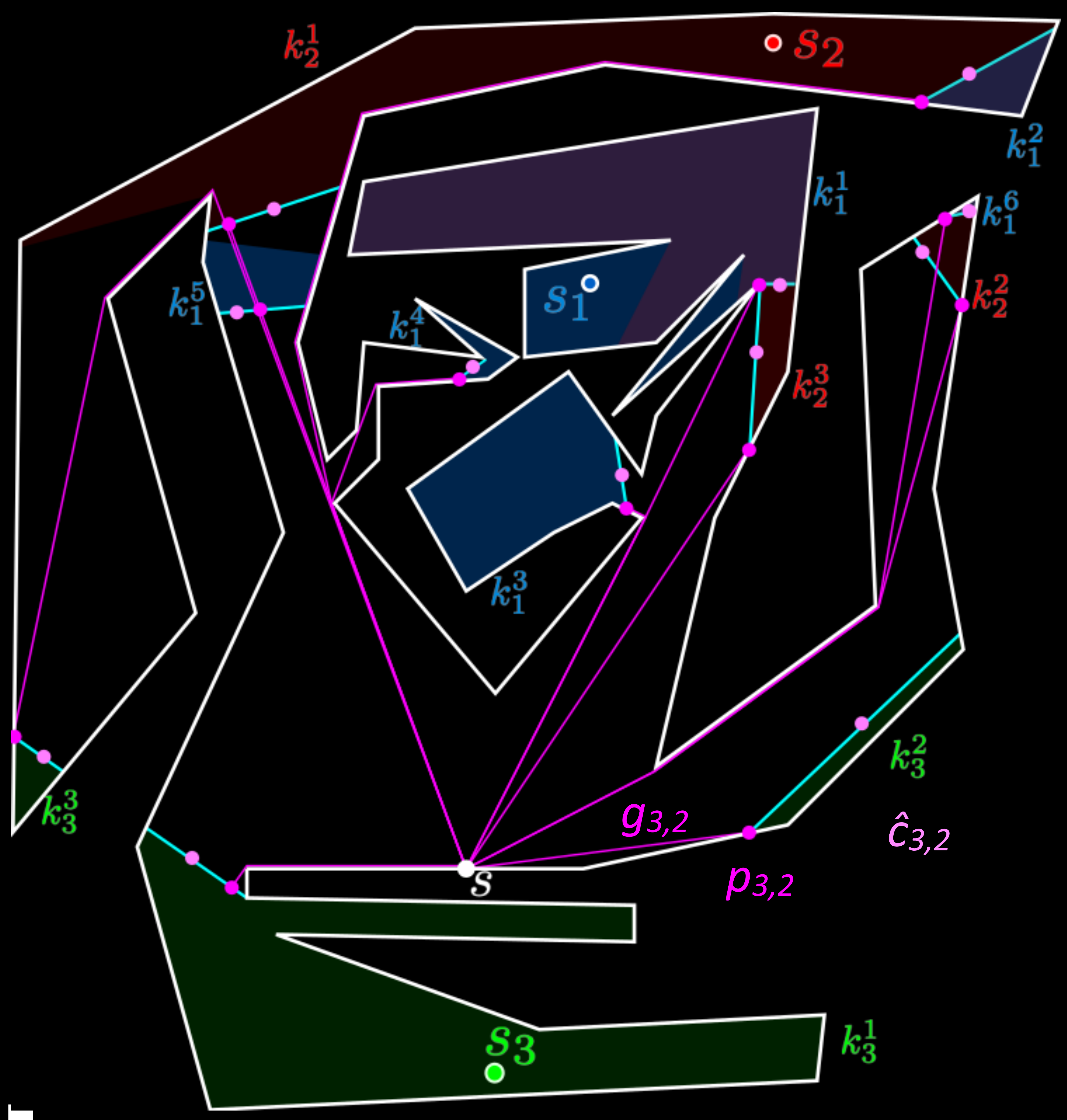

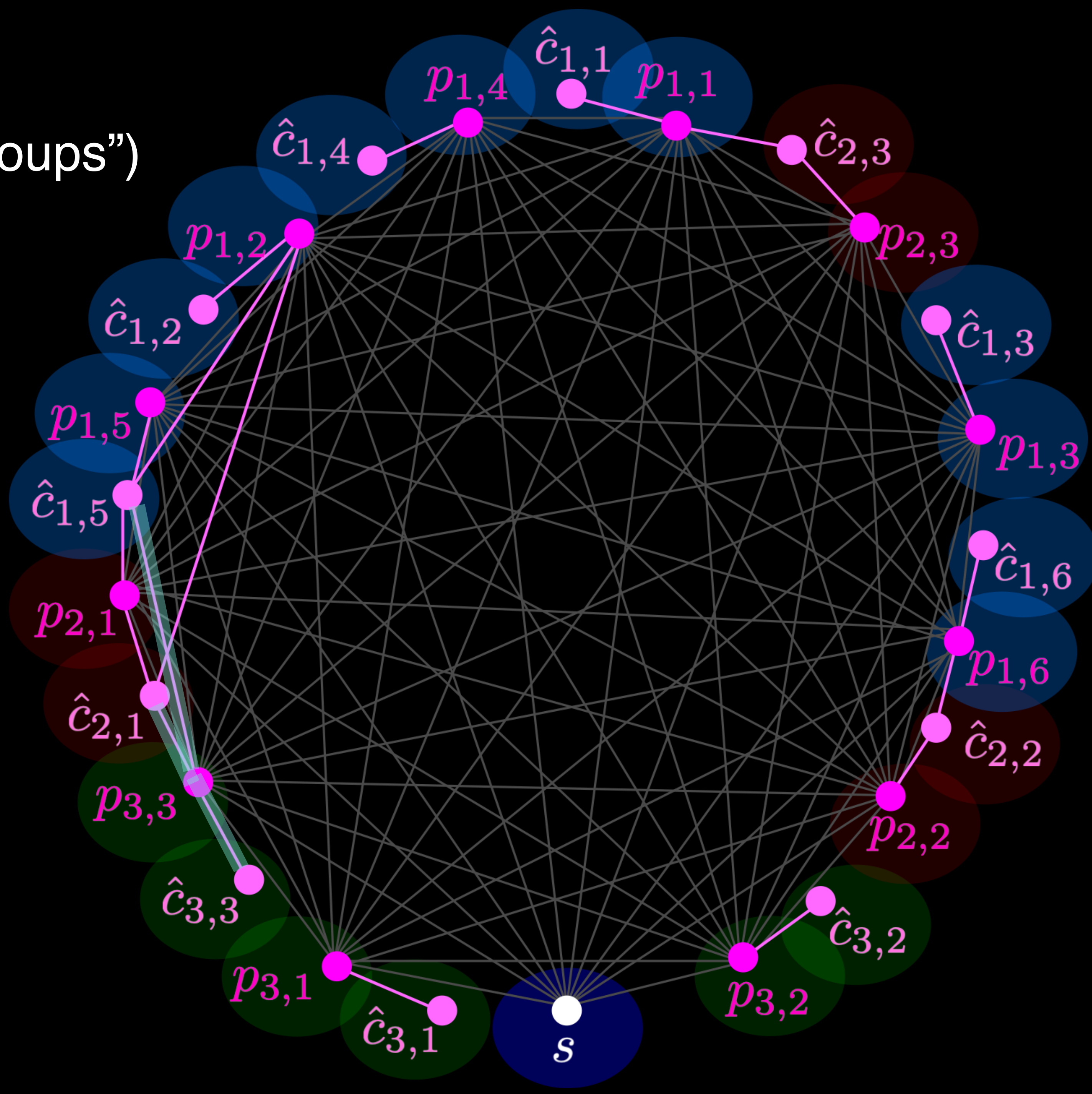

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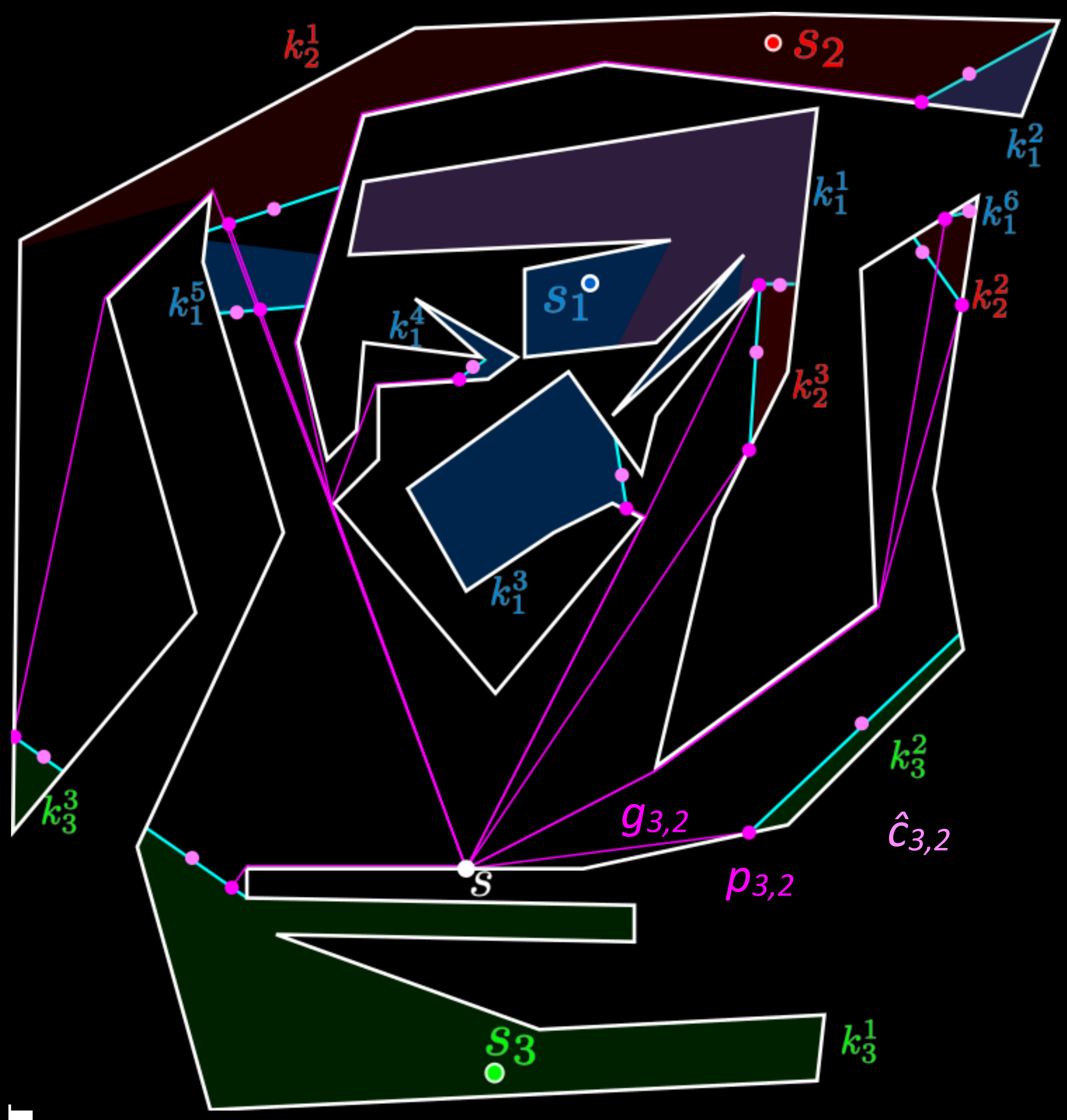

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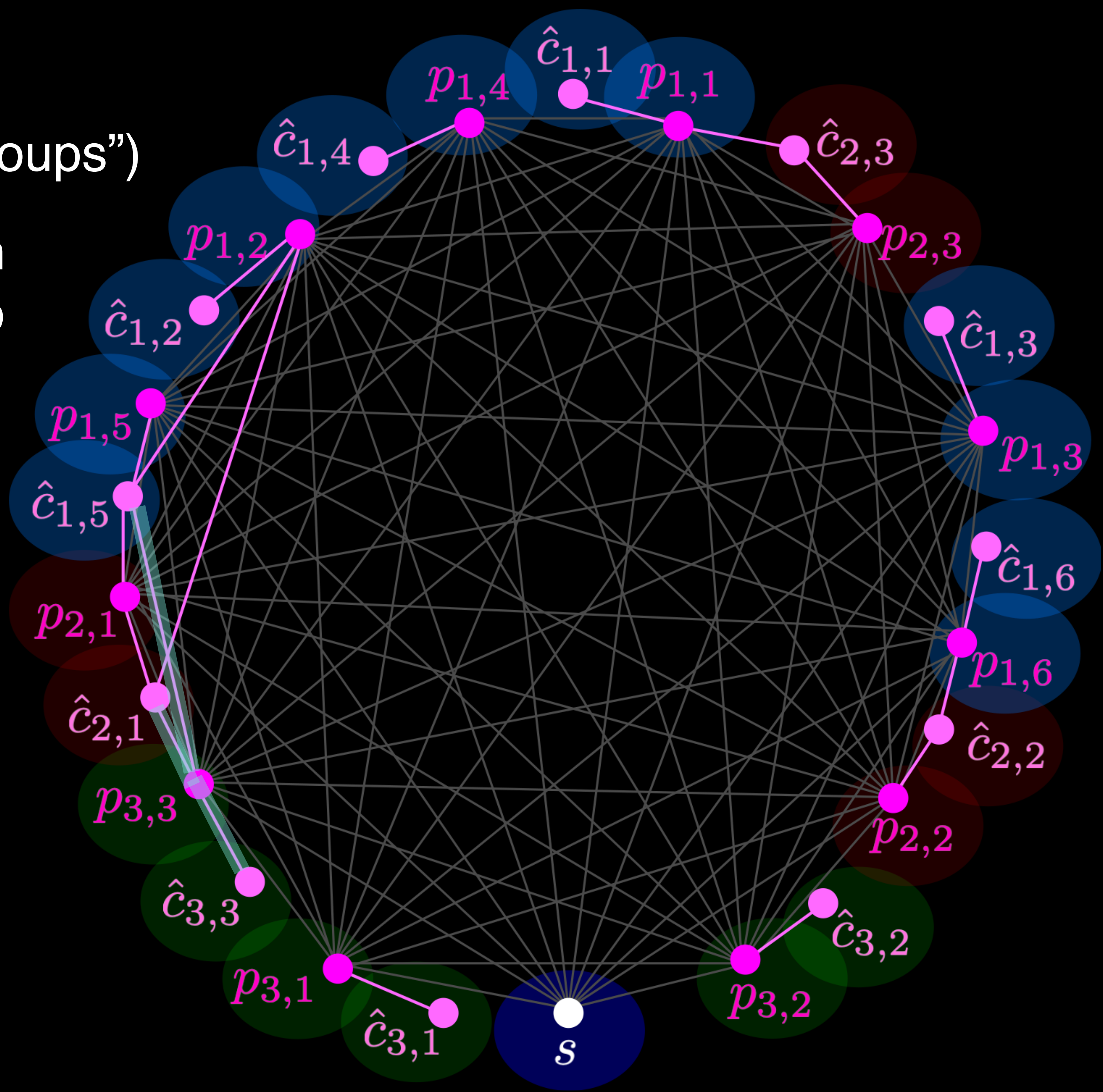

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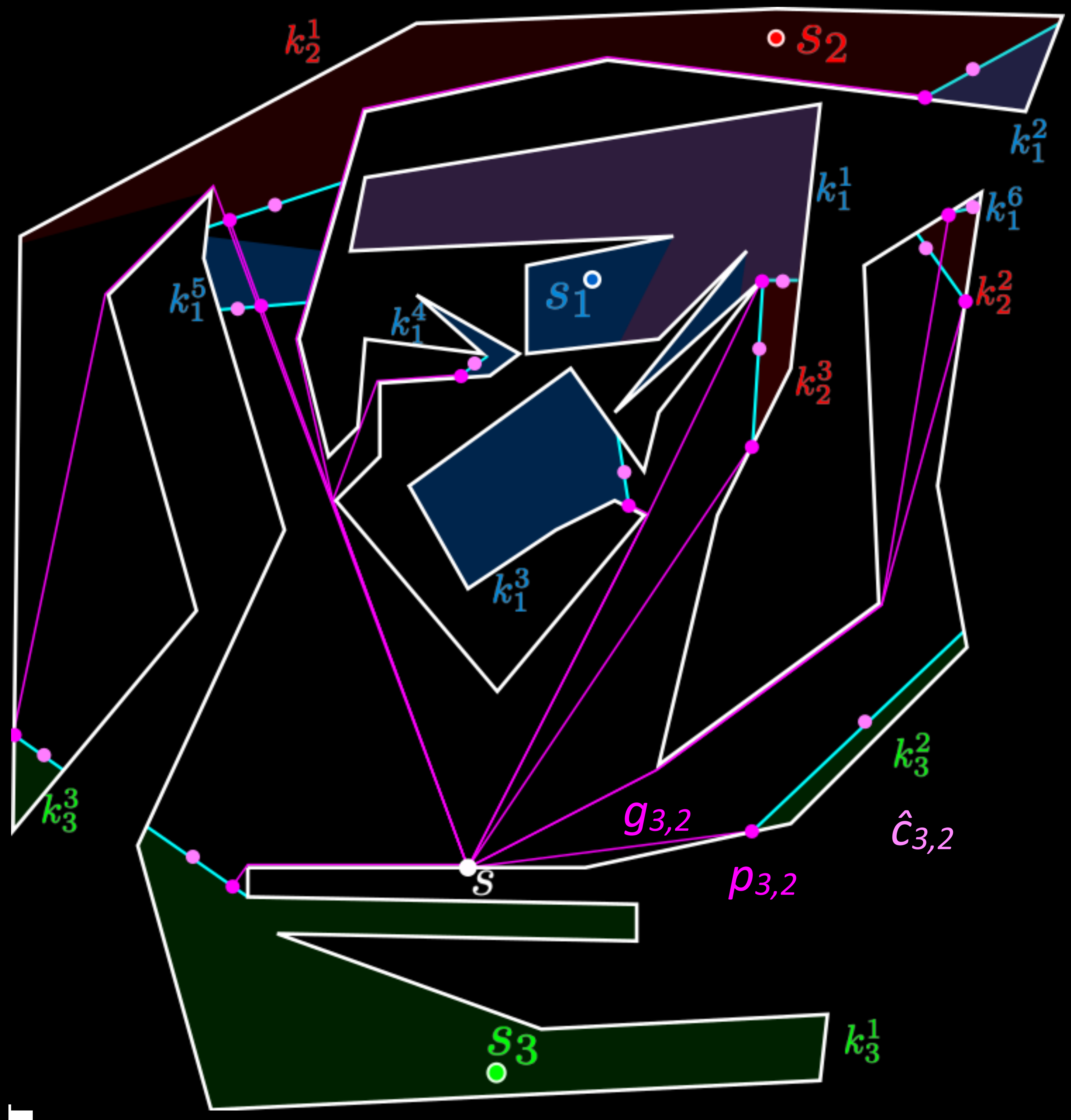

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- Build complete graph G on candidate points $p_{i,j}$:
  - Gray edges: length of geodesic
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  - Graph, with *m* vertices and *Q* vertex subsets (“groups”)
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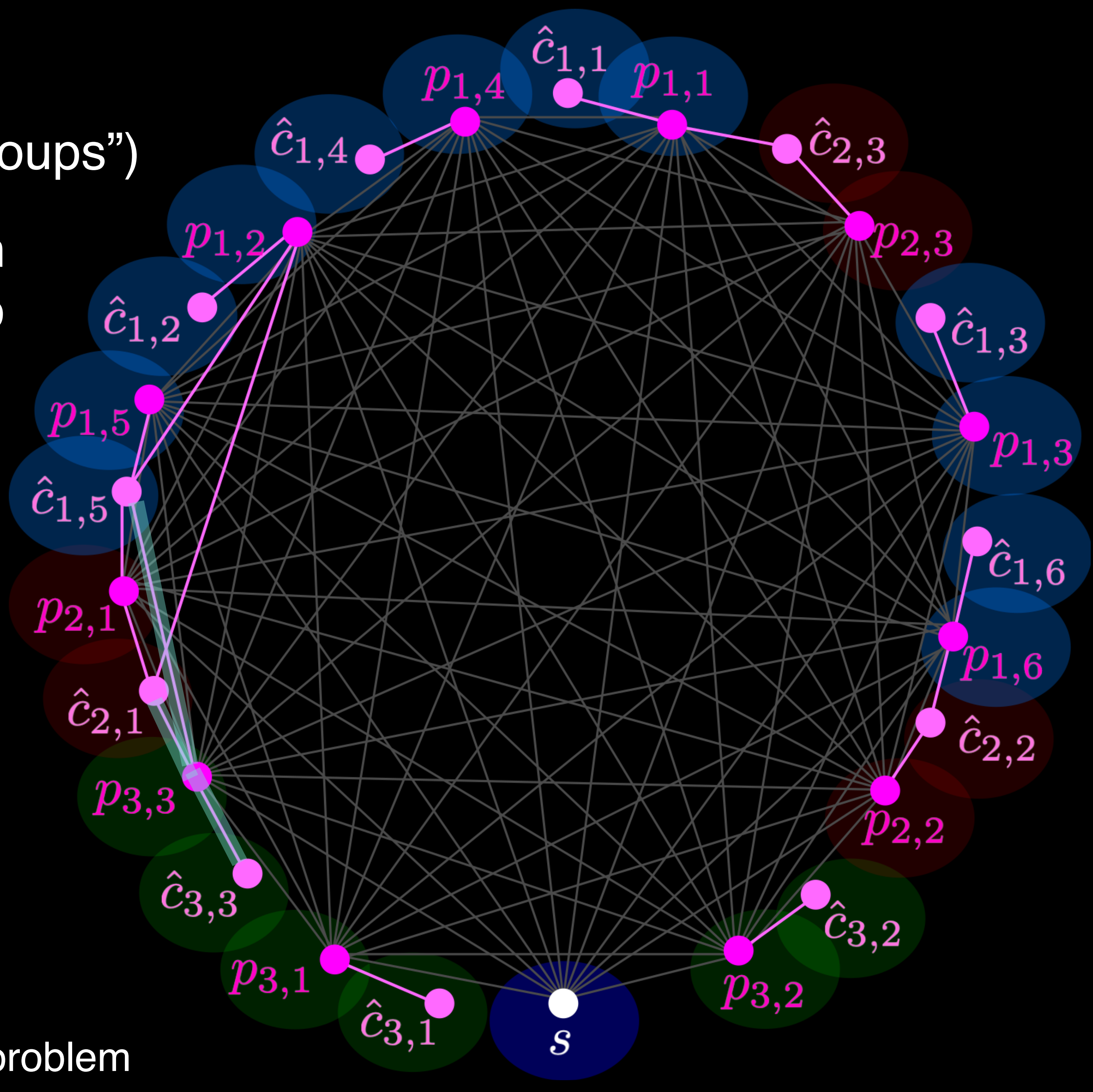
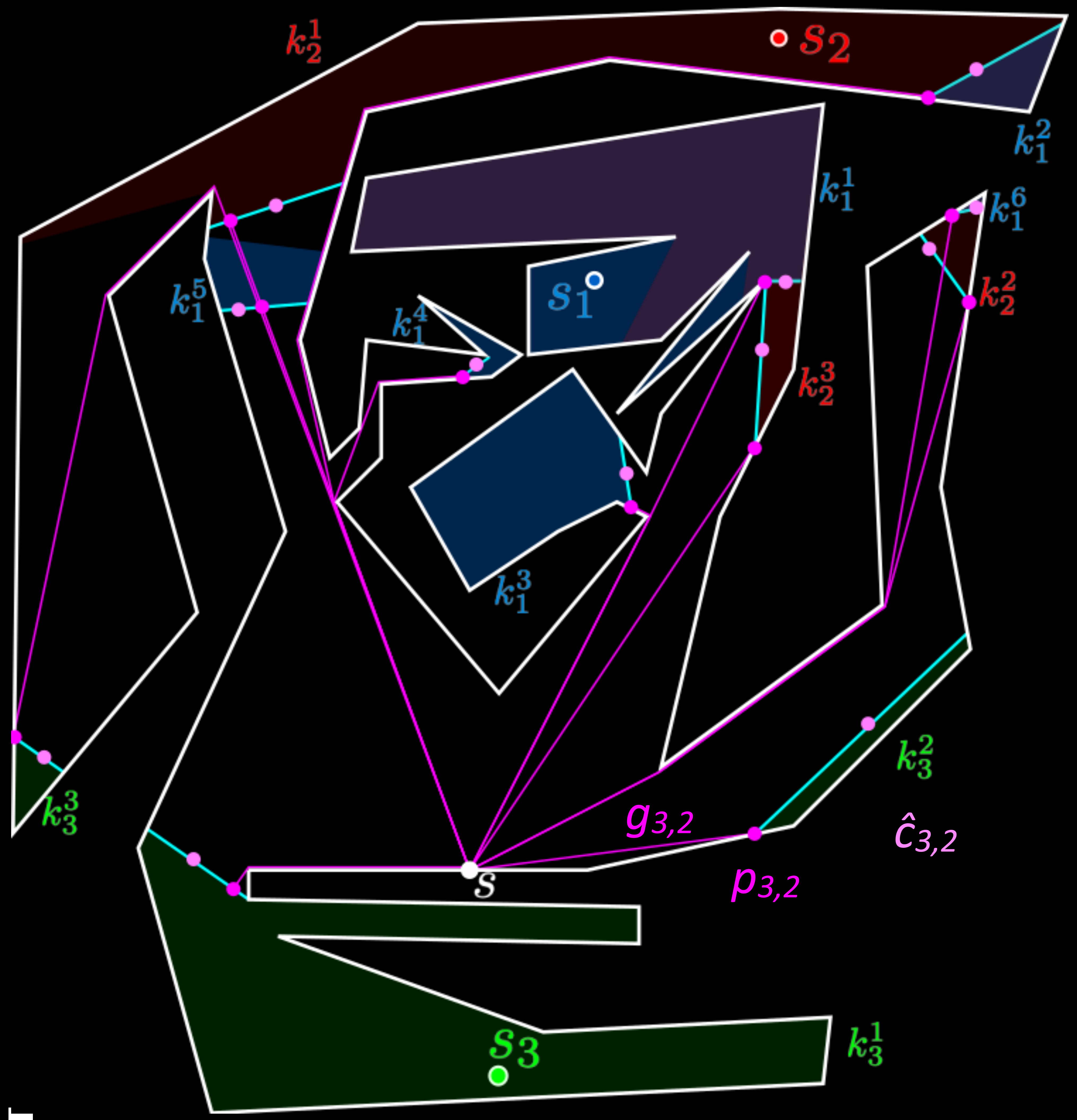


GKR00: Naveen Garg, Goran Konjevod, and R. Ravi. A polylogarithmic approximation algorithm for the group Steiner tree problem

Approximation Algorithm for *k*-TrWRP(*S*,*P*,*s*)



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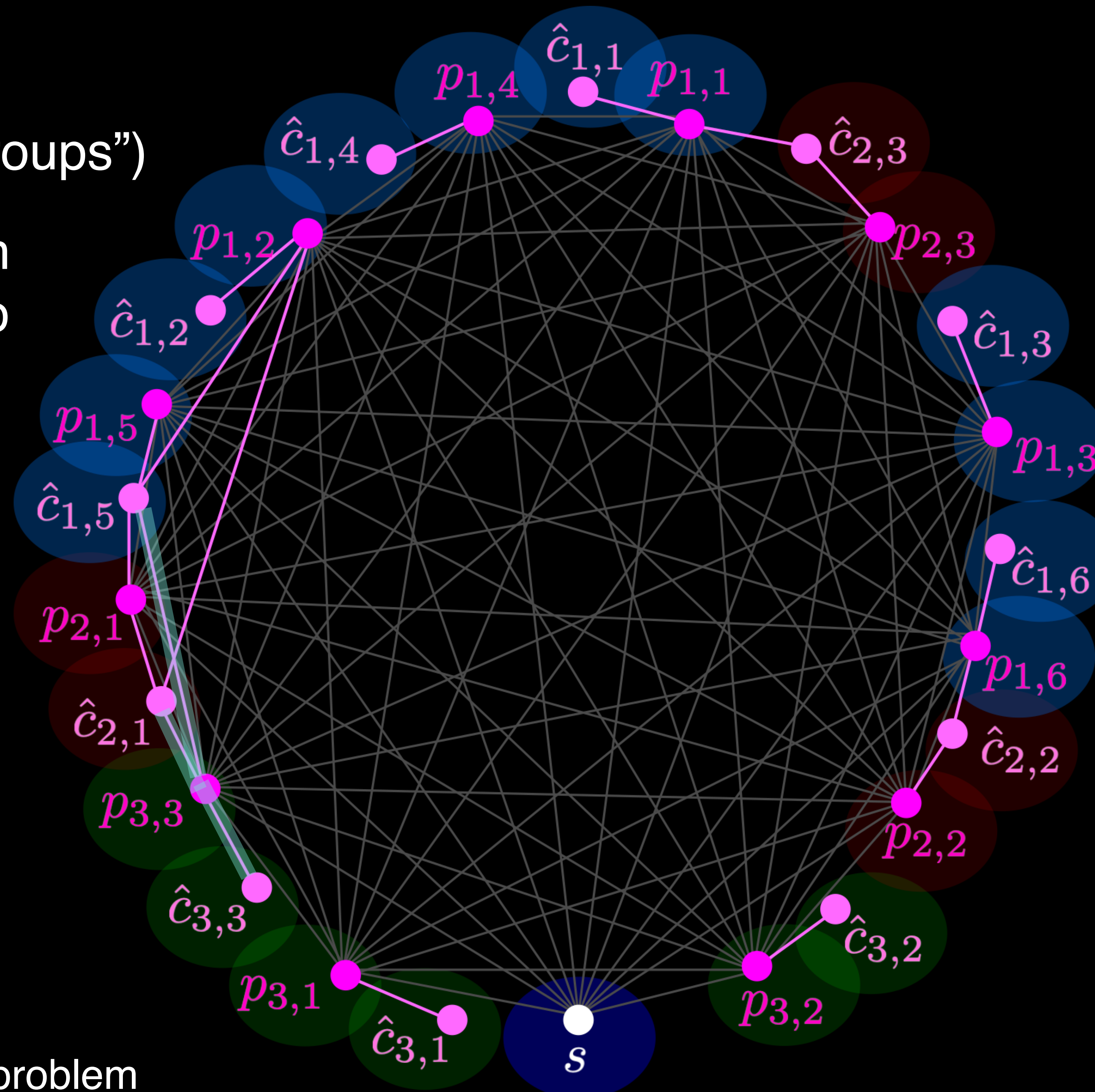

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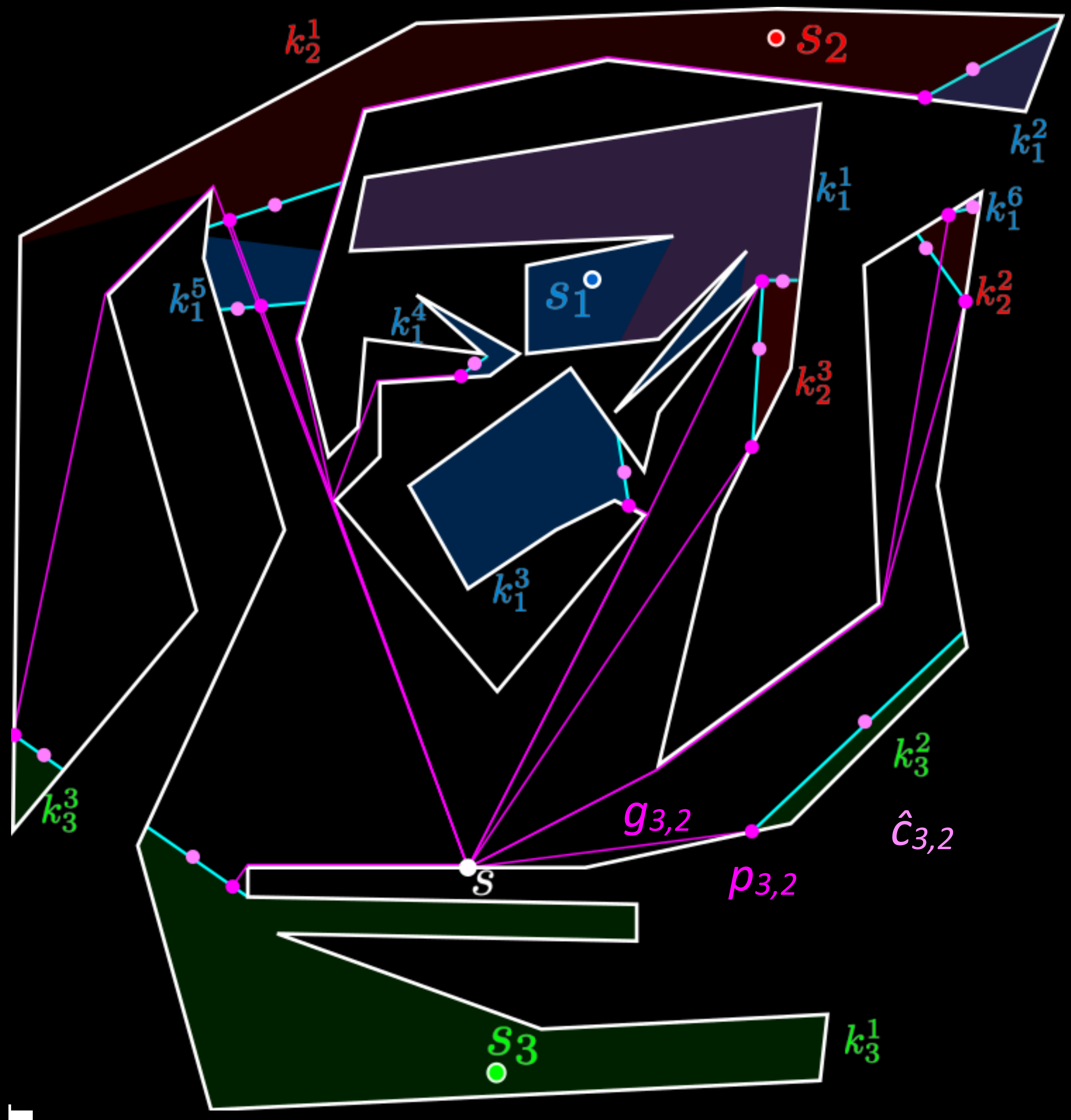

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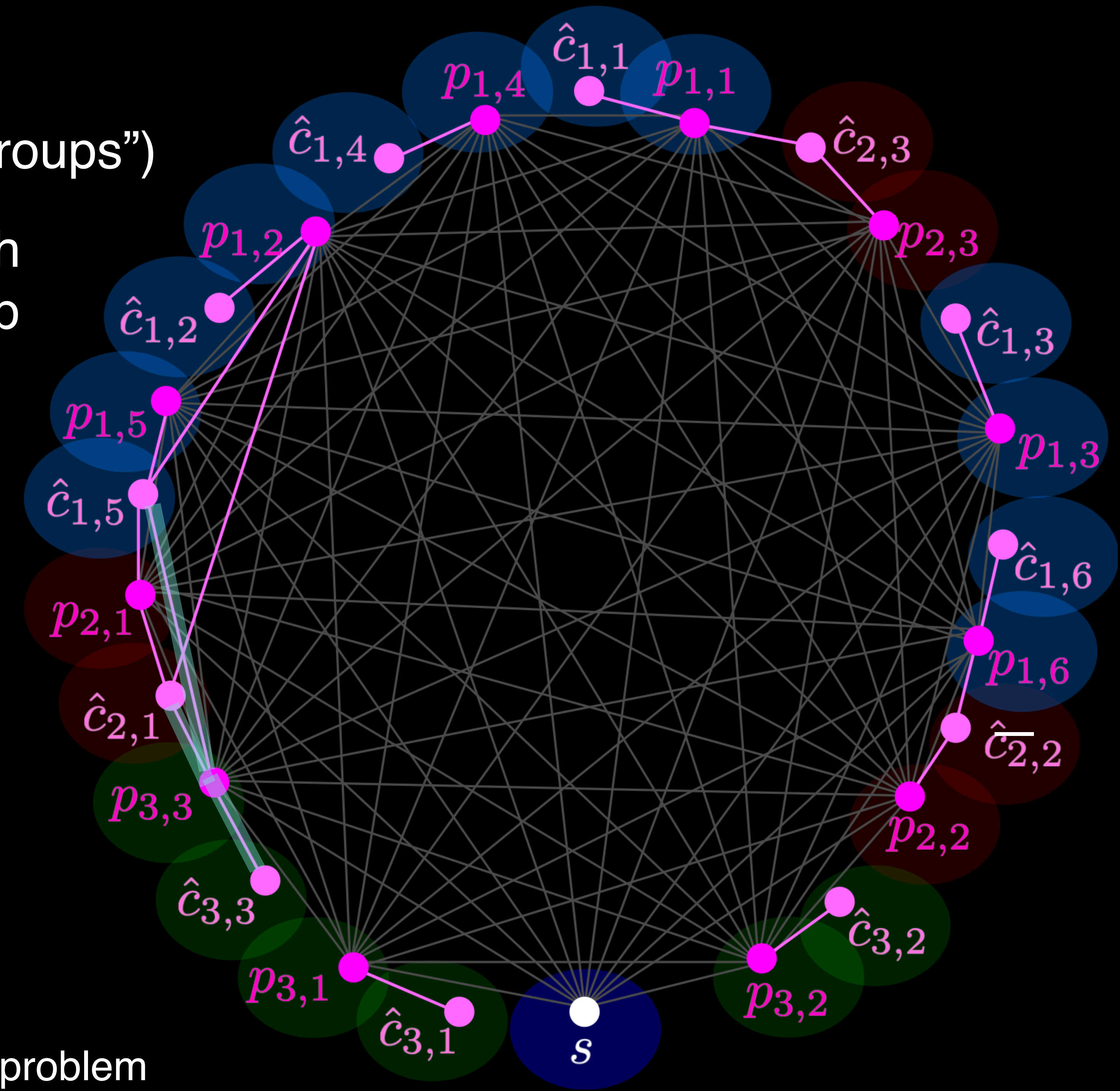

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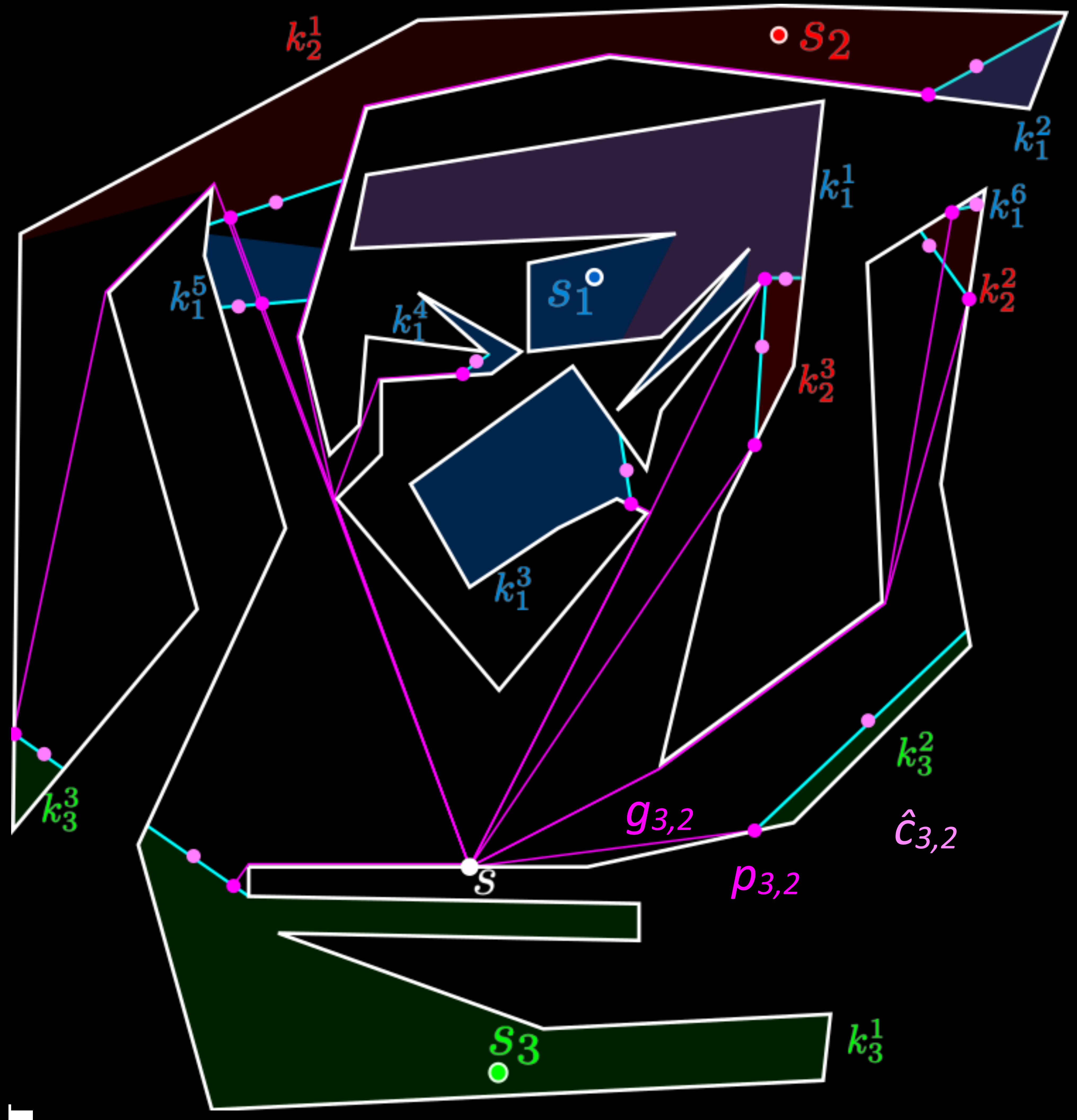

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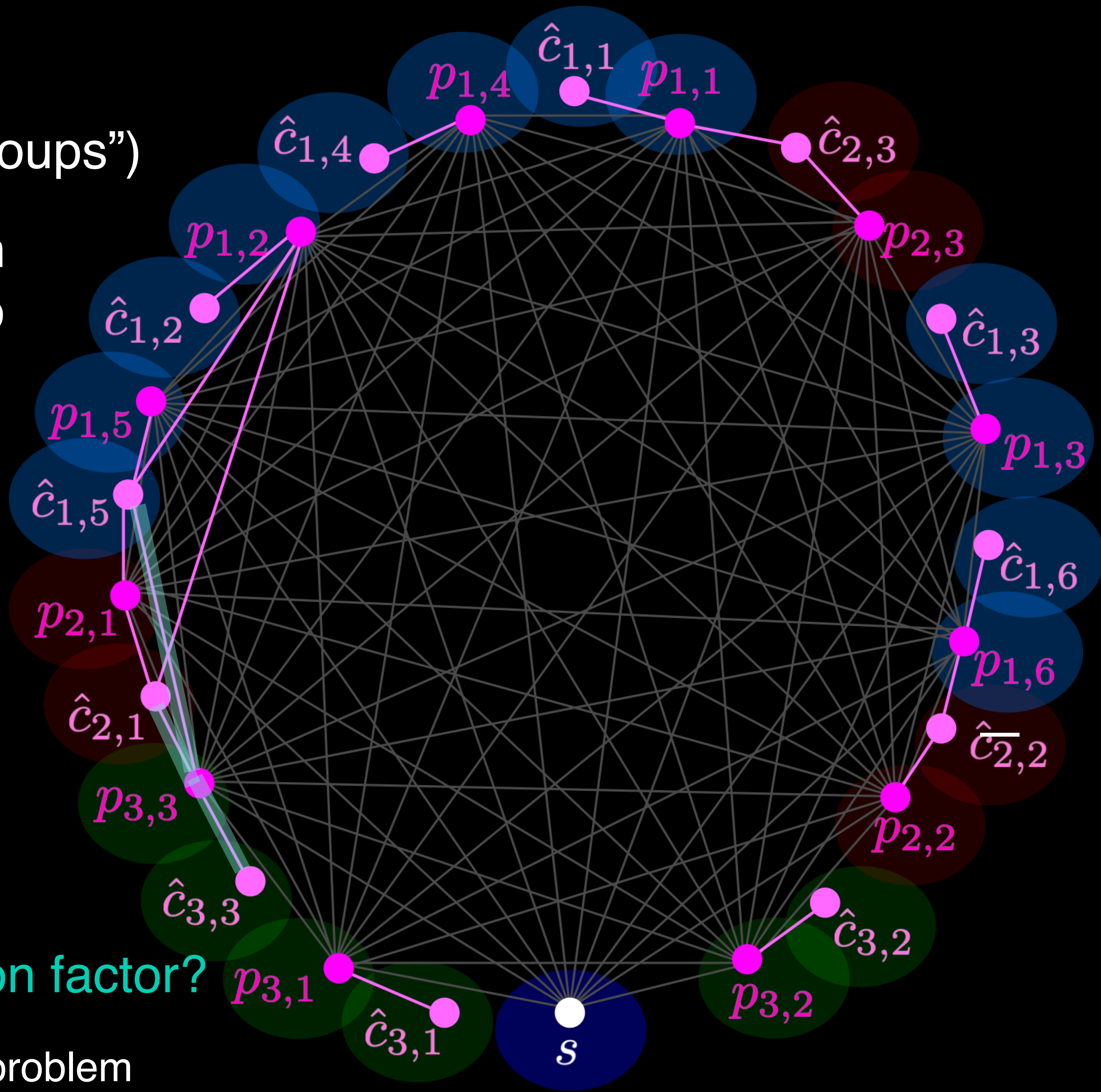

To do: why do we achieve the claimed approximation factor?

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GKR00: Naveen Garg, Goran Konjevod, and R. Ravi. A polylogarithmic approximation algorithm for the group Steiner tree problem

Proof idea: alter(unknown) optimal route OPT(S,P,s) to pass through points from $V(G)$, and new tour has length at most constant· OPT(S,P,s)

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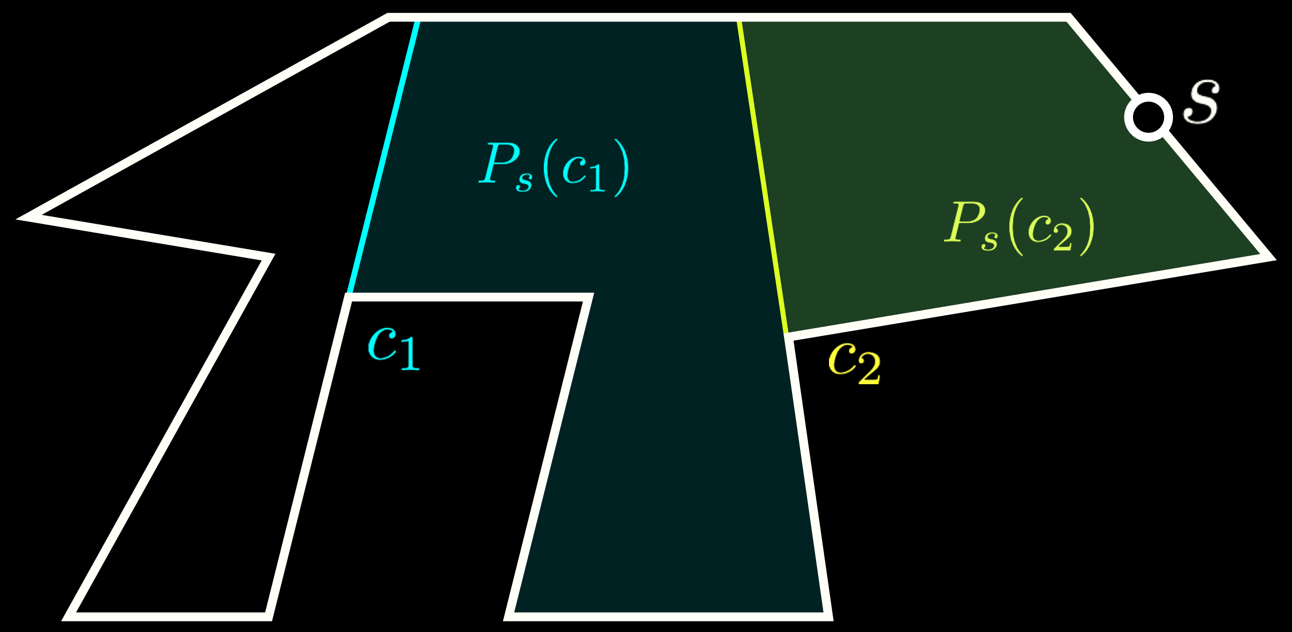
A cut c partitions polygon into two subpolygons:
$P_s(c)$—subpolygon that contains starting point s
A cut c_1 *dominates* c_2 if $P_s(c_2) \subseteq P_s(c_1)$
Essential cut: not dominated by other cut



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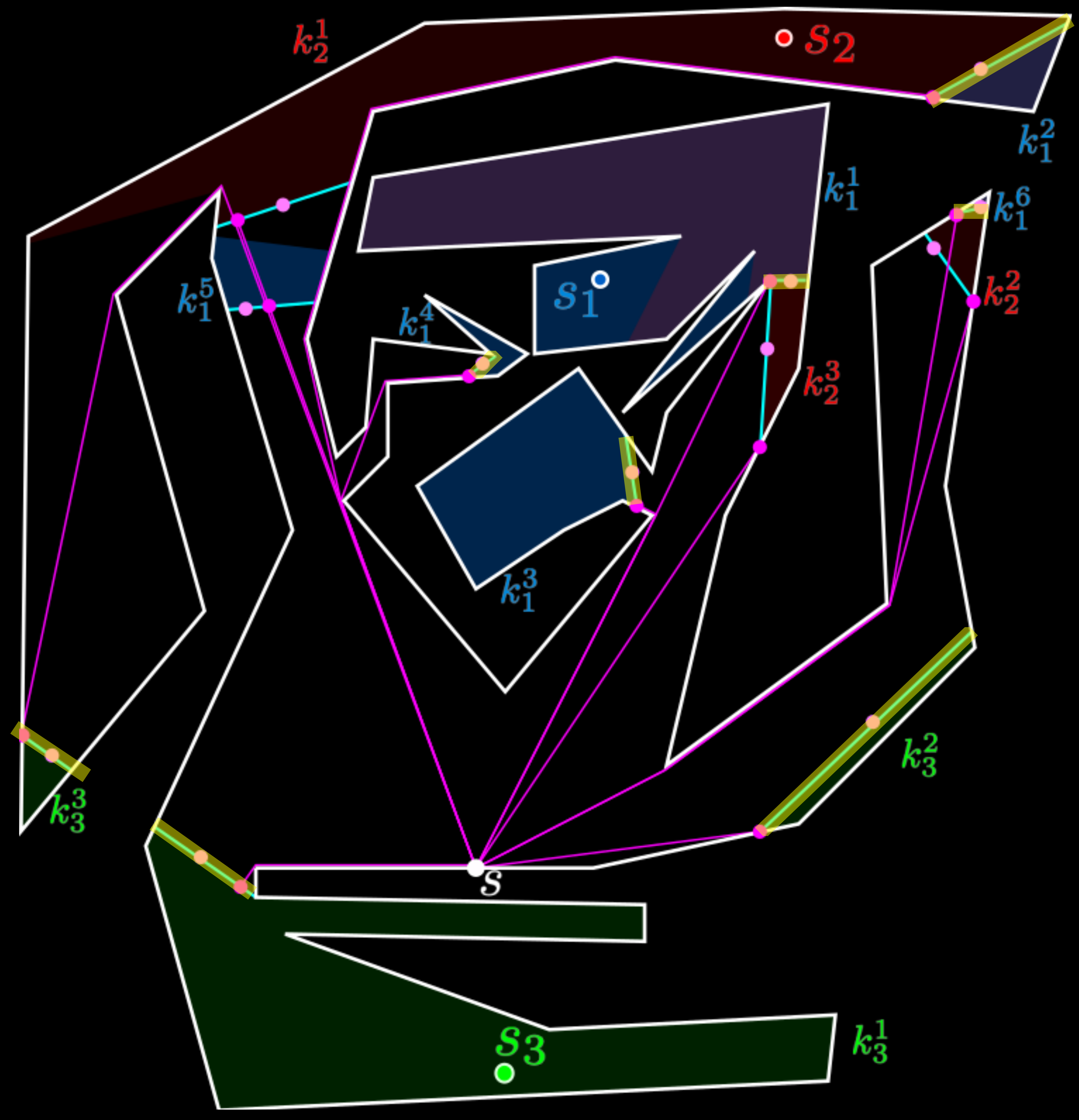

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- Claim 3: No geodesic can intersect CH_P(OPT, $\mathcal{P}_{C''}$) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most $3 \cdot \|OPT(S,P,s)\|$.

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- Claim 4: $CH_P(\mathcal{P}_{C''})$ is not longer than CH_P(OPT, $\mathcal{P}_{C''}$) and $CH_P(\mathcal{P}_{C''})$ visits one point per γ_i (except for γ_0).

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- To connect s (which may lie in the interior of $CH_P(\mathcal{P}_{C''})$, we need costs at most $\|OPT(S,P,s)\|$.

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- Build relative convex hull of all $o_{i,j}$ and all points in $\mathcal{P}_{C''}$ (relative w.r.t. polygon P): CH_P(OPT, $\mathcal{P}_{C''}$)
- Claim 3: No geodesic can intersect CH_P(OPT, $\mathcal{P}_{C''}$) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most 3·‖OPT(S,P,s)‖.
- Claim 4: $CH_P(\mathcal{P}_{C''})$ is not longer than CH_P(OPT, $\mathcal{P}_{C''}$) and $CH_P(\mathcal{P}_{C''})$ visits one point per γ_i (except for γ_0).
- To connect s (which may lie in the interior of $CH_P(\mathcal{P}_{C''})$, we need costs at most ‖OPT(S,P,s)‖.

$$\|R\| \leq \alpha_1 \cdot f(|V(G)|, |S|)\|\mathrm{OPT}_G(S,P,s)\| \leq \alpha_2 \cdot f(n|S|, |S|)\|\mathrm{CH}_P(\mathcal{P}_{C''})\| \leq \alpha_3 \cdot f(n|S|, |S|)\|\mathrm{CH}_P(\mathrm{OPT}, \mathcal{P}_{C''})\|$$
$$\leq \alpha_4 \cdot f(n|S|, |S|)\|\mathrm{OPT}(S,P,s)\|$$

with $f(N,M) = \log^2 N \log\log N \log M$

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➜ After last iteration, no two remaining geodesics visit the same cut in *C'*

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- If $\ell(g_{c'}) = \ell(g_c)$: Either $\ell(g_{c'[s,p_c]}) < \ell(g_{c'}) = \ell(g_c)$ or (if p_c on c') $p_{i,j} = p_c$ (claim holds)

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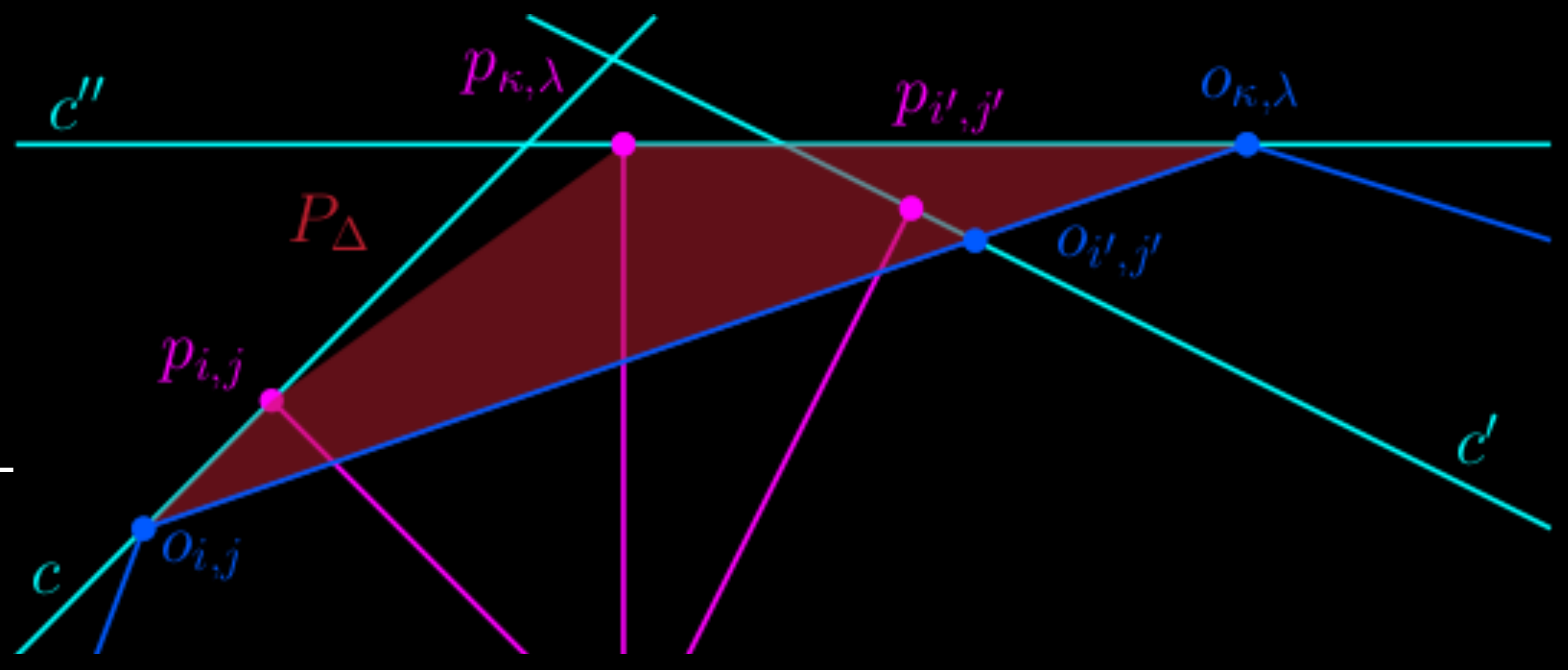

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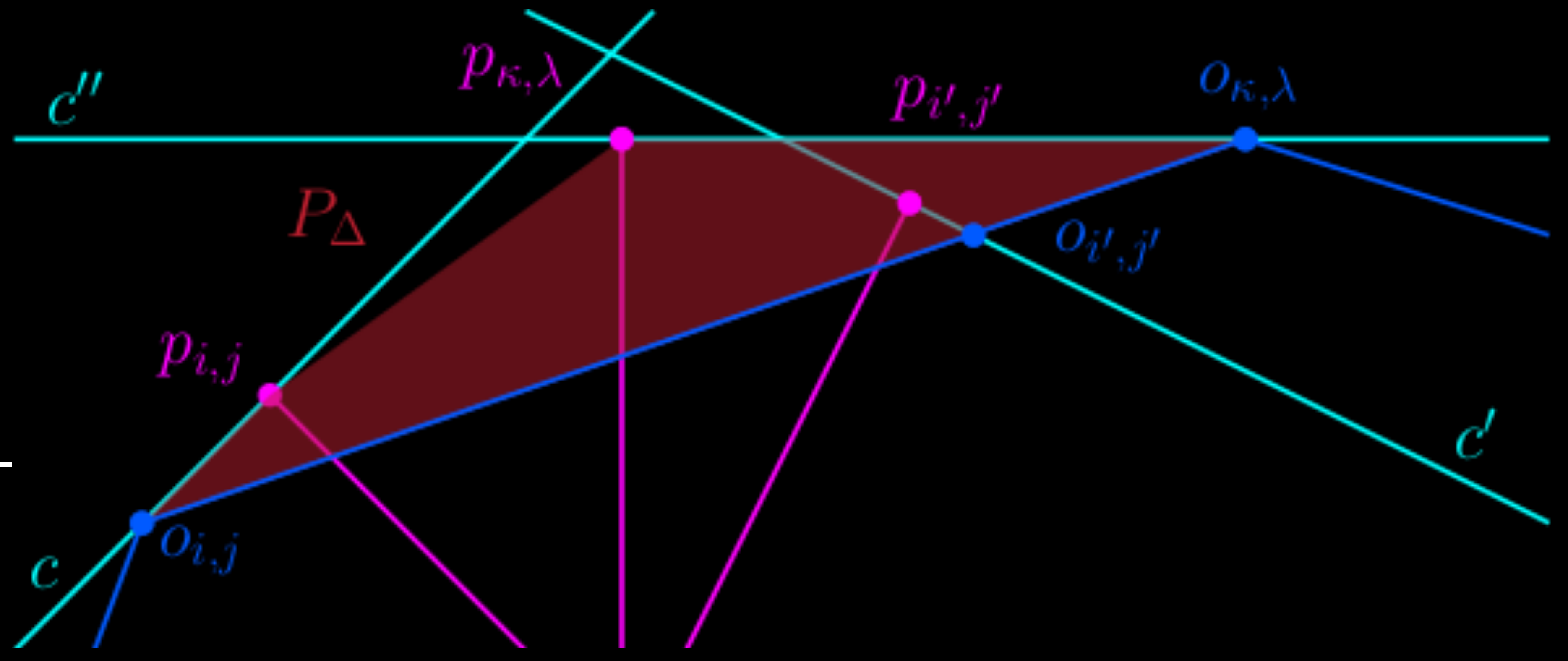

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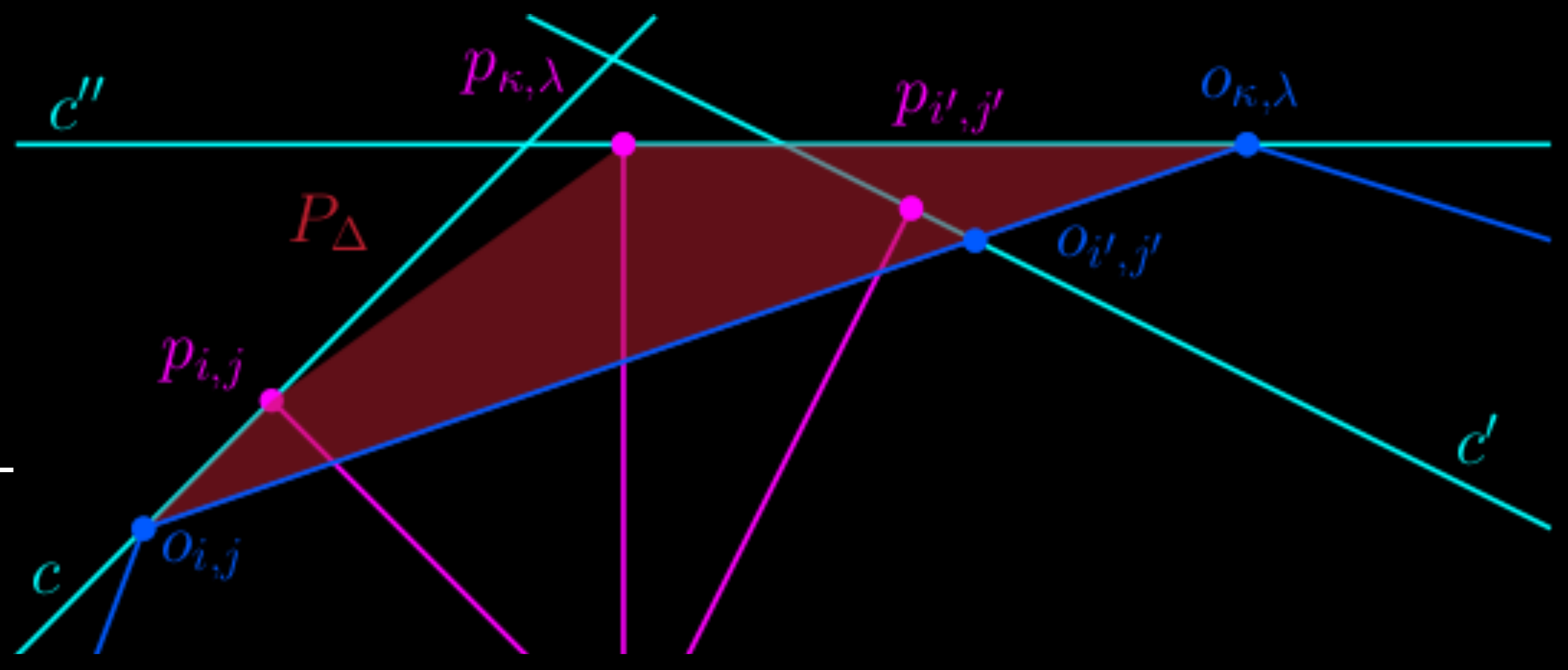

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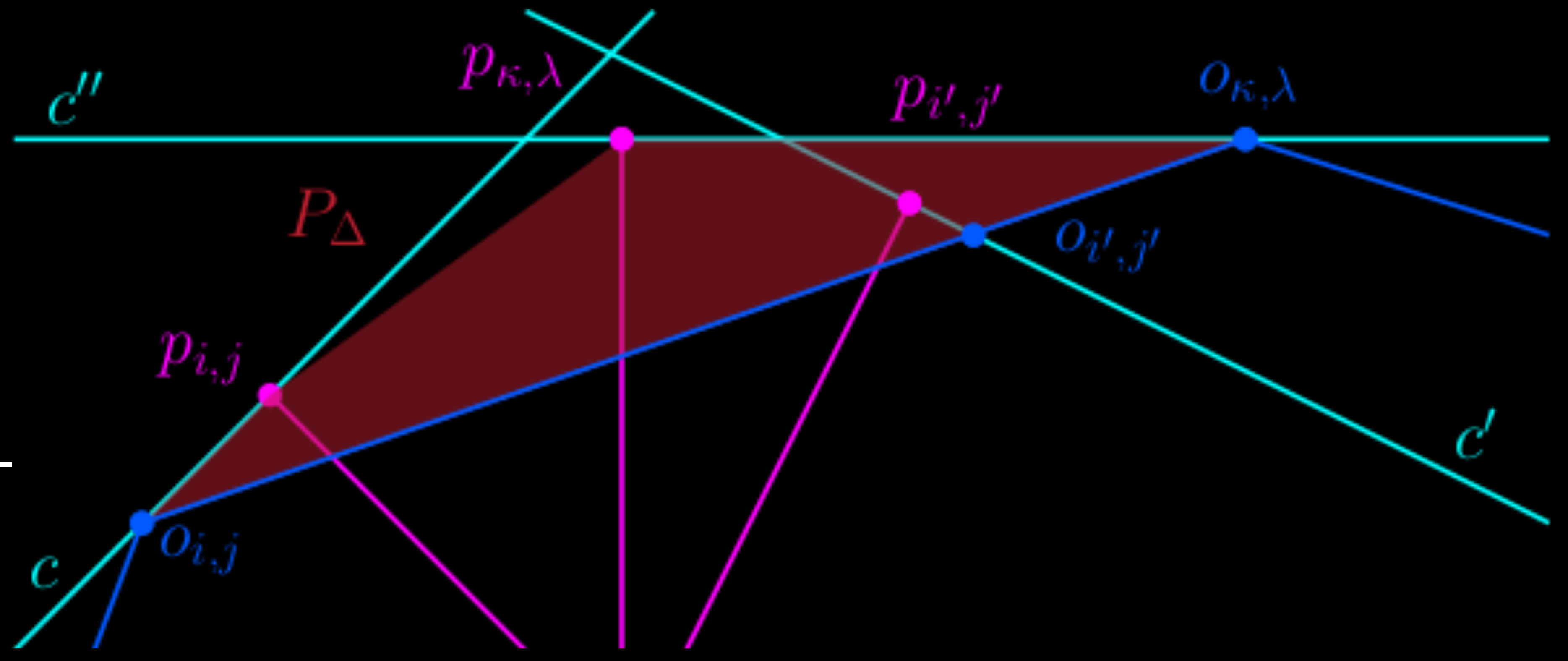

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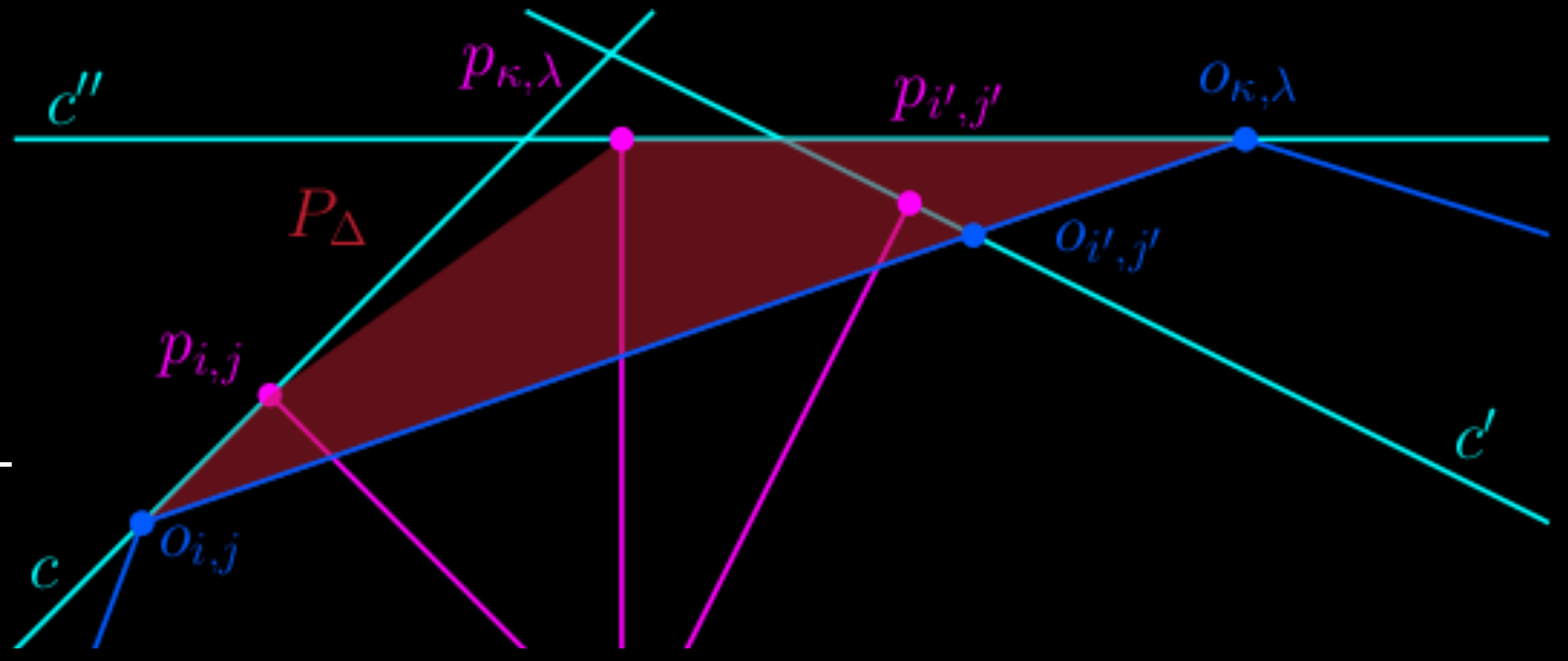

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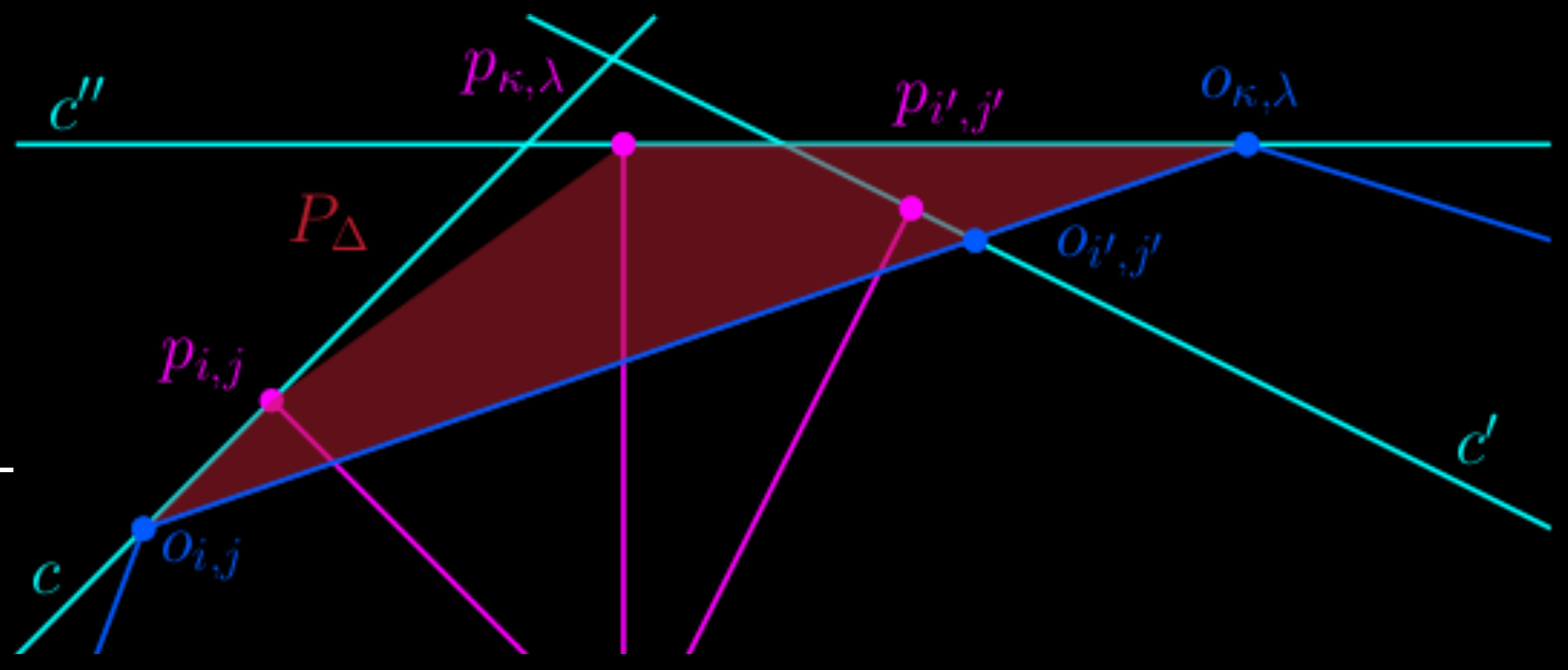

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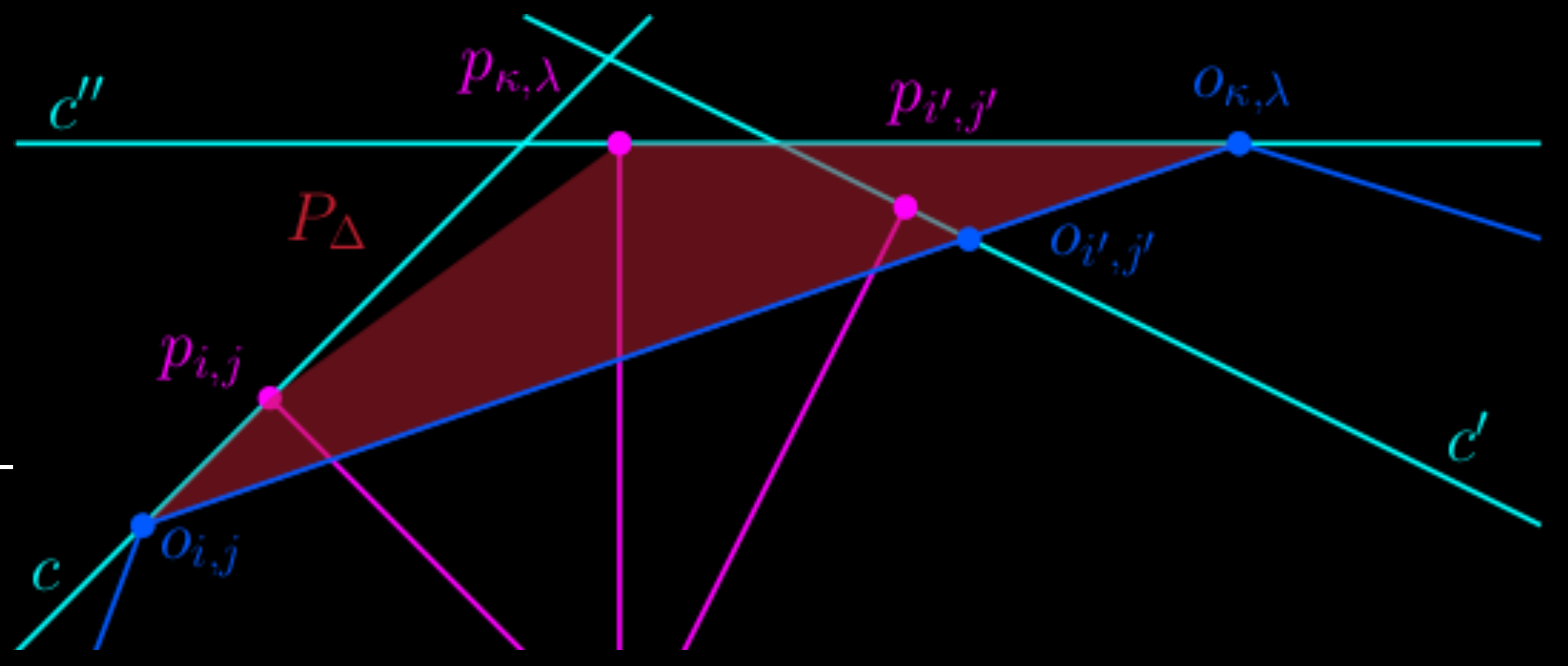

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Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot ||OPT(S,P,s)||$.

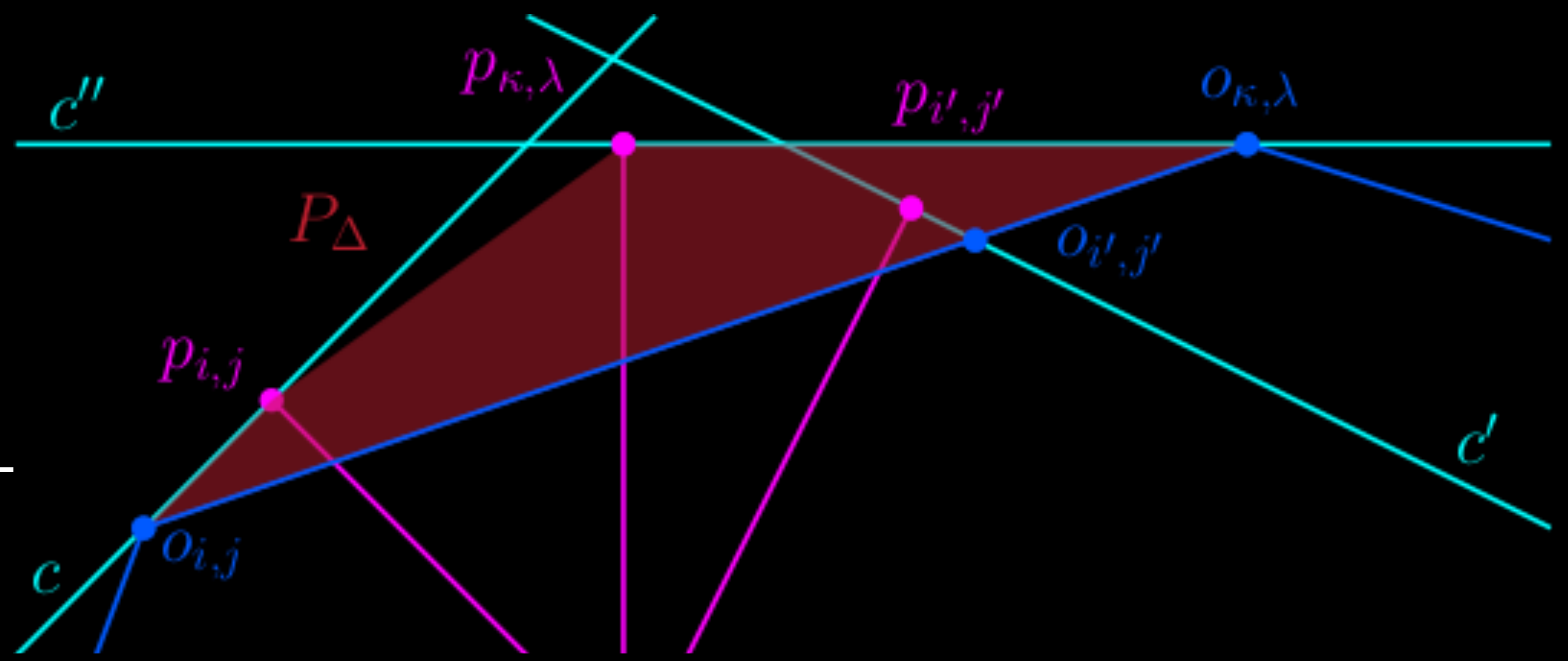
Lemma 2: Between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, P_{C''})$, we have at most two points in $\mathcal{P}_{C''}$.

Proof:

- Let $o_{i,j}$ and $o_{i',j'}$ be the two consecutive points from OPT on $CH_P(OPT, \mathcal{P}_{C''})$
- By Lemma 1, $p_{i,j}$ and $p_{i',j'}$ can lie between $o_{i,j}$ and $o_{i',j'}$
- BUT: we cannot have a point $p_{\kappa,\lambda}$ between $o_{i,j}$ and $p_{i,j}$ or between $o_{i',j'}$ and $p_{i',j'}$
- Assume there is a point $p_{\kappa,\lambda}$ between on $p_{i,j}$ and $p_{i',j'}$ on $CH_P(OPT, \mathcal{P}_{C''})$ (on cuts *c”, c, c’*, resp.)
- OPT visits $o_{\kappa,\lambda}$ on *c”*
- $o_{i,j}$ and $o_{i',j'}$ are consecutive pts on $CH_P(OPT, \mathcal{P}_{C''})$

➡ Order of OPT $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$ or $o_{\kappa,\lambda}$, $o_{i,j}$ $o_{i',j'}$, w.l.o.g. $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$

- Cut *c”* is a line segment
- Consider polgyon P_Δ with vertices $o_{i,j}$ $p_{i,j}$, $p_{\kappa,\lambda}$, $o_{\kappa,\lambda}$, $o_{i',j'}$, $o_{i,j}$



Cuts, points of the type $p_{i,j}$,
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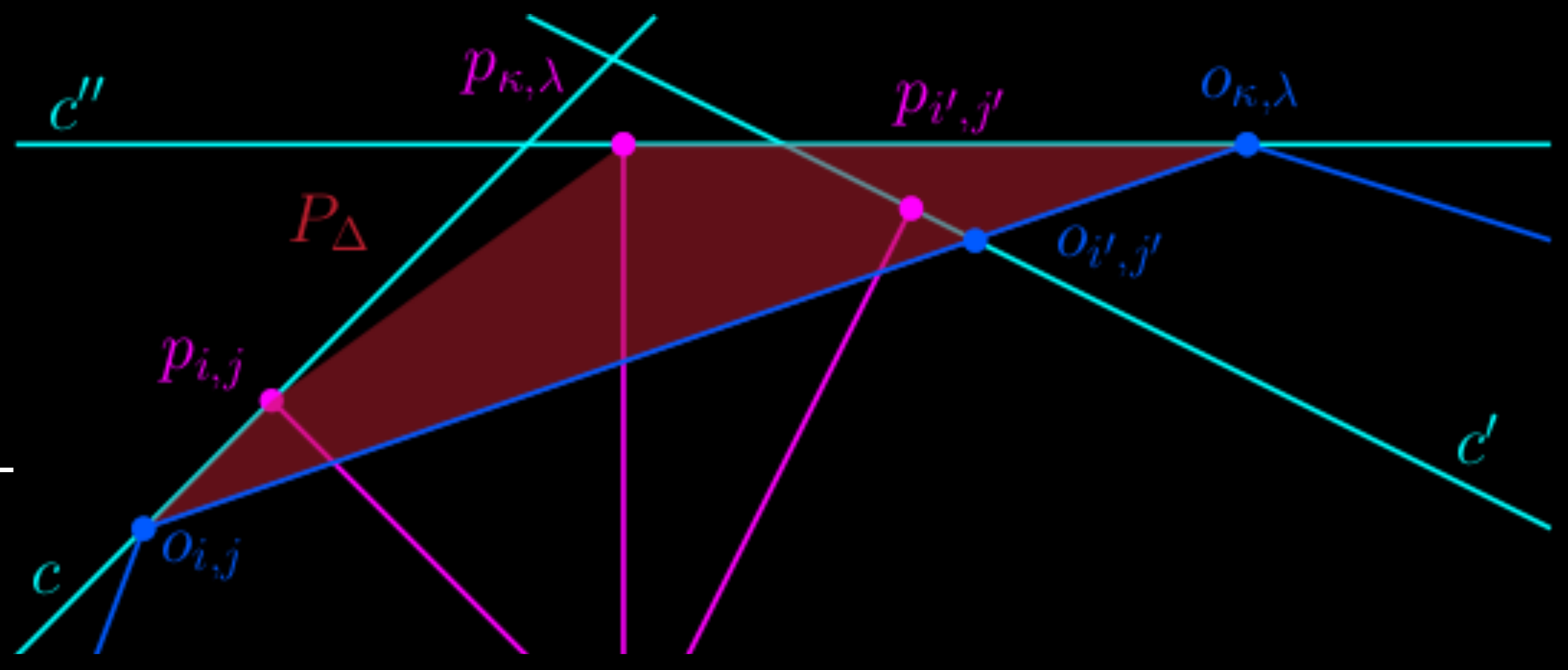
- OPT visits $o_{\kappa,\lambda}$ on c''
- $o_{i,j}$ and $o_{i',j'}$ are consecutive pts on $CH_P(OPT, \mathcal{P}_{C''})$

➡ Order of OPT $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$ or $o_{\kappa,\lambda}$, $o_{i,j}$ $o_{i',j'}$, w.l.o.g. $o_{i,j}$ $o_{i',j'}$, $o_{\kappa,\lambda}$

- Cut c'' is a line segment
- Consider polgyon P_Δ with vertices $o_{i,j}$ $p_{i,j}$, $p_{\kappa,\lambda}$, $o_{\kappa,\lambda}$, $o_{i',j'}$, $o_{i,j}$
- Point $p_{i',j'}$ must lie in P_Δ's interior + $o_{i',j'}$ cannot lie on $CH_P(OPT, \mathcal{P}_{C''})$ ⚡



Cuts, points of the type $p_{i,j}$,
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Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot \|OPT(S,P,s)\|$.

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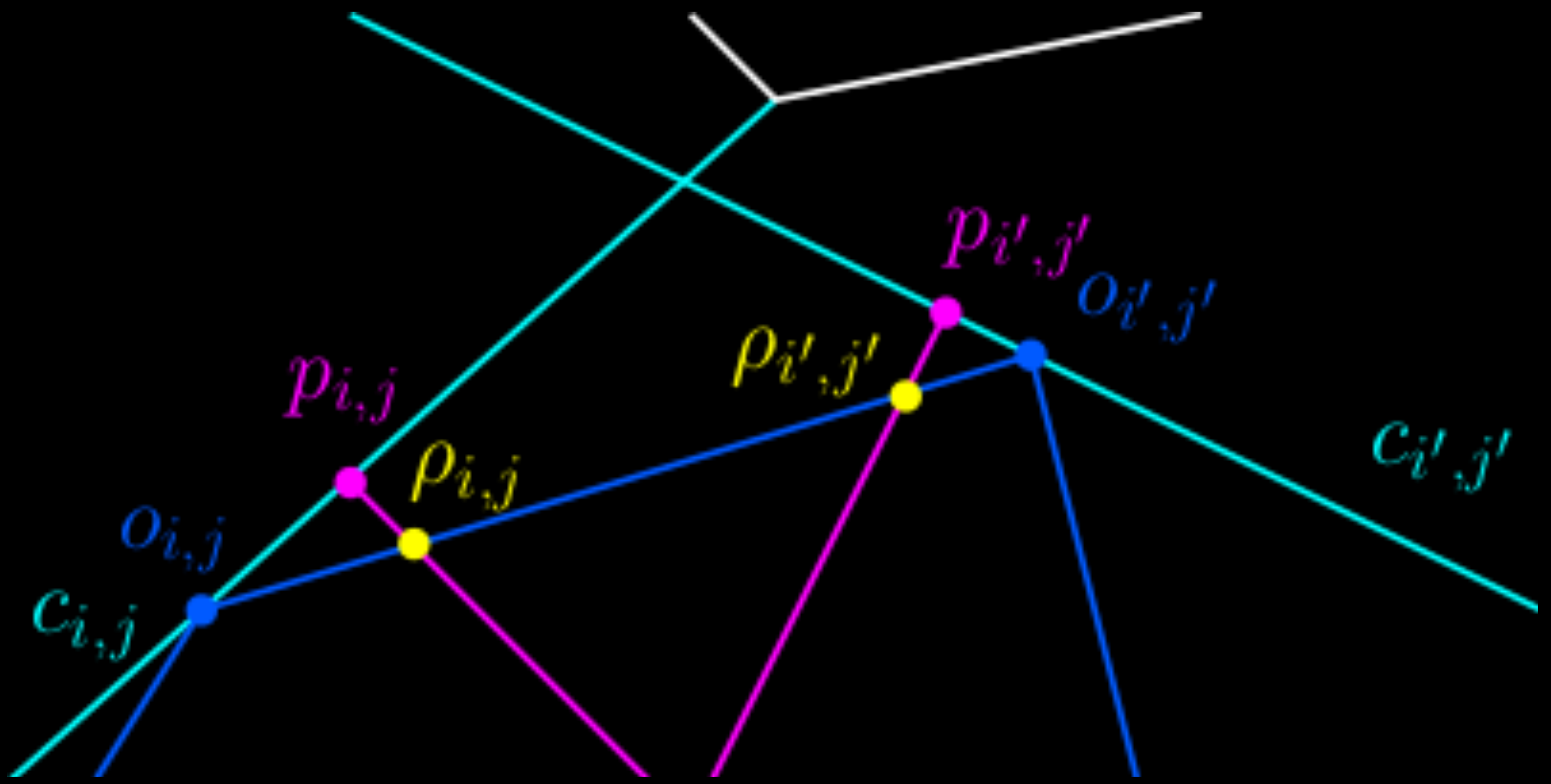
Proof:

Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot \|OPT(S,P,s)\|$.

Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$



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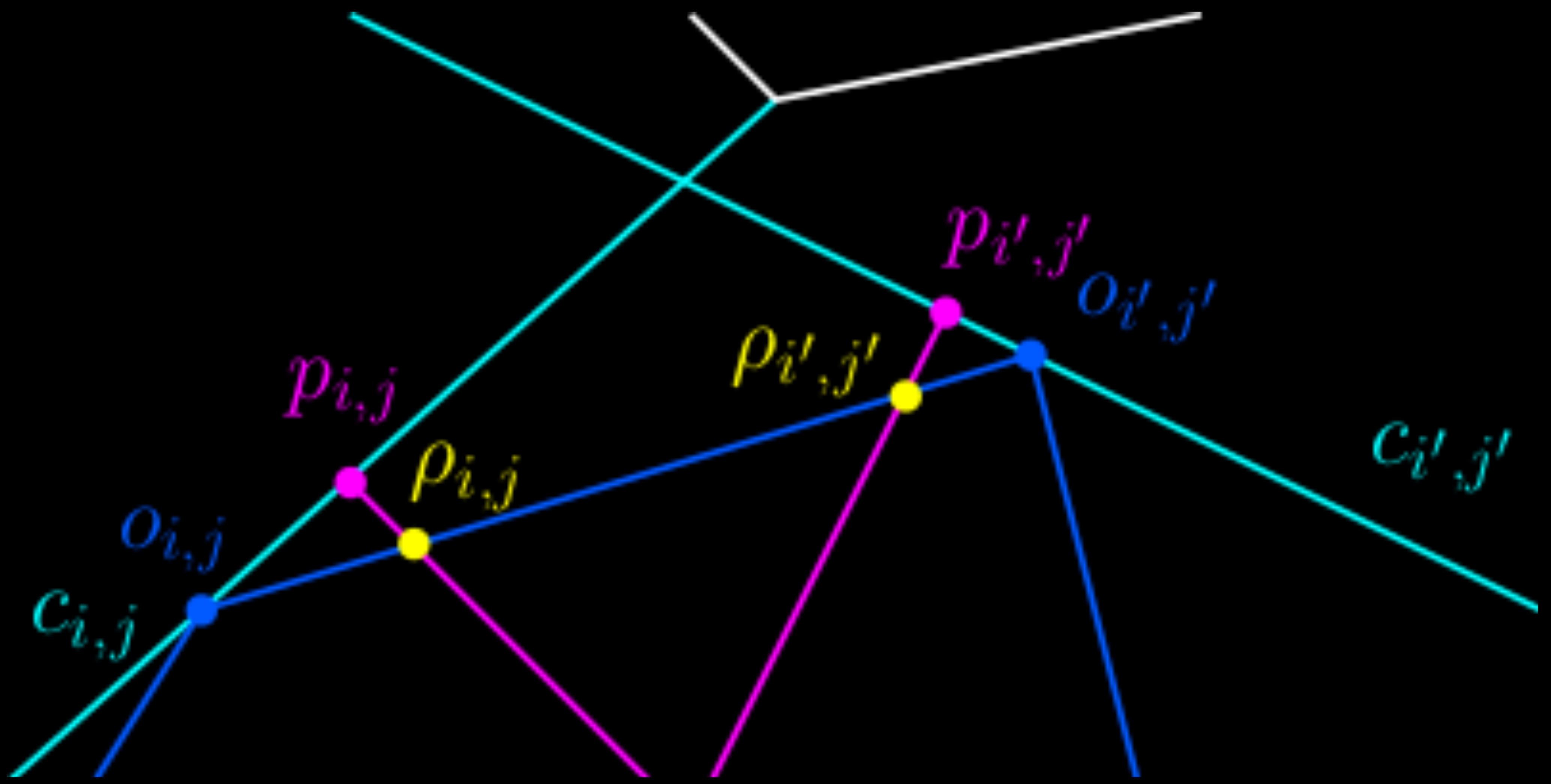

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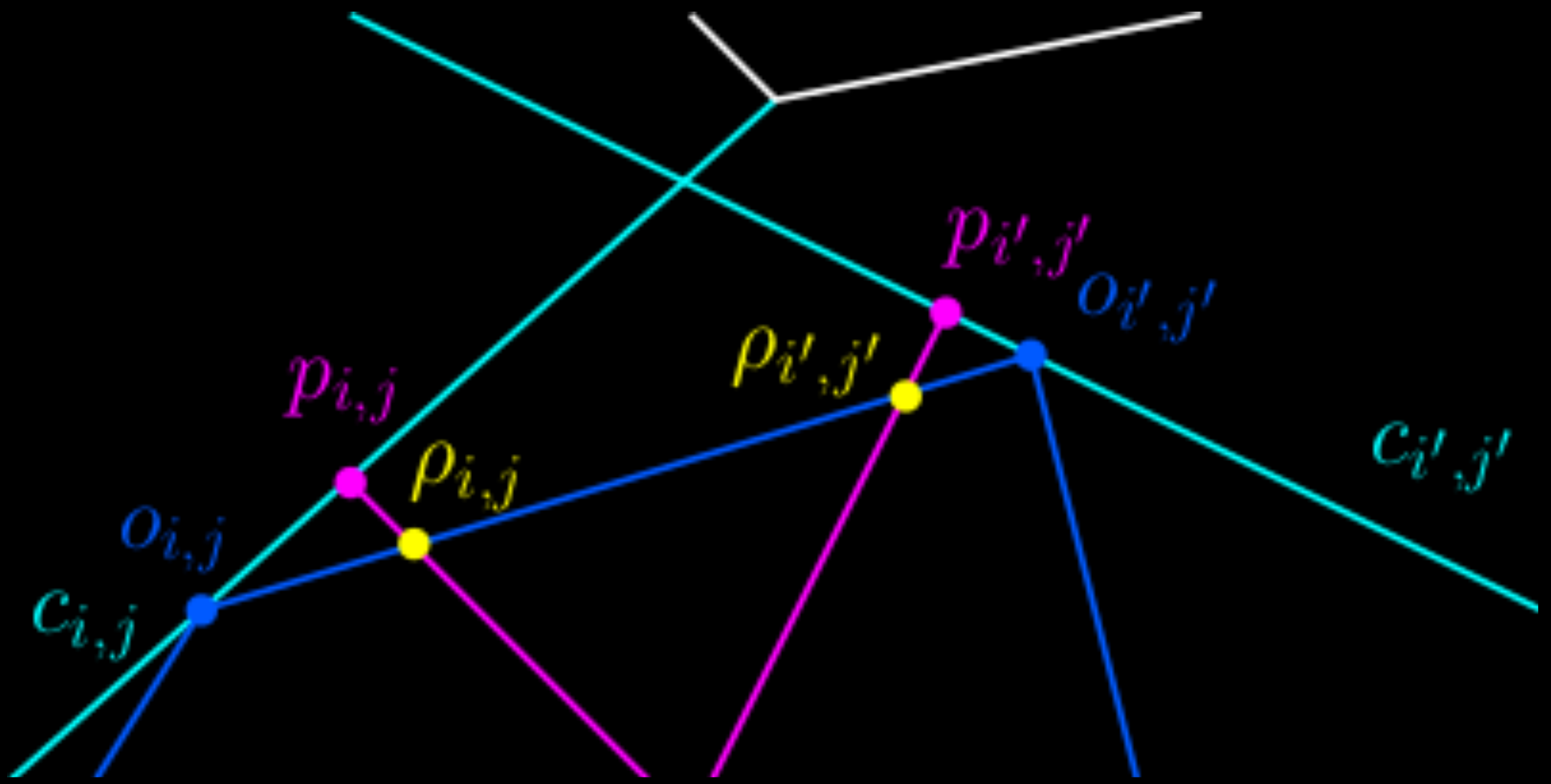
➡ $g_{i,j}$ intersects $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$



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Lemma 3: $||CH_P(OPT, \mathcal{P}_{C''})|| \leq 3 \cdot ||OPT(S,P,s)||$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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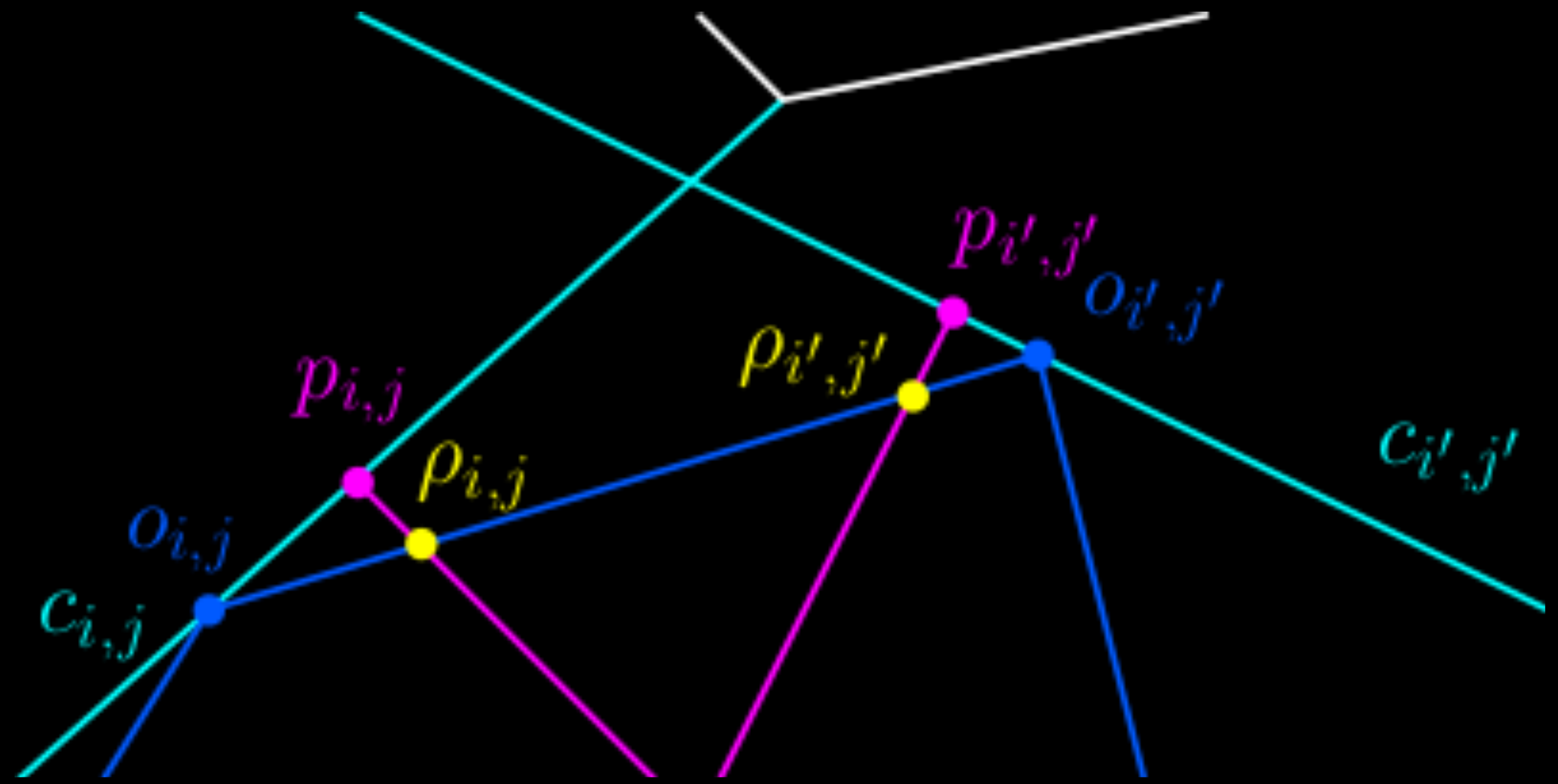
➡ $g_{i,j}$ intersects $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$

- $g_{i,j}$ is geodesic



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Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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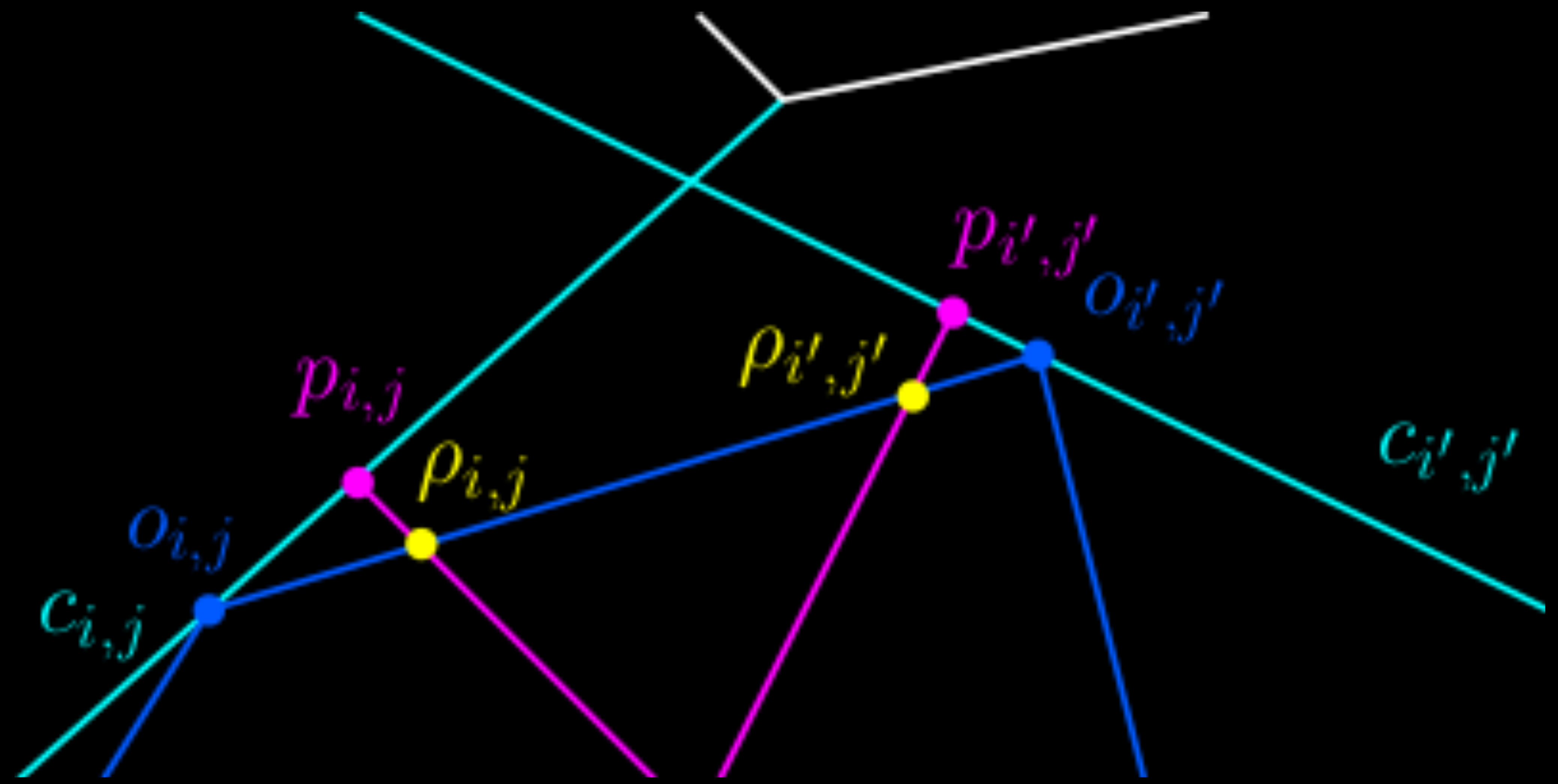
- $g_{i,j}$ is geodesic

➡ $\ell(\varrho_{i,j}, p_{i,j}) \leq \ell(\varrho_{i,j}, o_{i,j})$ (and $\ell(\varrho_{i',j'}, p_{i',j'}) \leq \ell(\varrho_{i',j'}, o_{i',j'})$)



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

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Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

Proof:

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➡ $g_{i,j}$ intersects $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$

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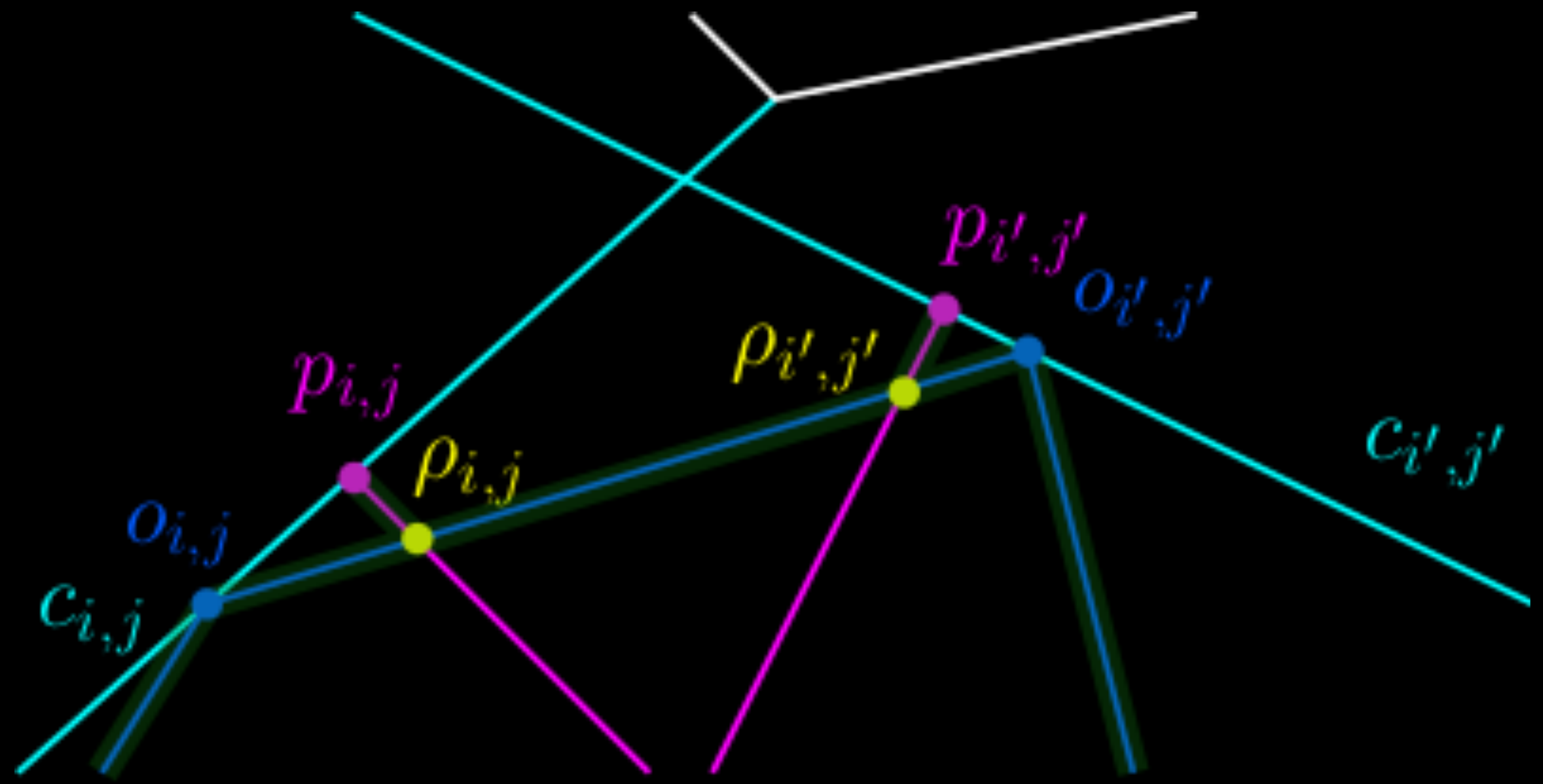
- Alter $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$: $o_{i,j}\ \varrho_{i,j}\ p_{i,j}\ \varrho_{i,j}\ \varrho_{i',j'}\ p_{i',j'}\ \varrho_{i',j'}\ o_{i',j'}$



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

Claim 3: No geodesic can intersect $CH_P(\text{OPT}, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(\text{OPT}, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(\text{OPT}, \mathcal{P}_{C''})$ has length at most $3 \cdot \|\text{OPT}(S,P,s)\|$.

Lemma 3: $\|CH_P(\text{OPT}, \mathcal{P}_{C''})\| \leq 3 \cdot \|\text{OPT}(S,P,s)\|$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of OPT(S,P,s) on $CH_P(\text{OPT}, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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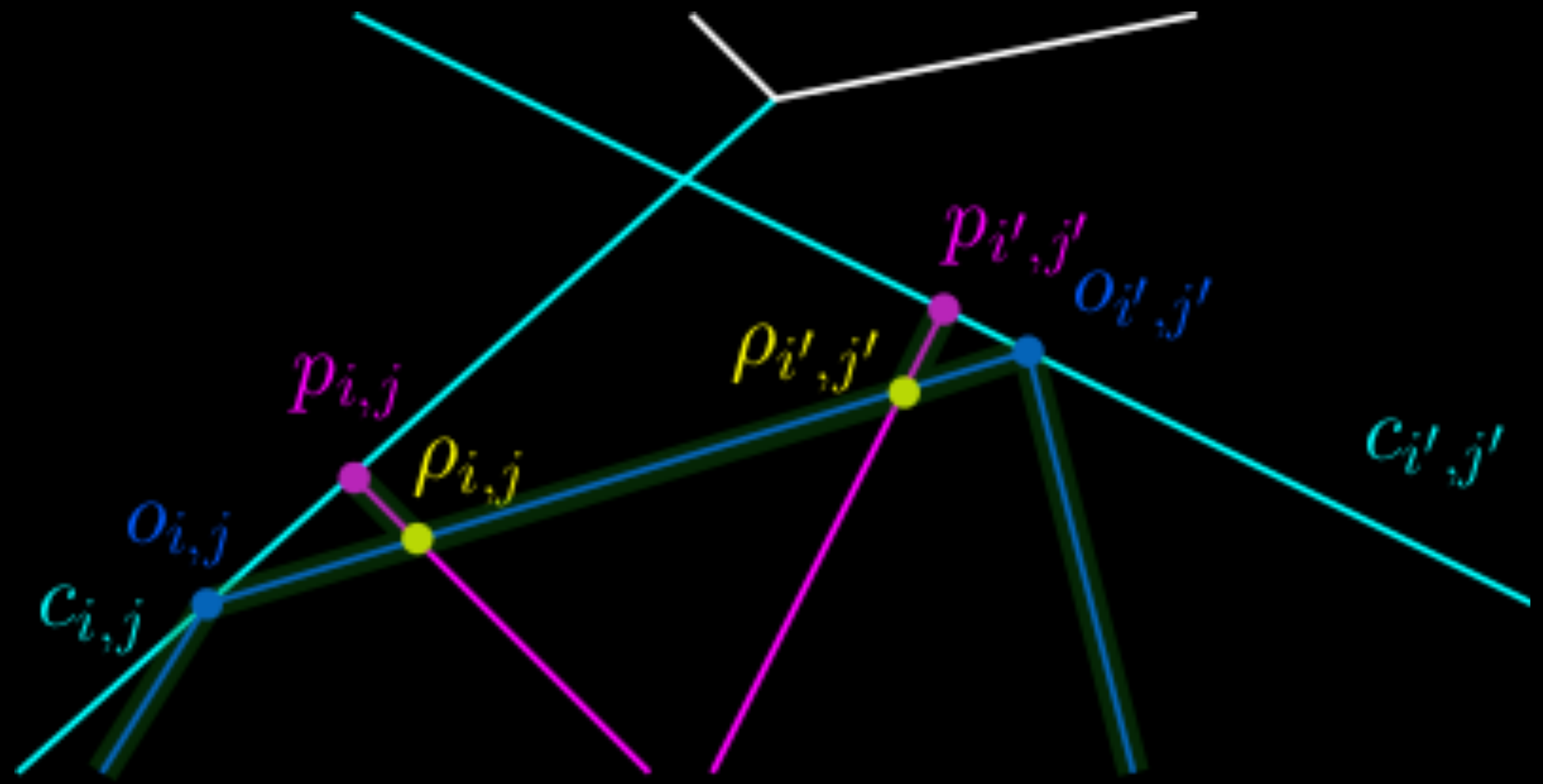
- Alter OPT(S,P,s) between $o_{i,j}$ and $o_{i',j'}$: $o_{i,j}\ \varrho_{i,j}\ p_{i,j}\ \varrho_{i,j}\ \varrho_{i',j'}\ p_{i',j'}\ \varrho_{i',j'}\ o_{i',j'}$

➡ New tour T: visits all points on $CH_P(\text{OPT}, \mathcal{P}_{C''})$



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Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of OPT(*S*,*P*,*s*) on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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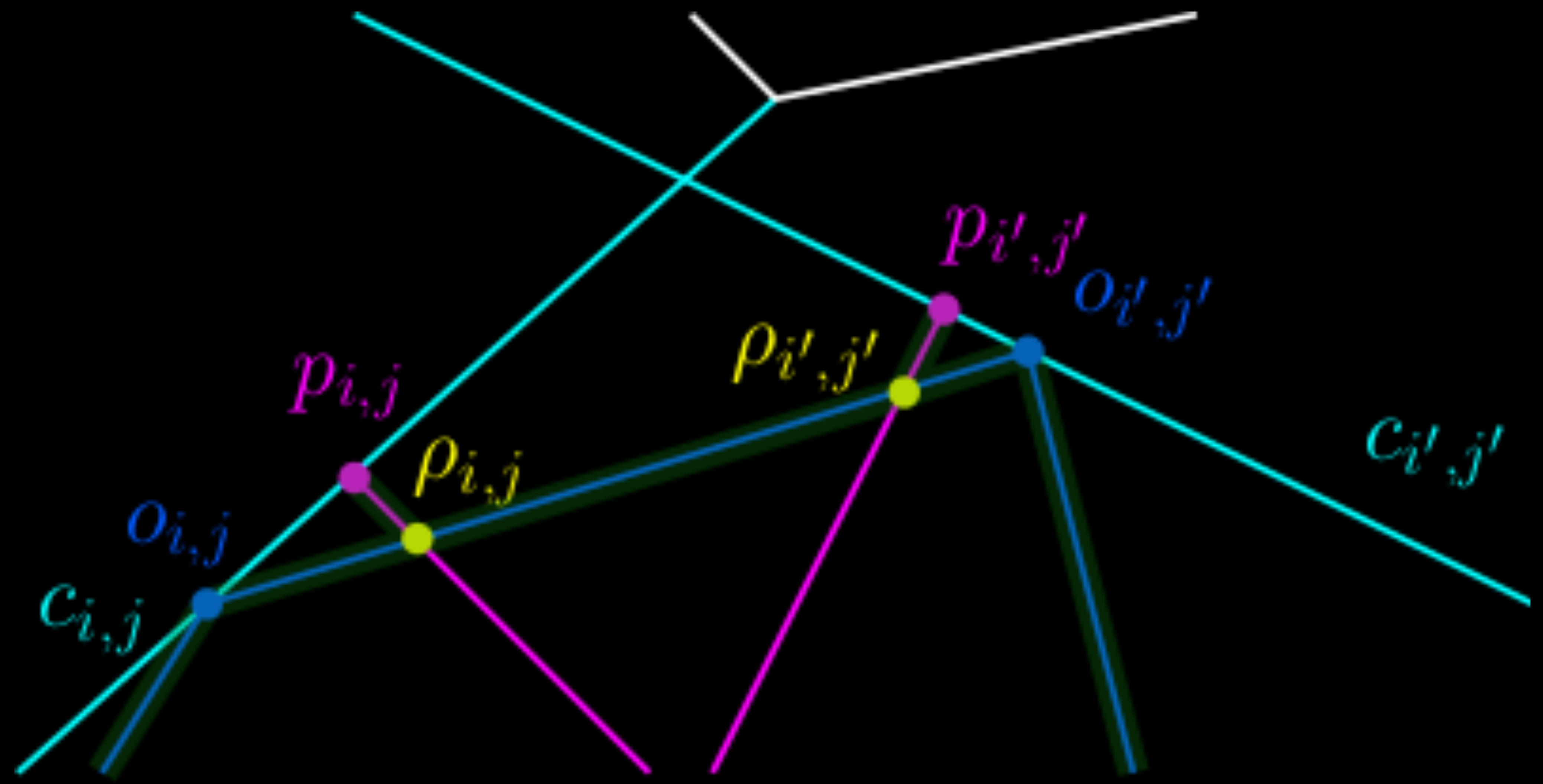
➡ New tour *T*: visits all points on $CH_P(OPT, \mathcal{P}_{C''})$

➡ $\|T\| \leq 3 \cdot \|OPT(S,P,s)\|$



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

Claim 3: No geodesic can intersect CH$_P$(OPT, $\mathcal{P}_{C''}$) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH$_P$(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH$_P$(OPT, $\mathcal{P}_{C''}$) has length at most 3·IIOPT(S,P,s)II.

Lemma 3: IICH$_P$(OPT, $\mathcal{P}_{C''}$)II ≤ 3·IIOPT(S,P,s)II.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of OPT(S,P,s) on CH$_P$(OPT, $\mathcal{P}_{C''}$), $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
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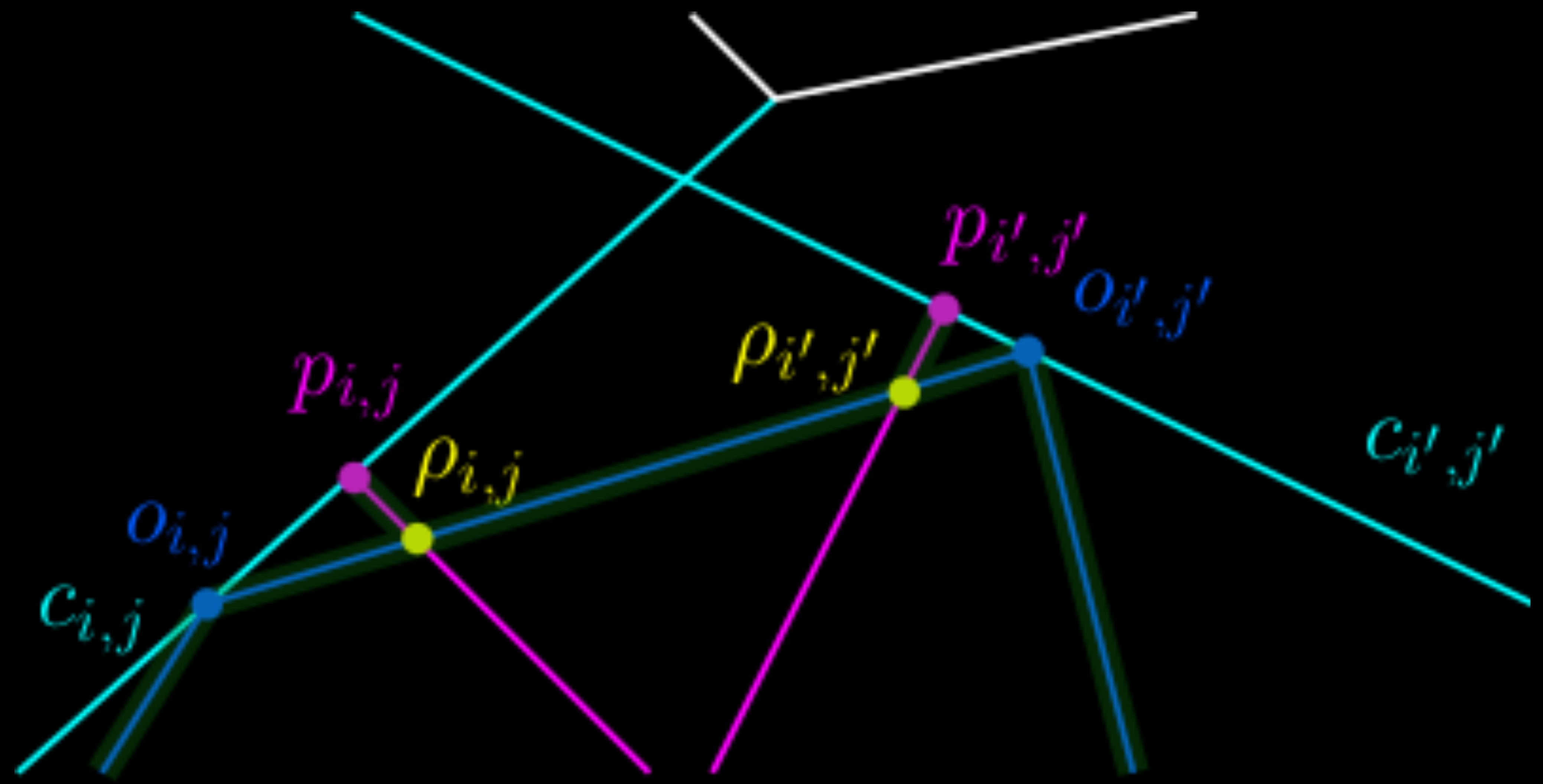
- Alter OPT(S,P,s) between $o_{i,j}$ and $o_{i',j'}$: $o_{i,j}\ \varrho_{i,j}\ p_{i,j}\ \varrho_{i,j}\ \varrho_{i',j'}\ p_{i',j'}\ \varrho_{i',j'}\ o_{i',j'}$

➡ New tour T: visits all points on CH$_P$(OPT, $\mathcal{P}_{C''}$)

➡ IITII ≤ 3·||OPT(S,P,s)||

- CH$_P$(OPT, $\mathcal{P}_{C''}$) is shortest tour to visit these points



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

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Claim 3: No geodesic can intersect $CH_P(OPT, \mathcal{P}_{C''})$ between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on $CH_P(OPT, \mathcal{P}_{C''})$, we have at most two points of $\mathcal{P}_{C''}$. $CH_P(OPT, \mathcal{P}_{C''})$ has length at most $3 \cdot ||OPT(S,P,s)||$.

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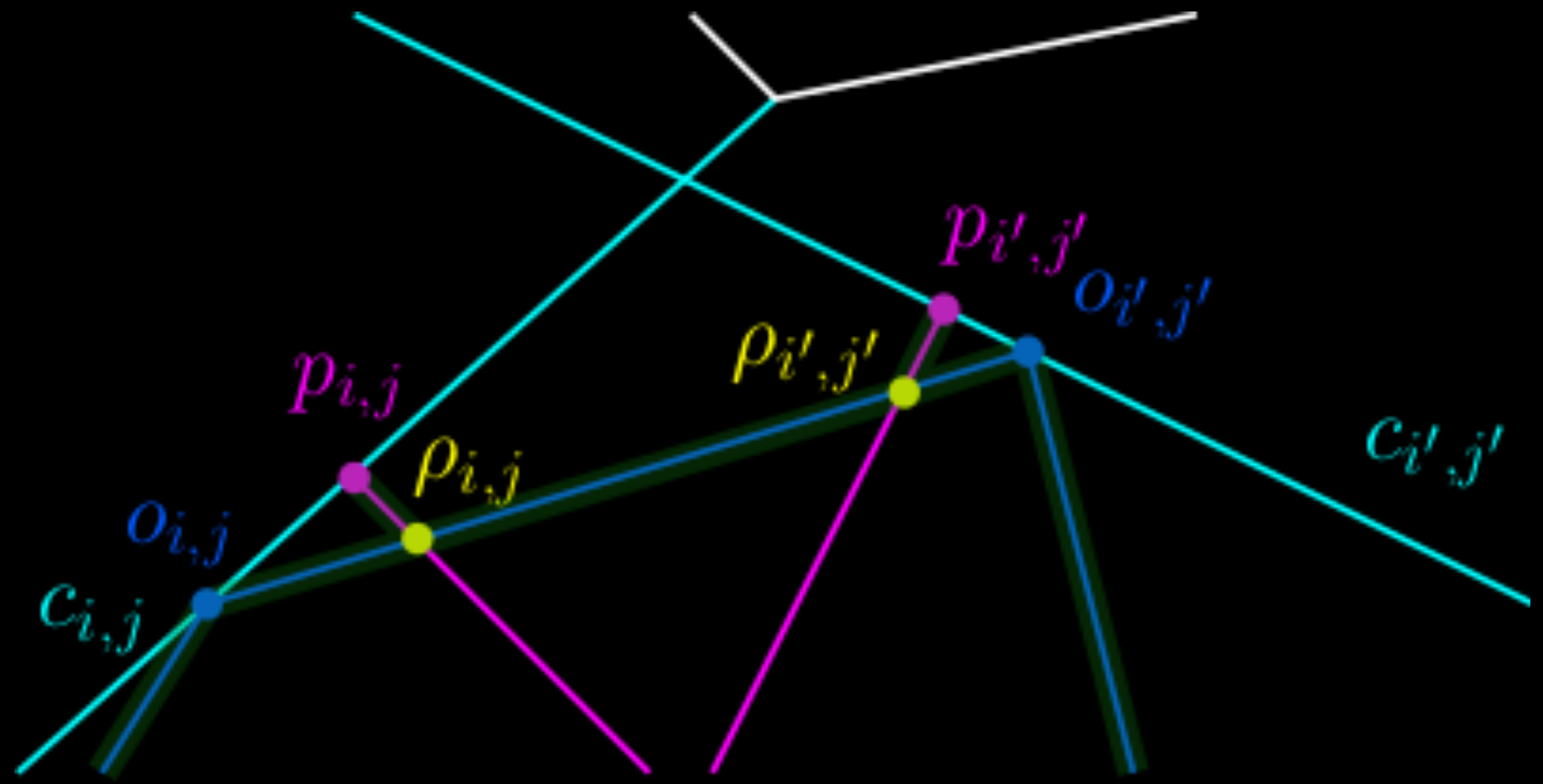
➡ New tour T: visits all points on $CH_P(OPT, \mathcal{P}_{C''})$

➡ $||T|| \leq 3 \cdot ||OPT(S,P,s)||$

- $CH_P(OPT, \mathcal{P}_{C''})$ is shortest tour to visit these points

➡ $||CH_P(OPT, \mathcal{P}_{C''})|| \leq ||T||$



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Lemma 3: $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$.

Proof:

- Lemmas 1,2 ➜ Between two consecutive points of $OPT(S,P,s)$ on $CH_P(OPT, \mathcal{P}_{C''})$, $o_{i,j}$ and $o_{i',j'}$, we hat at most two points where a geodesic visits a cut: $p_{i,j}$ and $p_{i',j'}$
- Points $o_{i,j}$ and $p_{i,j}$ both on $c_{i,j}$ / points $o_{i',j'}$ and $p_{i',j'}$ both on $c_{i',j'}$

➡ $g_{i,j}$ intersects $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$—in point: $\varrho_{i,j}$

- $g_{i,j}$ is geodesic

➡ $\ell(\varrho_{i,j}, p_{i,j}) \leq \ell(\varrho_{i,j}, o_{i,j})$ (and $\ell(\varrho_{i',j'}, p_{i',j'}) \leq \ell(\varrho_{i',j'}, o_{i',j'})$)

- Alter $OPT(S,P,s)$ between $o_{i,j}$ and $o_{i',j'}$: $o_{i,j}\ \varrho_{i,j}\ p_{i,j}\ \varrho_{i,j}\ \varrho_{i',j'}\ p_{i',j'}\ \varrho_{i',j'}\ o_{i',j'}$

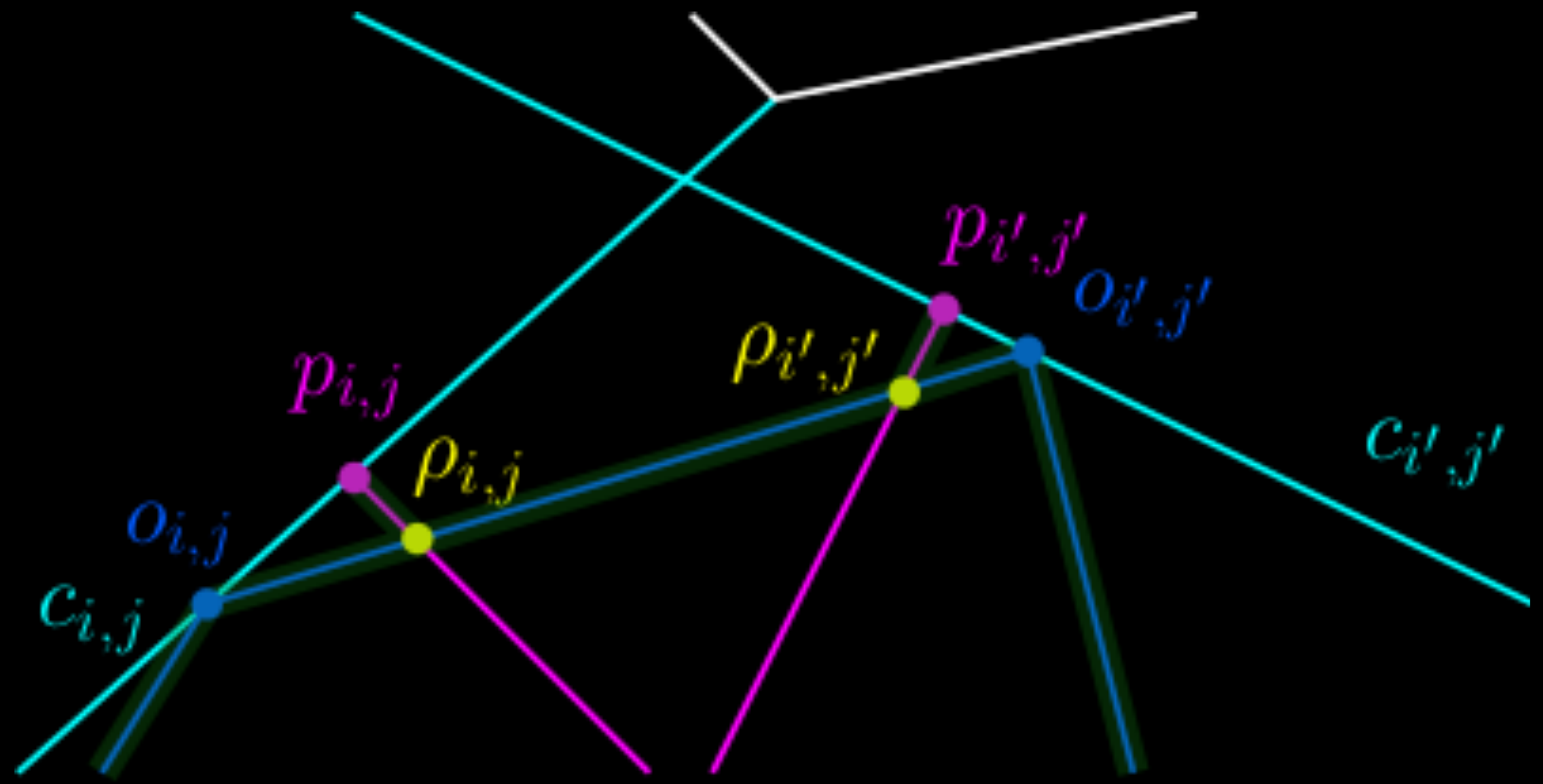
➡ New tour T: visits all points on $CH_P(OPT, \mathcal{P}_{C''})$

➡ $\|T\| \leq 3 \cdot \|OPT(S,P,s)\|$

- $CH_P(OPT, \mathcal{P}_{C''})$ is shortest tour to visit these points

➡ $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq \|T\|$

➡ $\|CH_P(OPT, \mathcal{P}_{C''})\| \leq 3 \cdot \|OPT(S,P,s)\|$



Cuts, points of the type $p_{i,j}$, polygon boundary, optimal route and points of the type $o_{i,j}$

Approximation Algorithm for k-TrWRP(S,P,s)

Theorem 2: Let P be a simple polygon with $n=|P|$. Let OPT(S,P,s) be the optimal solution for the k-TrWRP(S,P,s) and let R be the solution by our algorithm ALG(S,P,s). Then R yields an approximation ratio of $O(\log^2(|S|\ n) \log\log(|S|\ n) \log|S|)$.

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Outlook

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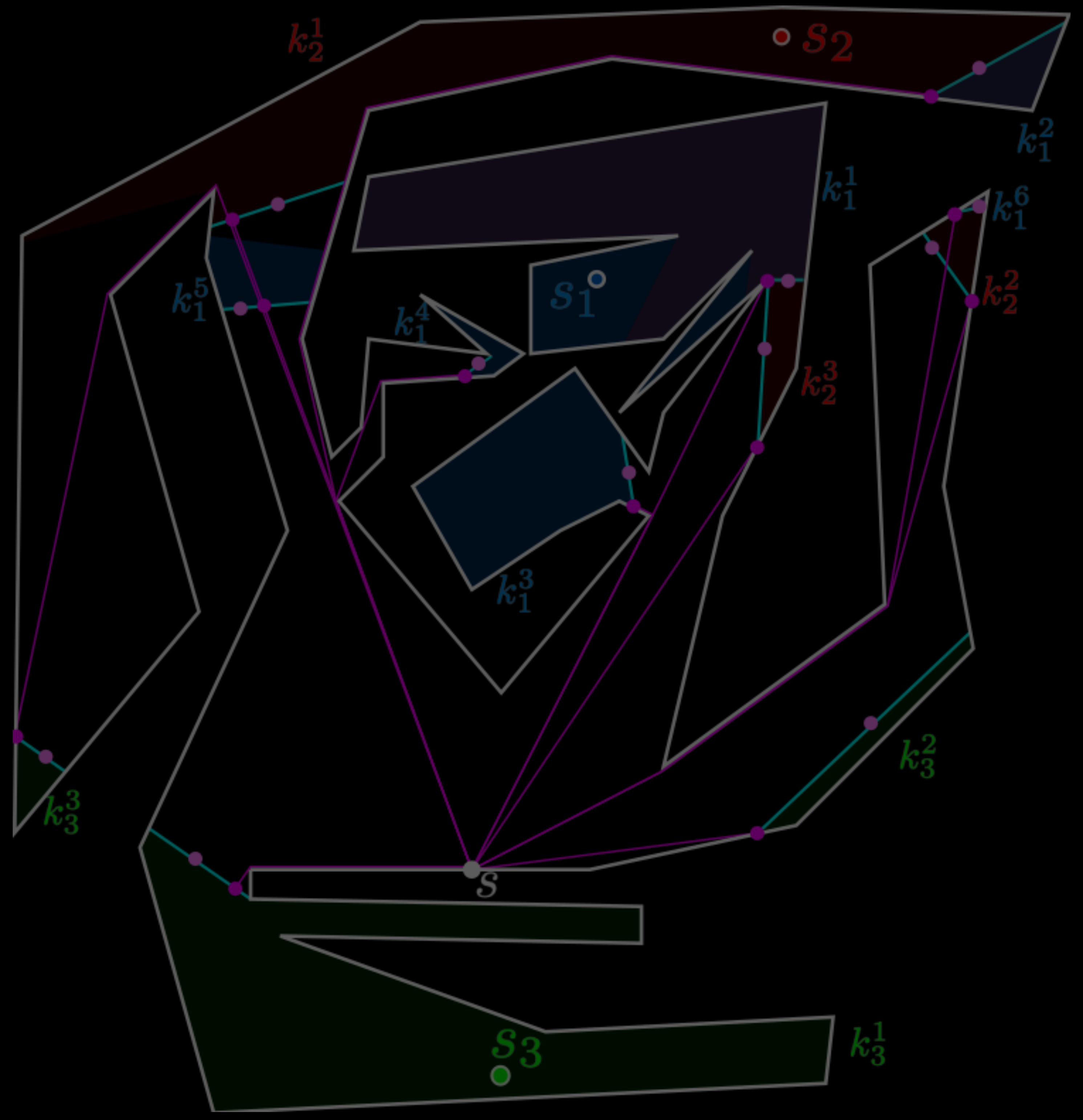
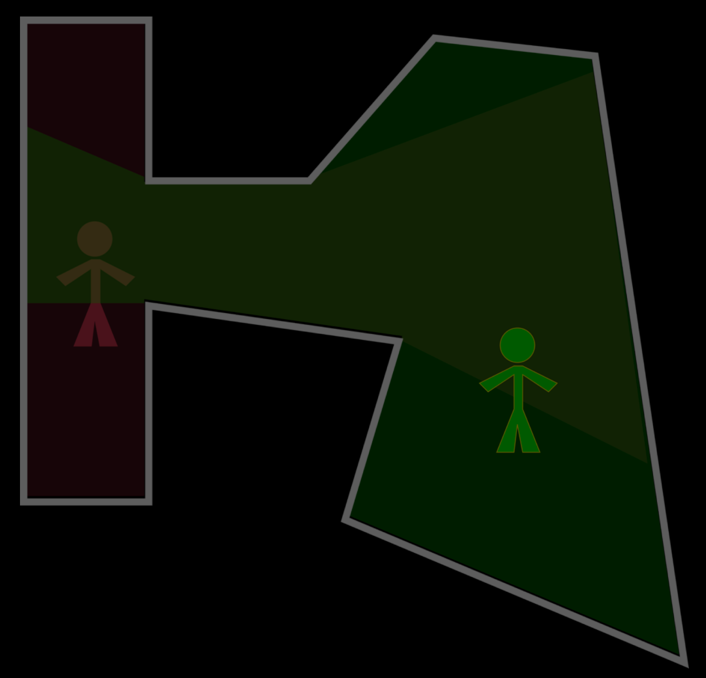
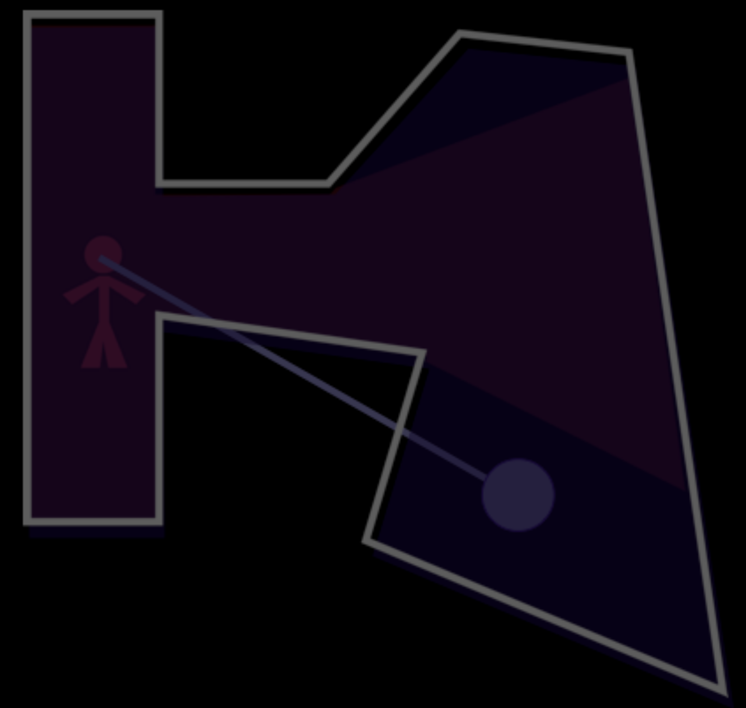
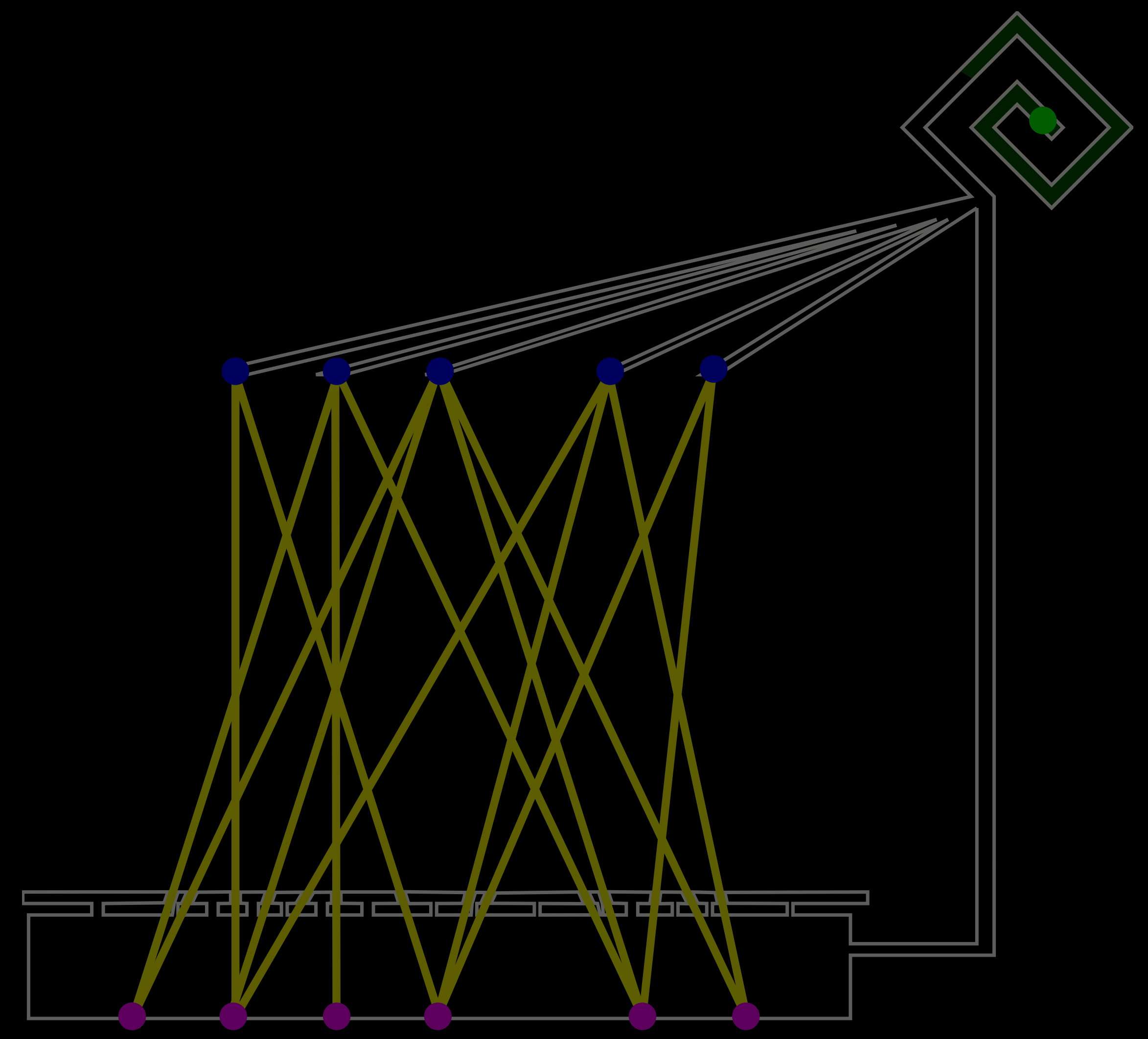
- Approximation for watchmen routes for k-transmitters without given starting point and/or when all of *P* should be monitored?
- Structural analogue for extensions for 0-transmitters?
- Improved combinatorial bounds for 2-/k-transmitter covers—in particular, better upper bounds for simple polygons than the one stemming from 0-transmitters





Thank you.





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