k-Transmitters/k-Modems

Christiane Schmidt XX Spanish Meeting on Computational Geometry 2023



Agenda

- The Art Gallery Problem
- k-Transmitters
- Art Gallery Theorems for *k*-Transmitters
- Computation of the k-Visibility Region
- Computational Complexity
- Sliding *k*-Transmitters
- The Watchman Route Problem (WRP)
- *k*-Transmitter Watchman Routes
- Outlook













Given: Polygon P







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→ Lower bound of 2

However, generally, the ratio between minim number of guards and maximum number of witnesses can be arbitrarily bad:

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Computational Complexity

- The AGP is NP-hard for point guards with holes [O'Rourke & Supowit 1983], vertex guards without holes [Lee & Lin 1986], point guards without holes [Aggarwal 1986]
- The AGP is $\exists \mathbb{R}$ -complete [Abrahamsen, Adamszek & Miltzow 2021]















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Other structural results

point

















We can alter:







We can alter:

• Capabilities of the guards





• Environment to be guarded



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Staircase visibility/ s-visiblity:



Two points are s-visible to each other if there exists a staircase path in P that connects them.





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Formally: a point p is **2(k)-visible** from a point q, if the line segment pq intersects P in at most two (k) connected components.



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k-Transmitters



k-/2-Transmitter





2VR(*p*)/*k*VR(*p*) can have O(n) connected components.




















Computational Geometry Column 52

Joseph O'Rourke*

Abstract

[Draft, January 27, 2012.] Two art-gallery-like problems of transmitters in





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Combinatorics and complexity of guarding polygons wi

Sarah Cannon^{a,1}, Thomas G. Fai^{b,2}, Justin Iwerks^c, Undine Leopold^d,

A Note on Approximating 2-Transmitters (about sliding k-Transmitters)

Saeed Mehrabi¹, Abbas Mehrabi²

An upper bound on the k-modem illumination problem.

Frank Duque * Carlos Hidalgo-Toscano *

Sliding k-Transmitters: Hardness and Approximation

Ziting Yu*

Sliding k-Transmitter arding Orthogonal Art Galleries with Sliding Fransmitters: Hardness and Approximation

erese Biedl¹ · Timothy M. Chan¹ · phanie Lee¹ · Saeed Mehrabi¹ · orizio Montecchiani² · mideh Vosoughpour¹ · Ziting Yu¹

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A time-space trade-off for computing the *k*-visibility region of a point in a polygon 🏾

Yeganeh Bahoo^a, Bahareh Banyassady^b, Prosenjit K. Bose^c, Stephane Durocher^a, Wolfgang Mulzer^b

Bengt J. Nilsson¹[™] and Christiane Schmidt²[™]

Art Gallery theorems **Computational complexity** (Approximation) algorithms k-visibility region

Watchman

Point and edge k-transmitters	Lower bound	Upper bound
Simple n-gons	[n/5]for k=2 [4]	<pre>[n/3]for k=2 * O(n/k) k-transmitters [5]</pre>
Monotone n-gons	[(n-2)/(2k+3)] [1]	[(n-2)/(2k+3)] [1]
Monotone orthogonal n-gons	[(n-2)/(2k+4)] for k=1, k even [1]	[(n-2)/(2k+4)] for k=1, k even [1]
	[(n-2)/(2k+6)] k≥3 odd [1]	[(n-2)/(2k+6)] k≥3 odd [1]
Ortogonal (2m)-gon		m even: Single (m-1)-transmitter; m odd: Single m-transmitte
Spiral n-gons		[n/4]for k=2 [3]
Arrangement of lines in the plane	Single [2n/3]-transmitter [2]	Single [2n/3]-transmitter [2]
	Two [n/2]-transmitters [2]	Two [n/2]-transmitters [2]
d-dim Euclidean space \w n convex obstacles		Single (dn+1)/(d+1)-transmitter [6]
Plane with obstacles		[(5n+6)/12] 1-tr for n disjoint line segments [3]
Simple n-gons	[n/6]for k=2 [4]	[3n/10]+1 for k=2 *
Monotone n-gons	[(n-2)/9]for k=2 [4]	[(n-2)/8]for k=2 [4]
Monotone orthogonal n-gons	[(n-2)/10]for k=2 [4]	[(n-2)/10]for k=2 [4]
Orthogonal n-gons	[(3n+4)/16] for k=2 [4]	[(n-2)/10]for k=2 *

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[6] Radoslav Foulen, Andreas F. Holmsen, János Pach. Intersecting Convex Sets by Rays

Tight bounds

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- $P \cap H_R(\ell_i)$ contains n-i+1 edges of P.
- If both $P \cap H_L(\ell_i)$ and $P \cap H_R(\ell_i)$ are illuminated, then *P* is illuminated

Open Problem: k-Transmitter Combinatorics

complexity of guarding polygons with edge and point 2-transmitters.

All other gadgets remain entirely below these lines

BUT: the best upper bound is (essentially) given by the bound for the AGP with "normal" visibility: $\lfloor n/3 \rfloor$ 2-transmitters

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- → OPEN PROBLEM #1: Close the gap between lower and upper bound for 2-transmitters (intuition: should be closer to [n/5]

Joe O'Rourke, Computational Geometry Column 52, 2012: The only upper bound I know is obtained by ignoring the extra power of the Art Gallery Theorems for k-Transmitters, when [n/3] suffice—the original art gallery theorem! - Lower bound of $\lfloor \frac{n}{5} \rfloor$ 2-transmitters to cover a simple *n*-gon

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cover a simple *n*-gon











• Simple polygon *P*, query point $q \in P$



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 - $\theta \in [0,2\pi)$: r_{θ} ray from *q* eminating in CCW angle θ with *x*-axis



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 - $\theta \in [0,2\pi)$: r_{θ} ray from *q* eminating in CCW angle θ with *x*-axis
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 - *Critical vertex*: incident edges on same side of $r_{\theta} c$ in total in *P*
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 - Critical vertex: two edges added/removed
 - •Otherwise: replace one edge by another



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• Simple polygon *P*, query point $q \in P$

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 - Based again on radial decomposition



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Computational Complexity Minimum Point 2-transmitter [*k*-transmitter] Cover (MP2TC) [MP*k*TC] Problem:





Minimum Point 2-transmitter [*k*-transmitter] Cover (MP2TC) [MP*k*TC] Problem: Given: Polygon P.

Task: Find the minimum cardinality point 2-transmitter [*k*-transmitter] cover of P.





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• MPkTC is NP-hard for simple polygons—reduction from Minimum Line Cover (MLCP)





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guarding polygons with edge and point 2-transmitters.

Usual spike box

- Sarah Cannon, Thomas G. Fai, Justin Iwerks, Undine Leopold, and Christiane Schmidt. Combinatorics and complexity of
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Usual spike box → Modify slightly by adding "crowns"



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- ME2TC is NP-hard for simple polygons—adapted version of the [Lee & Lin, 86] reduction from 3SAT for minimum edge guard cover



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Sliding k-Transmitters



Sliding k-Transmitters in Rectilinear Polygon

[Biedl, Chan, Lee, Mehrabi, Montecchiani, Vosoughpour, Yu, 2019] [Mahdavi, Seddighin, Ghodsi, 2020]



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• Axis-parallel line segment *s* in polygon *P*



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- Axis-parallel line segment *s* in polygon *P*
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- Axis-parallel line segment *s* in polygon *P*
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- k-transmitter can see a point in P if the perpendicular from p onto s intersects P's boundary at most k times



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Sliding 2-transmitter





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- Constant-factor approximation for any fixed non-negative k
- ST₀: NP-hard in rectilinear polygons with holes even if only horizontal o-transmitters
- ST_k, k>0: NP-hard even for simple, monotone polygons



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- [n/4] horizontal sliding k-transmitters sometimes necessary and always sufficient in rectilinear polygons
- Simple, rectilinear polygons: $\lfloor (n+1)/5 \rfloor$ sliding o-transmitters such that no two of them intersect each other are always sufficient



Orthogonal Art Galleries with Sliding k-Transmitters: Hardness and Approximation

Sliding 2-transmitter



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- NP-hard for *k*=2

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Orthogonal Art Galleries with Sliding k-Transmitters: Hardness and Approximation

Sliding 2-transmitter



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- Goal: minimize total length of sliding *k*-transmitters to guard *P*
- NP-hard for *k*=2
- 2-approximation



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Sliding 2-transmitter



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- Now: one guard (*watchman*) that can move





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What is the shortest tour for a watchman along which all points of *P* become visible?



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A cut c partitions polygon into two subpolygons:



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A cut c partitions polygon into two subpolygons: $P_{s}(c)$ —subpolygon that contains starting point s A cut c_1 dominates c_2 if $P_s(c_2) \subseteq P_s(c_1)$



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•Watchman route can be computed in polynomial time in a simple polygon with or without a given starting point on the boundary [Chin&Ntafos 1986] [Tan, Hirata, Inagaki 1999] [Dror, Efrat, Lubiw, Mitchell 2003] [Carlsson, Jonsson, Nilsson 1993] [Tan 2001]





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- Central concept: extensions
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- As for the AGP, we can alter the capabilities of the watchman or the area to be guarded





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k-Transmitter Watchman Routes [Bengt J. Nilsson, S., 2023]





25

• Mobile k-transmitter



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- Extensions do not translate to k-transmitters for k≥2 (no longer local!)





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Corollary: The same holds for *k*-TrWRP(*S*,*P*,*s*).



long

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- When we map a point (x, y) to (x, y+cx) for a large enough constant c, we obtain a x-y-monotone polygon for which the visibility properties are maintained
- We can even transform our histogram into a star-shaped polygon:
























Small detour for a recent result









Small detour for a recent result

Here, we need a starting point •





k-Transmitter Watchman Routes

for the k-TrWRP(S,P,s), this holds even for star-shaped polygons.



Theorem 2: For a discrete set of points S and a polygon P, the k-TrWRP(S,P) does not admit a polynomial-time approximation algorithm with approximation ratio c ln ISI unless P=NP, even for k=4 and for P being a histogram, or an x-y-monotone polygon;







Theorem 3: Let *P* be a simple polygon with n=|P|. Let OPT(*S*,*P*,*s*) be the optimal solution for the *k*-TrWRP(*S*,*P*,*s*) and let R be the solution by our algorithm ALG(*S*,*P*,*s*). Then R yields an approximation ratio of O(log² (|*S*| *n*) log log (|*S*| *n*) log |*S*|).









- Create a candidate point for each connected component of the k-visibility region of each point in S.



































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Starting point s ³⁴









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Cuts



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Starting point s ³⁴





























- 52 S 10 k_1^3 k_3^2 $/k_{3}^{3}$ **g**3,2 **Ĉ**3,2 **p**_{3,2} s k_3^1 *s*₃ LINKÖPING UNIVERSITY

- Create a candidate point for each connected component of the k-visibility region of each point in S. - Candidate points: intersection of geodesics from starting point *s* to cuts (*C*^{all} set of all cuts) - Build complete graph G on candidate points pi,j:



 $\langle \hat{c}_{1,3} \rangle$ $\hat{c}_{1,6}$

 2,2

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 - Example: When we visit k_{3^3} (in point p_{3^3}), we also visit the cuts of k_{3^3} , k_{2^1} and k_{1^5} . Thus, we have edges from p_3^3 to \hat{c}_3^3 , \hat{c}_2^1 , and \hat{c}_1^5 .



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 γ_1 candidate points that belong to $s_{1,1}$ γ_2 candidate points that belong to $s_{2,1}$ γ_3 candidate points that belong to $s_{3,1}$





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- Approximate a group Steiner tree:





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 - Graph, with *m* vertices and *Q* vertex subsets ("groups")



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Naveen Garg, Goran Konjevod, and R. Ravi. A polylogarithmic approximation algorithm for the group Steiner tree problem

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 $p_{2},$

 \hat{c}_2

 $p_{3,3}$

 $\hat{c}_{3,3}$

 $p_{3,1}$

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- We have m = O(n |S|), Q = |S| + 1



 $p_{3,2}$

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- Double this tree and obtain a route R the route is feasible as we visit one point per γ_i





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To do: why do we achieve the claimed approximation factor? $p_{3,1}$



 $p_{3,2}$

 \boldsymbol{S}

Proof idea: alter(unknown) optimal route OPT(S,P,s) to pass through points from V(G), and new tour has length at most constant OPT(S,P,s)





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- Identify all cuts of the $kVR(s_i)$ that OPT(S,P,s) visits—set $C(C \subseteq C^{all})$





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- Identify all cuts of the $kVR(s_i)$ that OPT(S, P, s) visits—set $C(C \subseteq C^{all})$

- Let $o_{i,j}$ denote the point where OPT(S, P, s) visits $c_{i,j}$ (first time)





Proof idea: alter(unknown) optimal route OPT(S,P,s) to pass through points from V(G), and new tour has length at most constant OPT(S,P,s)

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- Let $o_{i,j}$ denote the point where OPT(S, P, s) visits $c_{i,j}$ (first time)
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A cut c partitions polygon into two subpolygons: P_s(c)—subpolygon that contains starting point s A cut c_1 dominates c_2 if $P_s(c_2) \subseteq P_s(c_1)$ Essential cut: not dominated by other cut





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Claim 1: The geodesics in $G_{C'}$ are a set of *independent* geodesics, i.e., no essential cut is visited by two of these geodesics.





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- $G_{C''}$ set of geodesics that end at cuts in C''
- 1: The geodesics in $G_{C^{n}}$ are a set of *independent* geodesics, i.e., no essential cut is visited by two of these geodesics. Claim
- Claim 2: Each essential cut visited by OPT(S,P,s) (each cut in C') is touched by exactly one of the geodesics.





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- The geodesics in $\mathcal{G}_{C''}$ intersect the cuts in C'' in points of the type $p_{i,j}$ —set $\mathcal{P}_{C''}$





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- Claim 1: The geodesics in $G_{C^{n}}$ are a set of *independent* geodesics, i.e., no essential cut is visited by two of these geodesics.
- Claim 2: Each essential cut visited by OPT(S,P,s) (each cut in C') is touched by exactly one of the geodesics.
- The geodesics in $\mathcal{G}_{C''}$ intersect the cuts in C'' in points of the type $p_{i,j}$ —set $\mathcal{P}_{C''}$
- -Build relative convex hull of all $o_{i,j}$ and all points in $\mathcal{P}_{C''}$ (relative w.r.t. polygon P): CH_P(OPT, $\mathcal{P}_{C''}$)





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- Identify subset C' of essential cuts ($C' \subseteq C$)
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- The geodesics in $\mathcal{G}_{C''}$ intersect the cuts in C'' in points of the type $p_{i,j}$ —set $\mathcal{P}_{C''}$
- -Build relative convex hull of all $o_{i,j}$ and all points in $\mathcal{P}_{C''}$ (relative w.r.t. polygon P): CH_P(OPT, $\mathcal{P}_{C''}$)
- -Claim 3: No geodesic can intersect CH_P(OPT, \mathcal{P}_{C}) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most 3·IIOPT(S,P,s)II.





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Claim 1: The geodesics in $\mathcal{G}_{C^{p}}$ are a set of *independent* geodesics, i.e., no essential cut is visited by two of these geodesics. Claim 2: Each essential cut visited by OPT(S,P,s) (each cut in C') is touched by exactly one of the geodesics. - The geodesics in $\mathcal{G}_{C''}$ intersect the cuts in C'' in points of the type $p_{i,j}$ —set $\mathcal{P}_{C''}$ -Build relative convex hull of all $o_{i,j}$ and all points in $\mathcal{P}_{C''}$ (relative w.r.t. polygon P): CH_P(OPT, $\mathcal{P}_{C''}$) -Claim 3: No geodesic can intersect CH_P(OPT, \mathcal{P}_{C}) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most 3·IIOPT(S,P,s)II. -Claim 4: $CH_P(\mathcal{P}_{C'})$ is not longer than $CH_P(OPT, \mathcal{P}_{C'})$ and $CH_P(\mathcal{P}_{C'})$ visits one point per γ_i (except for γ_0).





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Claim 1: The geodesics in $G_{C^{n}}$ are a set of *independent* geodesics, i.e., no essential cut is visited by two of these geodesics. Claim 2: Each essential cut visited by OPT(S,P,s) (each cut in C') is touched by exactly one of the geodesics. - The geodesics in $\mathcal{G}_{C^{n}}$ intersect the cuts in C^{n} in points of the type $p_{i,j}$ —set $\mathcal{P}_{C^{n}}$ -Build relative convex hull of all $o_{i,j}$ and all points in $\mathcal{P}_{C''}$ (relative w.r.t. polygon P): CH_P(OPT, $\mathcal{P}_{C''}$) -Claim 3: No geodesic can intersect CH_P(OPT, \mathcal{P}_{C}) between a point $o_{i,j}$ and a point $p_{i,j}$ on the same cut. Thus, between any pair of points of the type $o_{i,j}$ on CH_P(OPT, $\mathcal{P}_{C''}$), we have at most two points of $\mathcal{P}_{C''}$. CH_P(OPT, $\mathcal{P}_{C''}$) has length at most 3·IIOPT(S,P,s)II. -Claim 4: $CH_P(\mathcal{P}_{C''})$ is not longer than $CH_P(OPT, \mathcal{P}_{C''})$ and $CH_P(\mathcal{P}_{C''})$ visits one point per γ_i (except for γ_0). - To connect s (which may lie in the interior of $CH_P(\mathcal{P}_{C^{\nu}})$, we need to connect s, which costs at most IIOPT(S, P, s) II.





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 - $\leq \alpha_4 \cdot f(n|S|, |S|) \| \text{OPT}(S, P, s) \|$



with $f(N, M) = \log^2 N \log \log N \log M$



Open Problem: k-Transmitter Watchmen





Structural analogue for extensions, which we have for 0-transmitters?



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A cut c partitions polygon into two subpolygons: P_s(c)—subpolygon that contains starting point s A cut c_1 dominates c_2 if $P_s(c_2) \subseteq P_s(c_1)$ Essential cut: not dominated by other cut





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• We see all of *P* iff we visit all essential cuts.





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→ OPEN PROBLEM #2: Is there a structure like essential cuts that guarantees k-visibility of P when visited?











stemming from 0-transmitters



• Improved combinatorial bounds for 2-/k-transmitter covers—in particular: Open Problem #1. Better upper bounds for simple polygons than the one



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Open Problem #2: Structural analogue for extensions for 0-transmitters?



- Improved combinatorial bounds for 2-/k-transmitter covers—in particular: Open Problem #1. Better upper bounds for simple polygons than the one stemming from 0-transmitters
- Open Problem #2: Structural analogue for extensions for 0-transmitters? • Approximation for watchmen routes for k-transmitters without given starting point and/or when all of *P* should be monitored?





- Improved combinatorial bounds for 2-/k-transmitter covers—in particular: Open Problem #1. Better upper bounds for simple polygons than the one stemming from 0-transmitters
- Open Problem #2: Structural analogue for extensions for 0-transmitters? • Approximation for watchmen routes for k-transmitters without given starting
- point and/or when all of *P* should be monitored?
- Generally: More structural insights for k-transmitters





