Applying Geometric Thick Paths to Compute the Number of Additional Train Paths in a Railway Timetable

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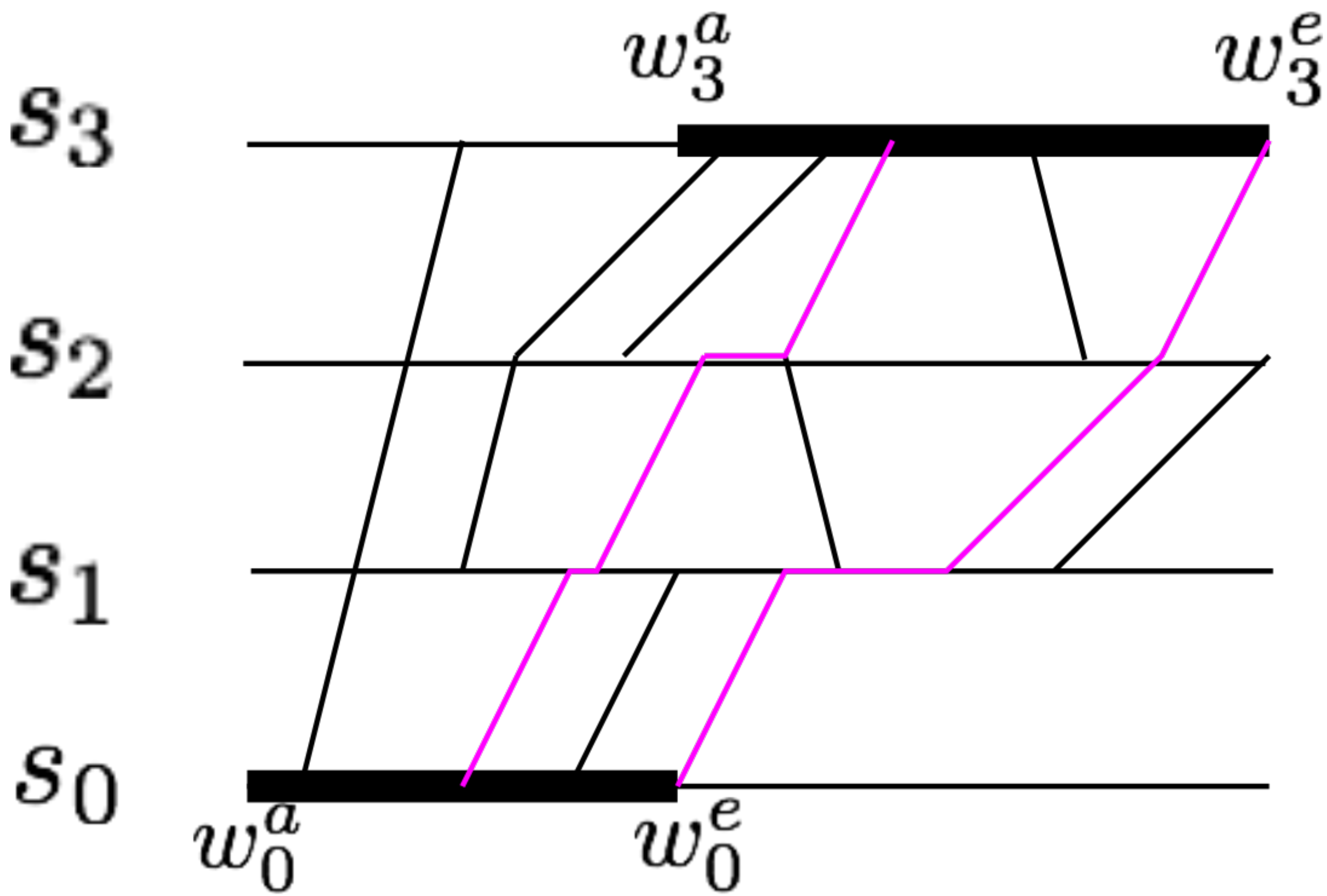
  - We present optimal solution

Existing trains

Additional trains: need to keep temporal distance



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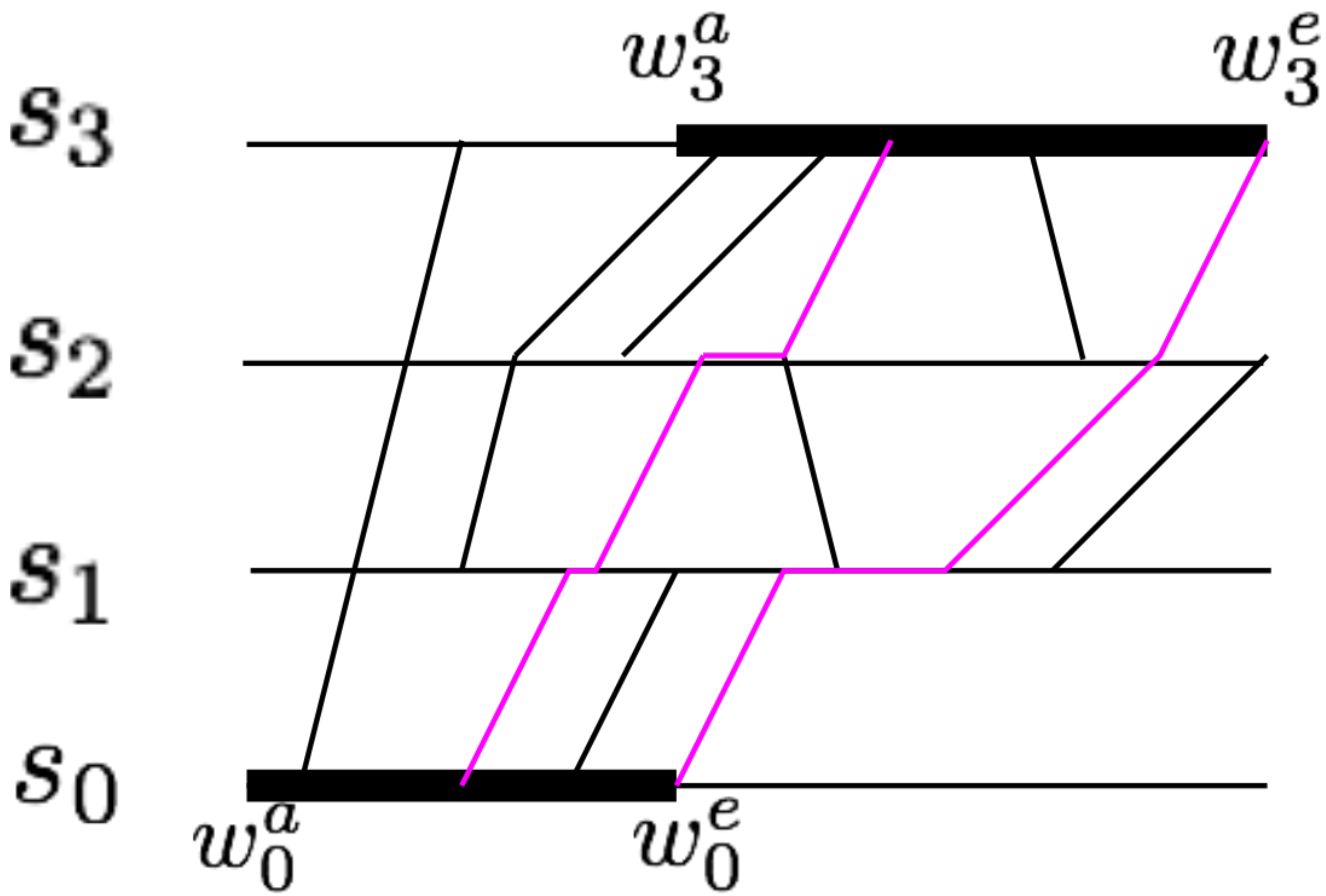

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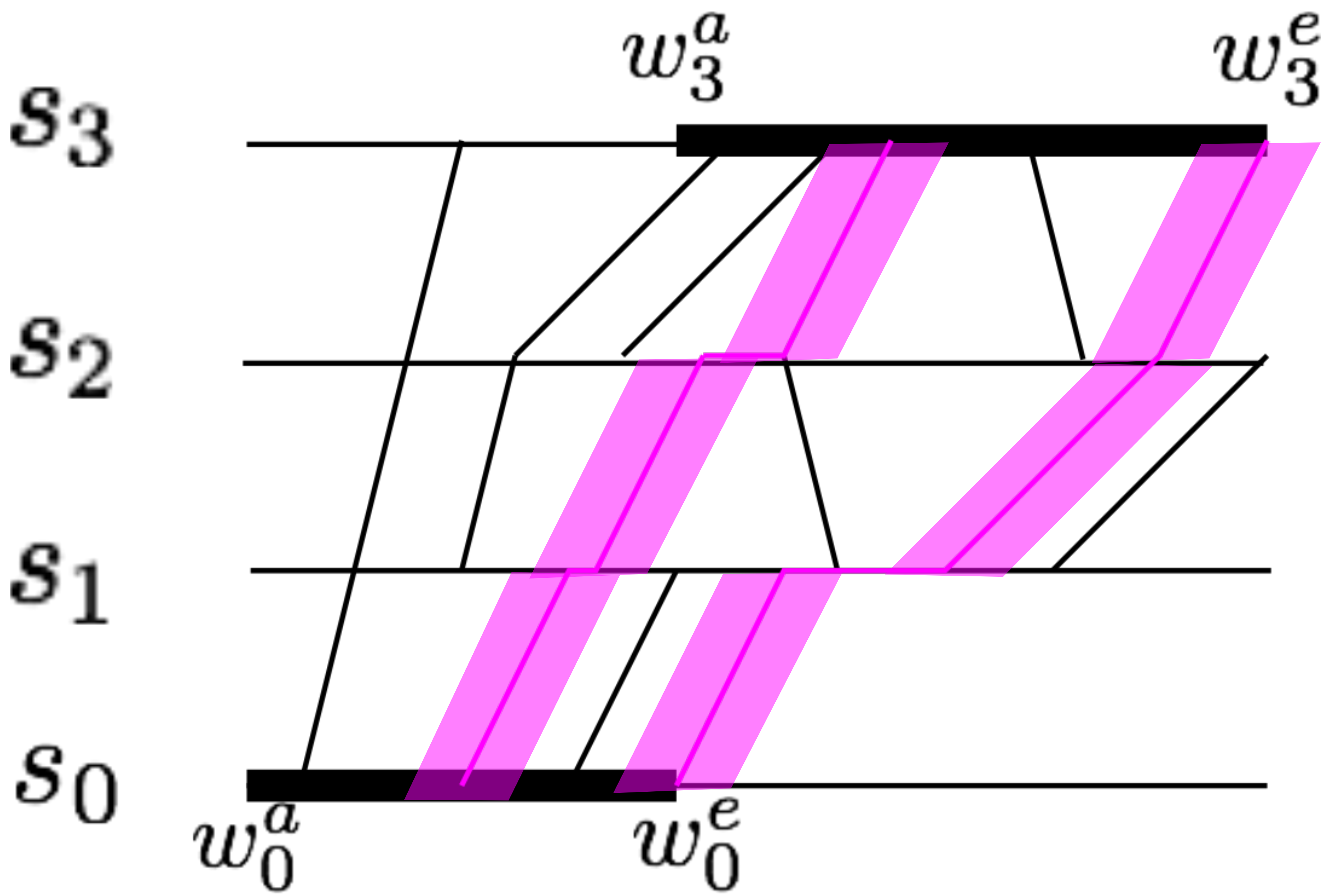

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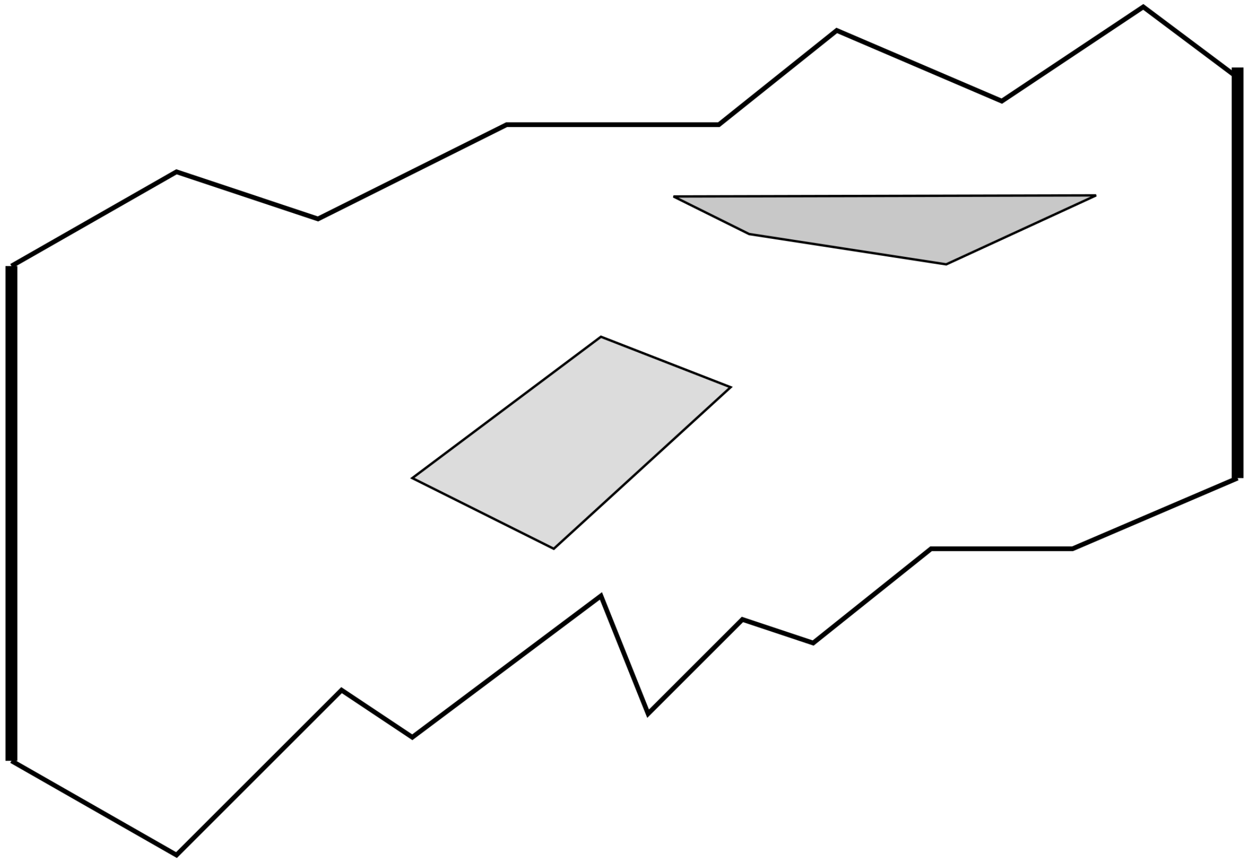
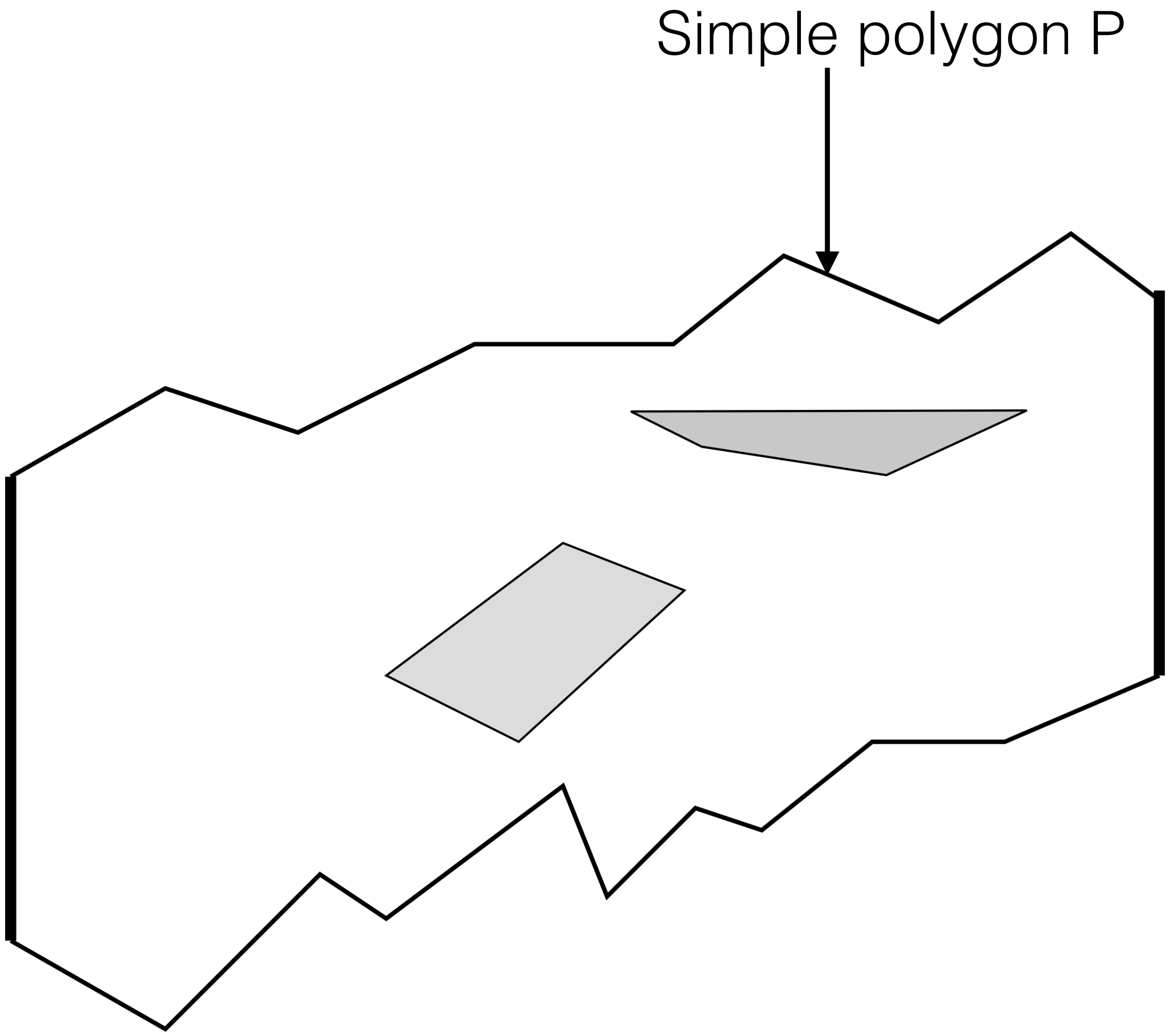
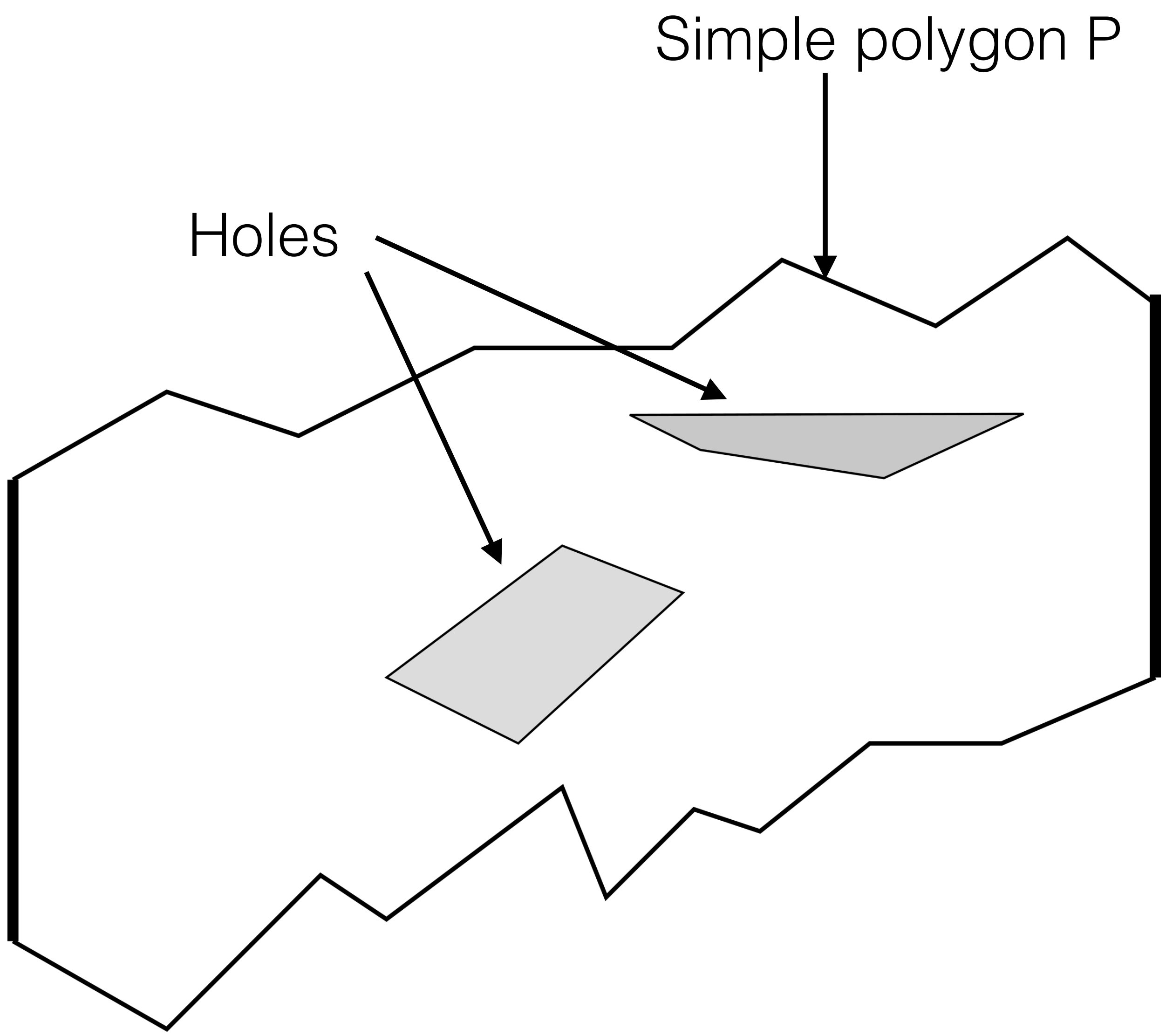
We start with 2

Routing a Maximum Number of Thick Paths through a Polygonal Domain




Simple polygon P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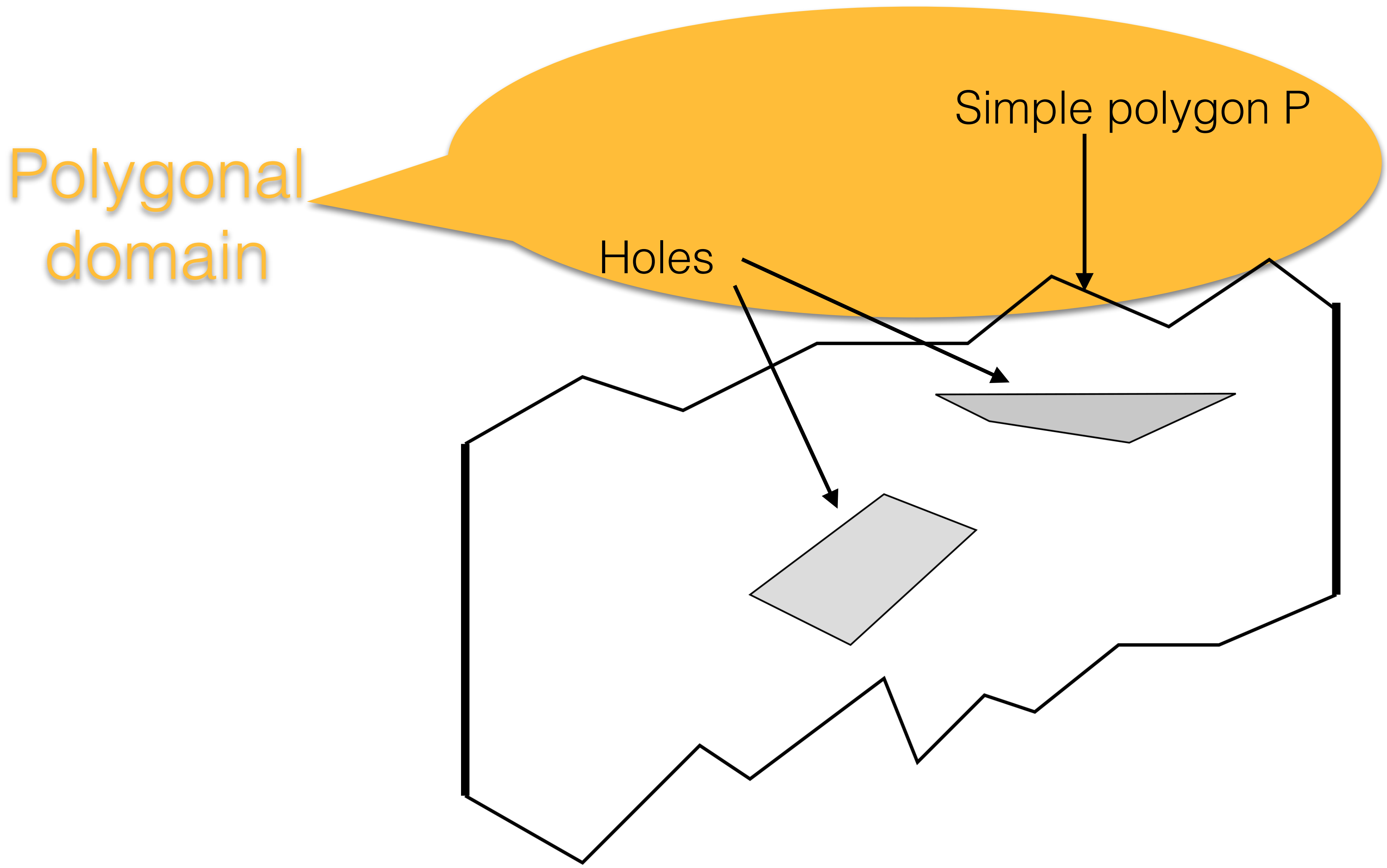
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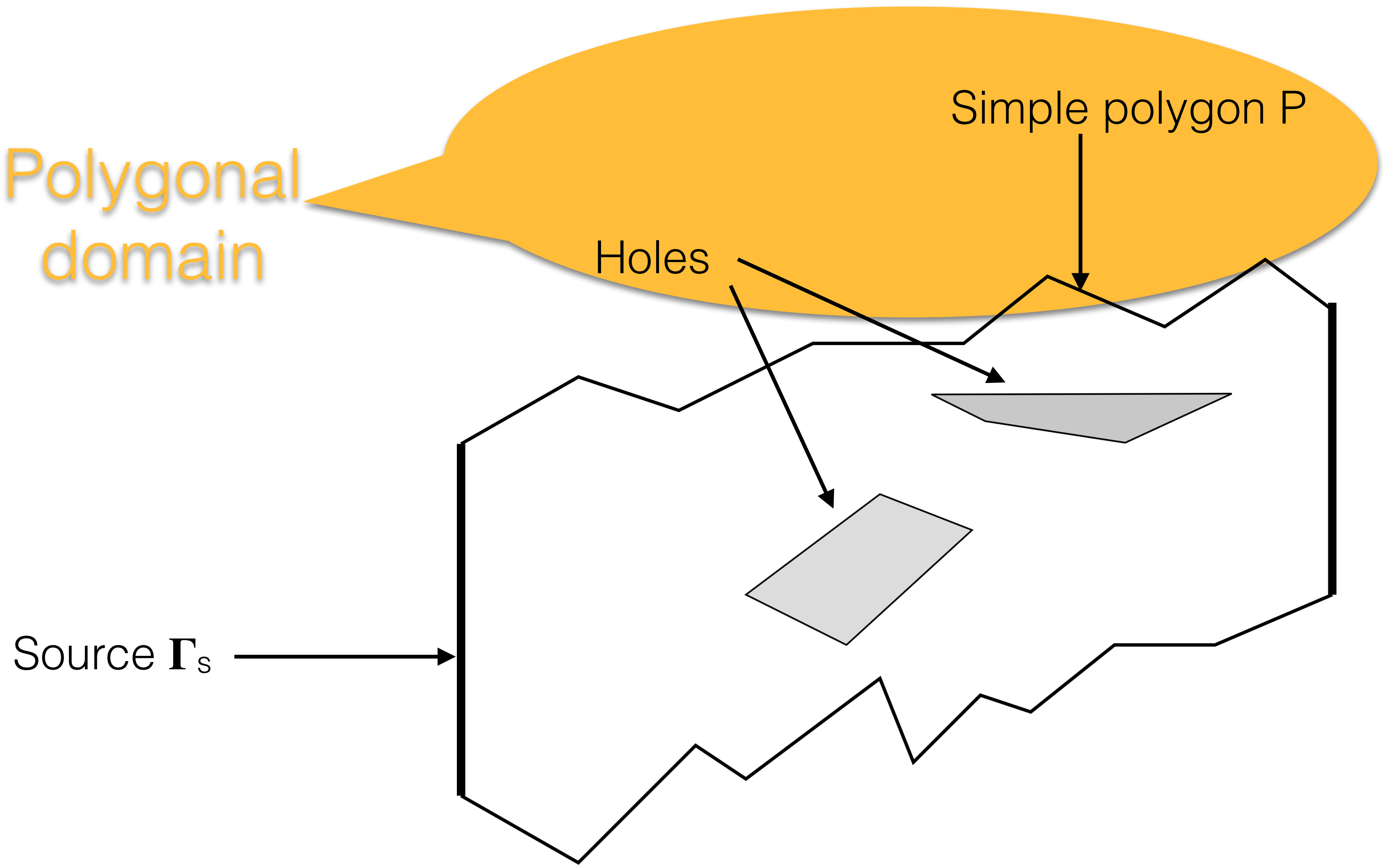

Polygonal domain
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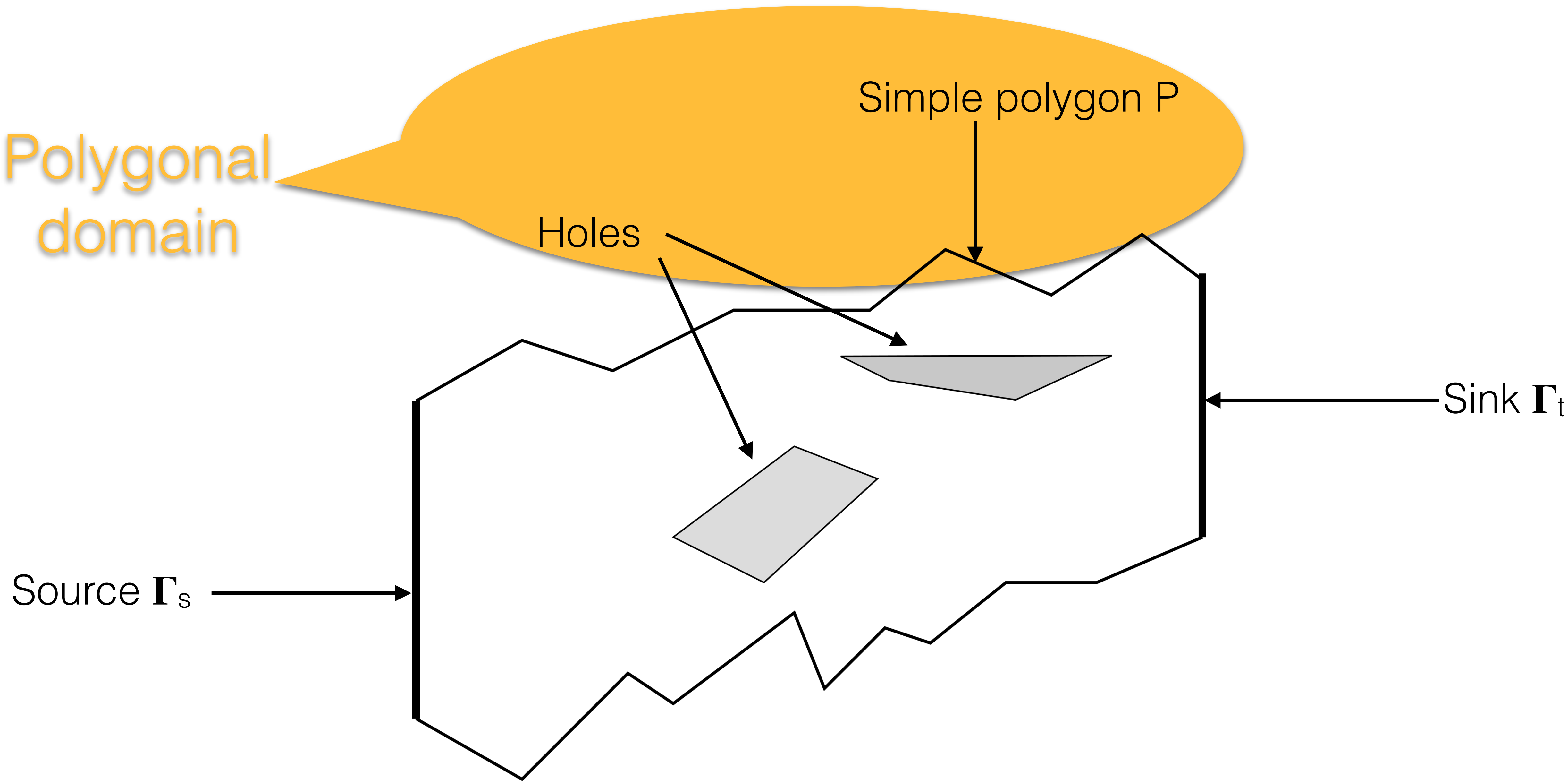
Source Γ_s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Sink Γ_t
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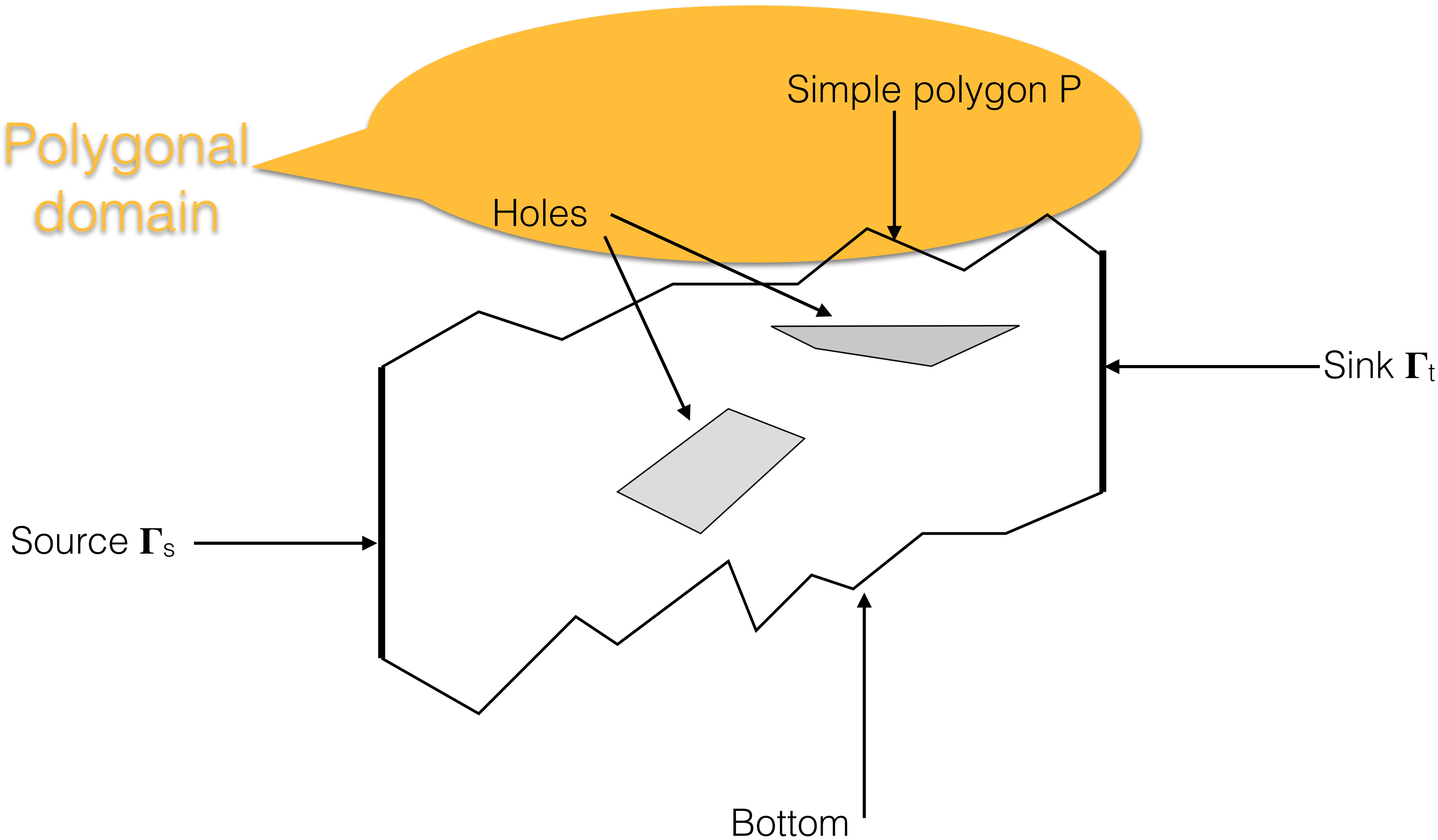
Polygonal domain
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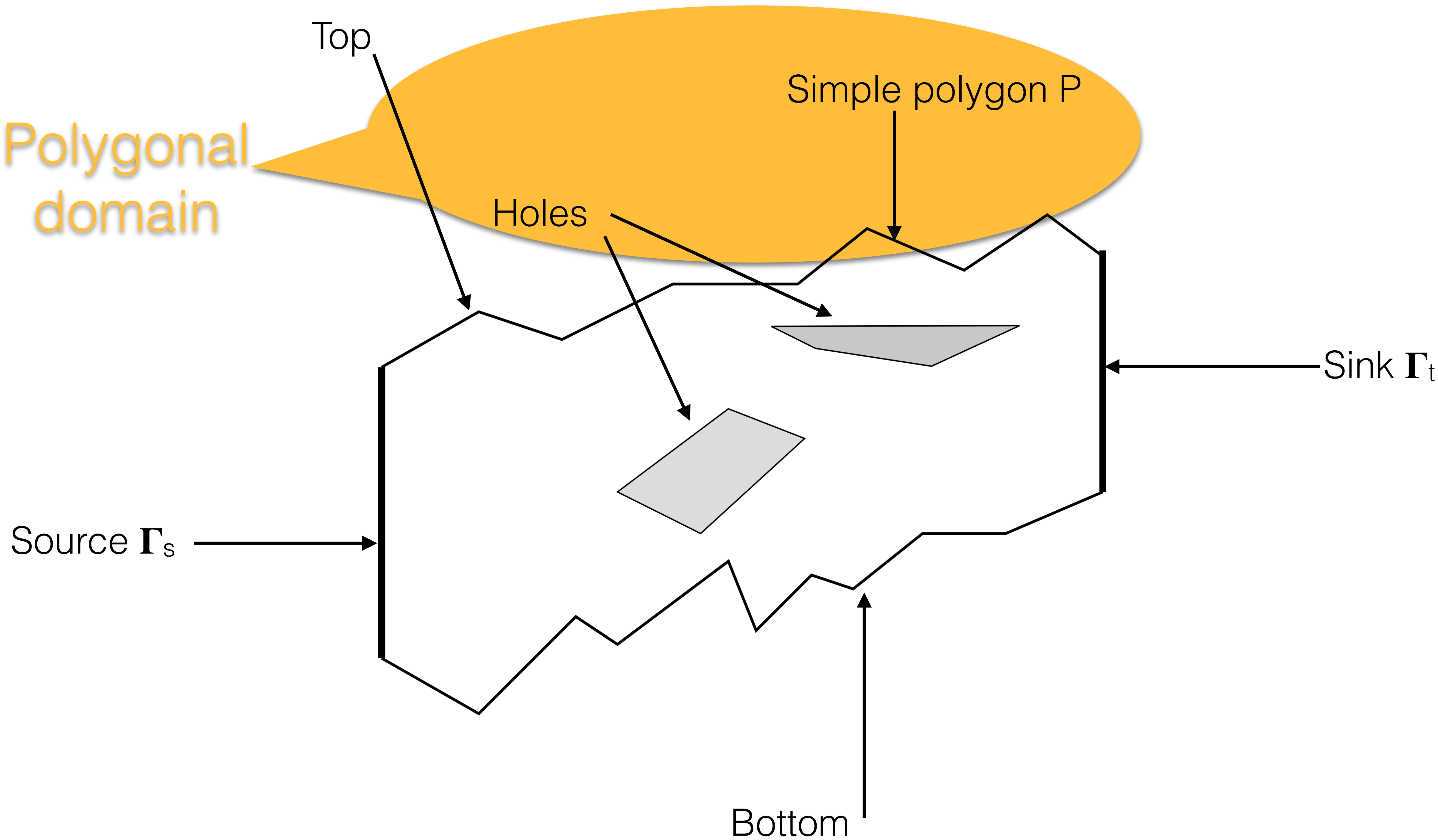
Holes
Sink $\mathbf{\Gamma}_t$
Source $\mathbf{\Gamma}_s$
Bottom


Top
Simple polygon P
Polygonal domain
Holes
Sink $\mathbf{\Gamma}_t$
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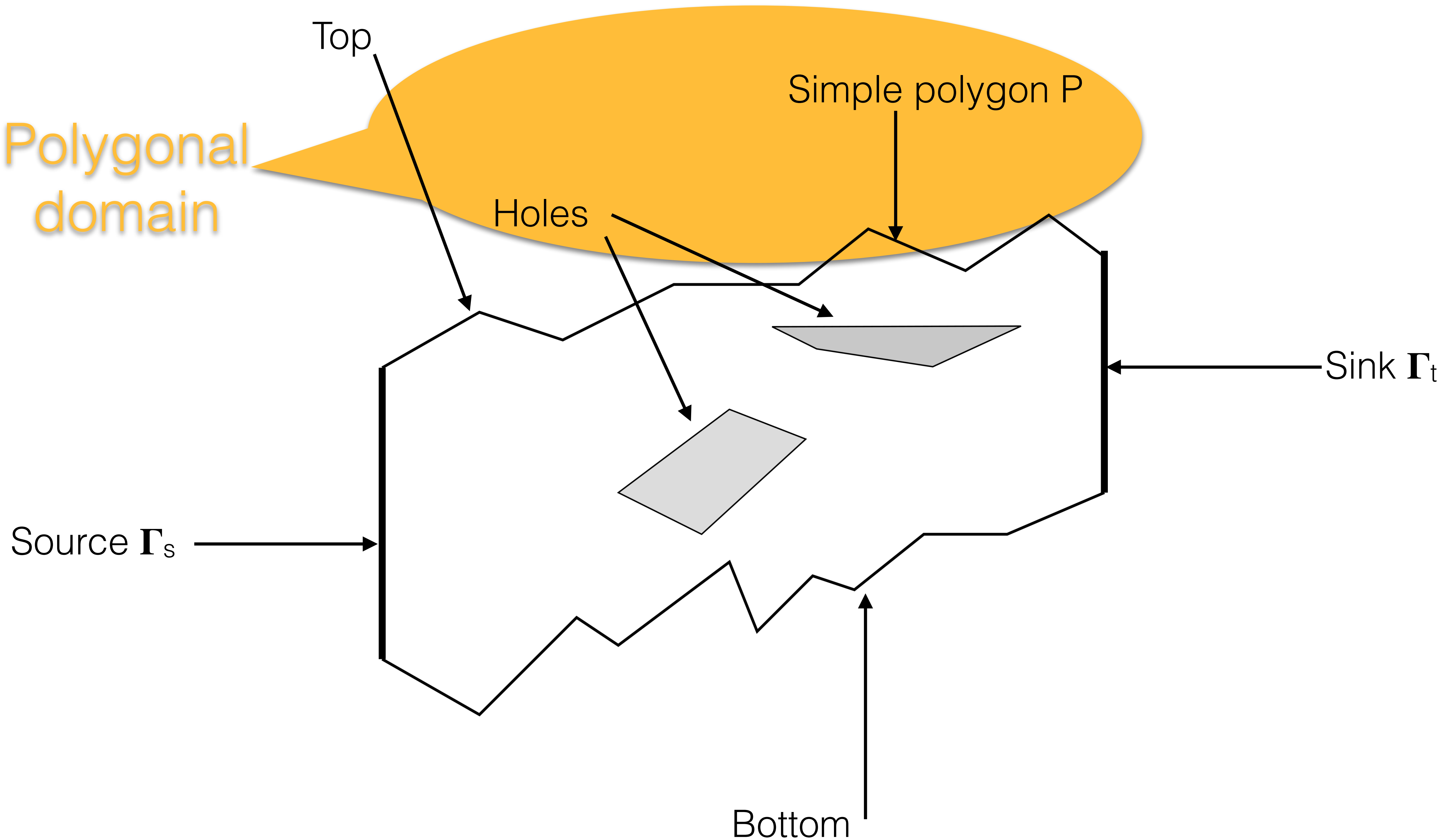
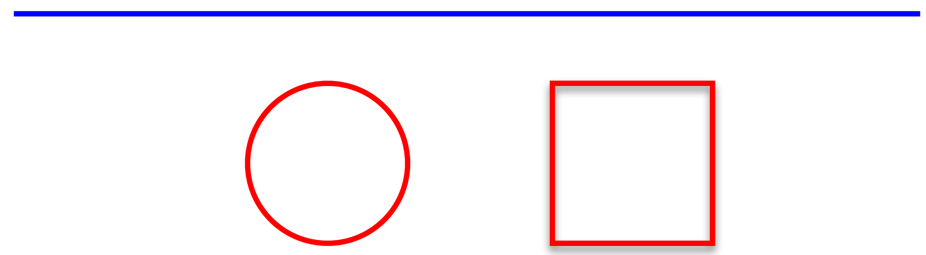
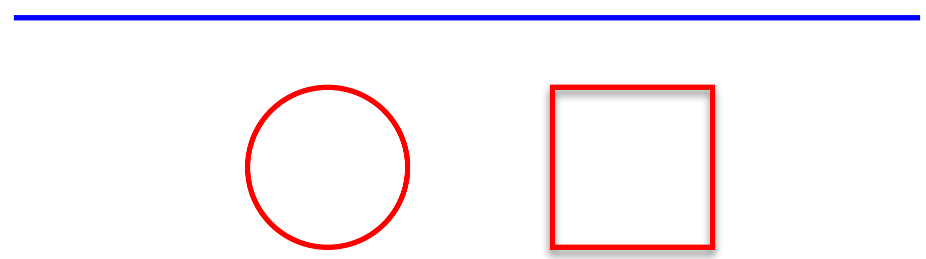


Route thick paths from the source to the sink, avoiding all holes (=obstacles)

Thin path π: simple curve



Thin path π: simple curve
Let C_r denote the open desk of radius r centered at the origin



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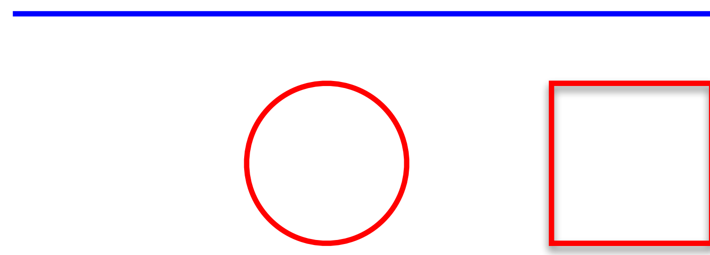
Let C_r denote the open desk of radius r centered at the origin
For $S \subset R^2$: $(S)^r = S \oplus C_r = \{x+y \mid x \in S, y \in C_r\}$ - Minkowski sum



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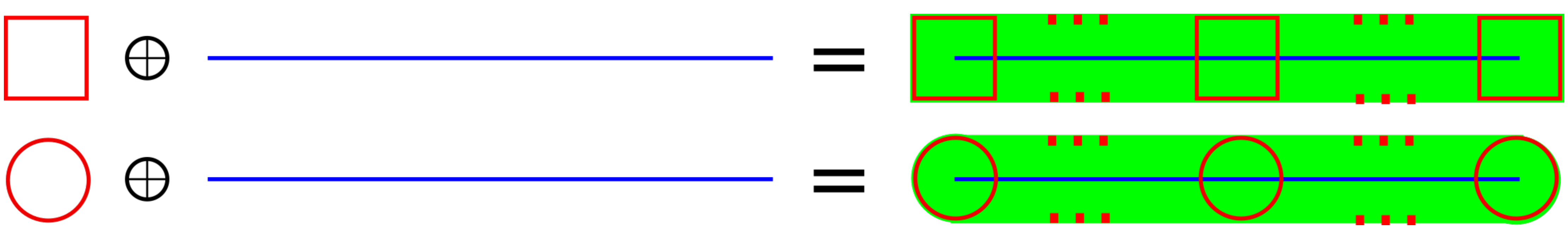
Thick path $\mathbf{\Pi}$: Minkowski sum of a thin path and a unit disk $\mathbf{\Pi}=(\pi)^1$



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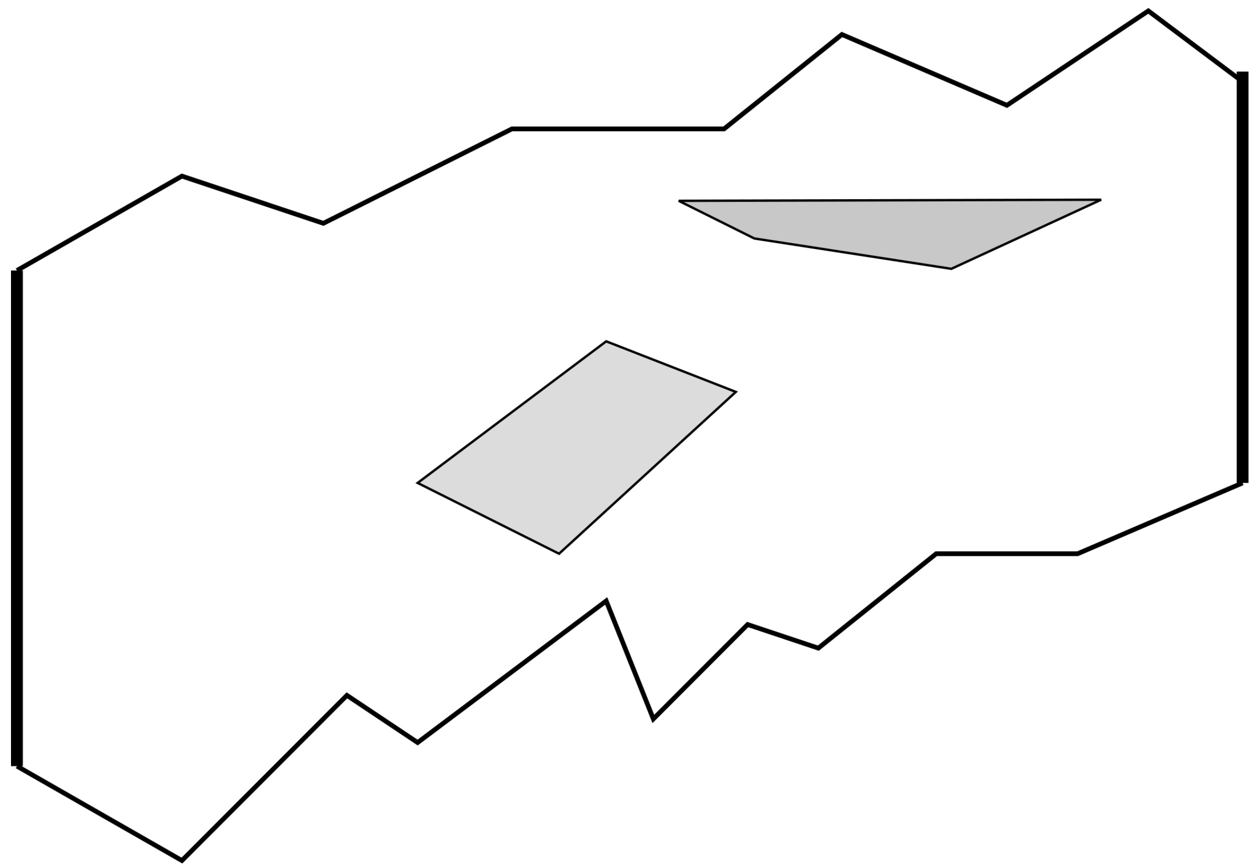

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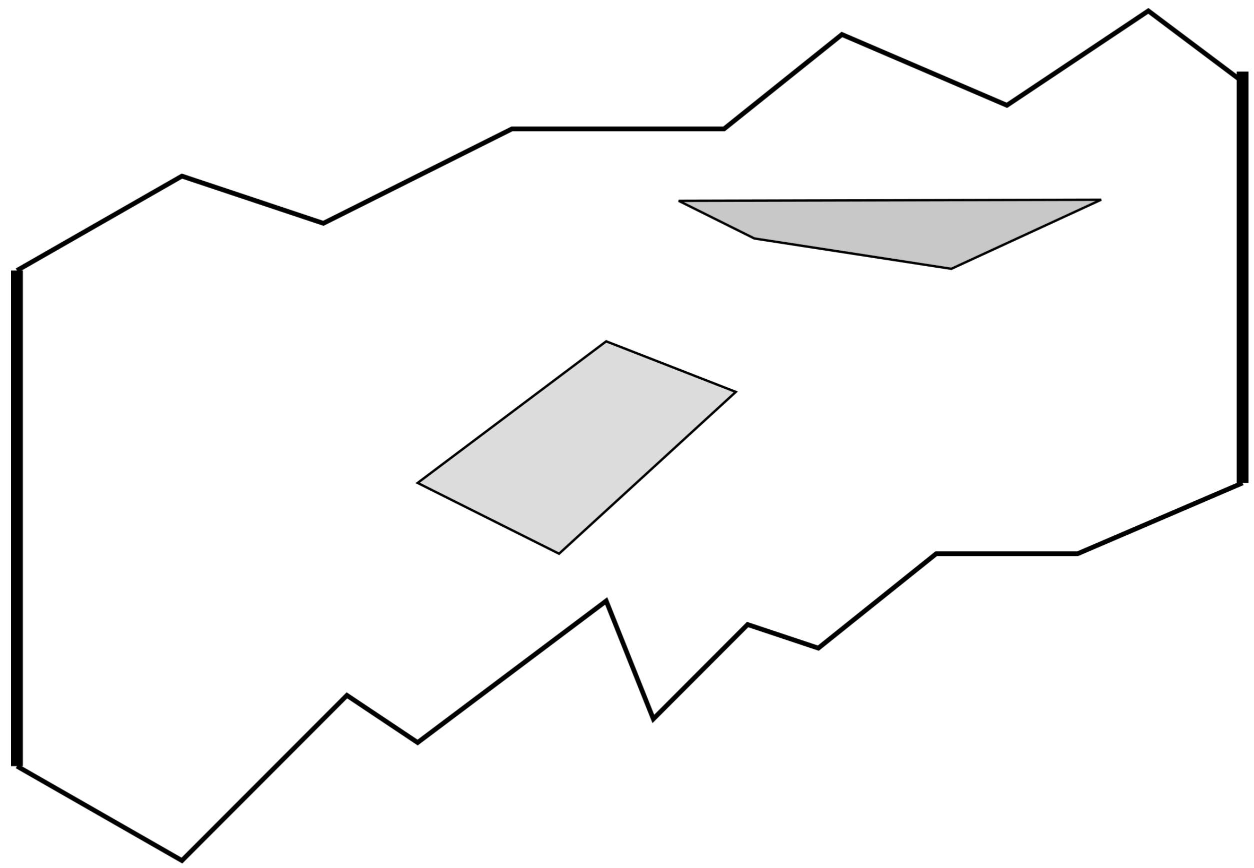
- Maximum number of non crossing thick paths from source to sink



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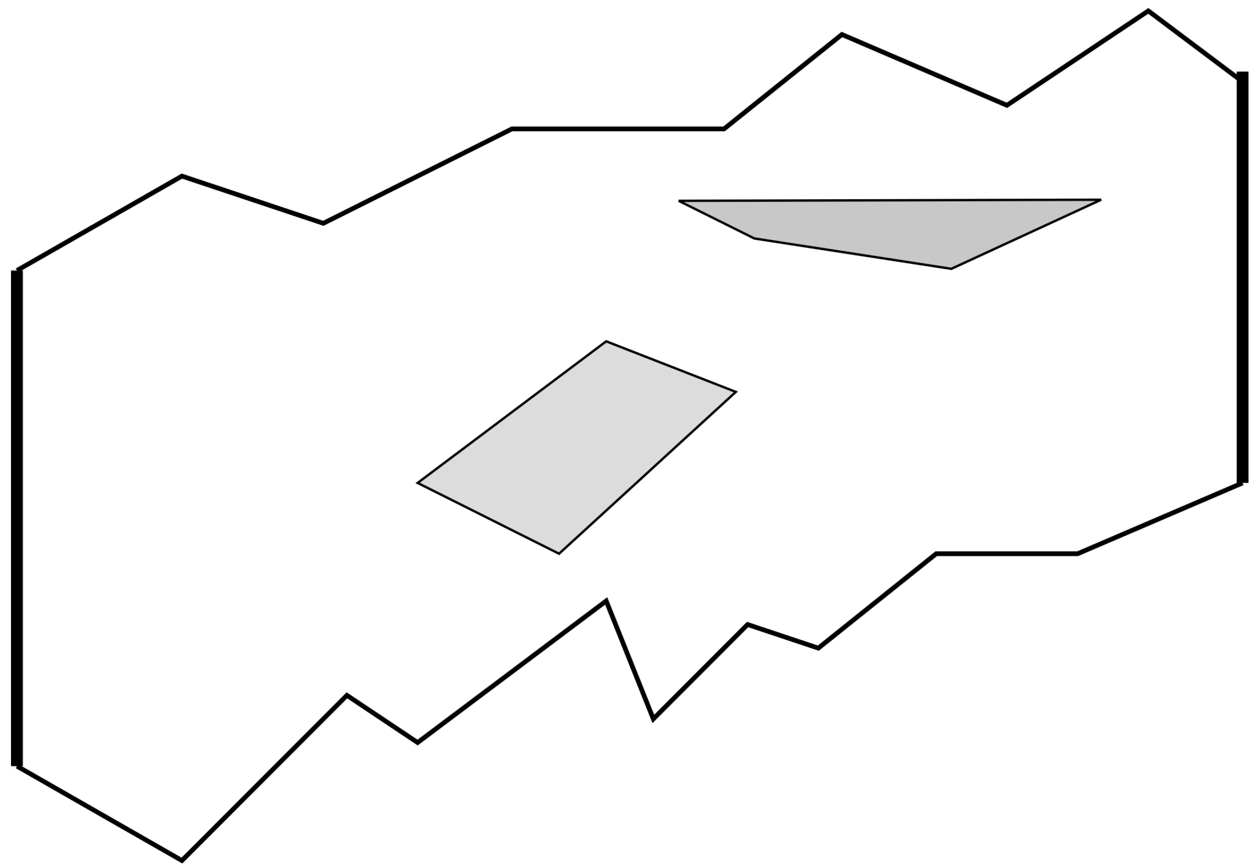

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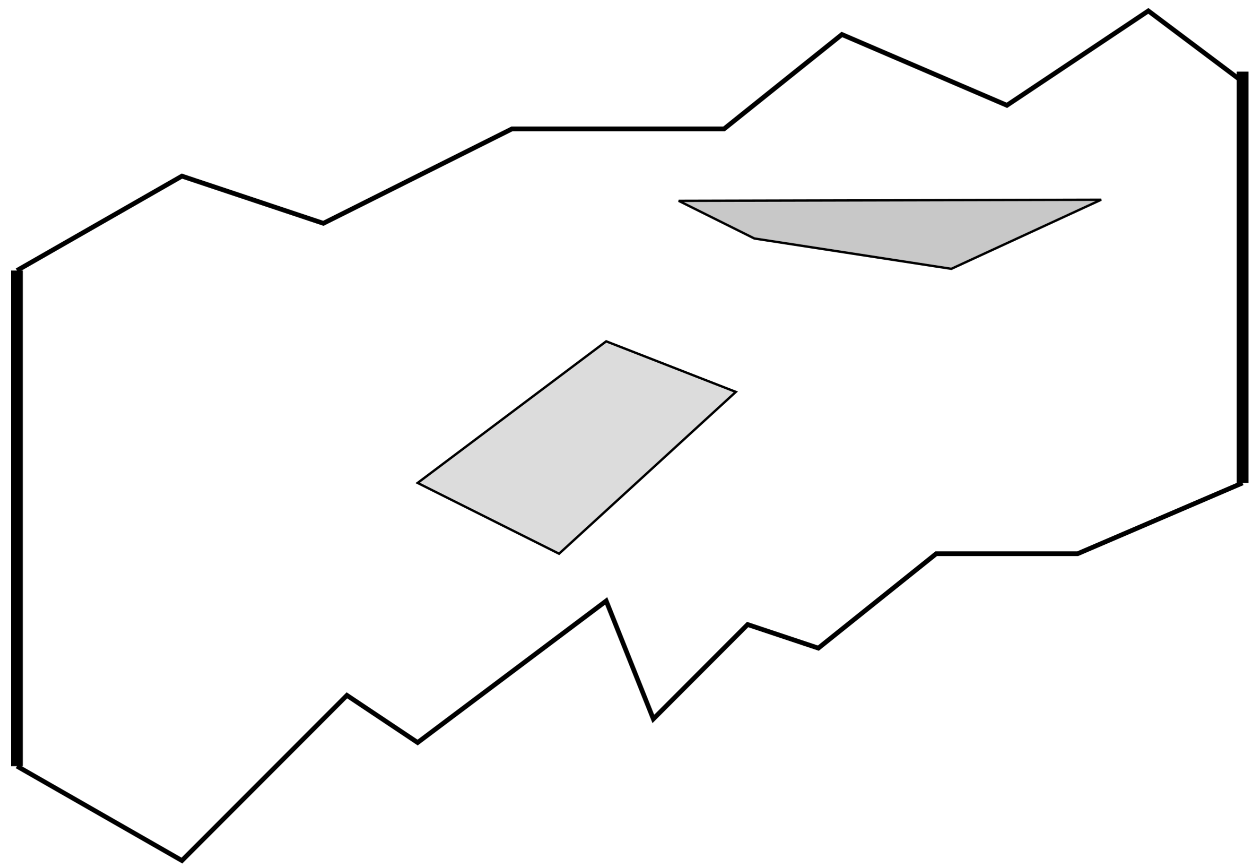

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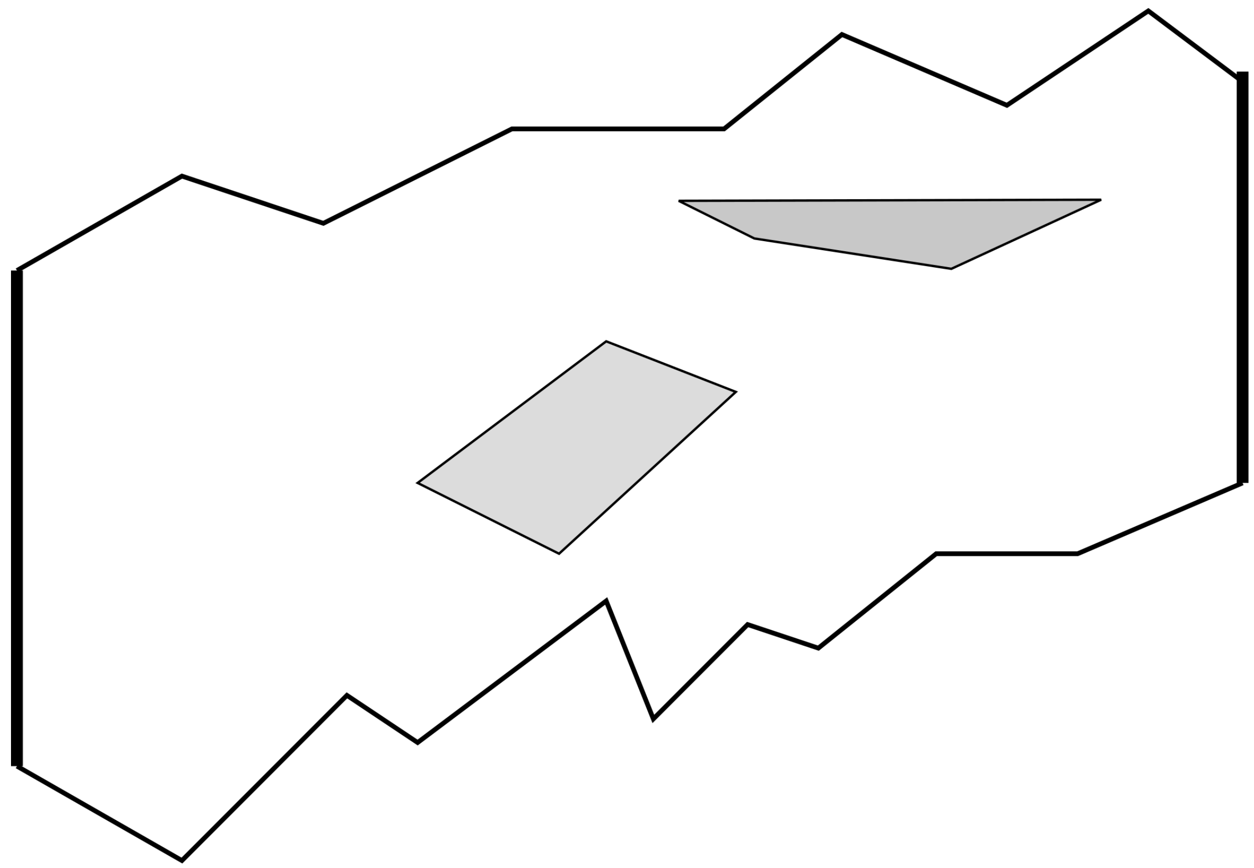

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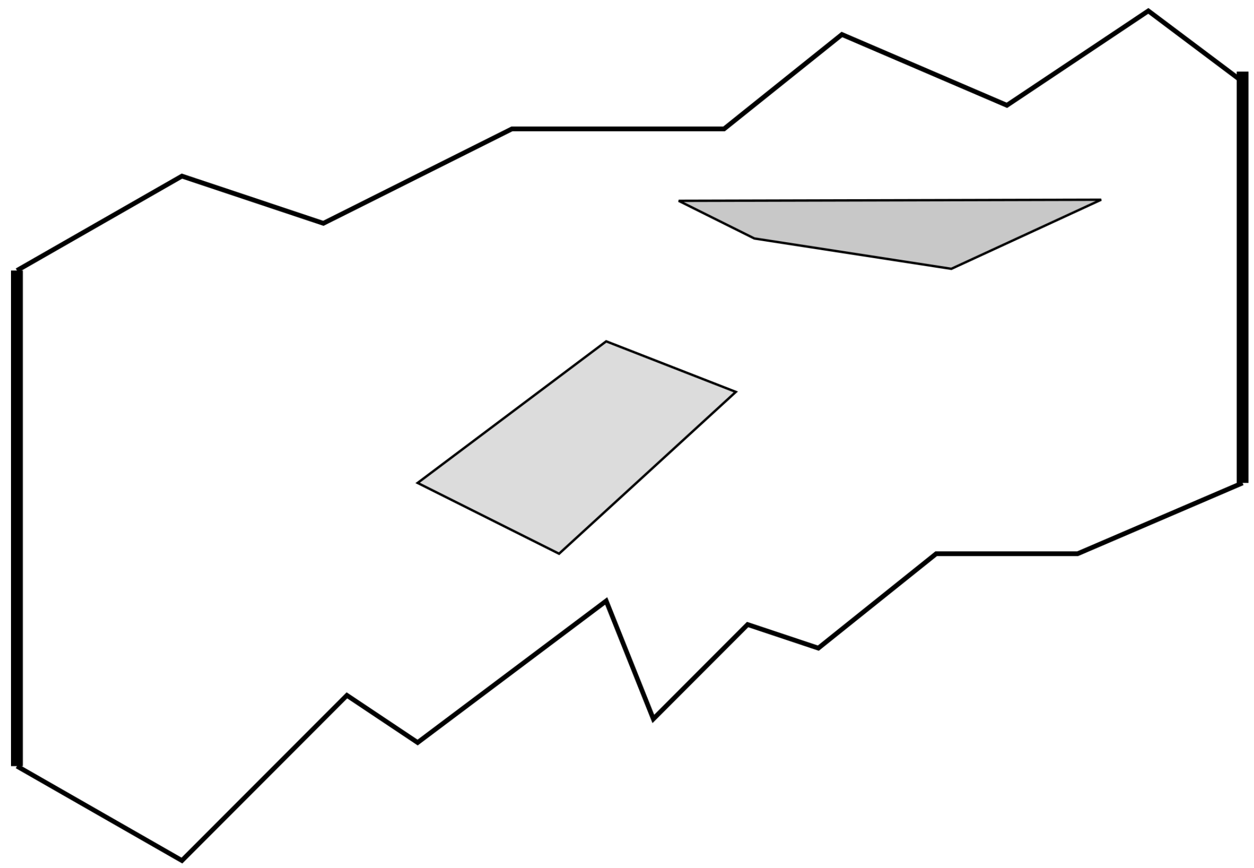
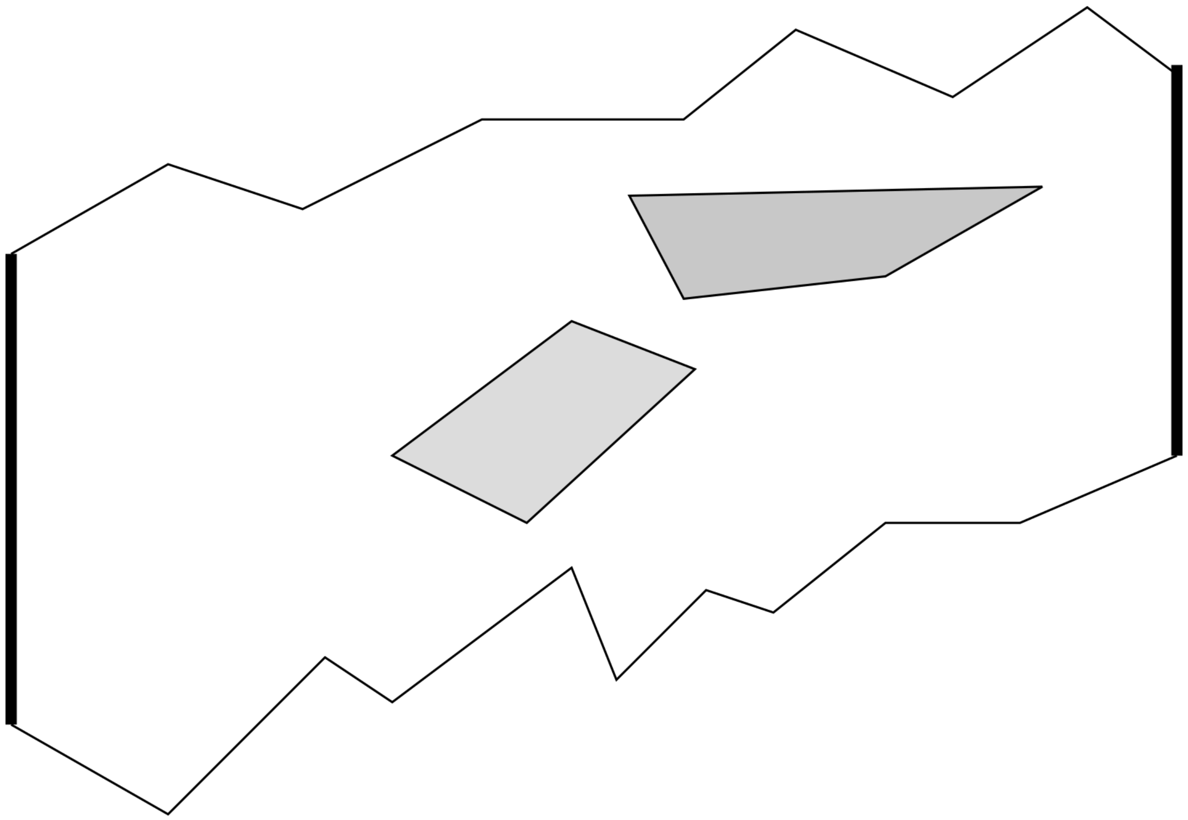
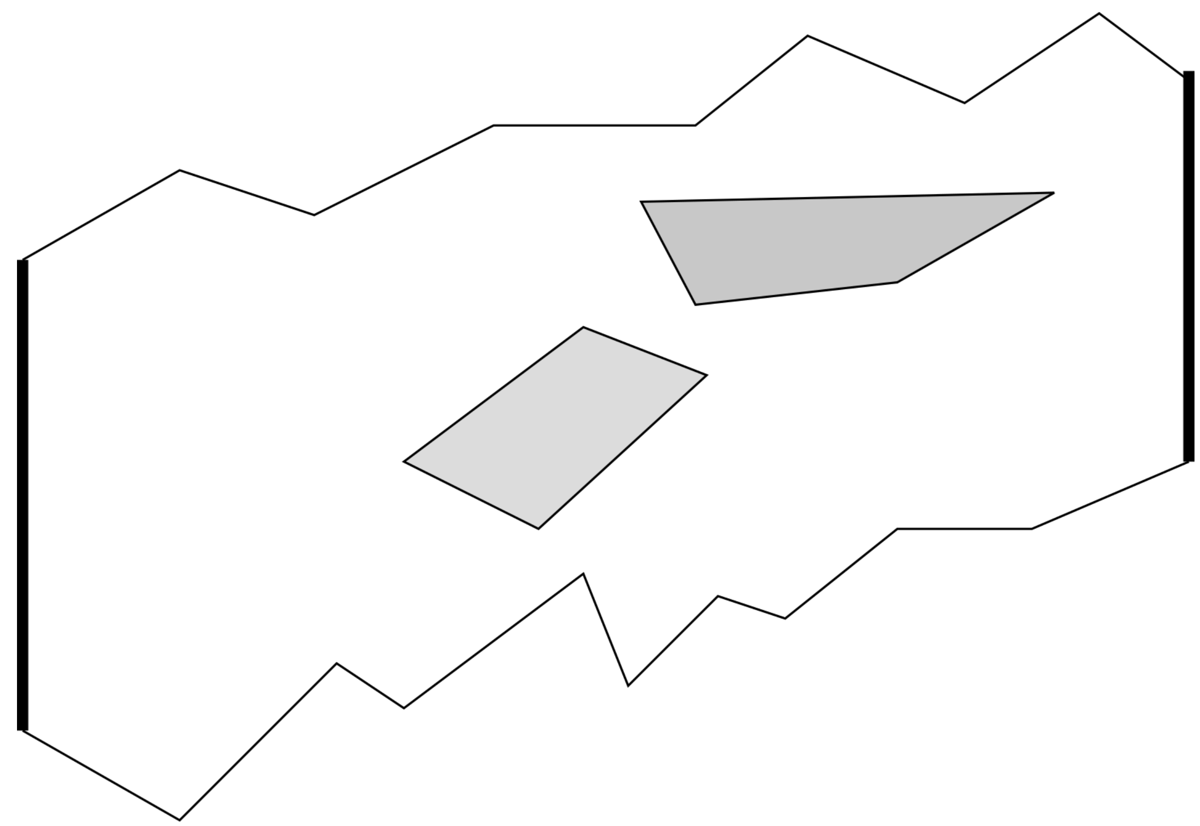
- Need some more concepts (Ω perforated at the source and sinks and Riemann flaps glued to Ω, …)





Algorithm by Arkin et al. (2010) to compute maximum number of thick paths:



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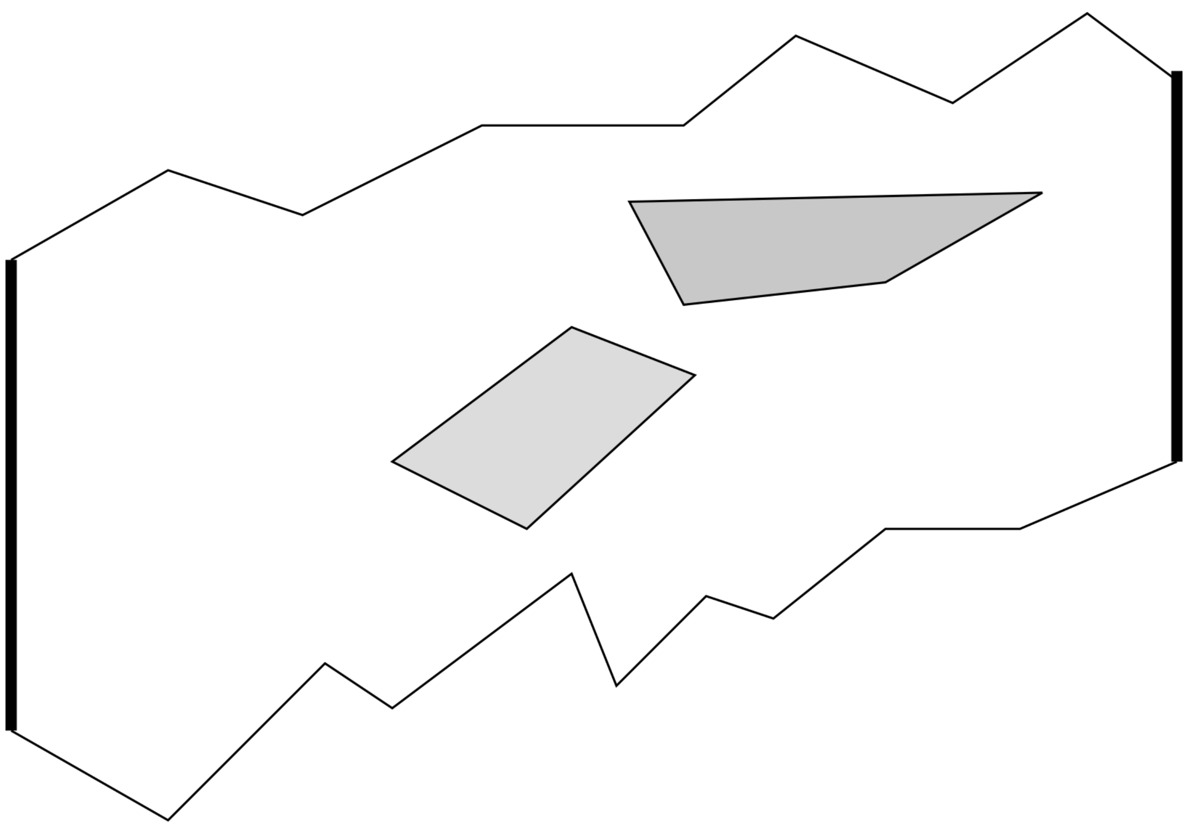
- Grass-fire analogy



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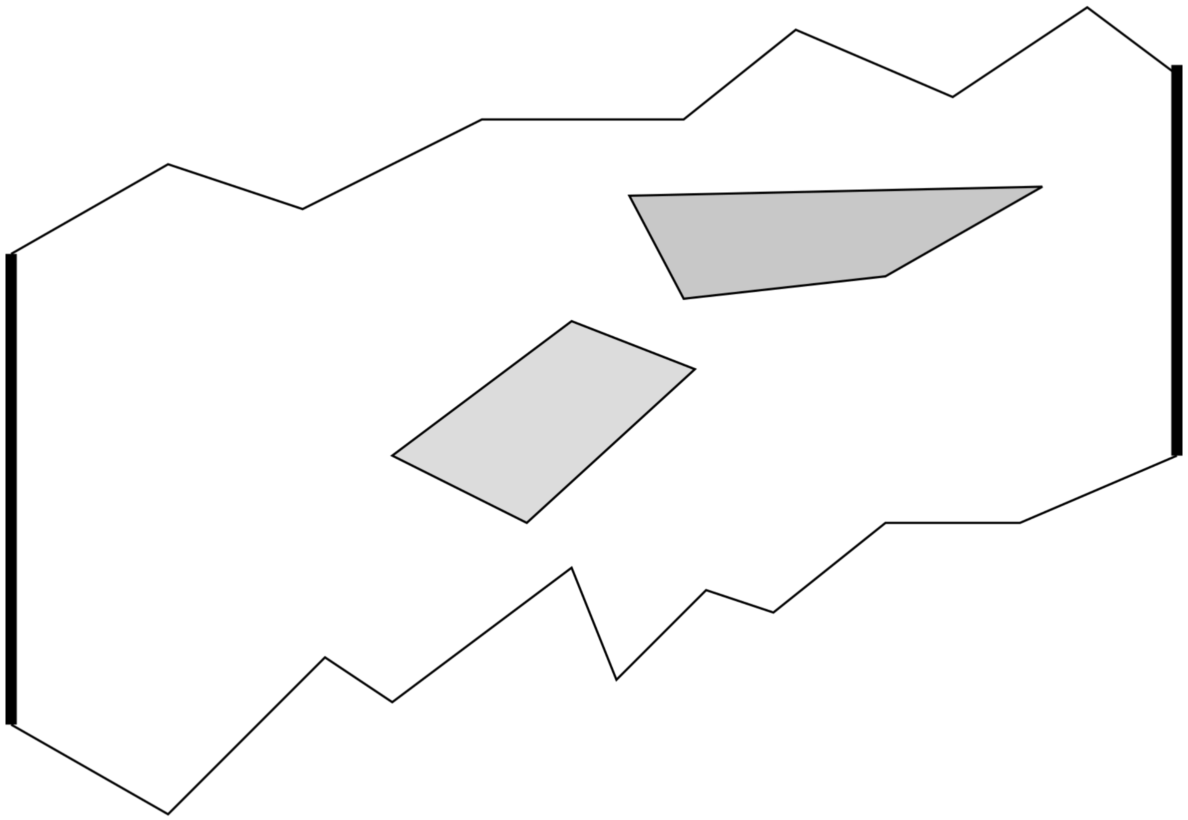

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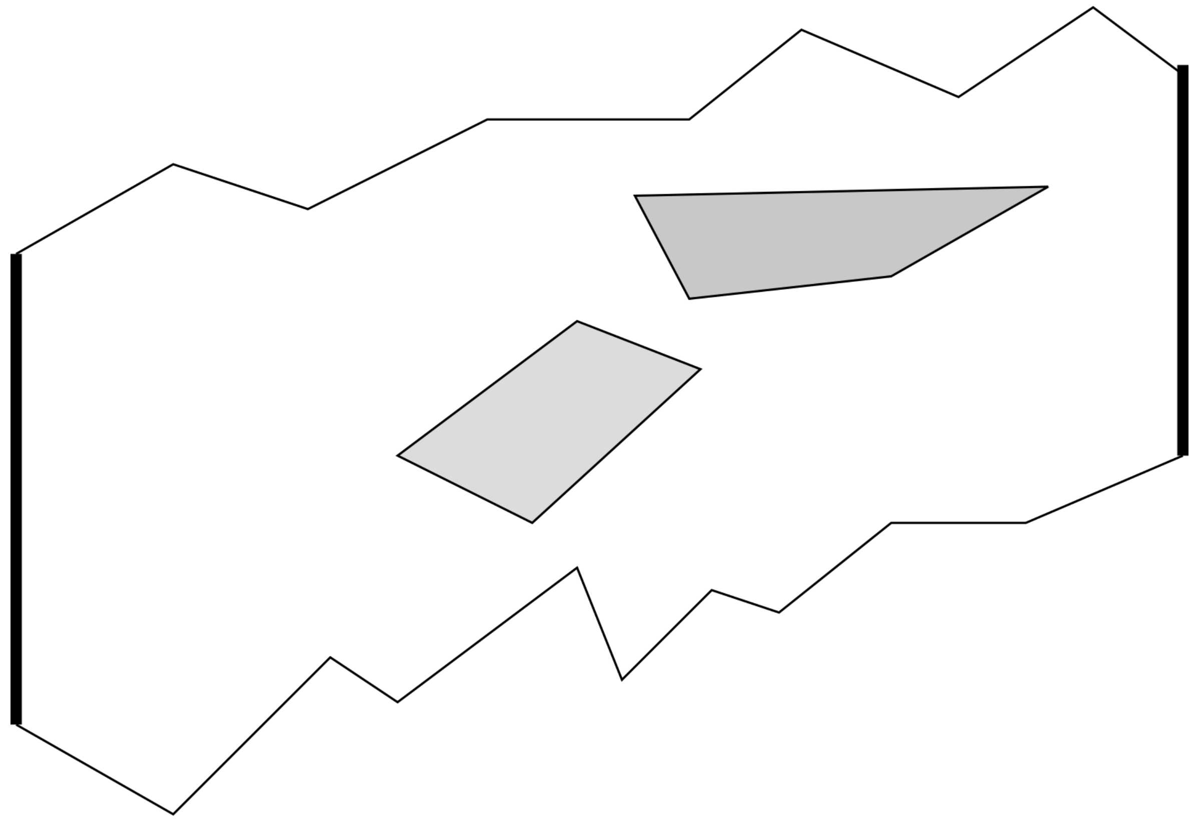

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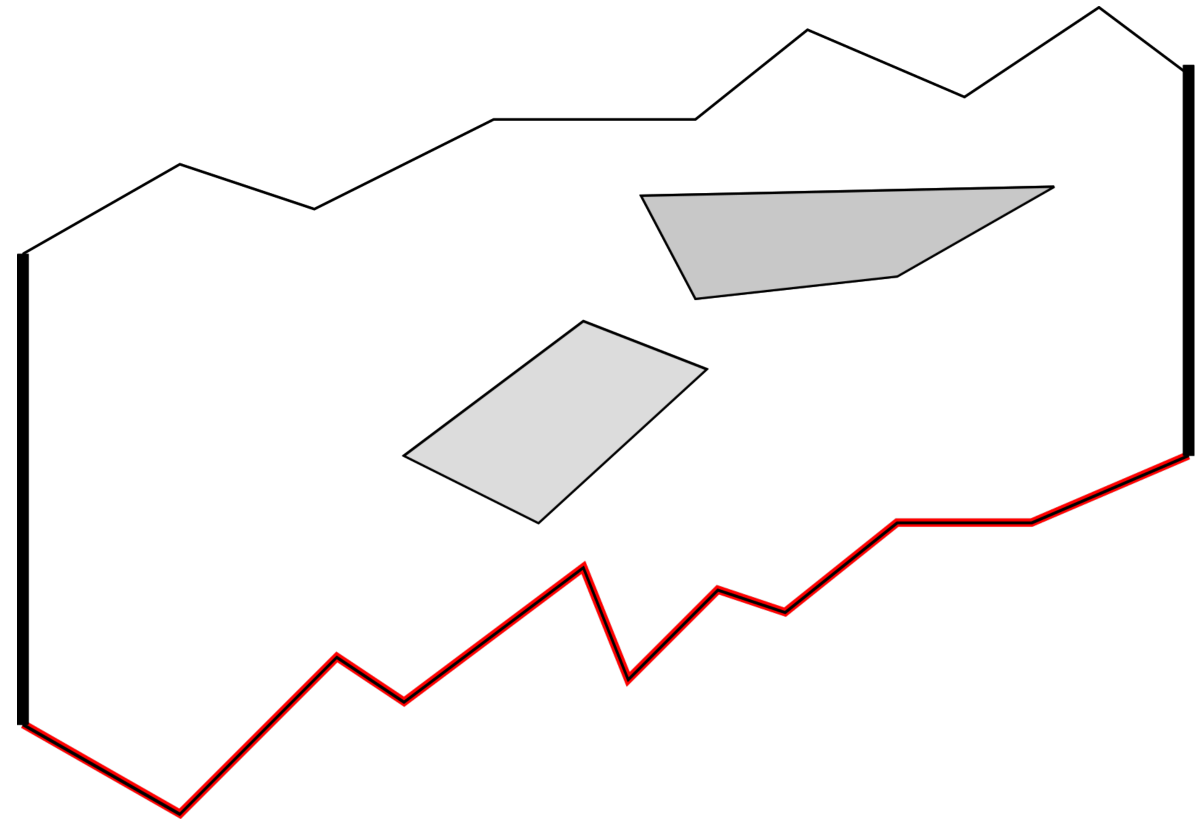
- We start setting the bottom on fire.



Algorithm by Arkin et al. (2010) to compute maximum number of thick paths:

- Grass-fire analogy
- Free space is grass over which fire travels with speed 1
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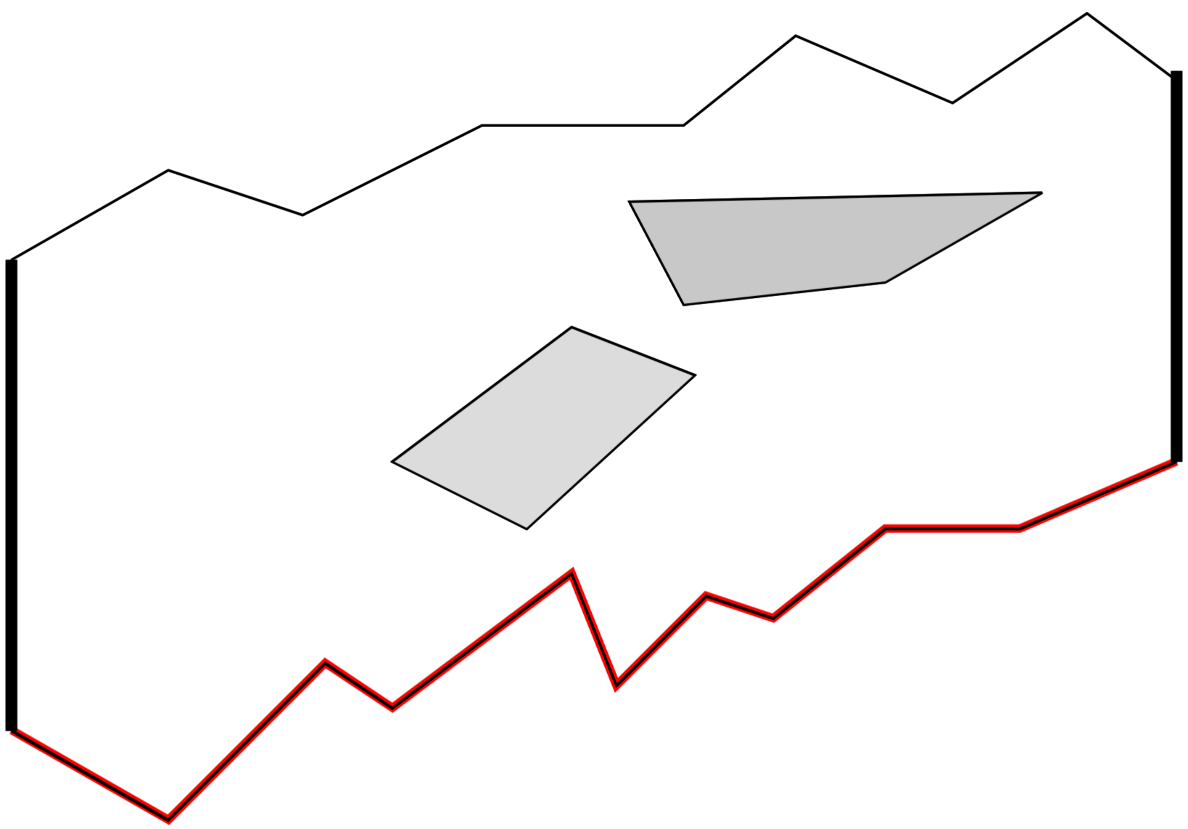
- Wavefront at time τ: boundary of burnt grass by time τ



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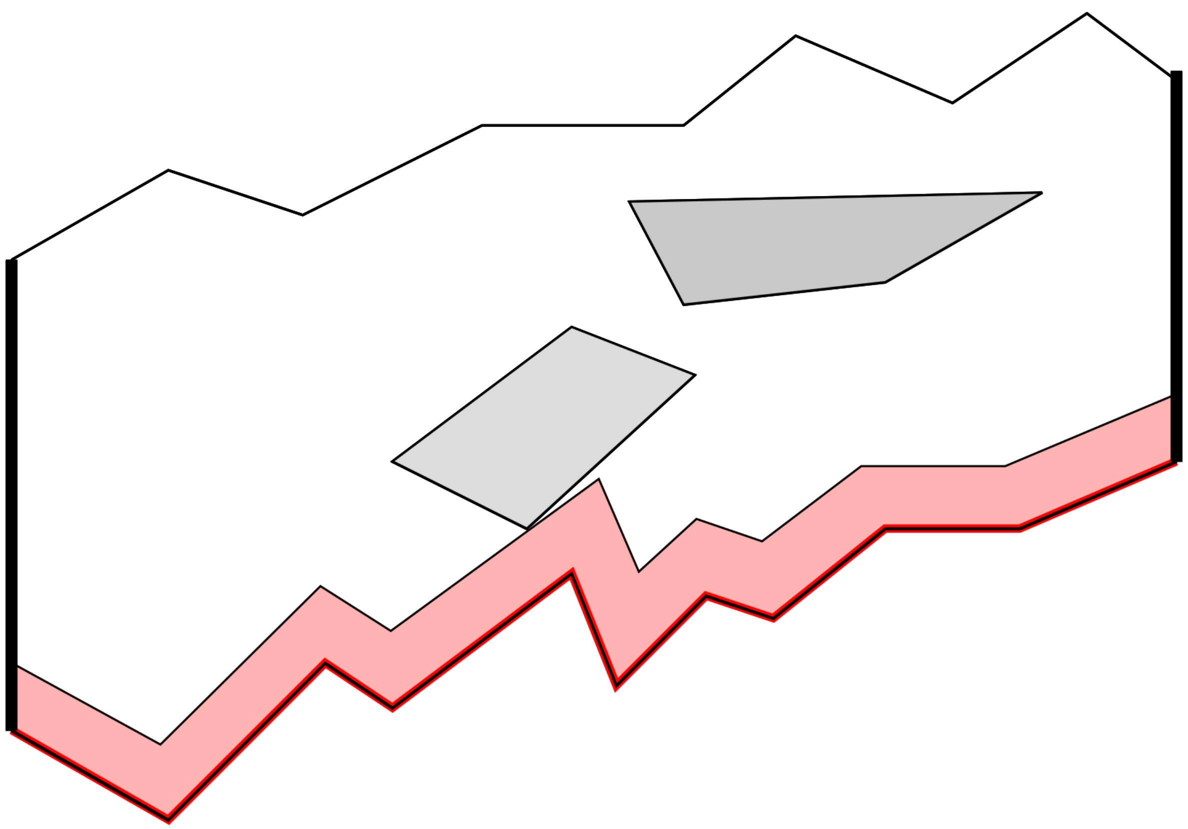

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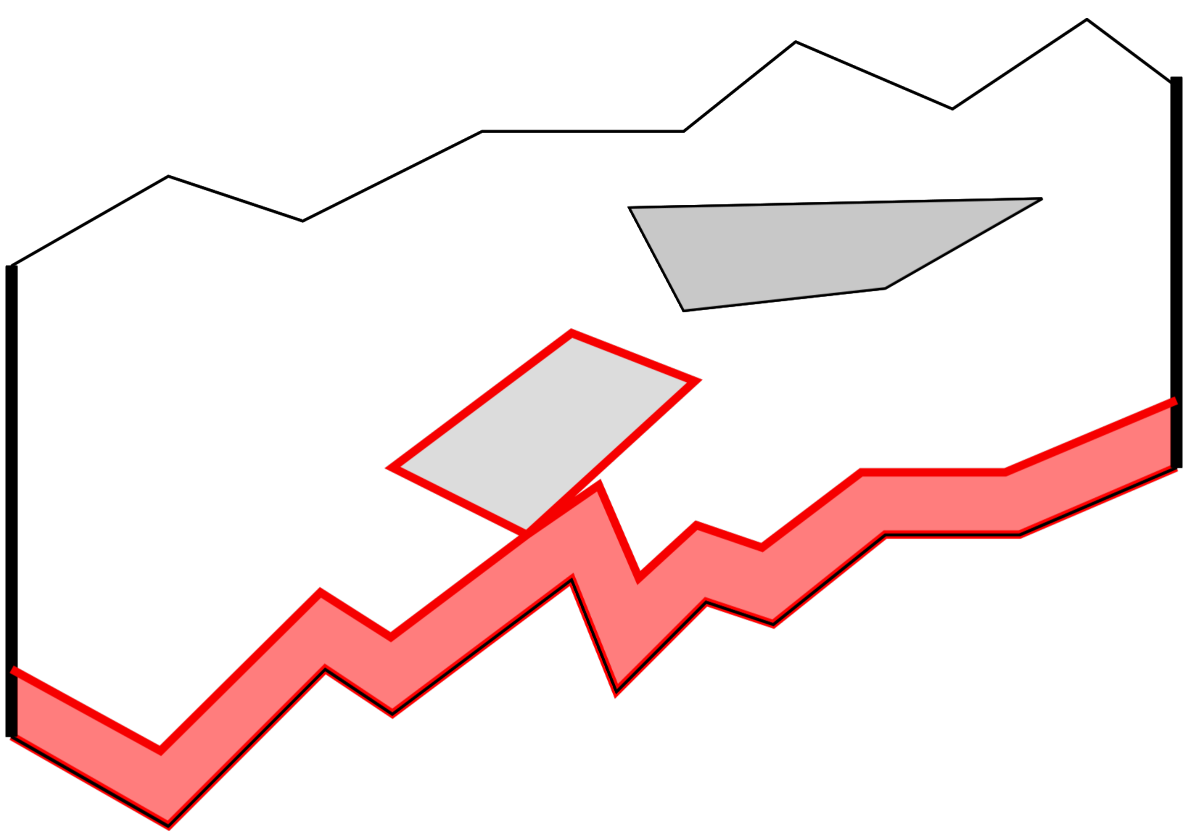

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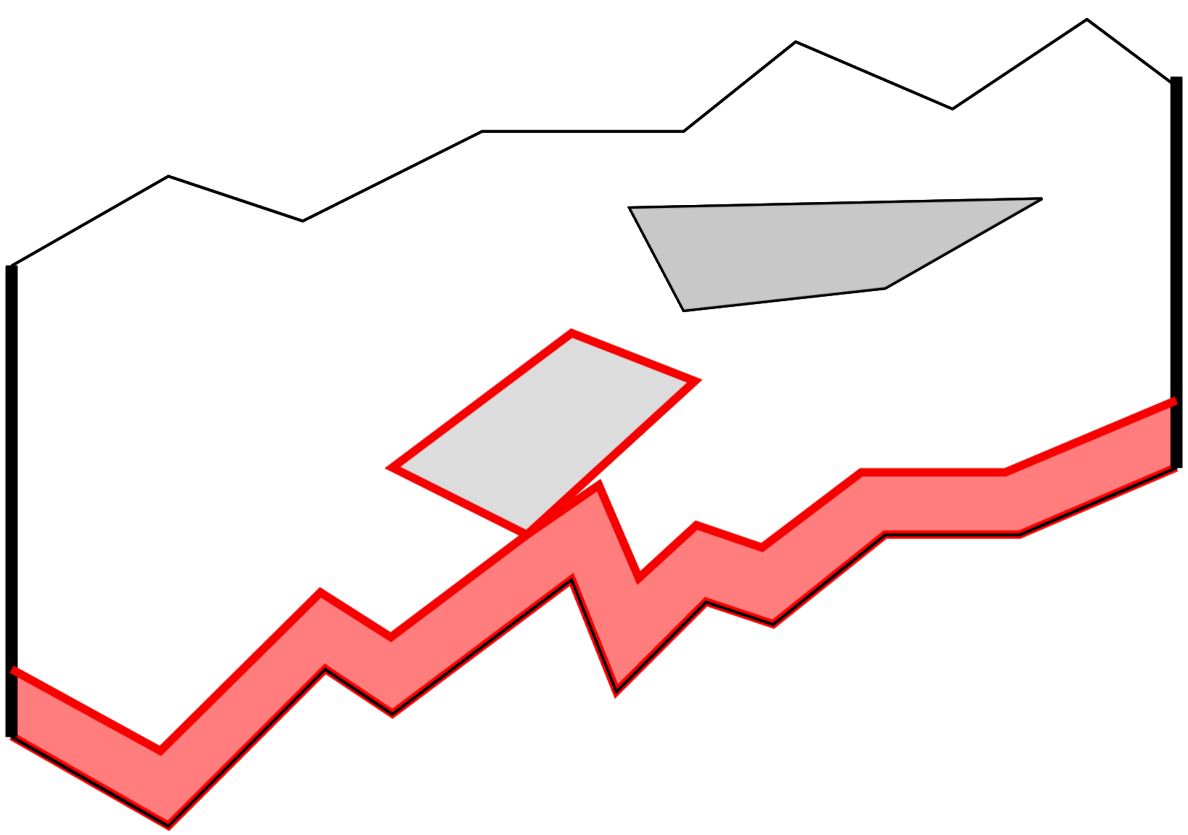
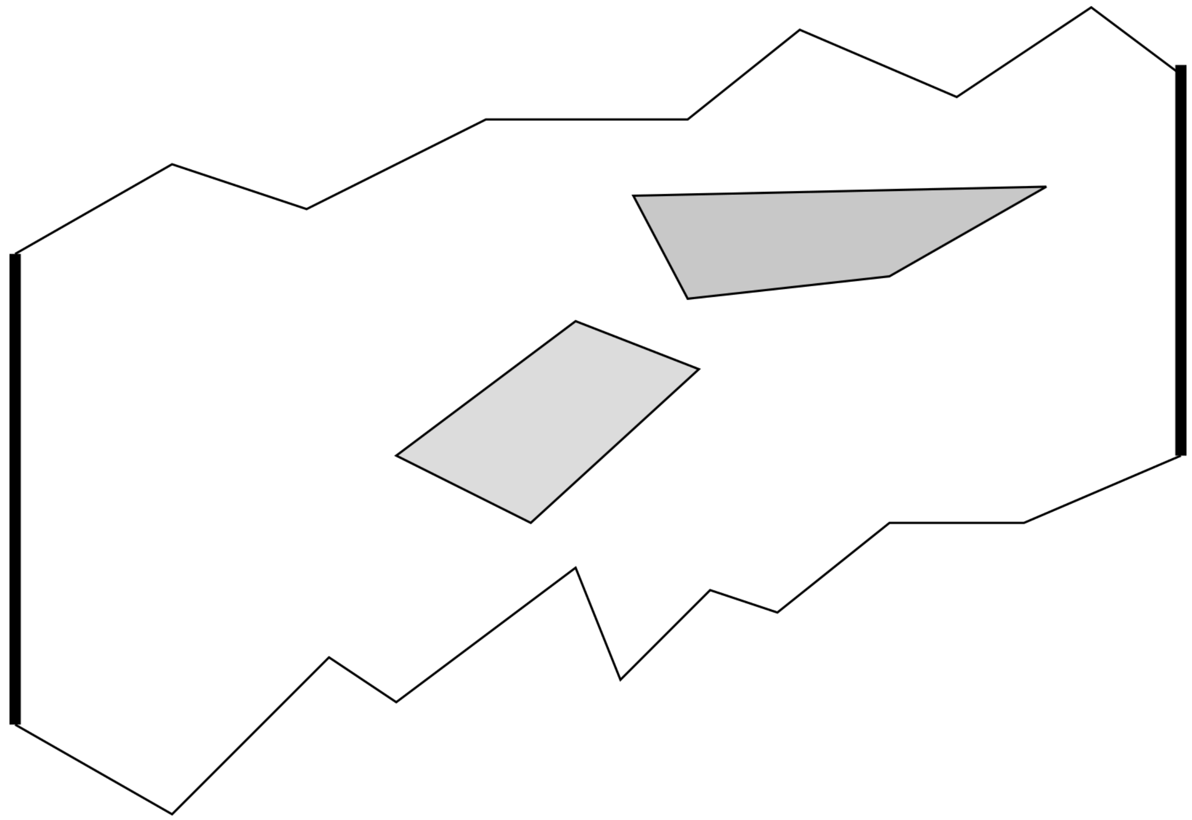
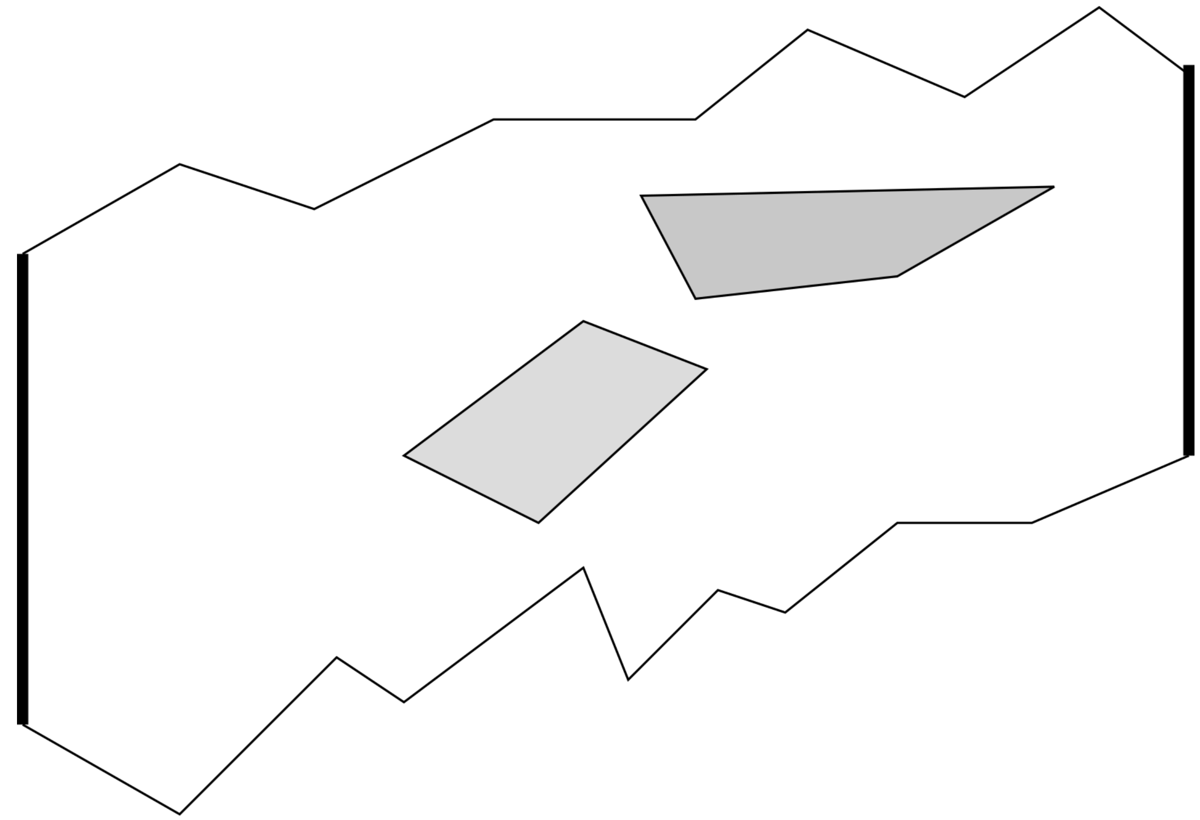
- Some additional tweaks when we hit a hole after $\tau<2$





Polishchuk (2007) extended this to x-monotone paths:



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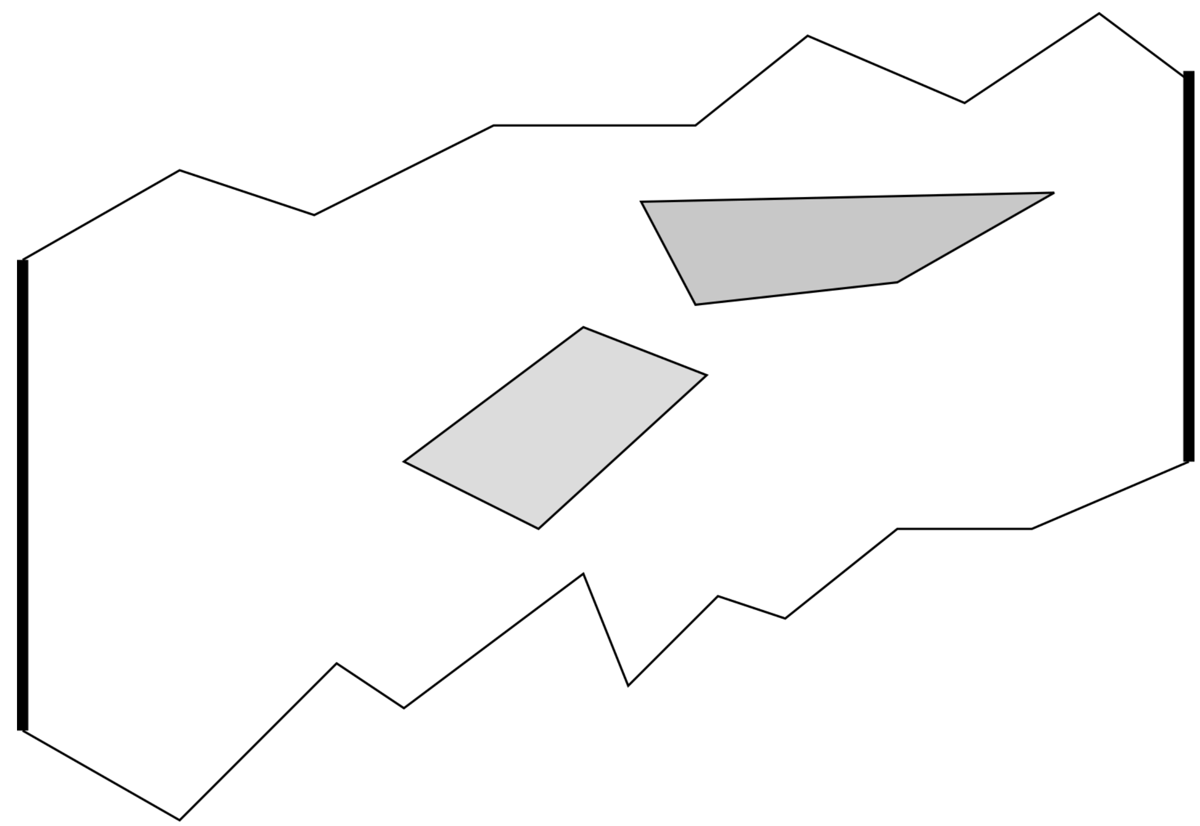

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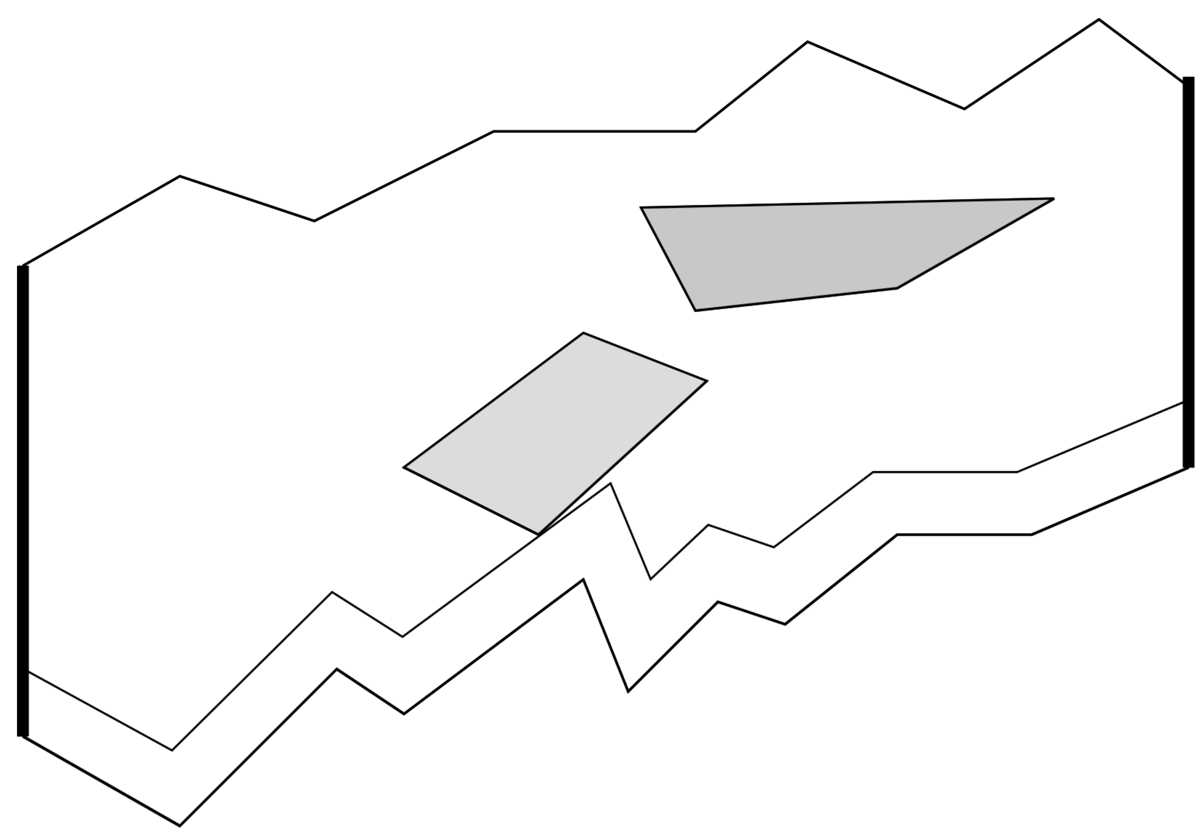

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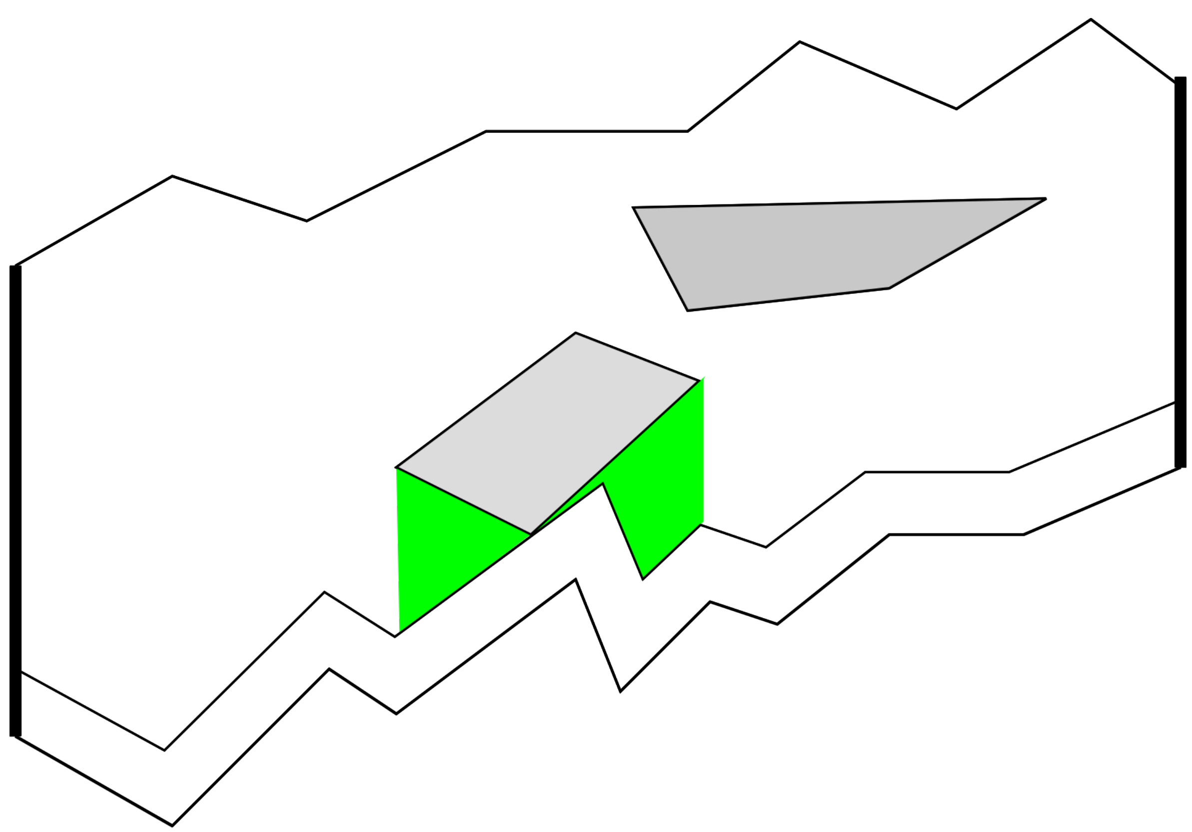

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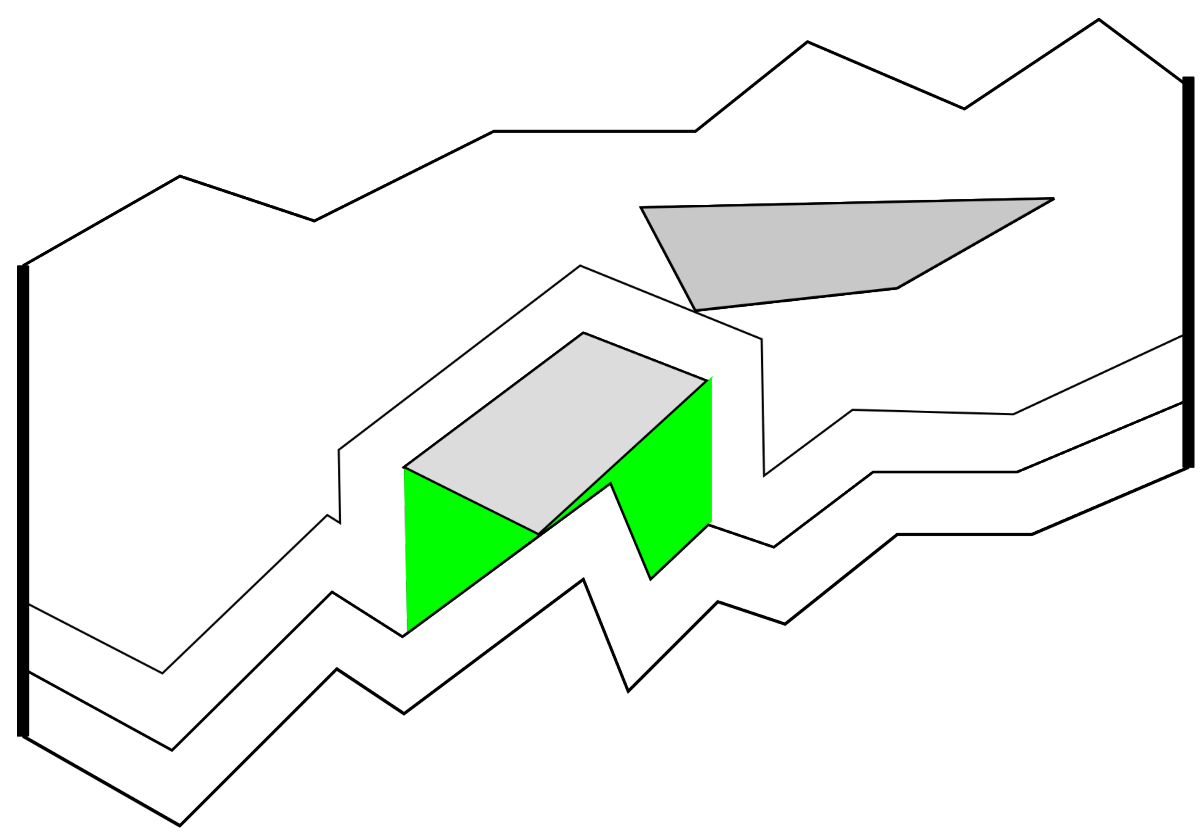

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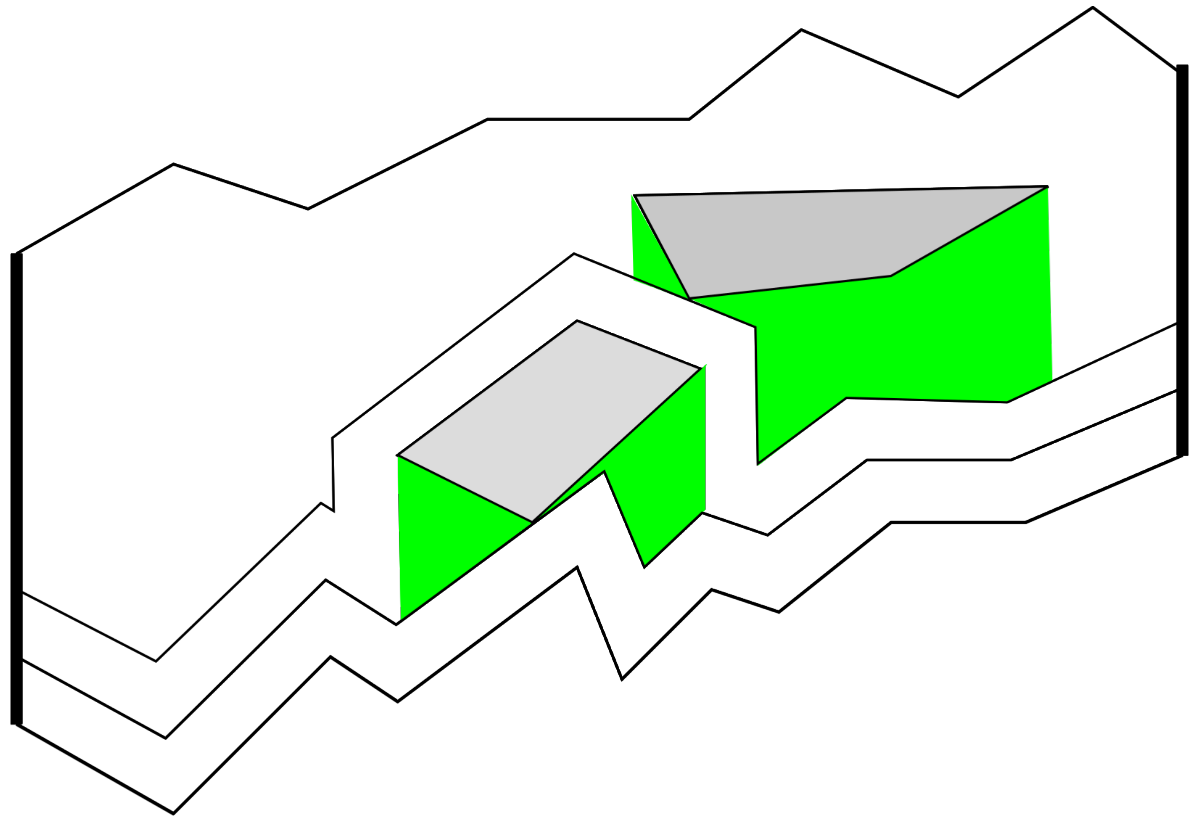

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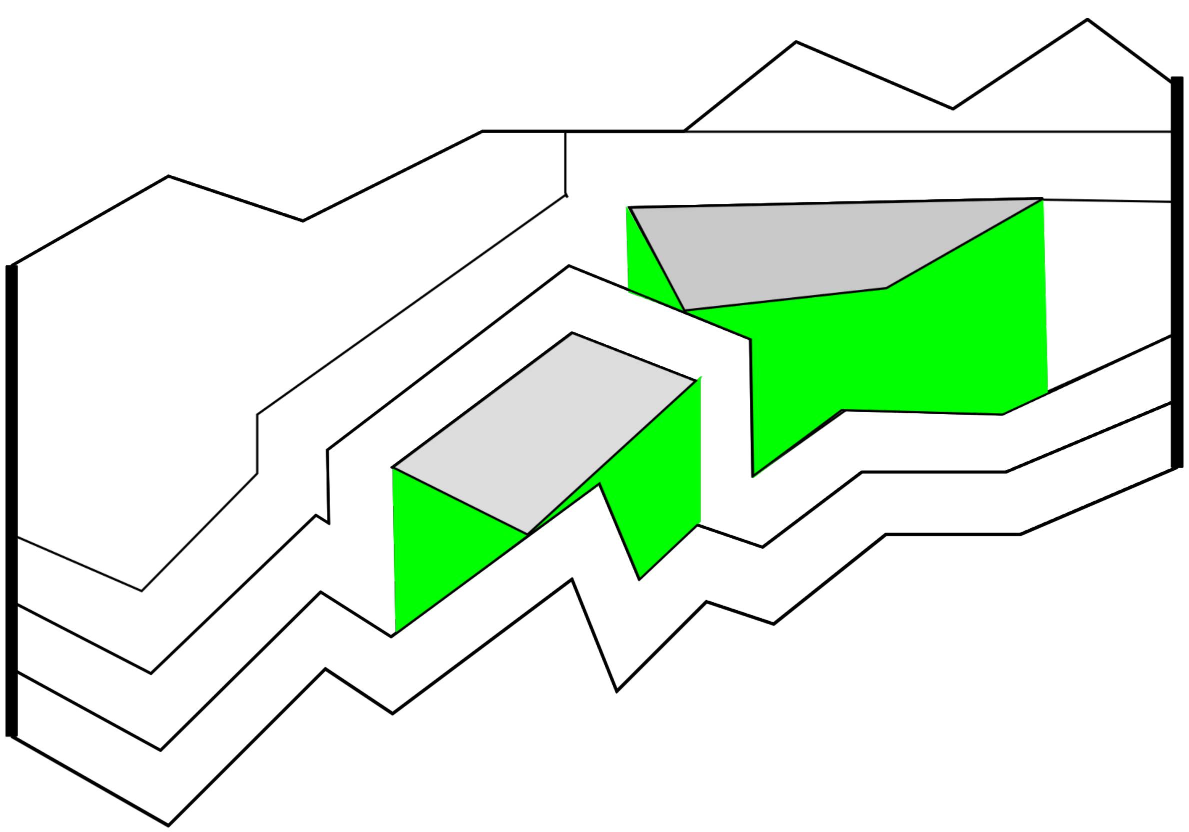

Thick Paths with Limited Slope

We want:

We want:

- Maximum number of non crossing thick paths from source to sink

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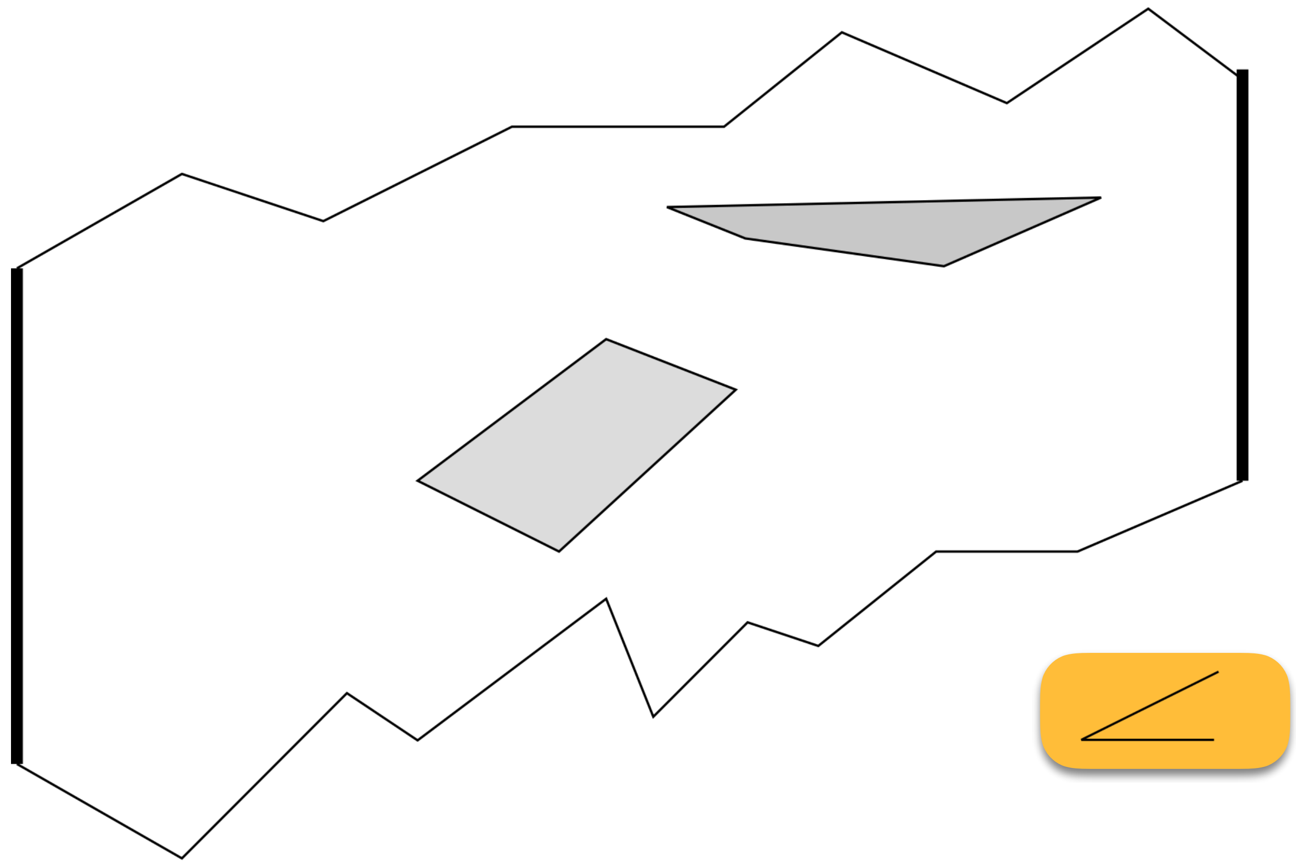
- Slope should be within a given cone C



We want:

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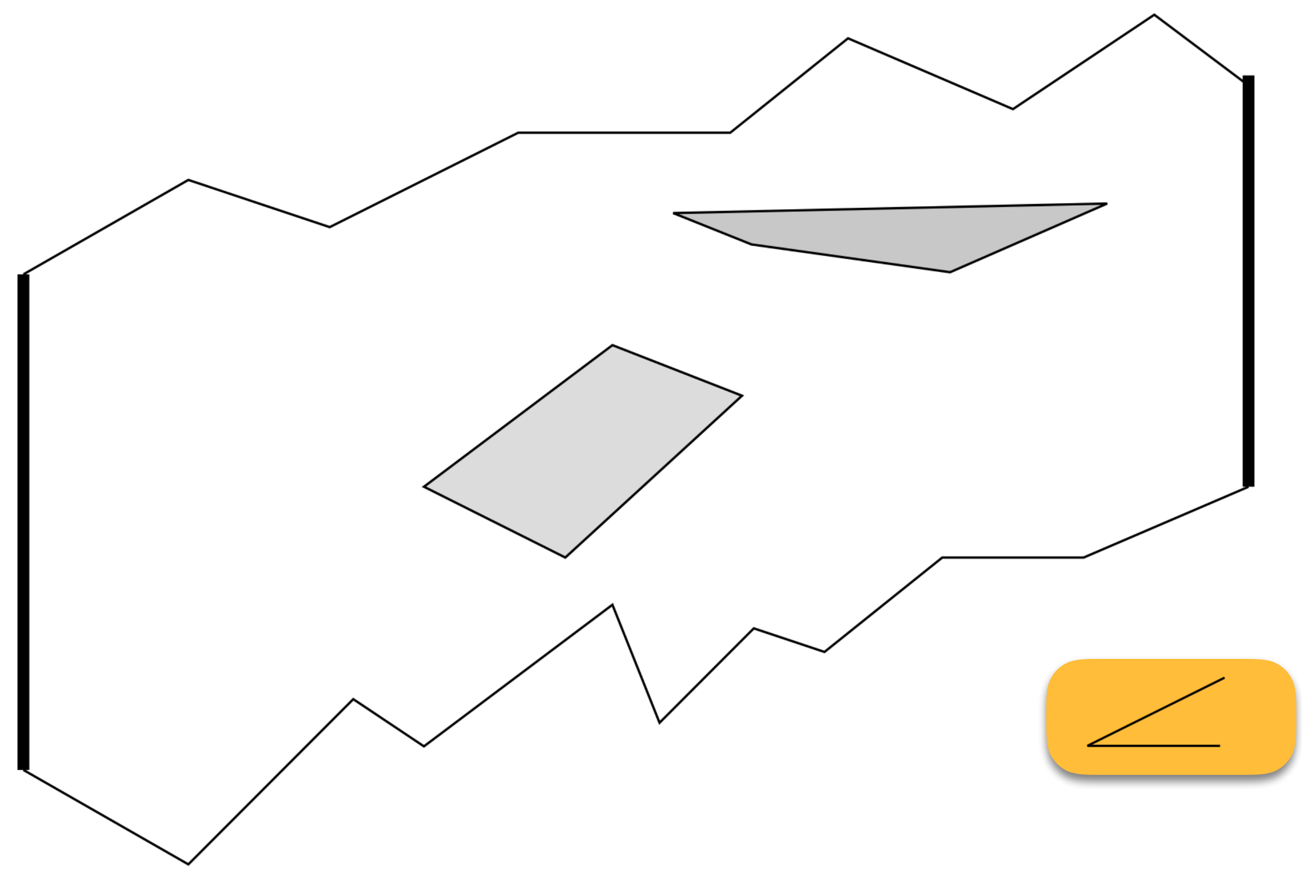
    - X-monotone



We want:

- Maximum number of non crossing thick paths from source to sink
- Slope should be within a given cone C
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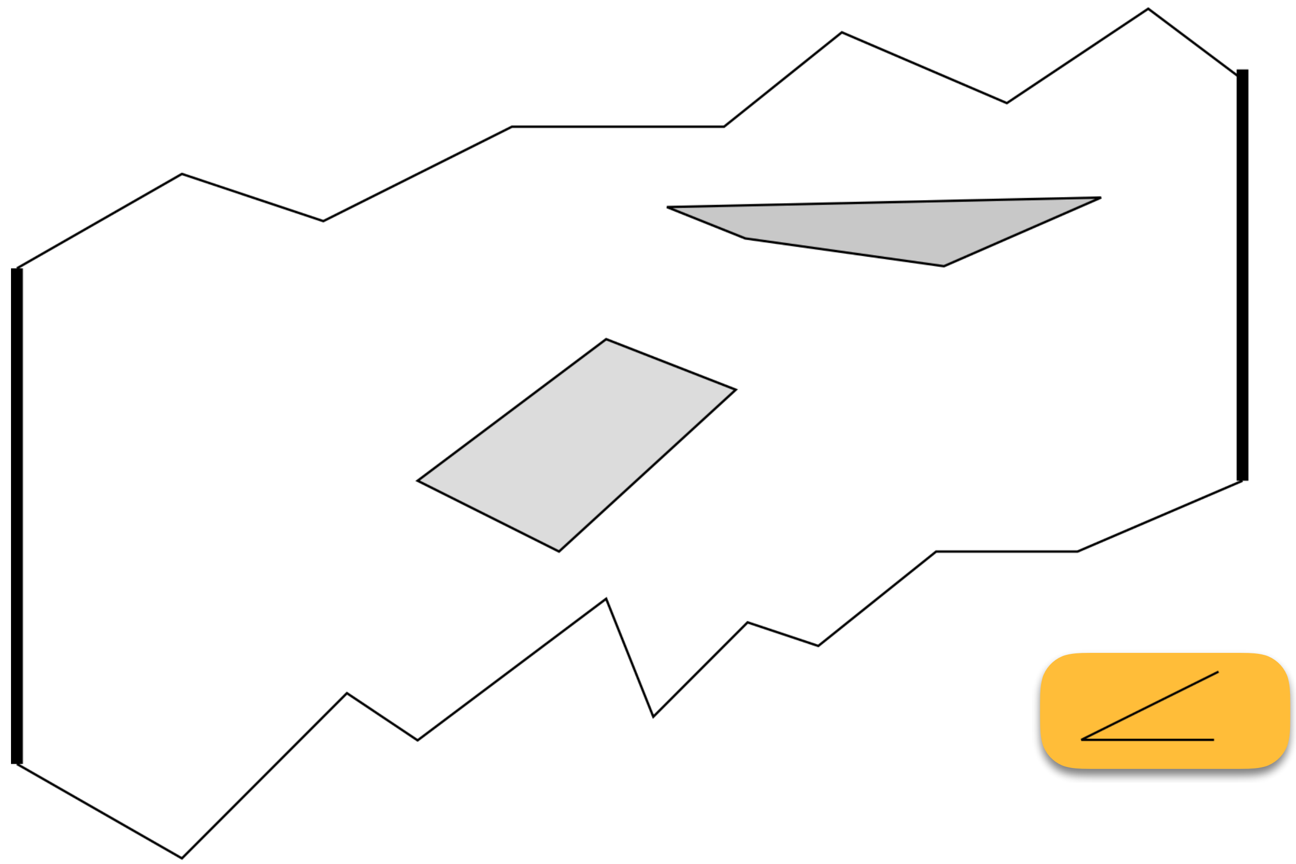
  - Limited speed ⇨ Limited slope



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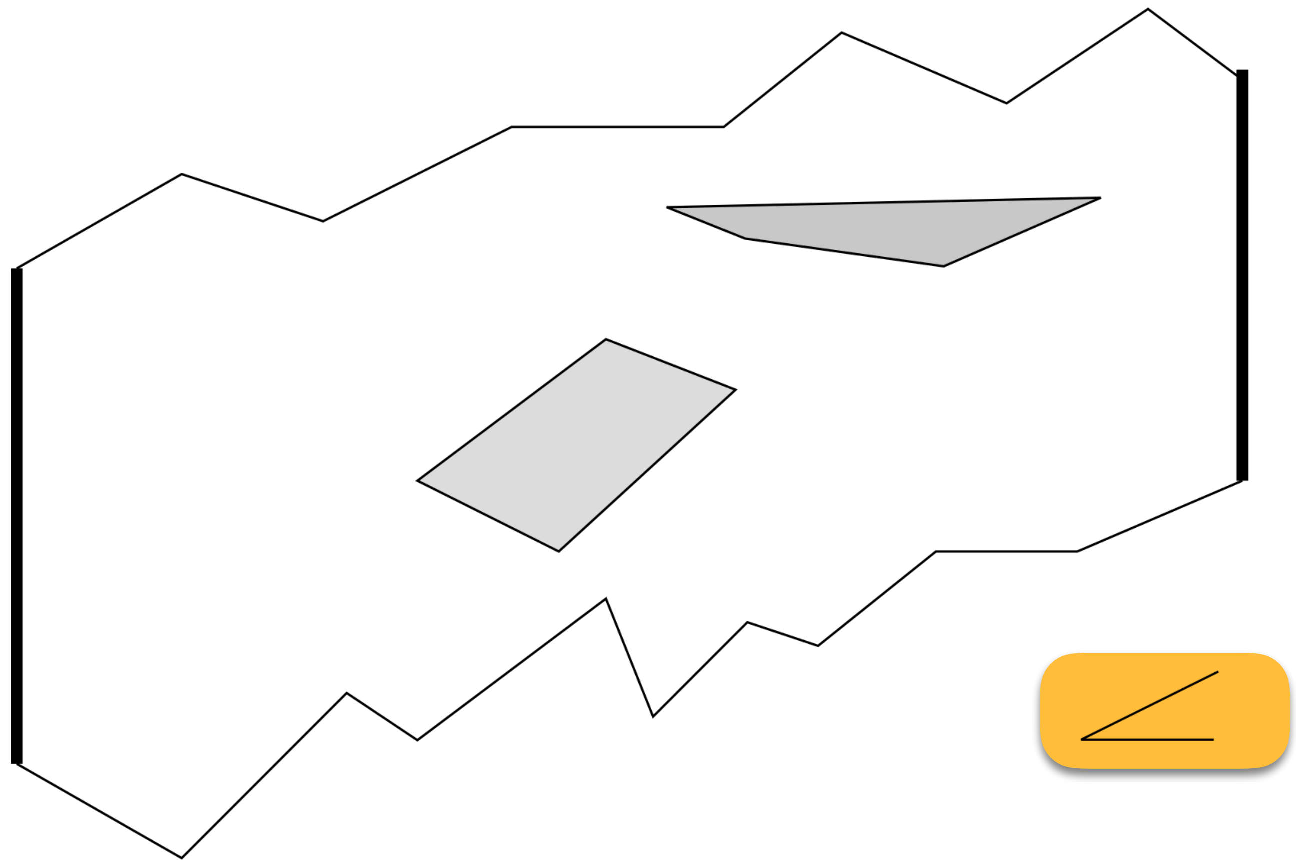
- We showed how to adapt the waterfall construction to compute the maximum number of thick non-crossing paths with a given slope range (≙C-respecting)



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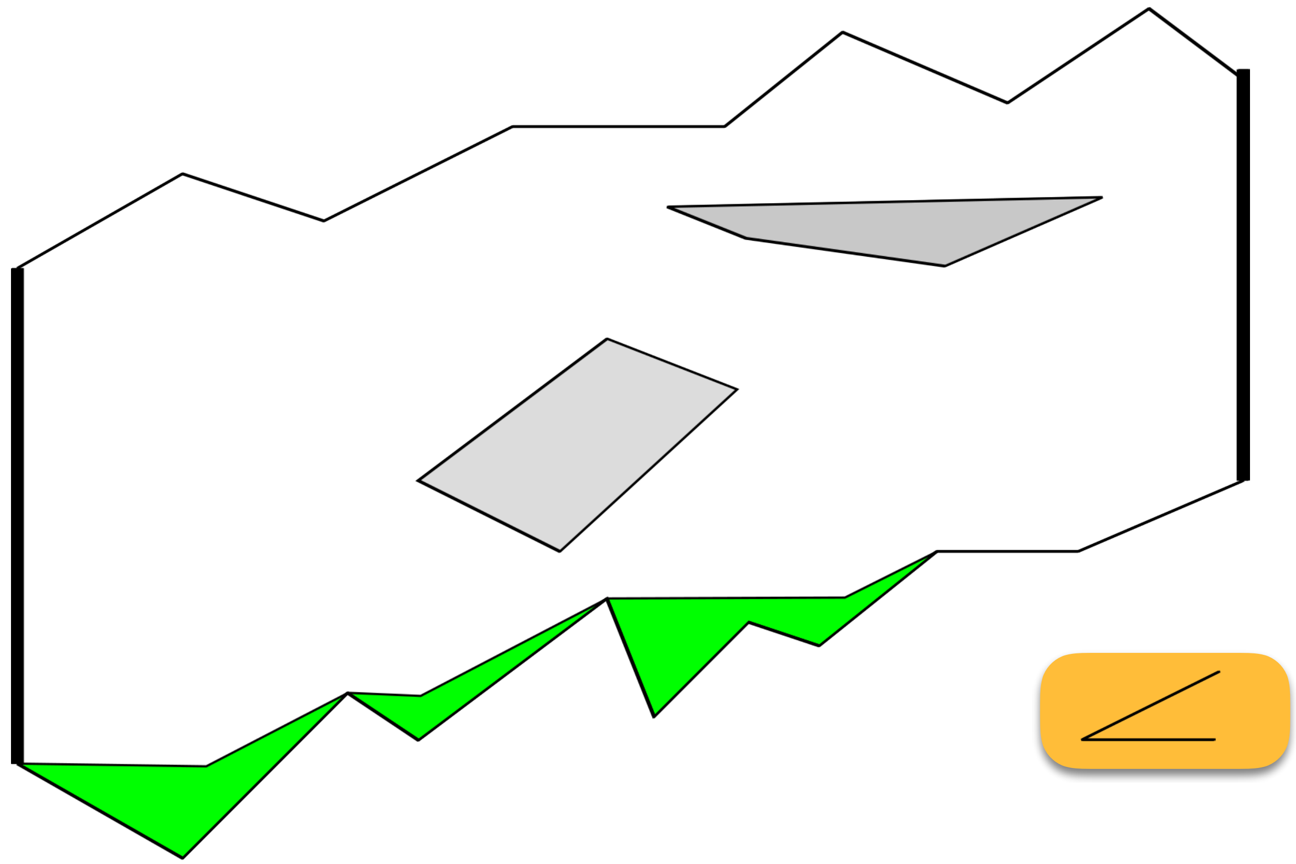

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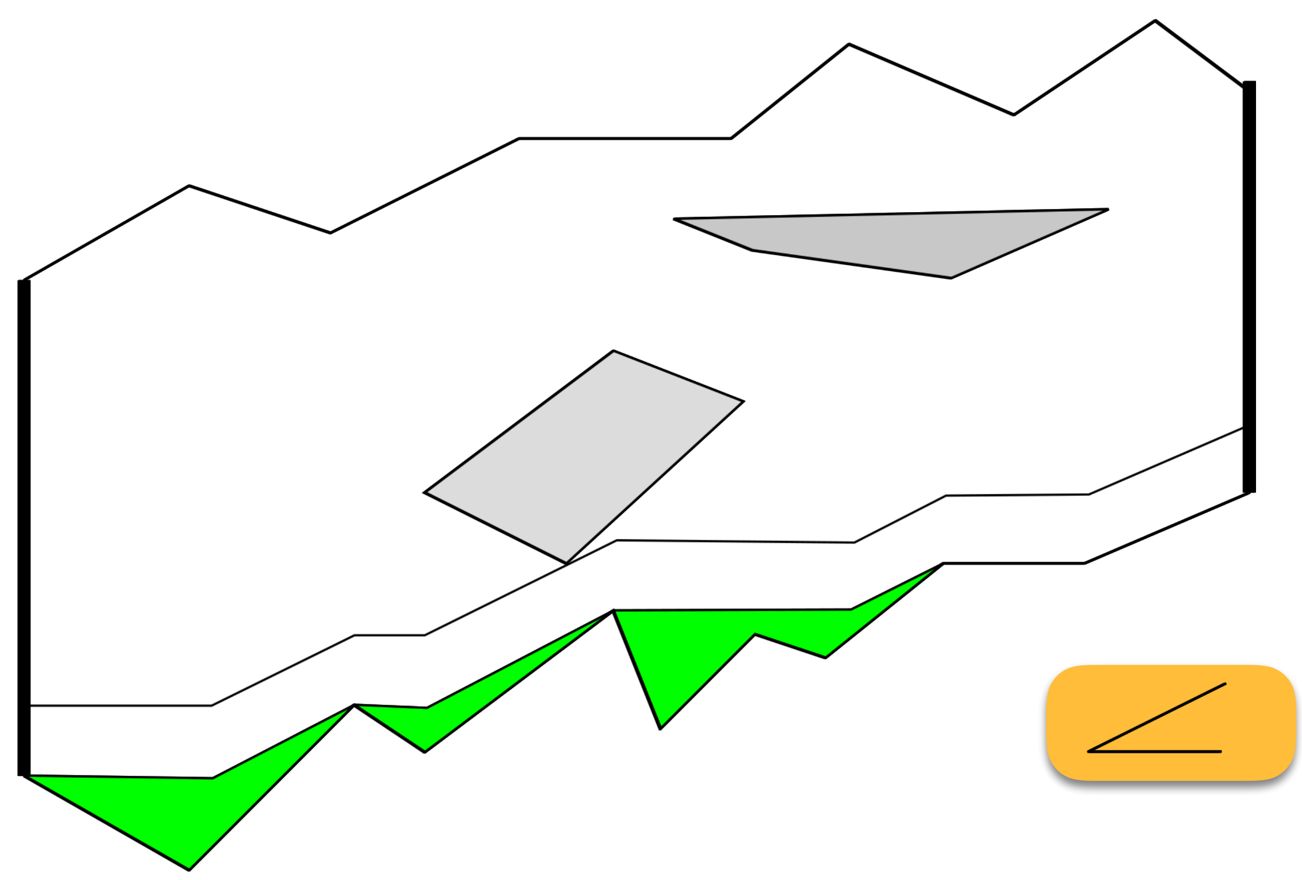

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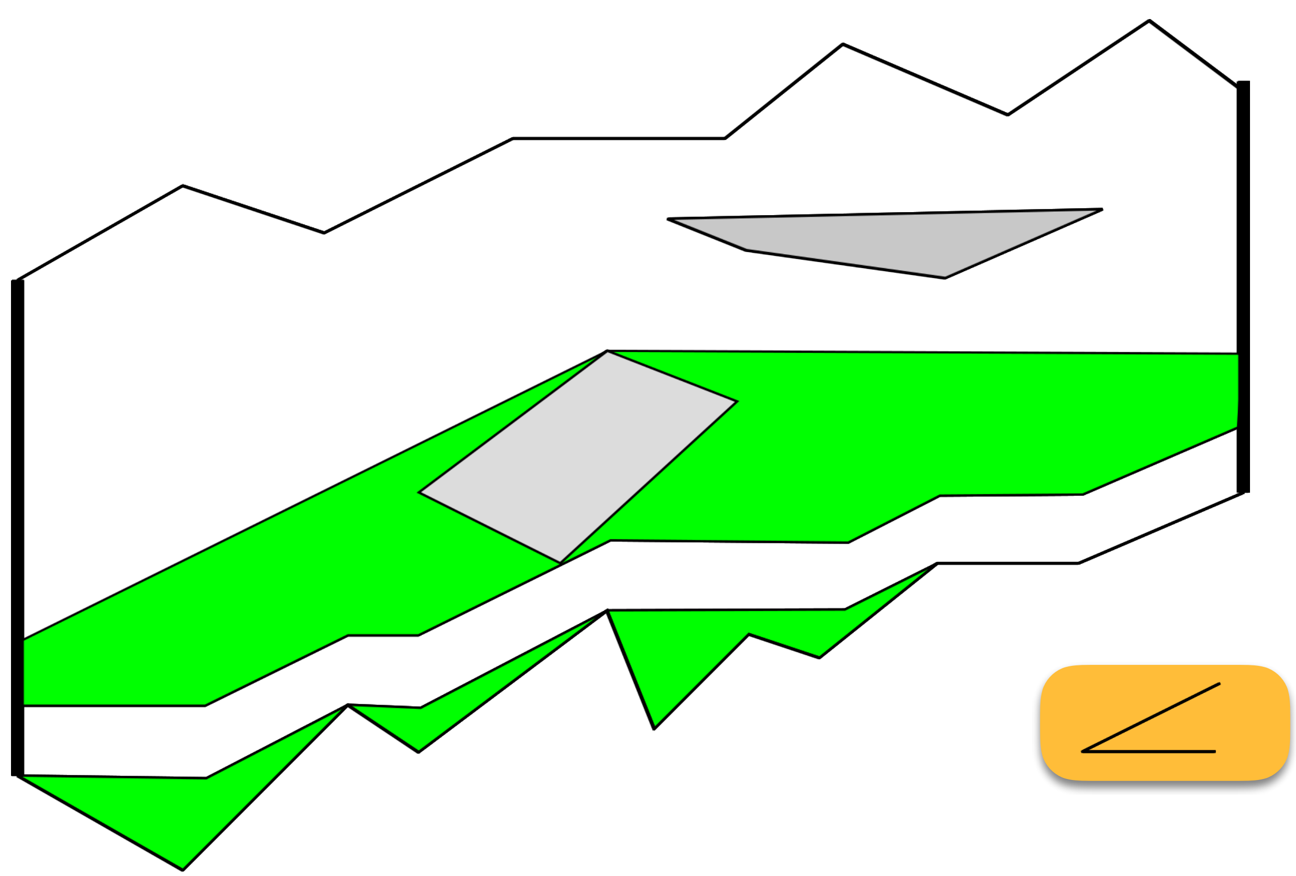

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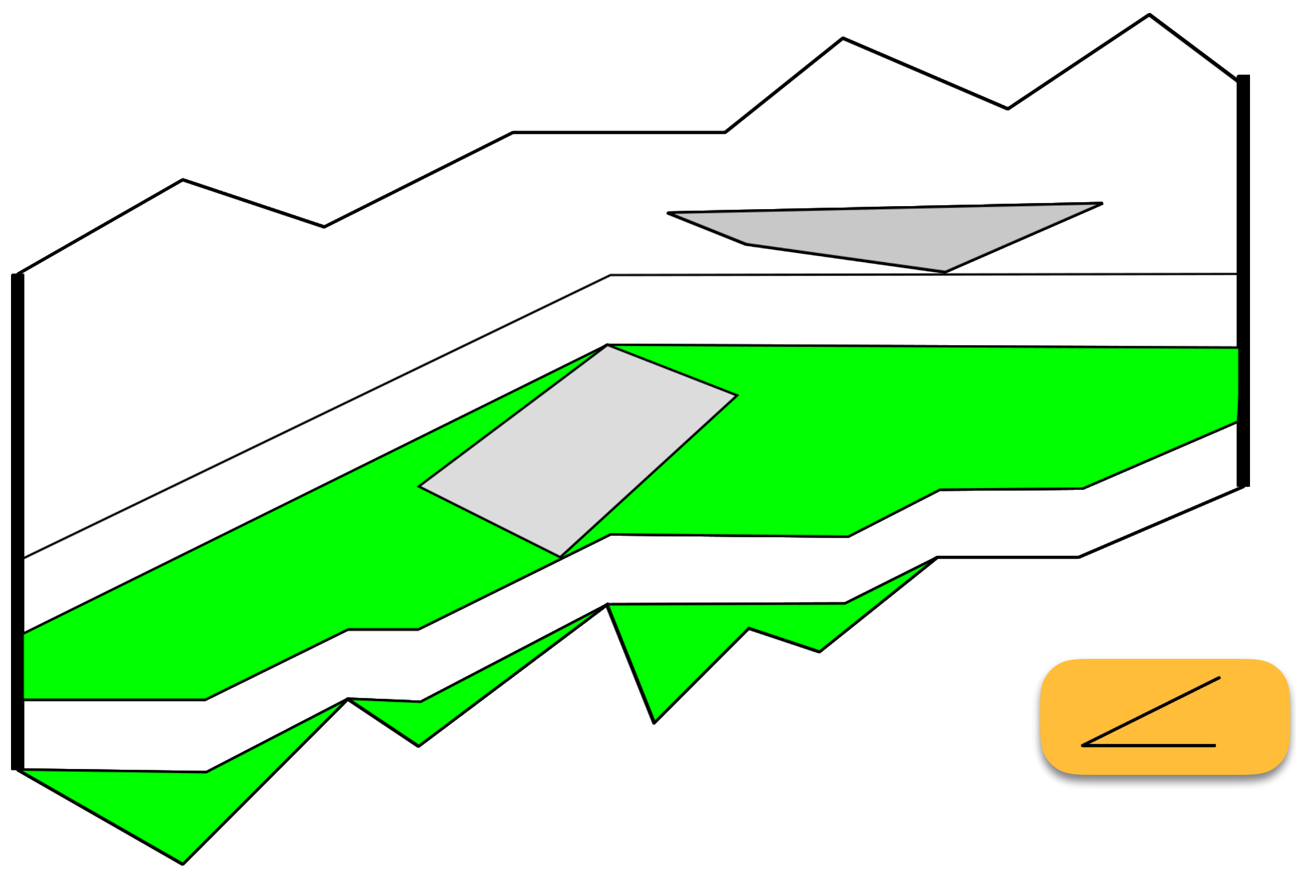

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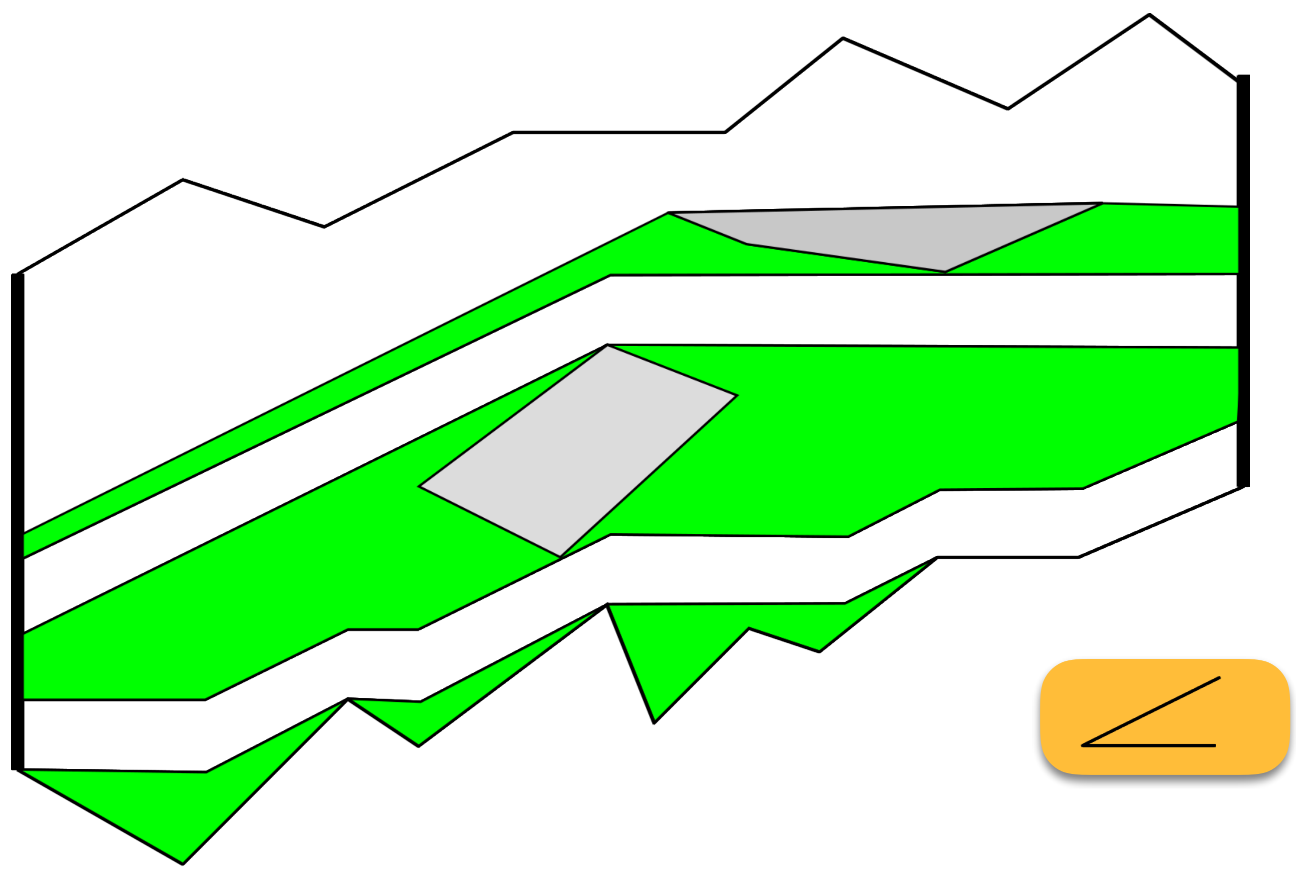

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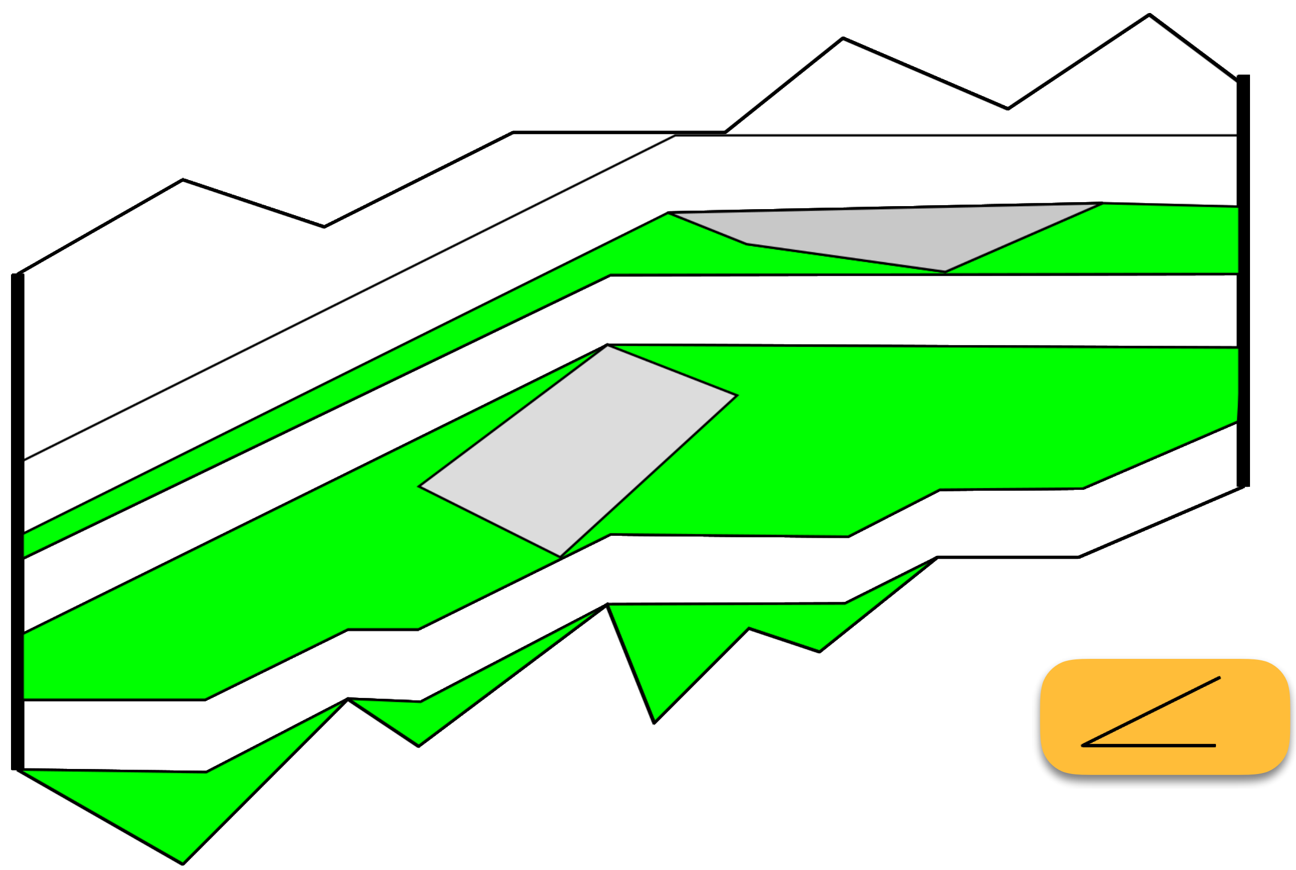

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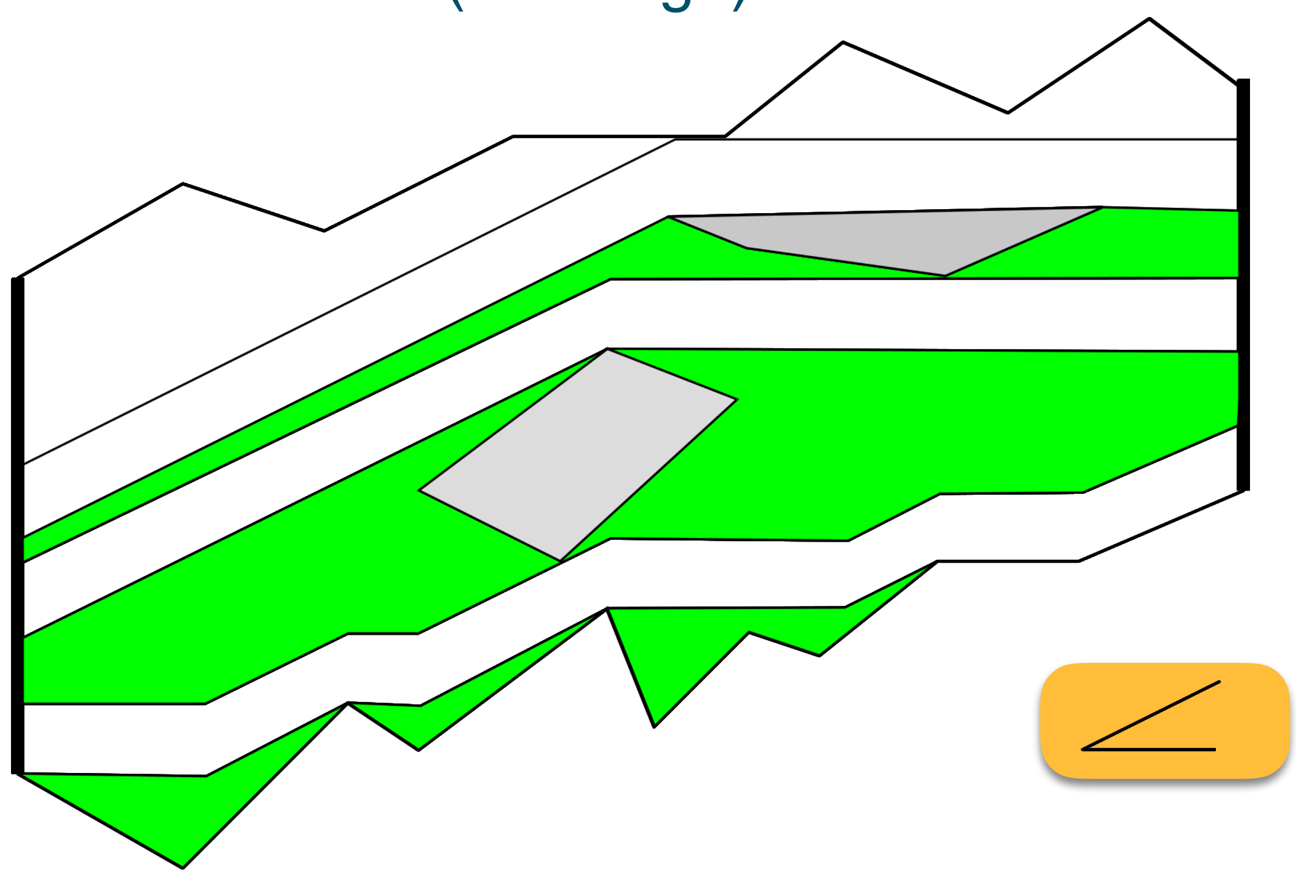
  Theorem: A representation of the maximum number of C-respecting thick-non-crossing paths can be found in O(nh+nlogn) time.



- We consider the time-space diagram—the geometric representation
- Inserting a new train: Route path from start to end station
- Paths not arbitrarily close ⇨ temporal distance (different to trains running in same or opposite direction)
- We think of train paths as “blown-up” line segments = thick paths
- Blown up by temporal distance (can be minimum, or more)
- How to route those thick paths? Concepts from Computational Geometry
- Need to make some adaptations, for example, if stations are lines, no path could cross these

➡ 1. Show how to construct the appropriate polygonal domain

➡ 2. Show how to route the maximum number of thick non-crossing paths in that domain:

  - Paths should be x-monotone (we cannot go back in time)
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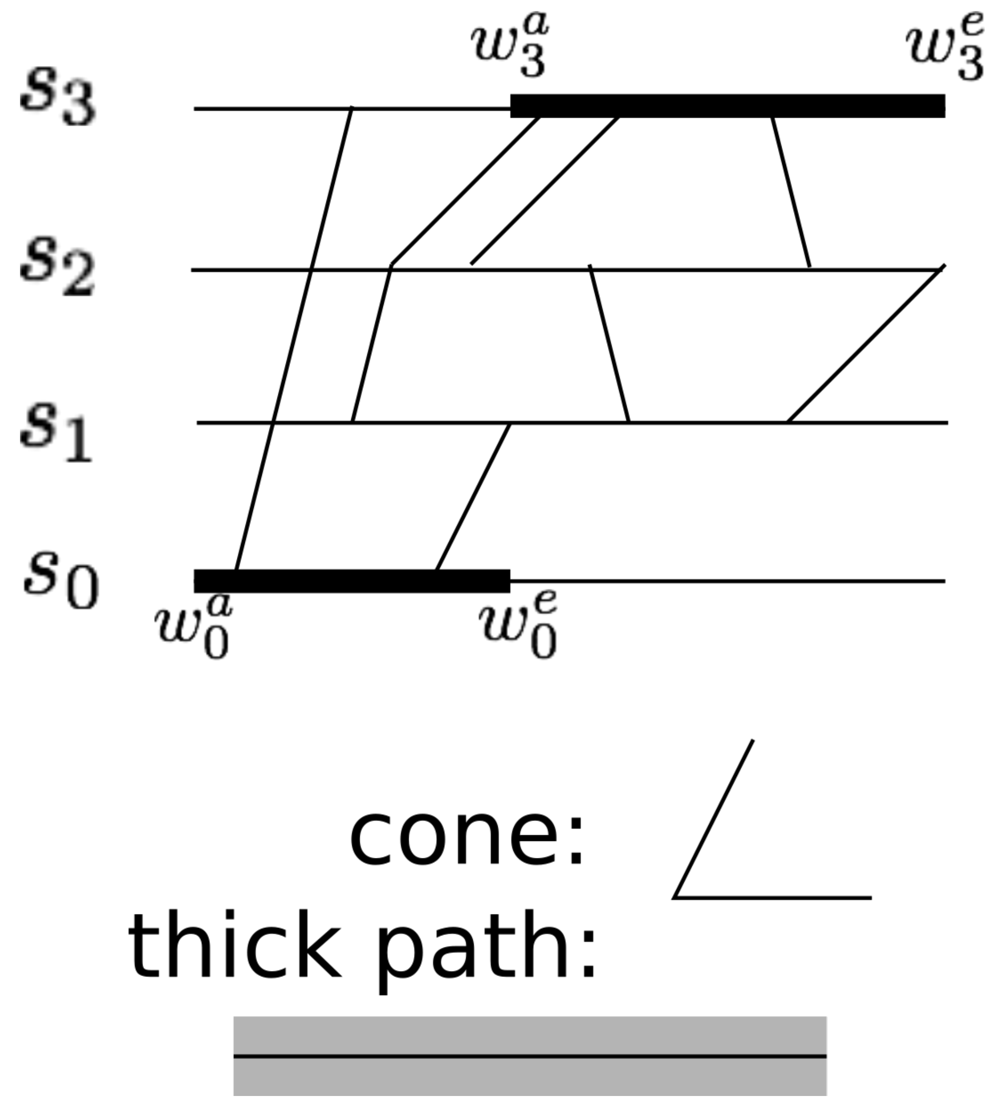
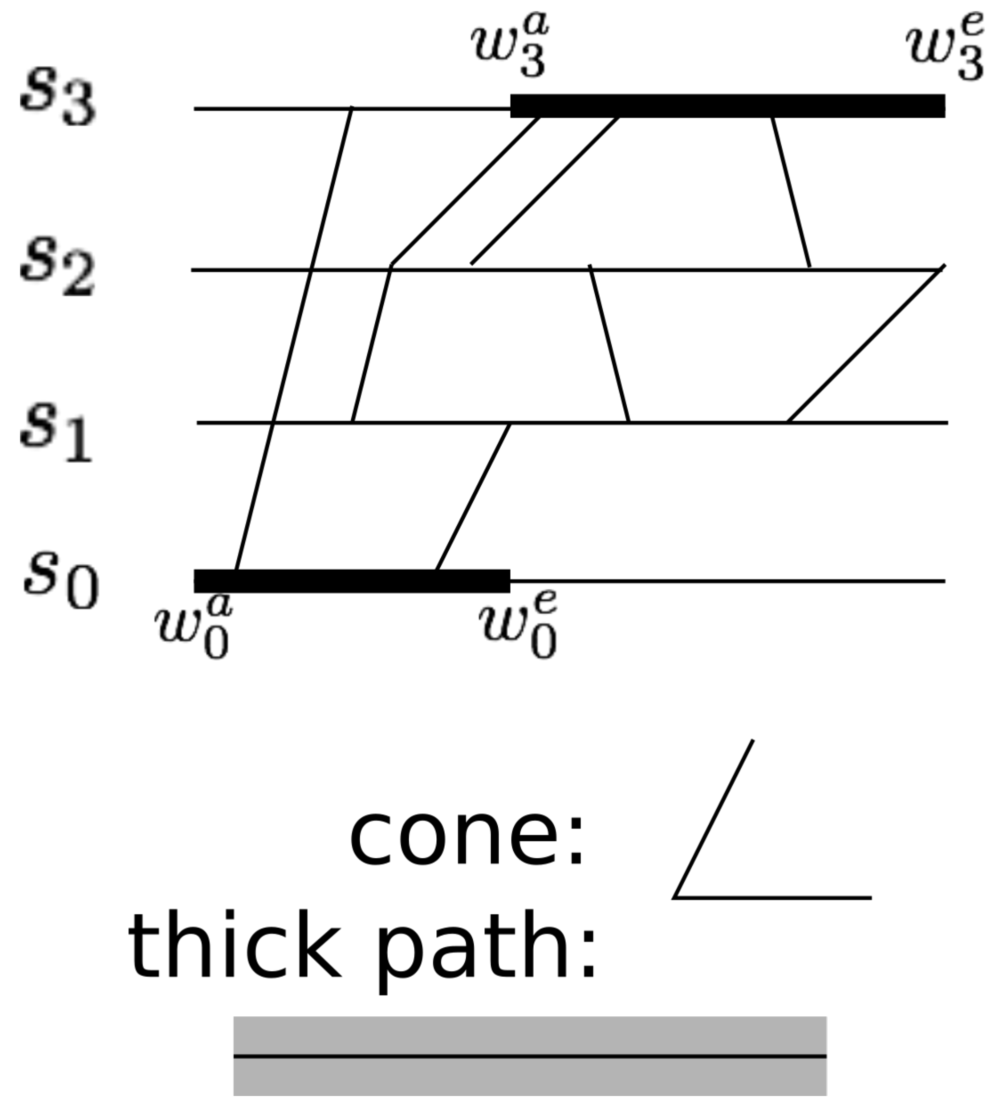
Still left to do

Construction of Polygonal Domain from the Timetable



s_3
s_2
s_1
s_0
w_3^a
w_3^e
w_0^a
w_0^e
cone:
thick path:

If we would define the time windows as source and sink



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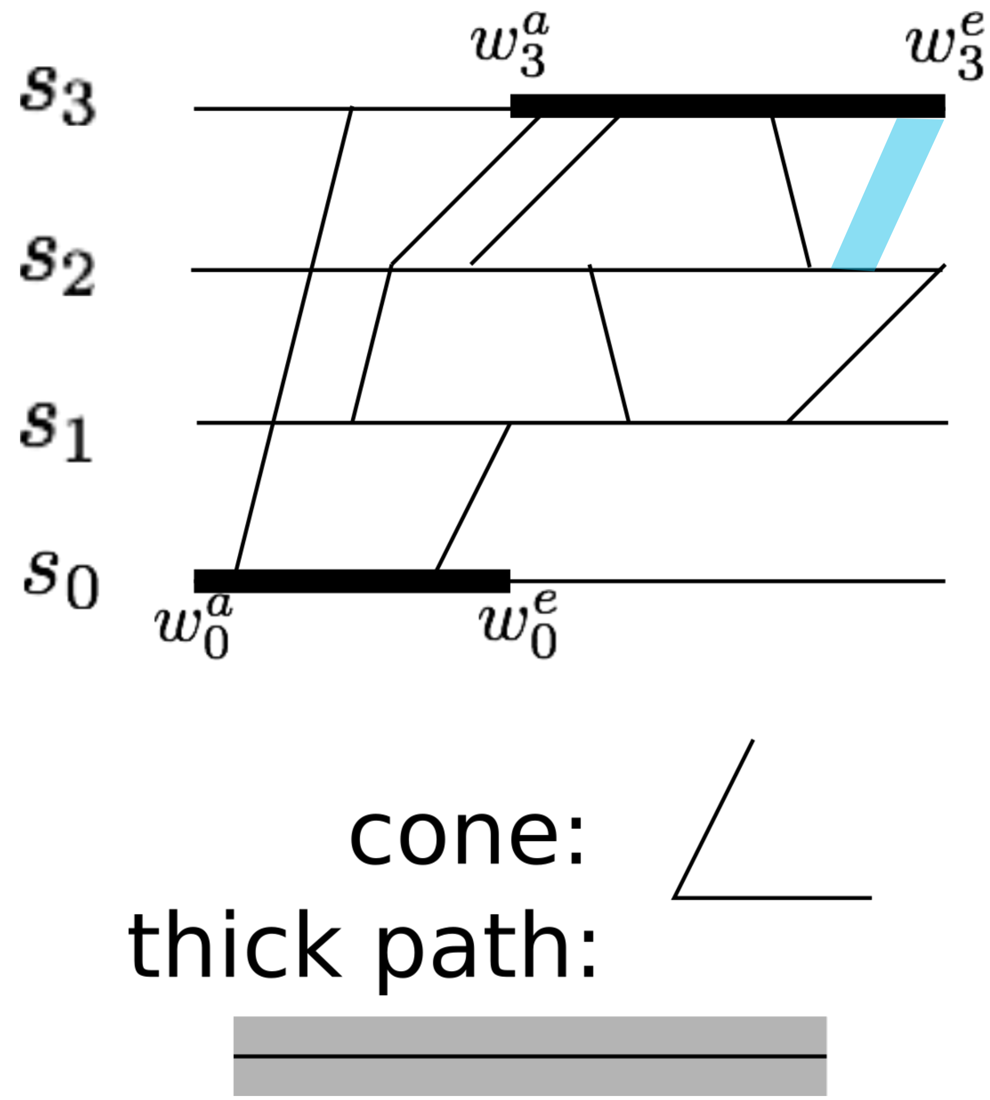

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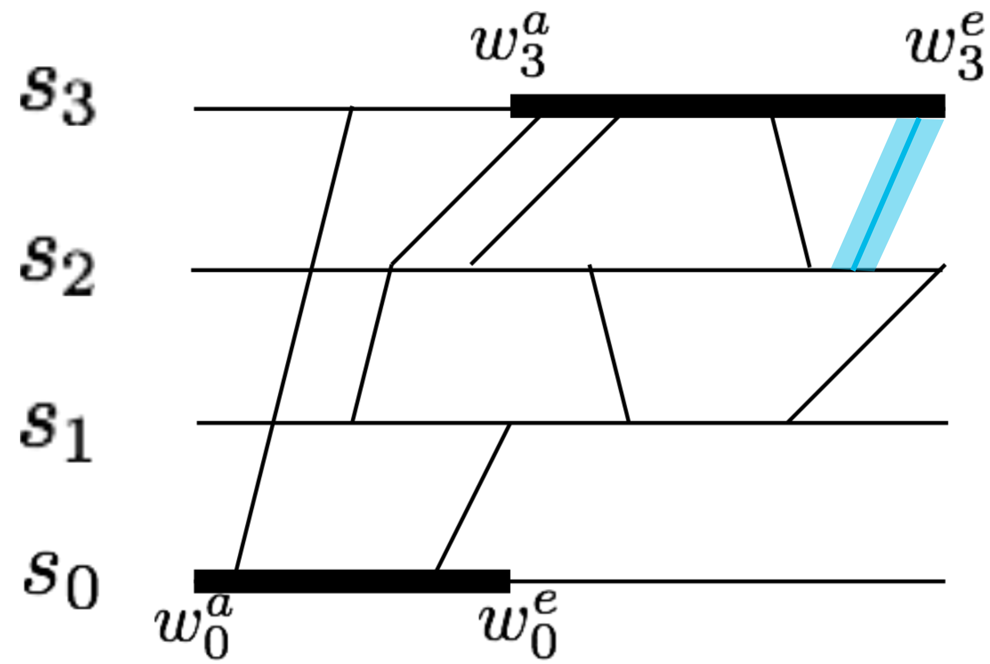
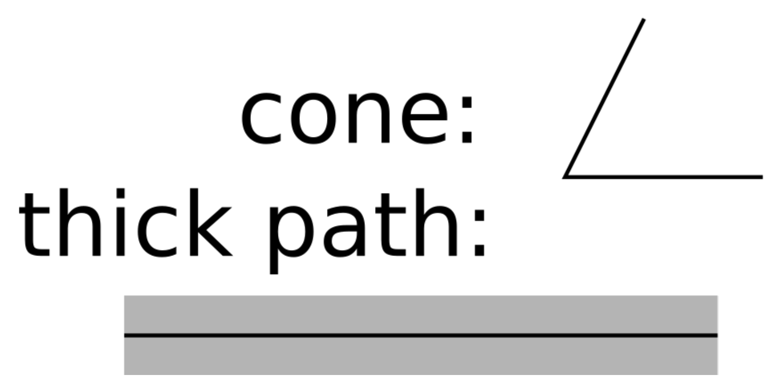
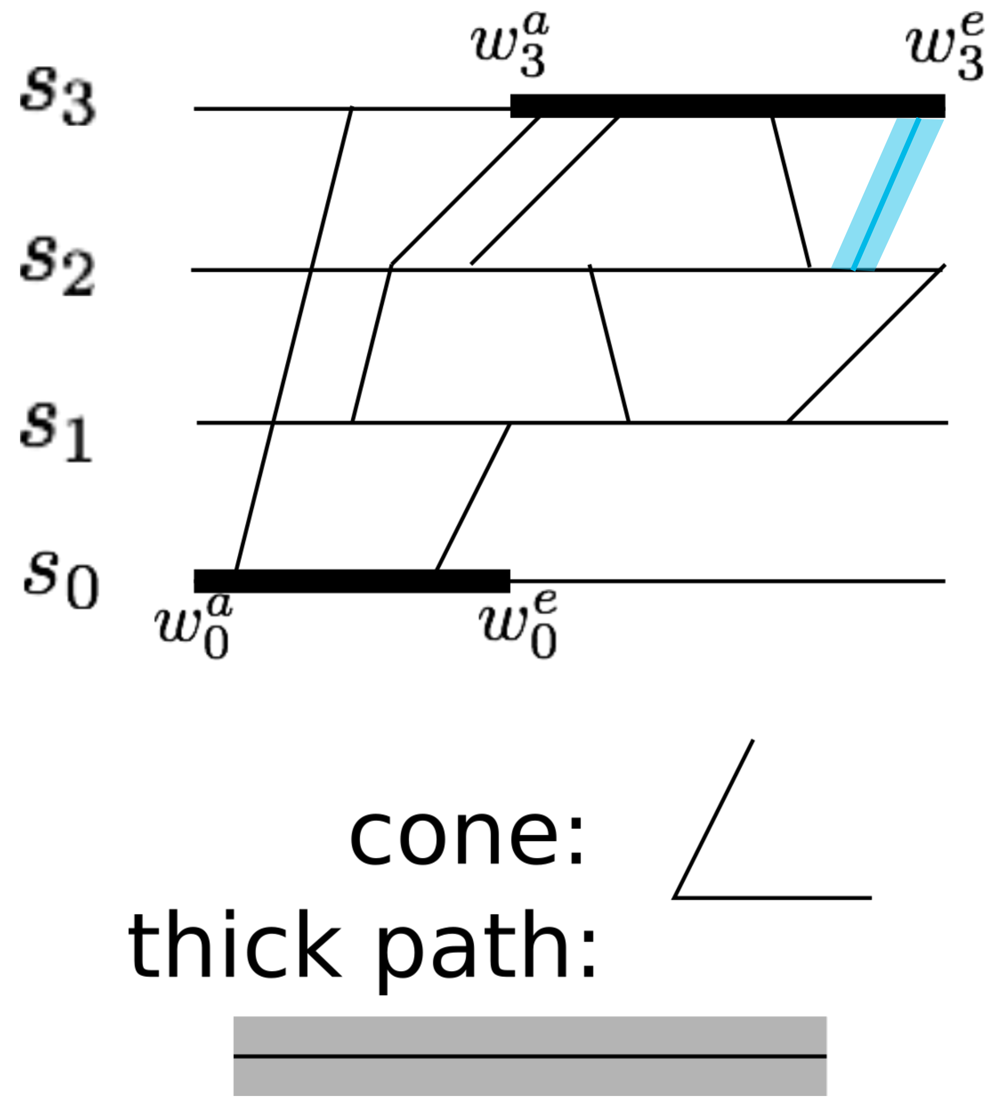




If we would define the time windows as source and sink
⇨ Possible thick paths would correspond to train paths in a smaller time interval

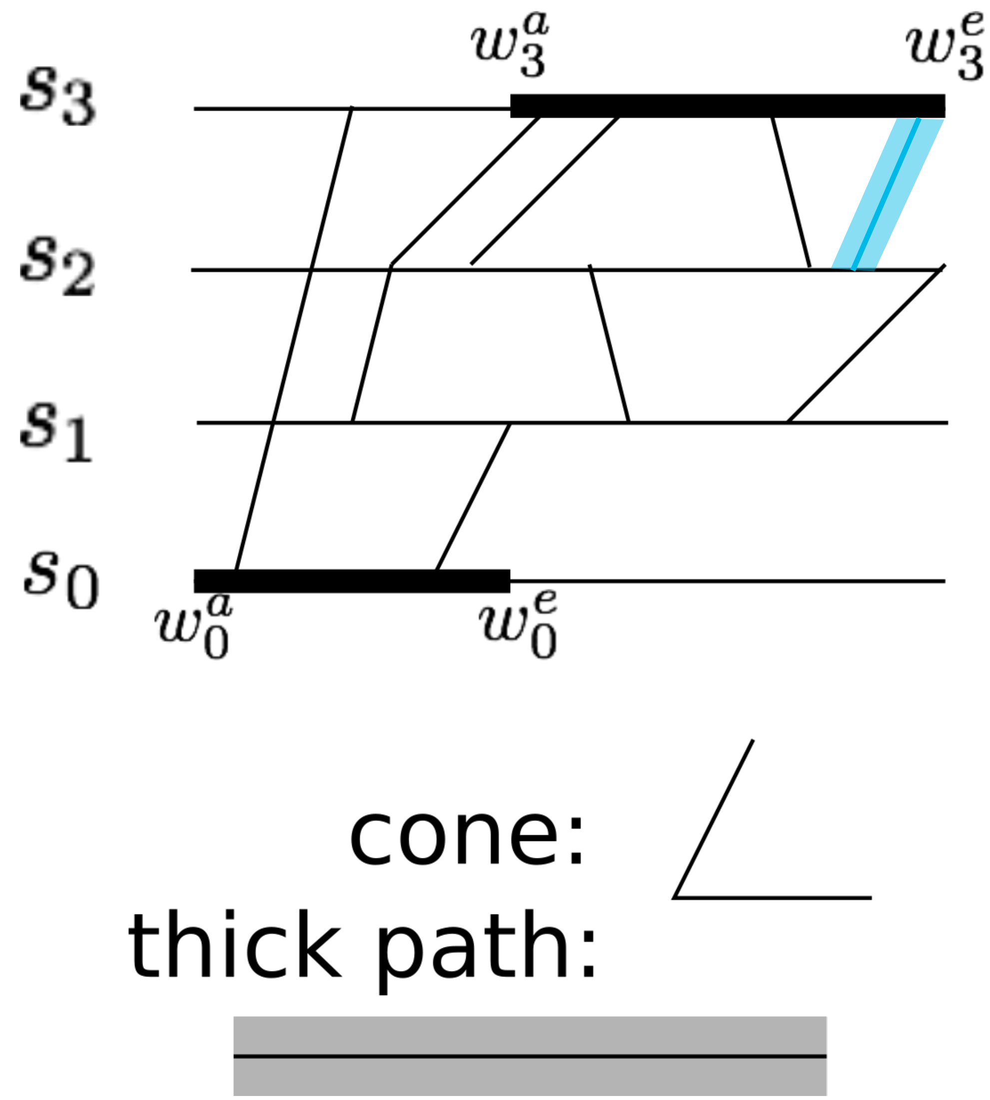


If we would define the time windows as source and sink

⇨ Possible thick paths would correspond to train paths in a smaller time interval

⇨ Extend the time windows by d/2to both sides to create $\mathbf{\Gamma}_s$ and $\mathbf{\Gamma}_t$

($\mathbf{\Gamma}_s=[p_1,p_2]$, $\mathbf{\Gamma}_t=[p_3,p_4]$)



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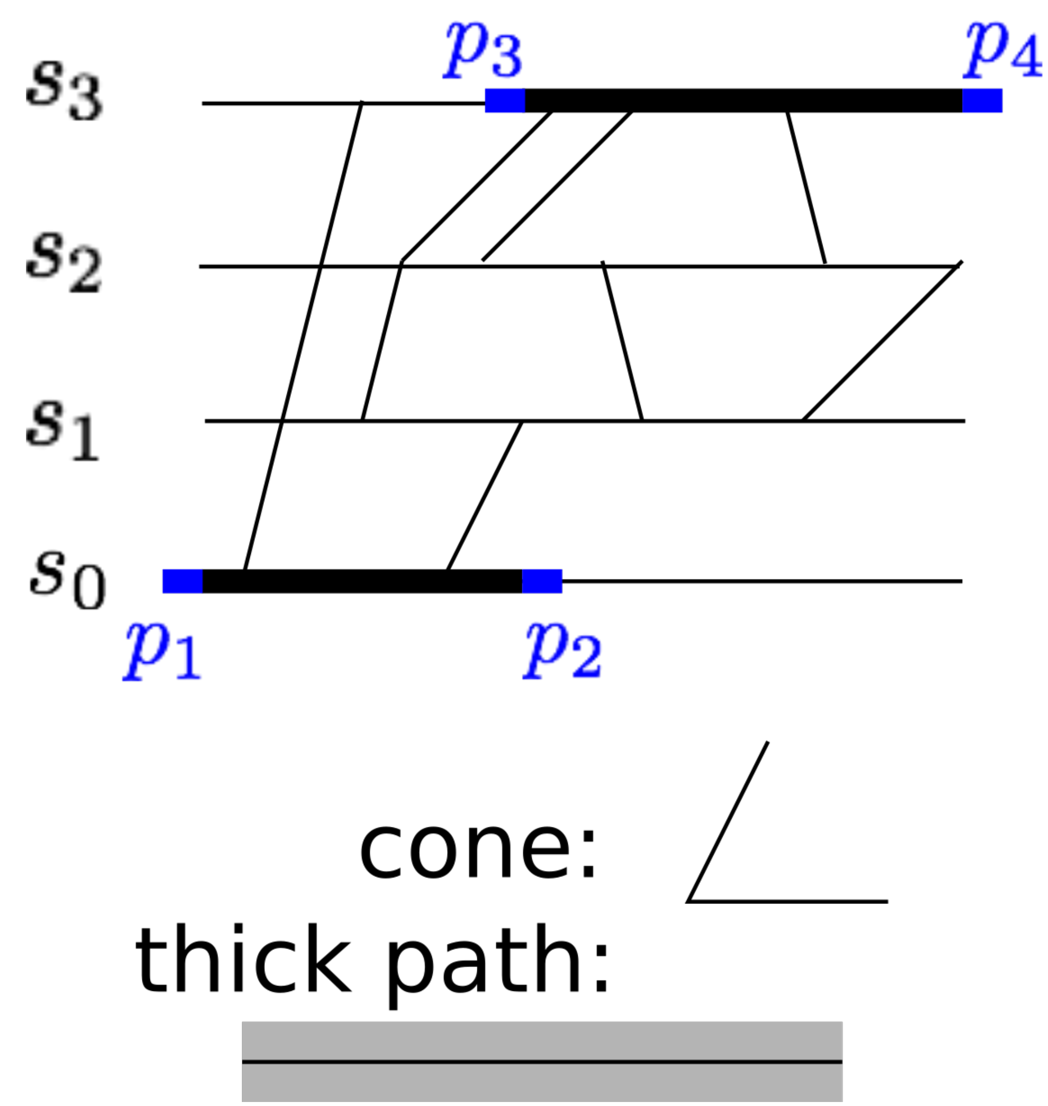
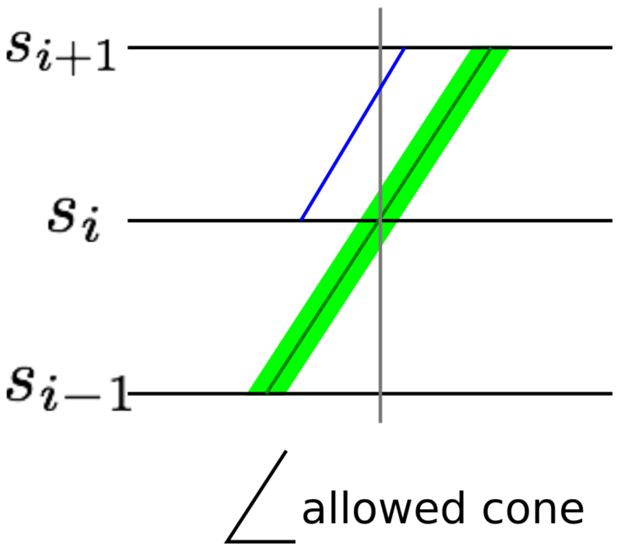
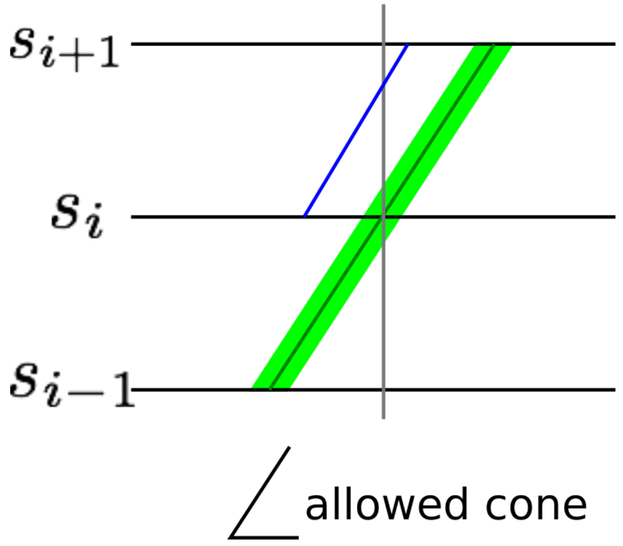
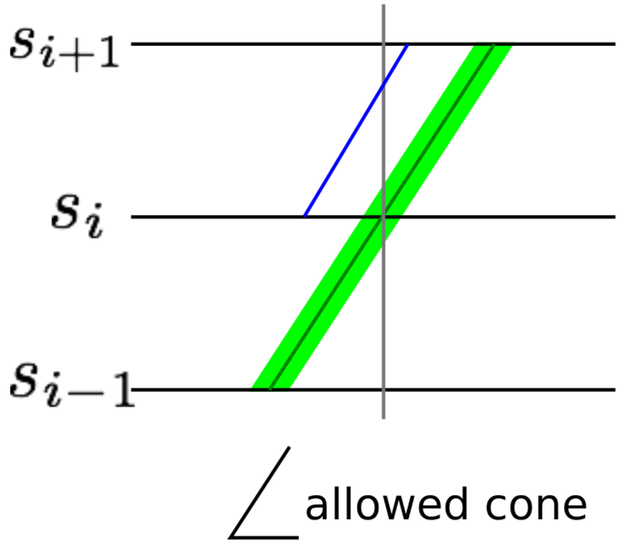



s_{i+1}
s_i
s_{i-1}
allowed cone

Vertical lines at stations are obstacles



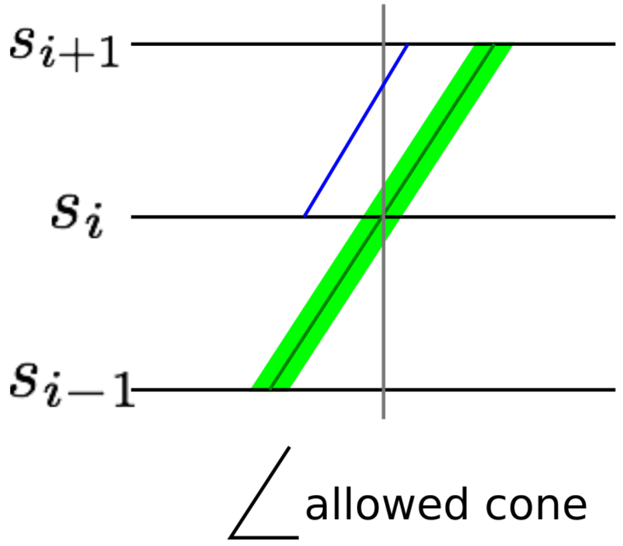
Vertical lines at stations are obstacles
⇨ We need to delete them



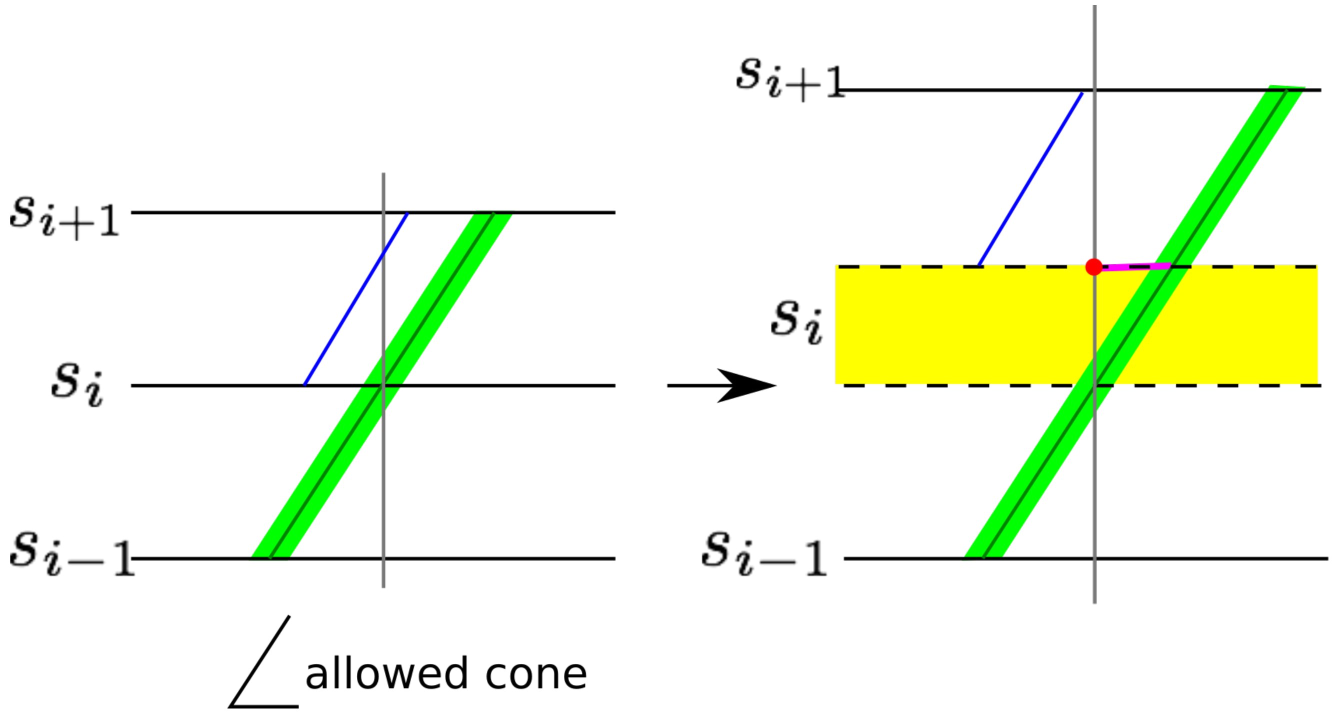
Vertical lines at stations are obstacles

⇨ We need to delete them

We need to be able to spend some time at a station



Vertical lines at stations are obstacles
⇨ We need to delete them
We need to be able to spend some time at a station
⇨ “Cut” each station open and blow up by vertical distance:



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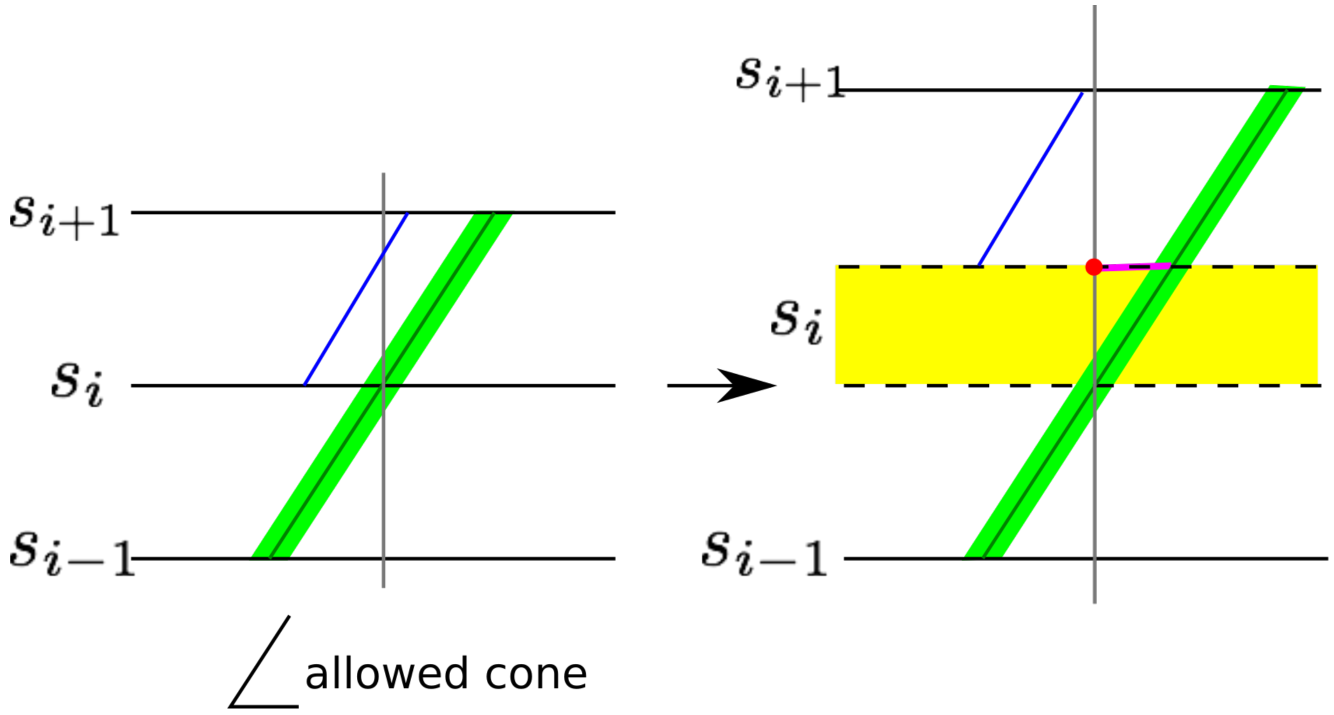

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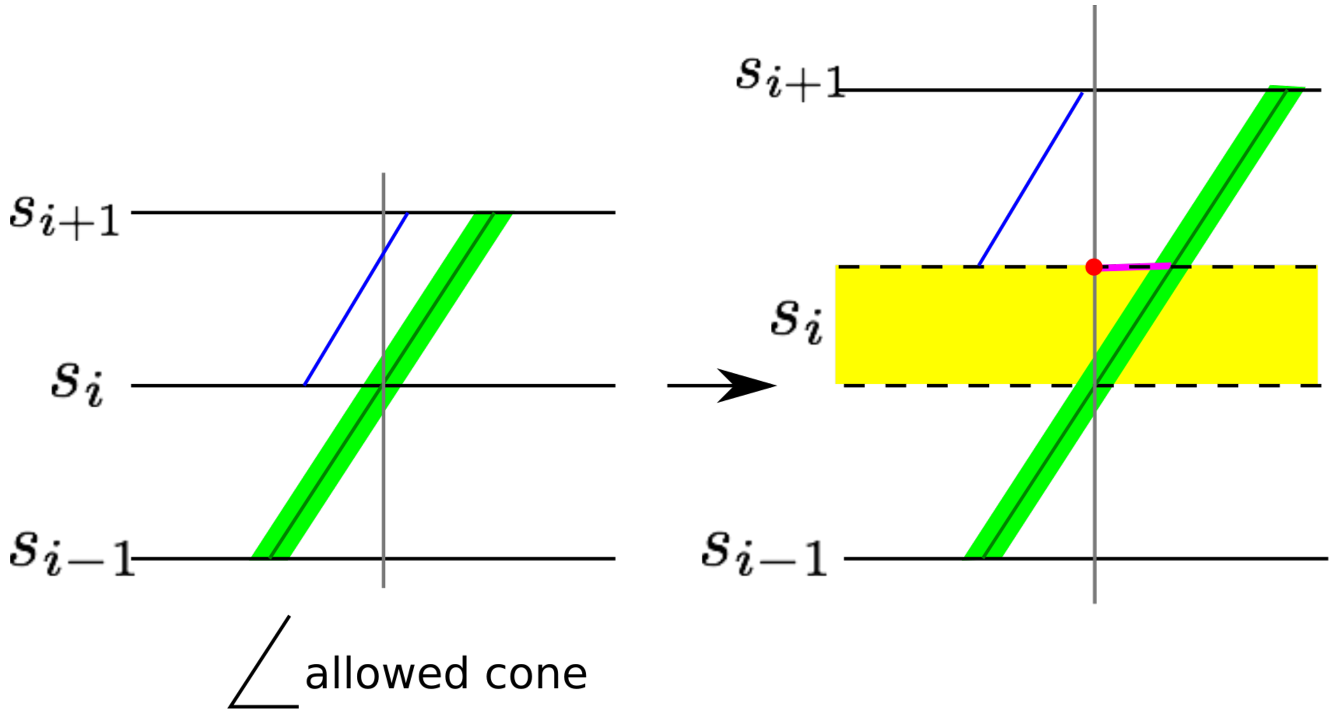

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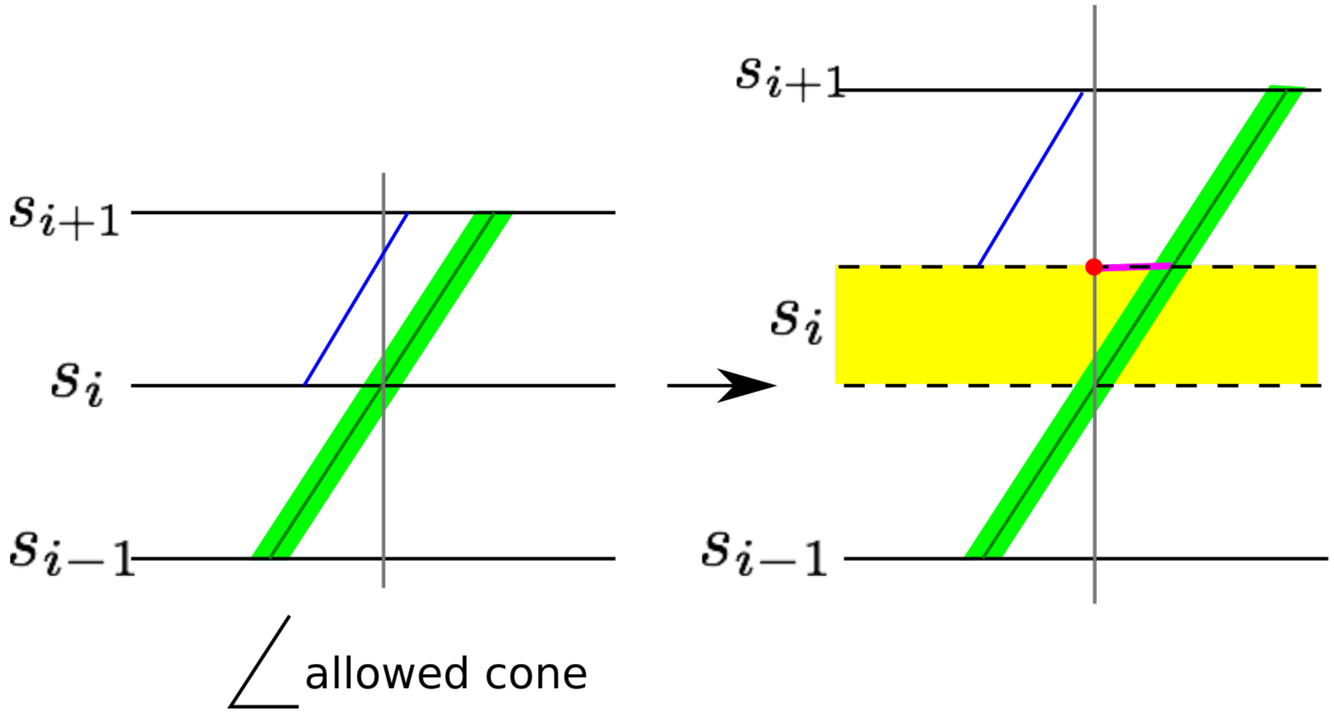

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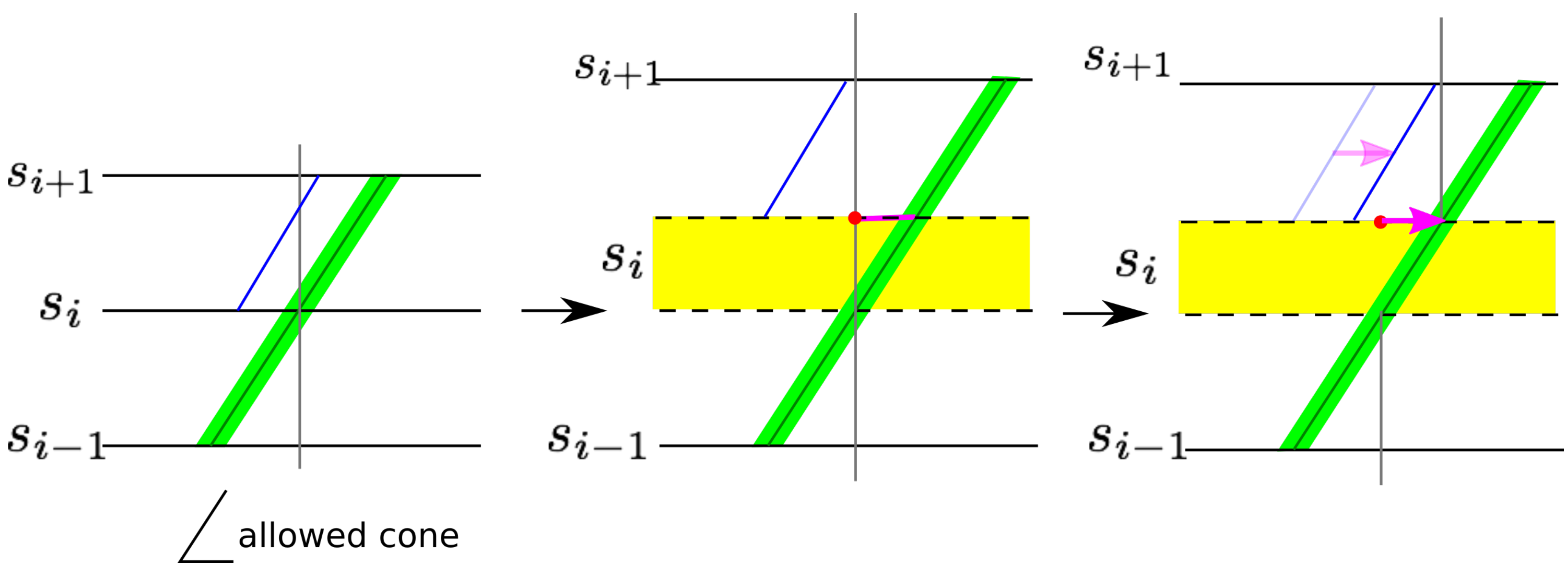
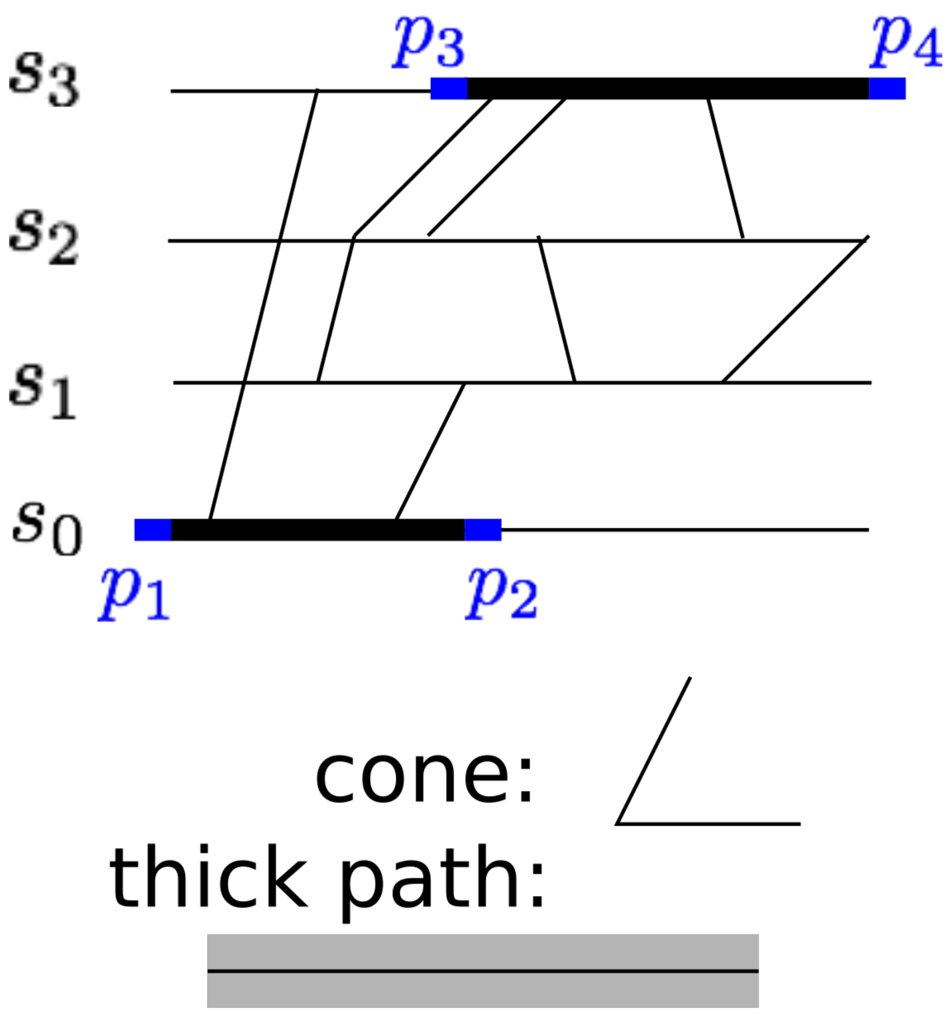
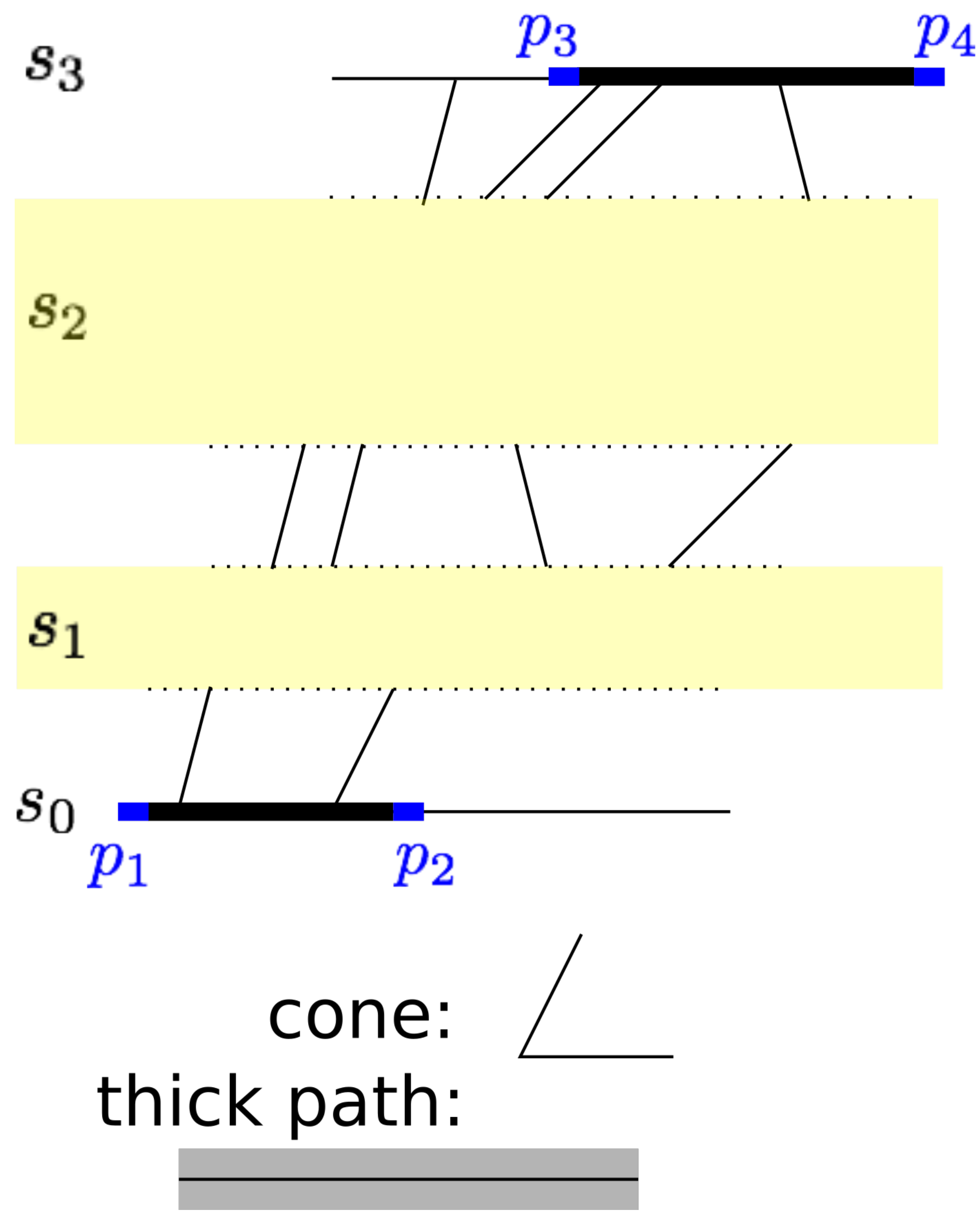
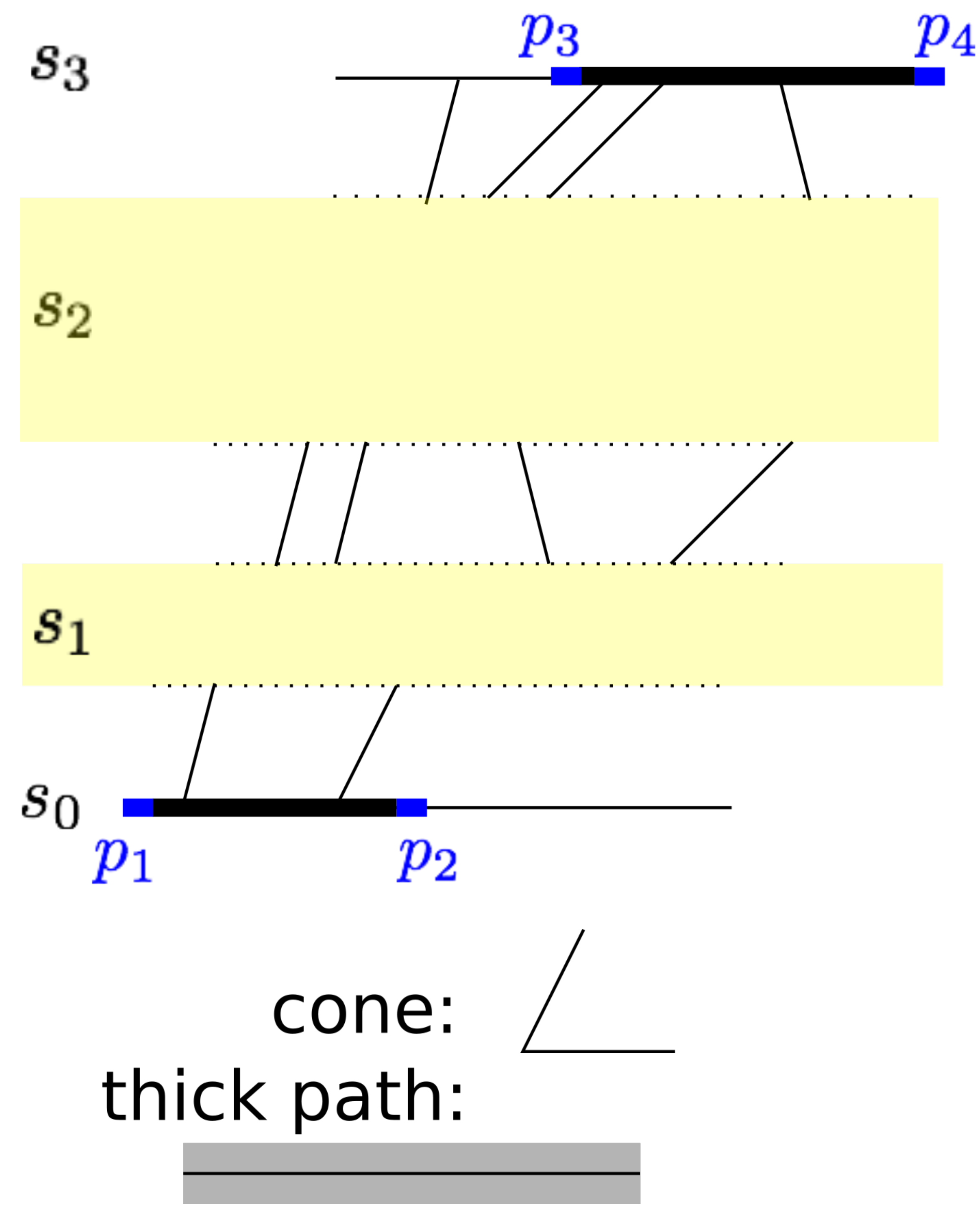
But now the time of departure cannot be reached by our paths with limited slope

⇨We need to shift the consecutive stations to the right, such that this path can be reached with limited slope



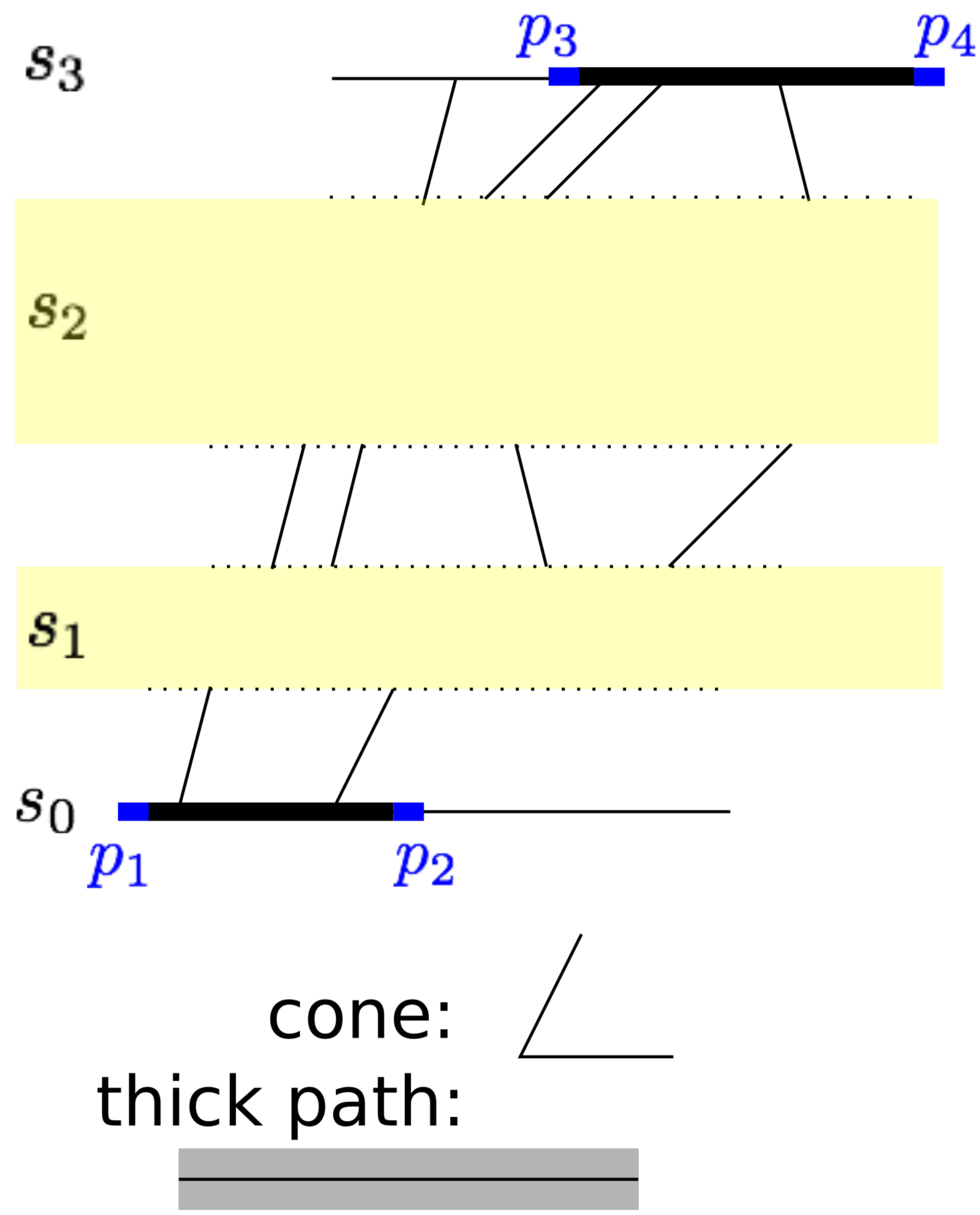

s_3
p_3
p_4
s_2
s_1
s_0
p_1
p_2
cone:
thick path:


s_3
p_3
p_4
s_2
s_1
s_0
p_1
p_2
cone:
thick path:

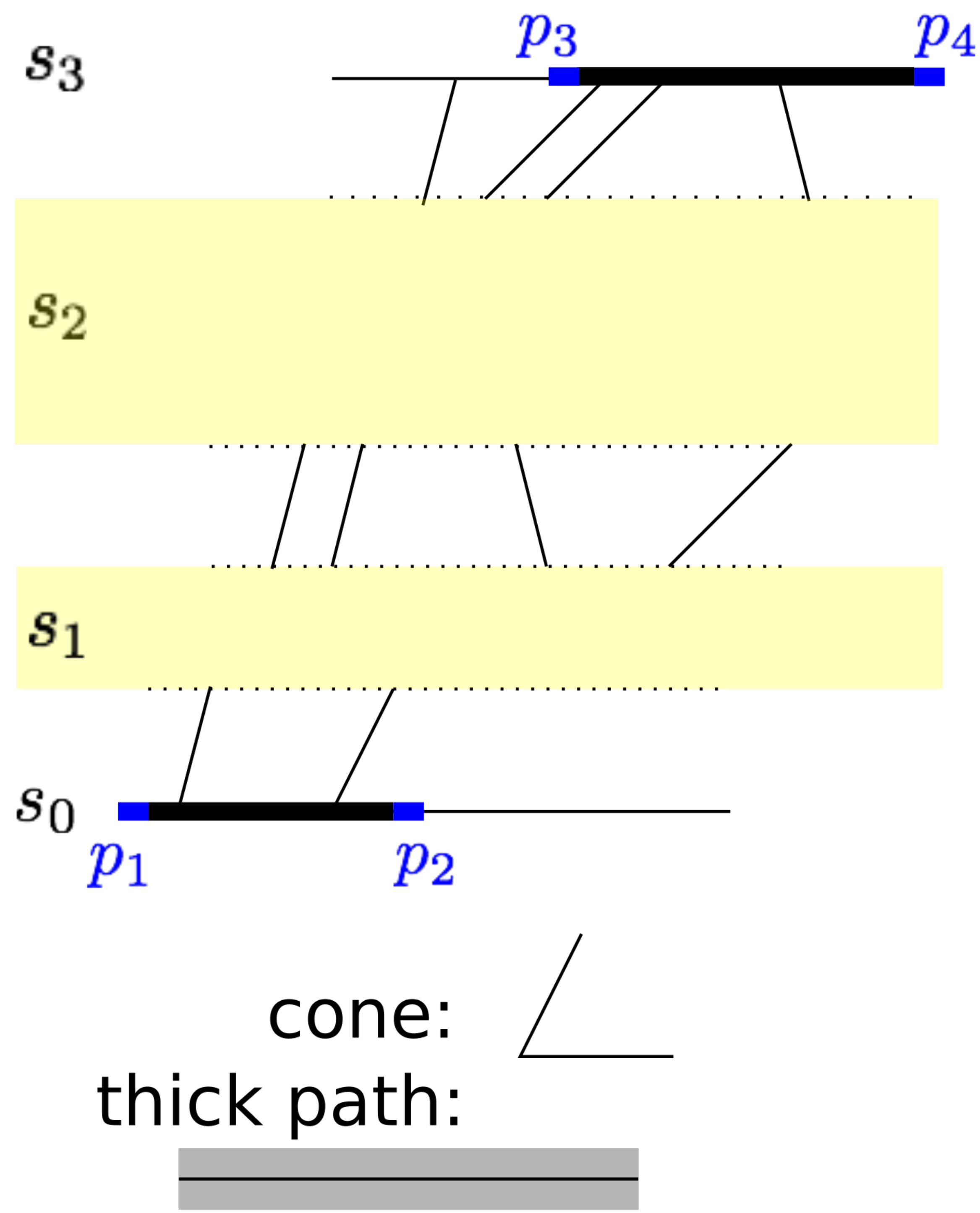


s_3
p_3
p_4
s_2
s_1
s_0
p_1
p_2
cone:
thick path:

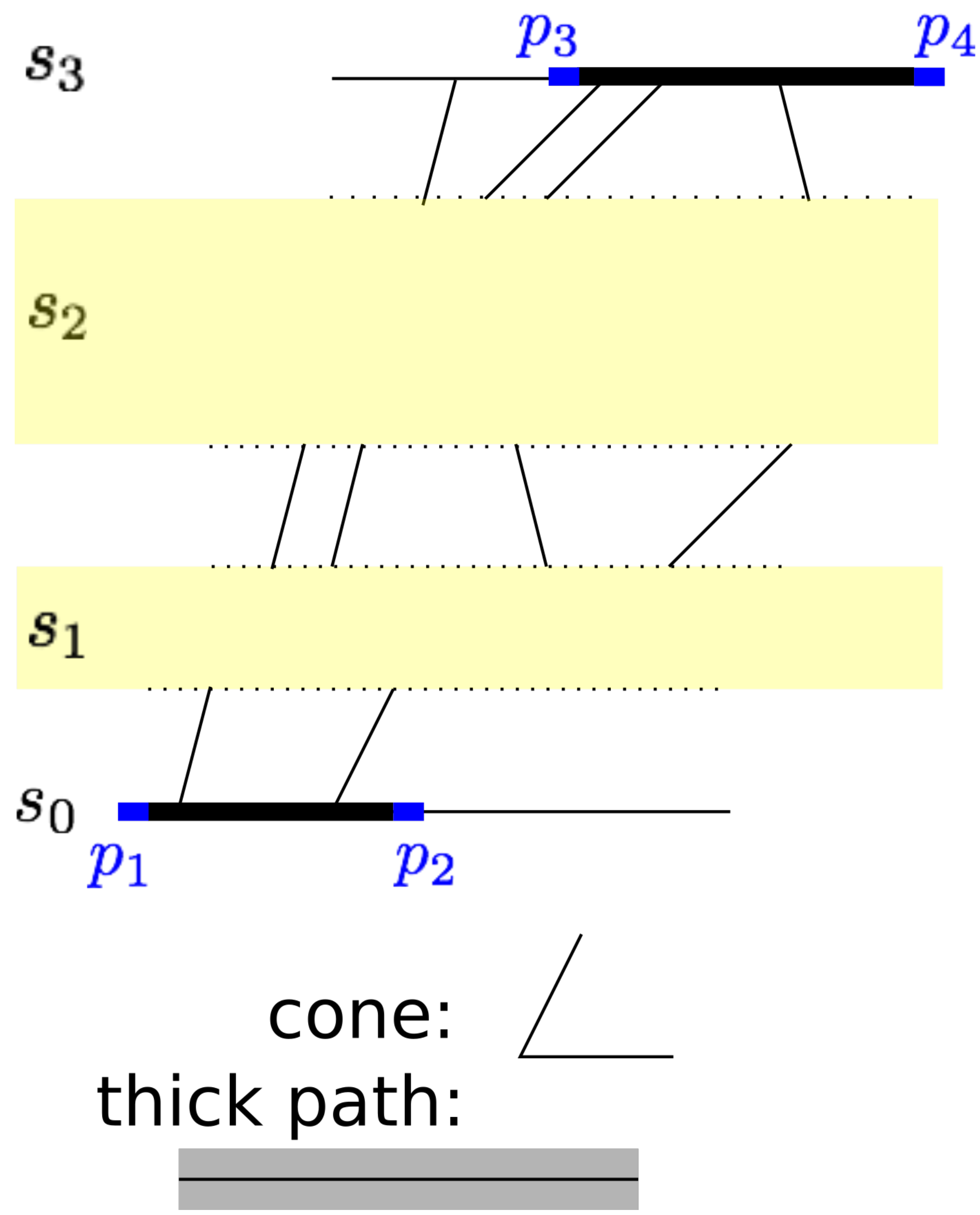
We need to keep a temporal distance to the existing trains in the timetable



We need to keep a temporal distance to the existing trains in the timetable
⇨ “Blow them up” as polygonal obstacles:



We need to keep a temporal distance to the existing trains in the timetable
⇨ "Blow them up" as polygonal obstacles: Insert the security distance (d_s, d_o)



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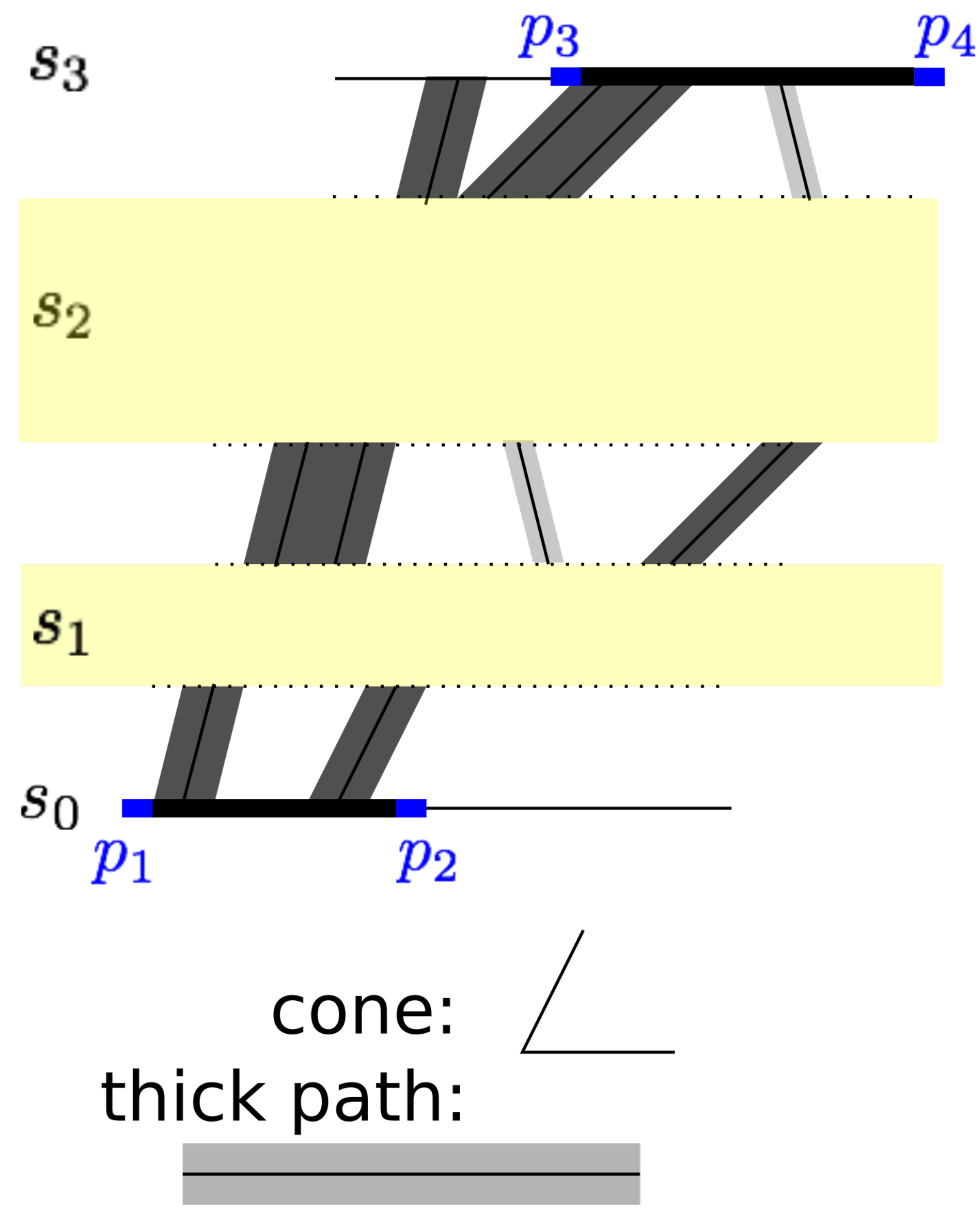
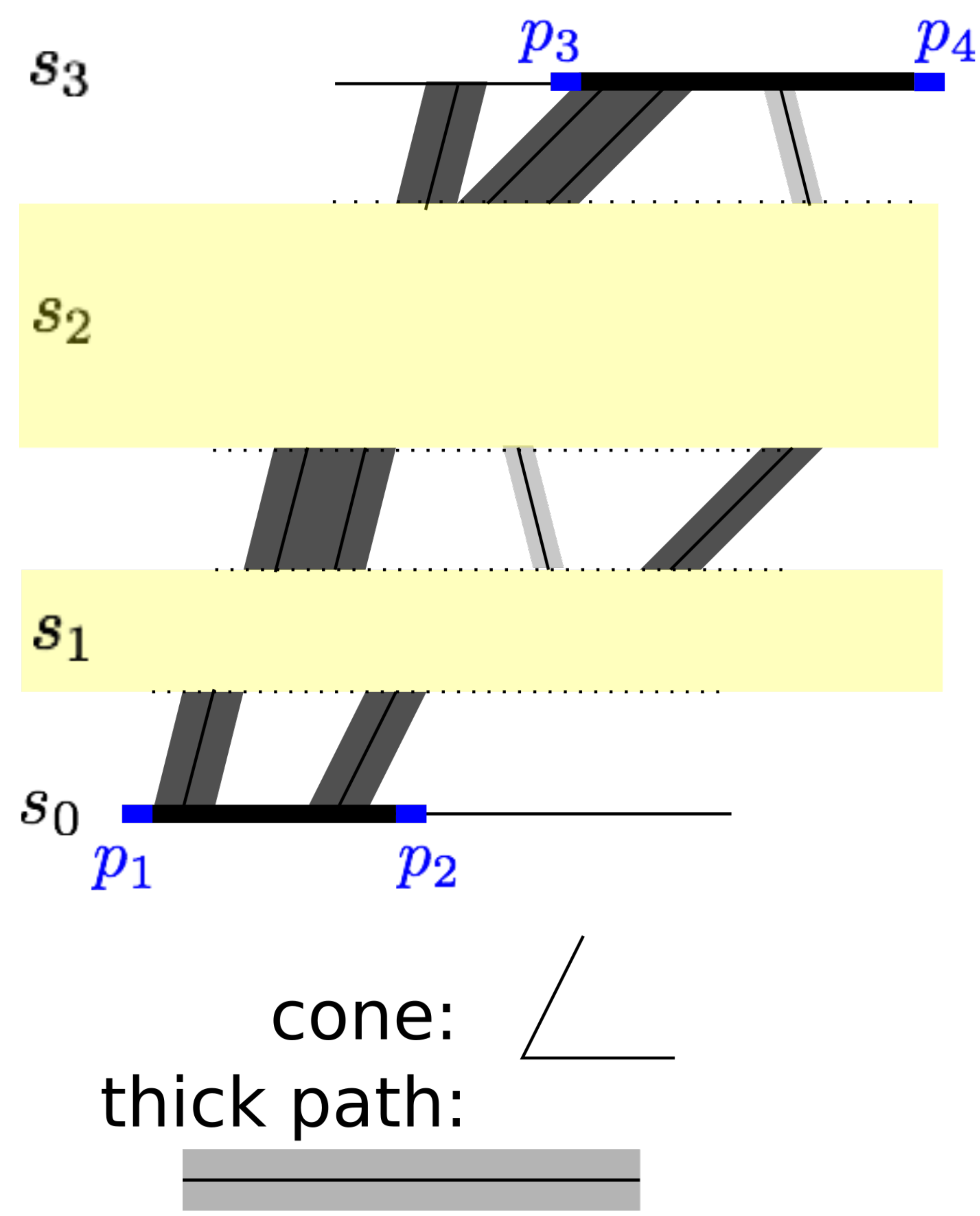
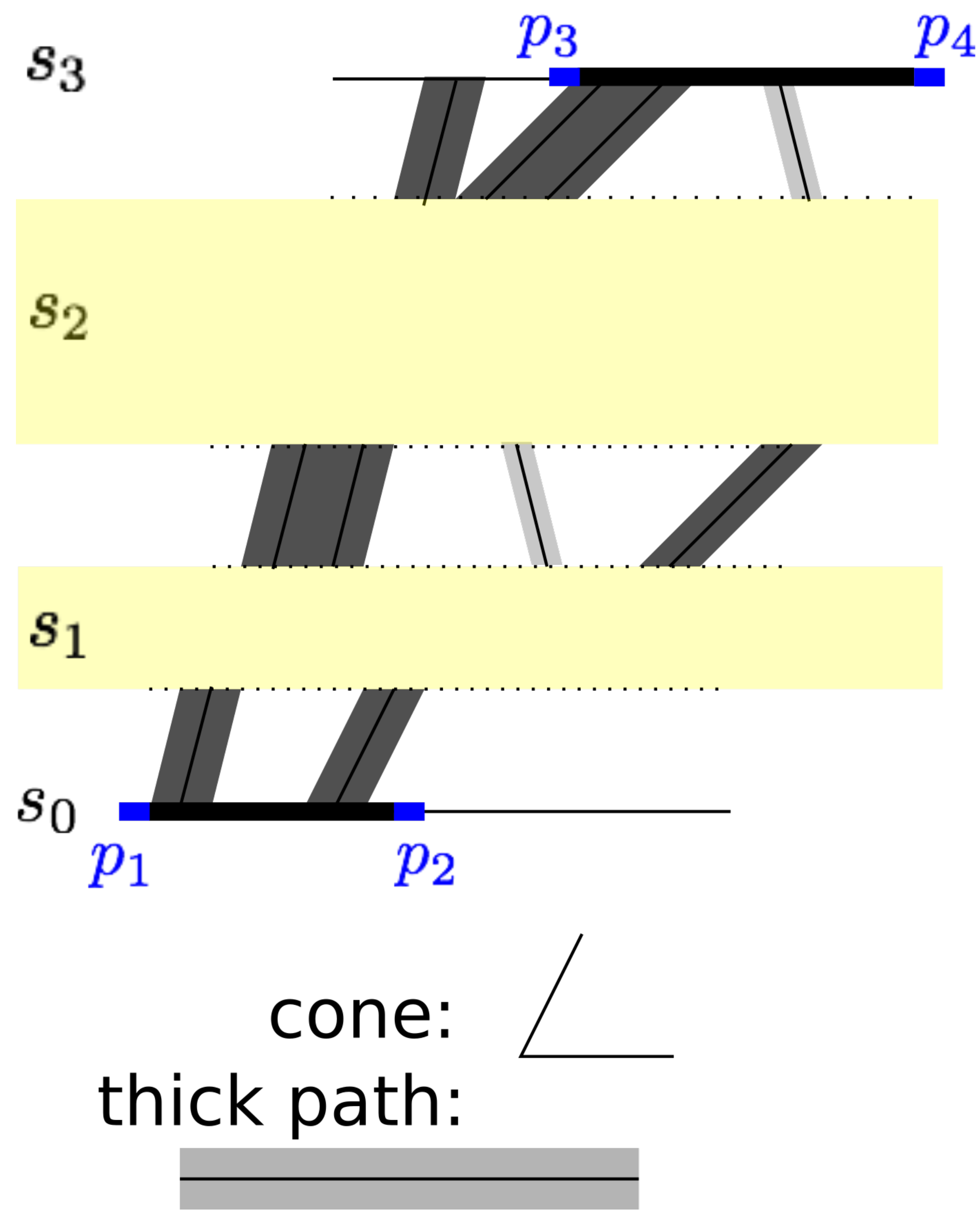


In the example we used d_s=d, d_0=d/2

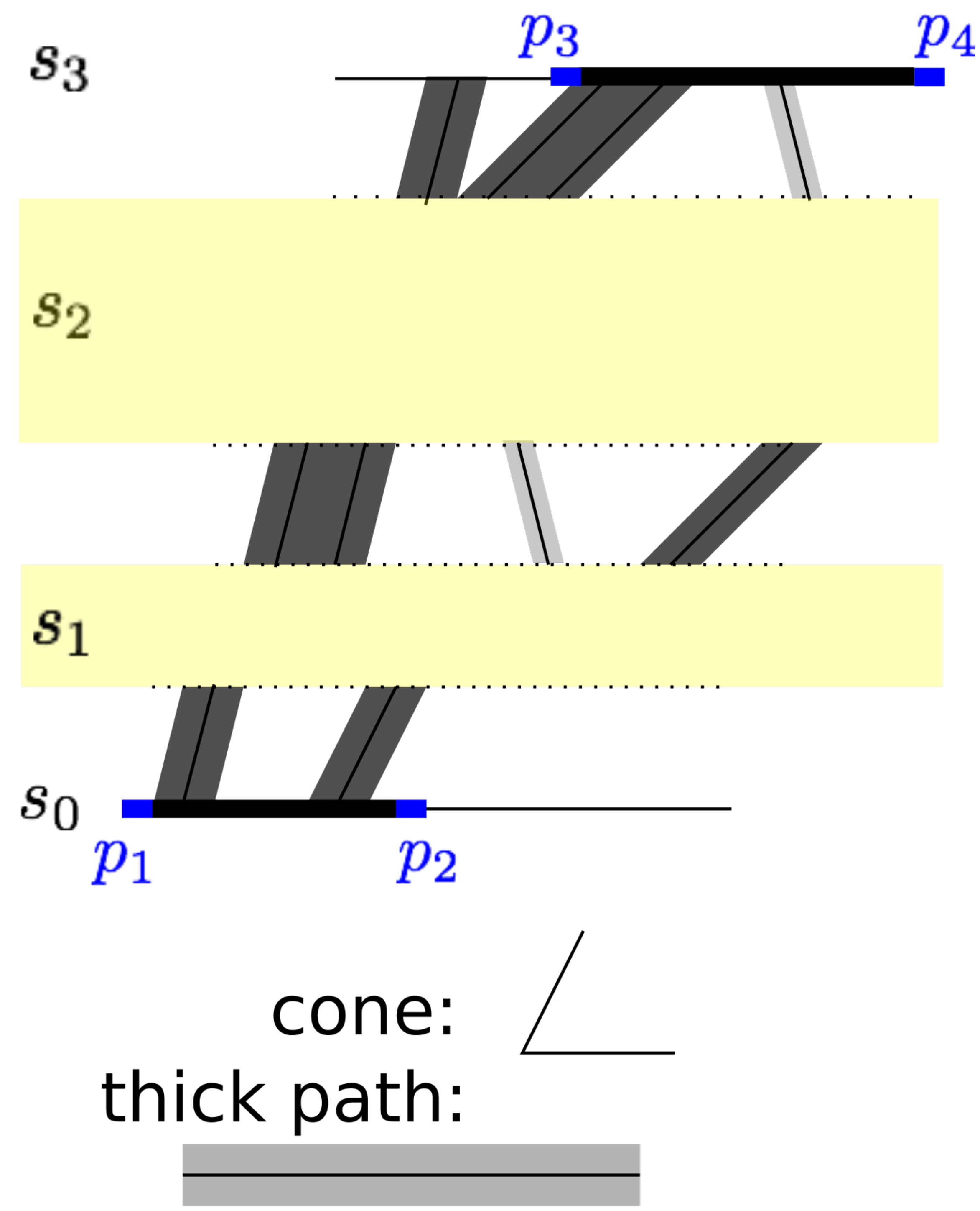

s_3
p_3
p_4
s_2
s_1
s_0
p_1
p_2
cone:
thick path:

We need to limit our outer polygon:



We need to limit our outer polygon:

- No train can run earlier than departing earliest with highest speed



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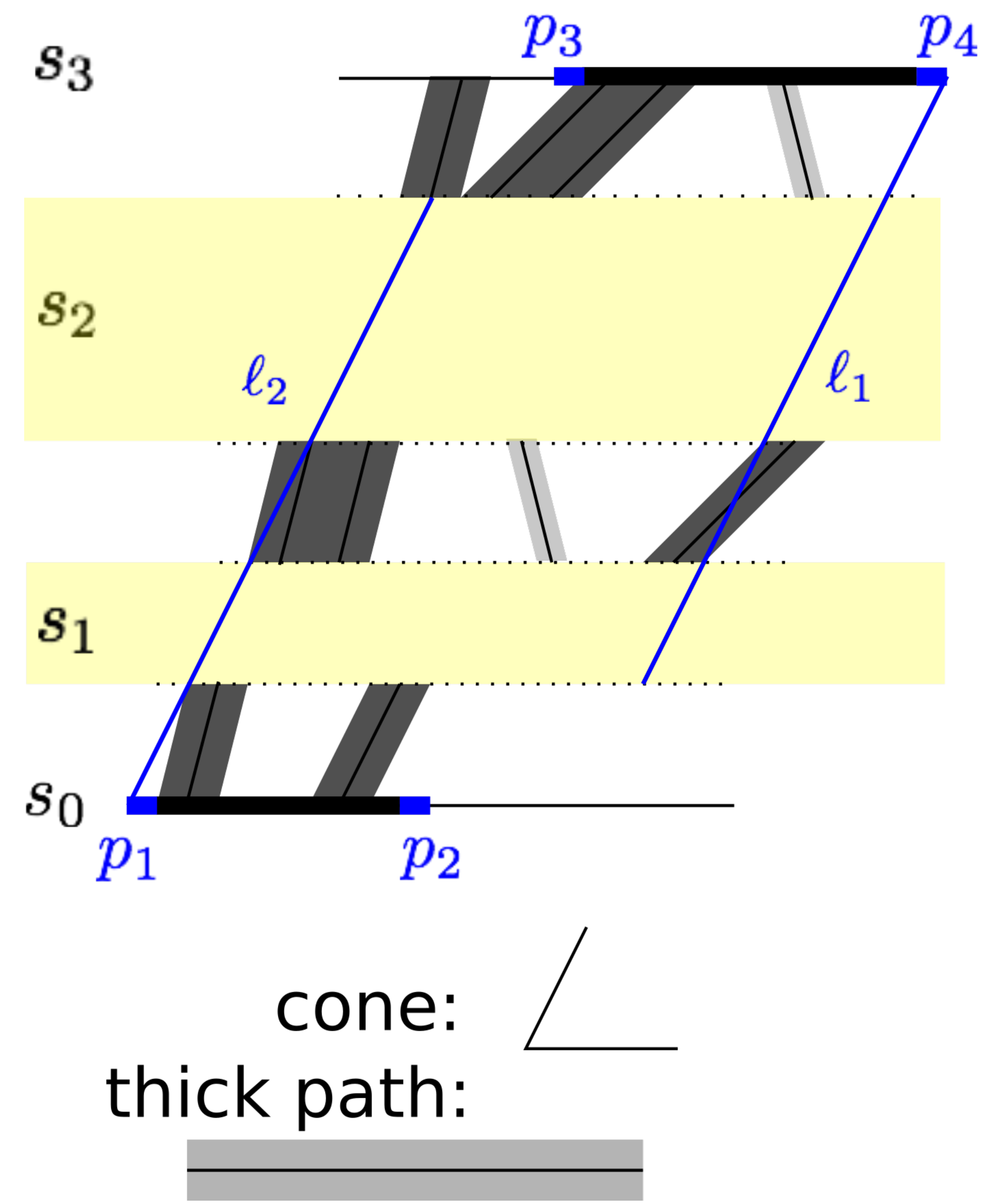
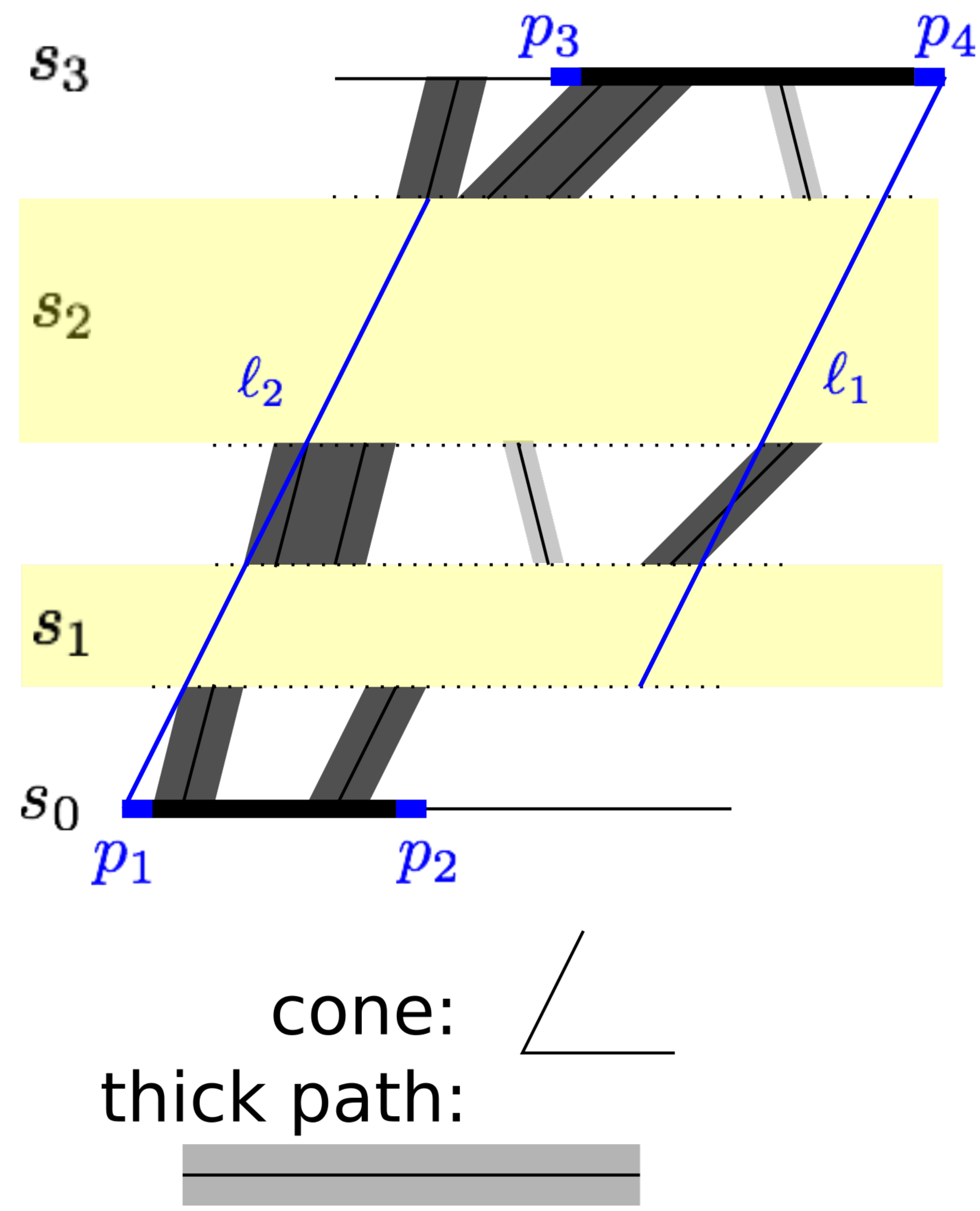
  ⇨ ℓ_2



We need to limit our outer polygon:

- No train can run earlier than departing earliest with highest speed
  ⇨ ℓ_2
- No train can run later than arriving latest with highest speed



We need to limit our outer polygon:

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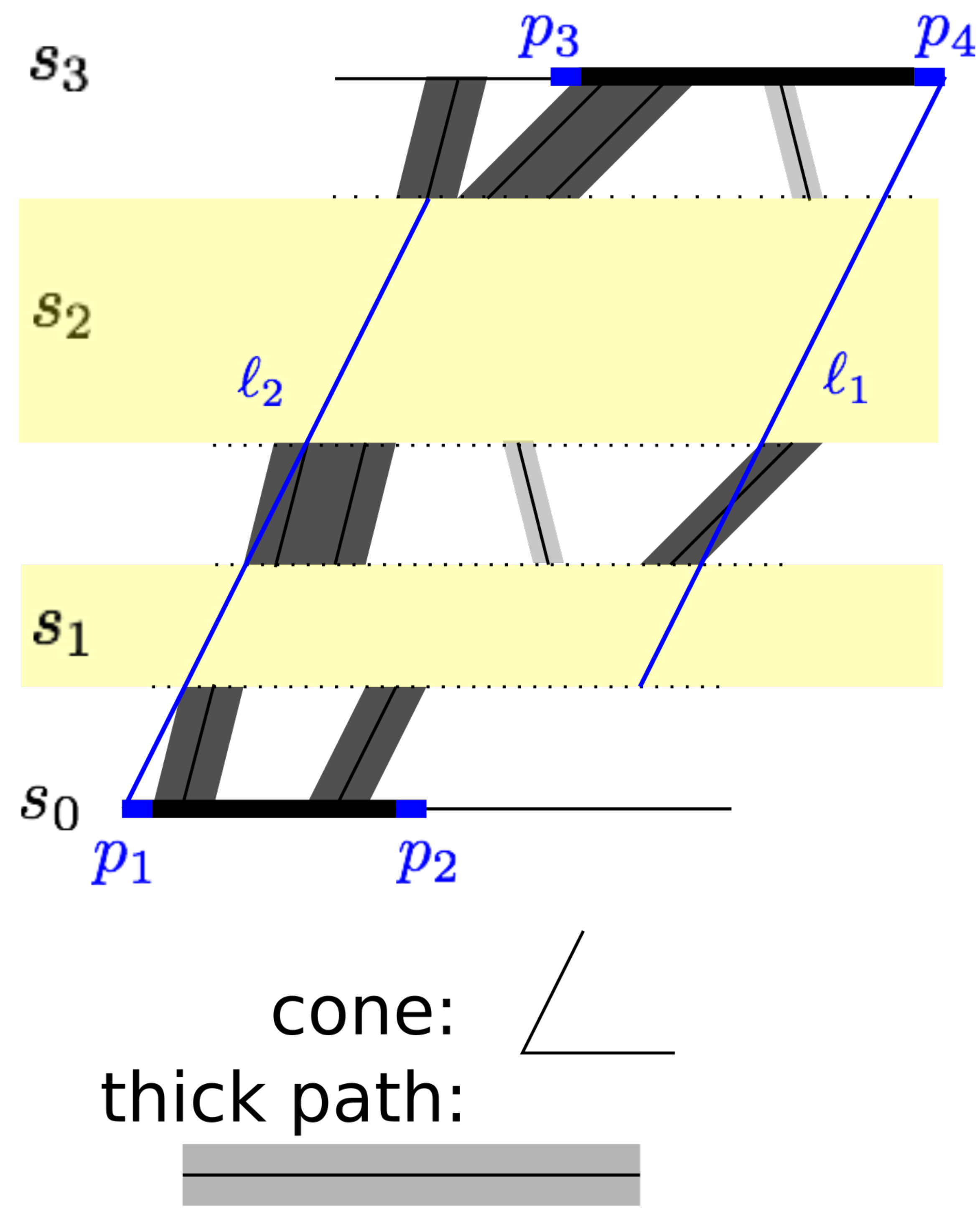
  ⇨ ℓ_2
- No train can run later than arriving latest with highest speed
  ⇨ ℓ_1



We need to limit our outer polygon:

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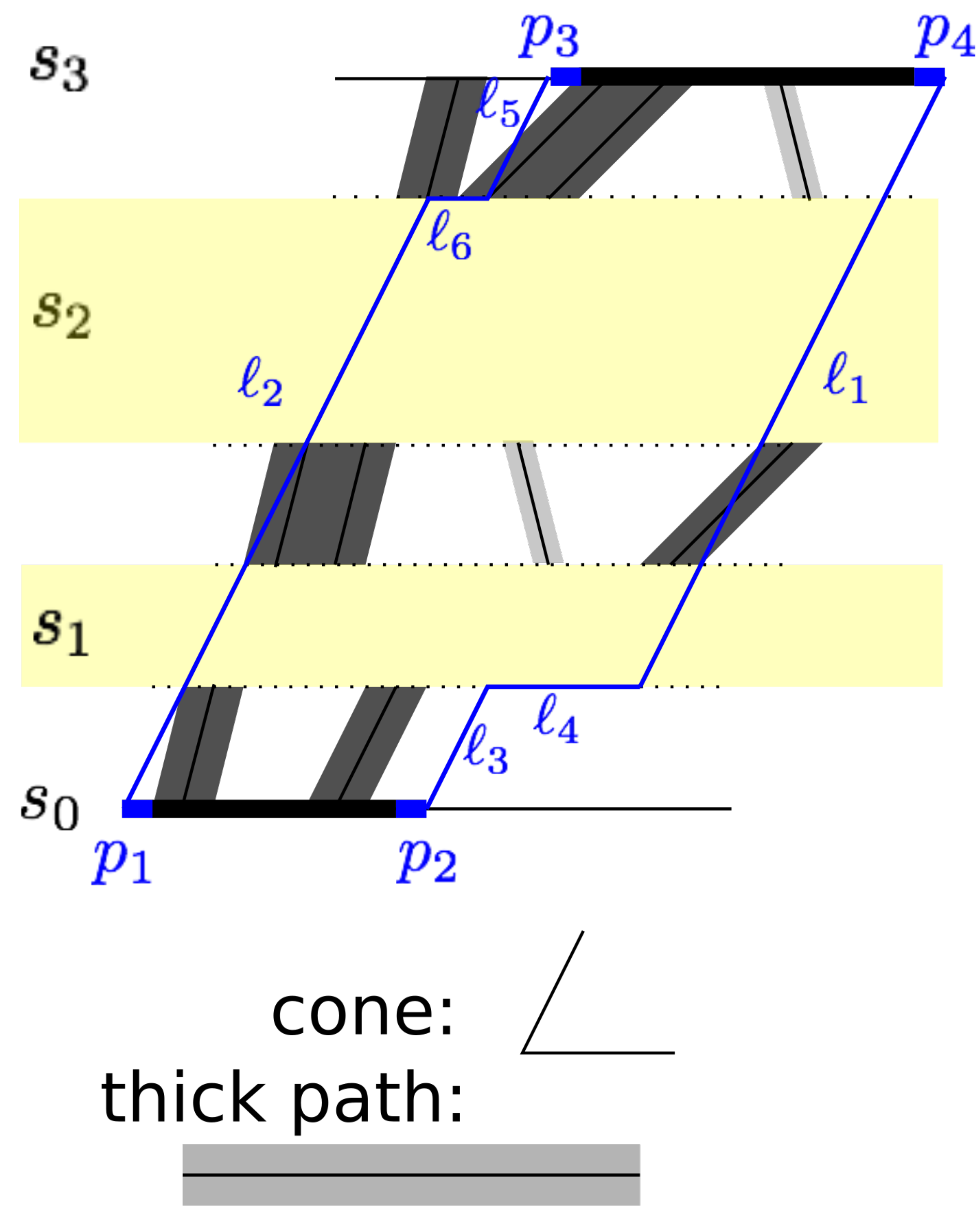
  ⇨ ℓ_2
- No train can run later than arriving latest with highest speed
  ⇨ ℓ_1
- Some further boundary parts



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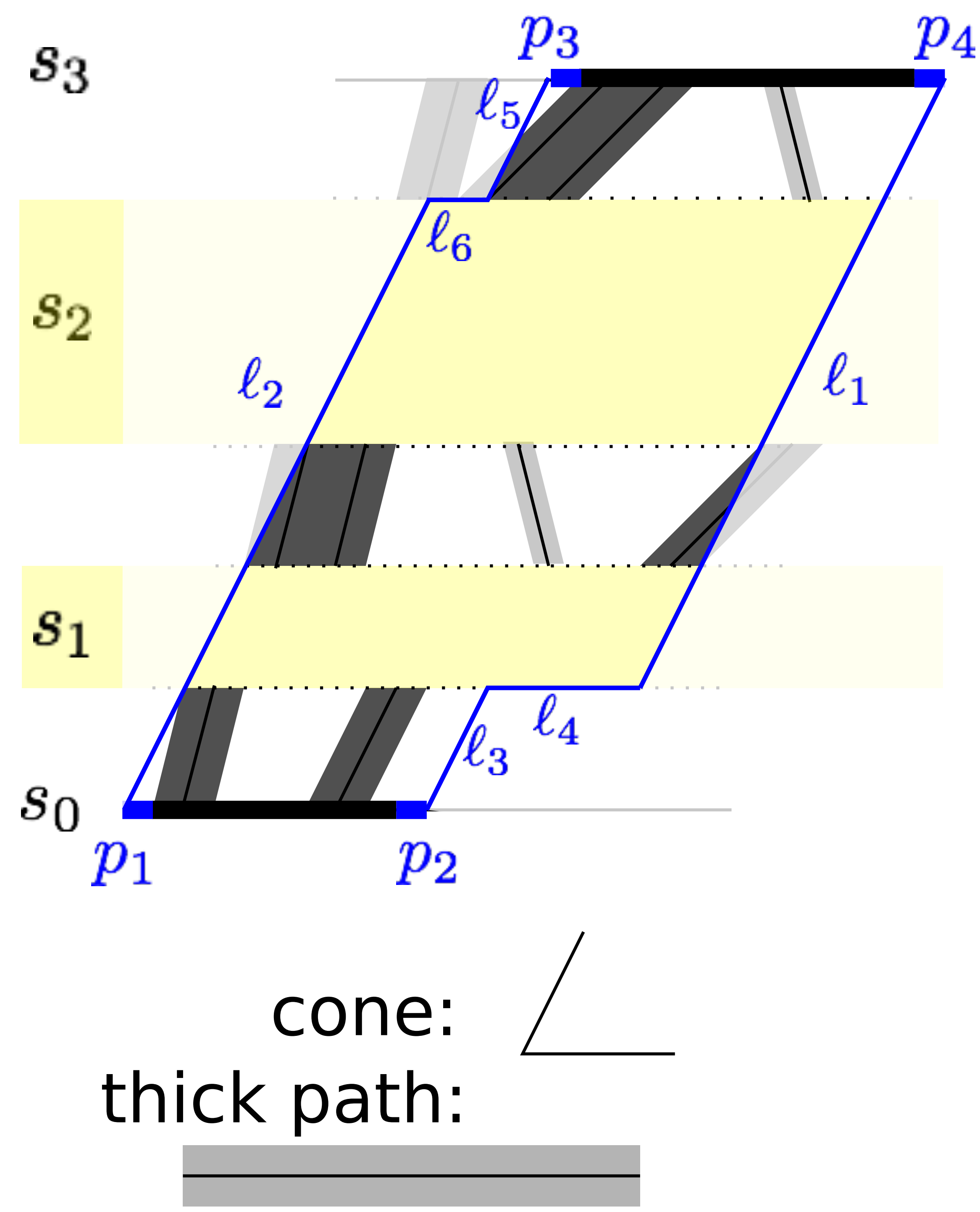
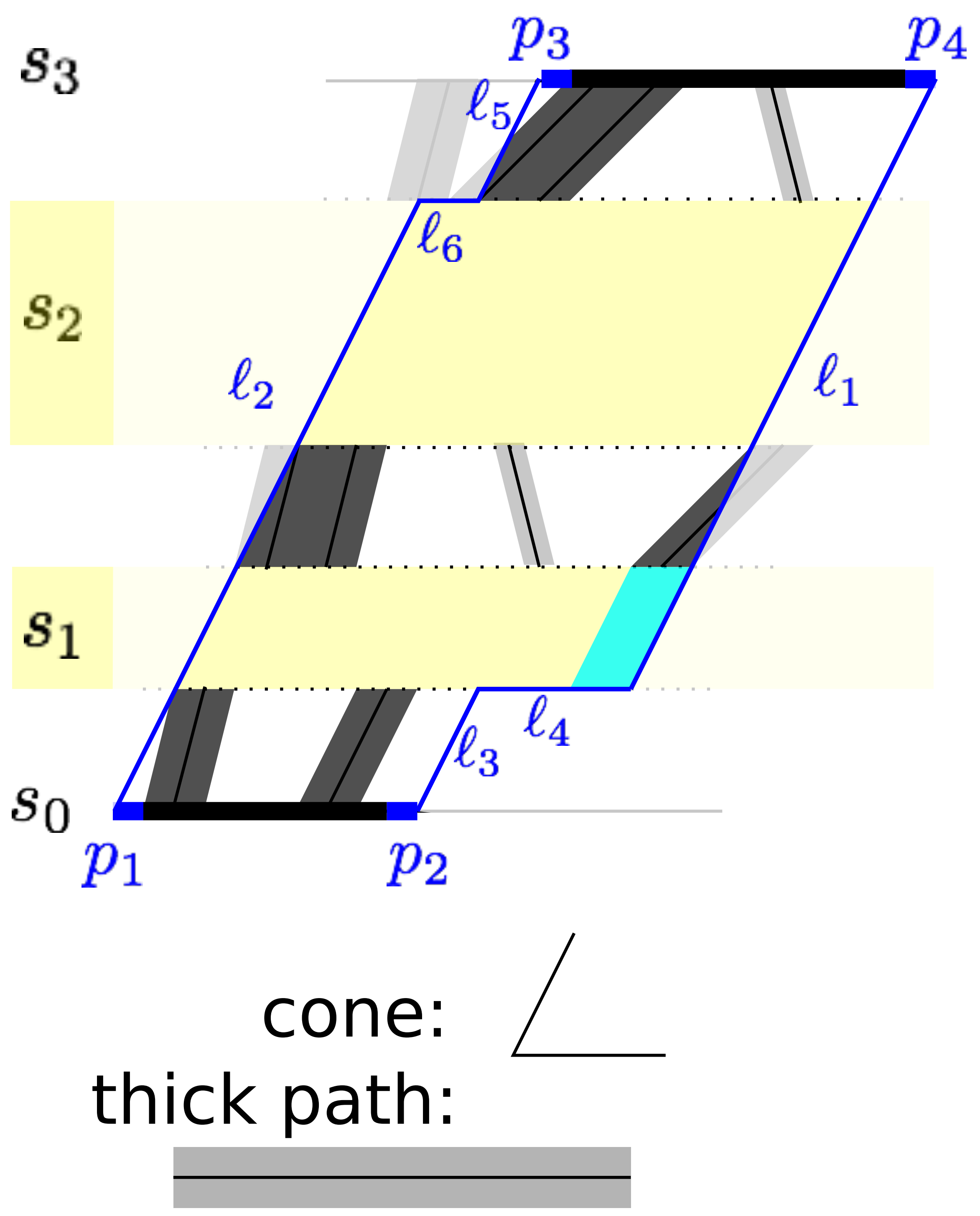
  ⇨ ℓ_2
- No train can run later than arriving latest with highest speed

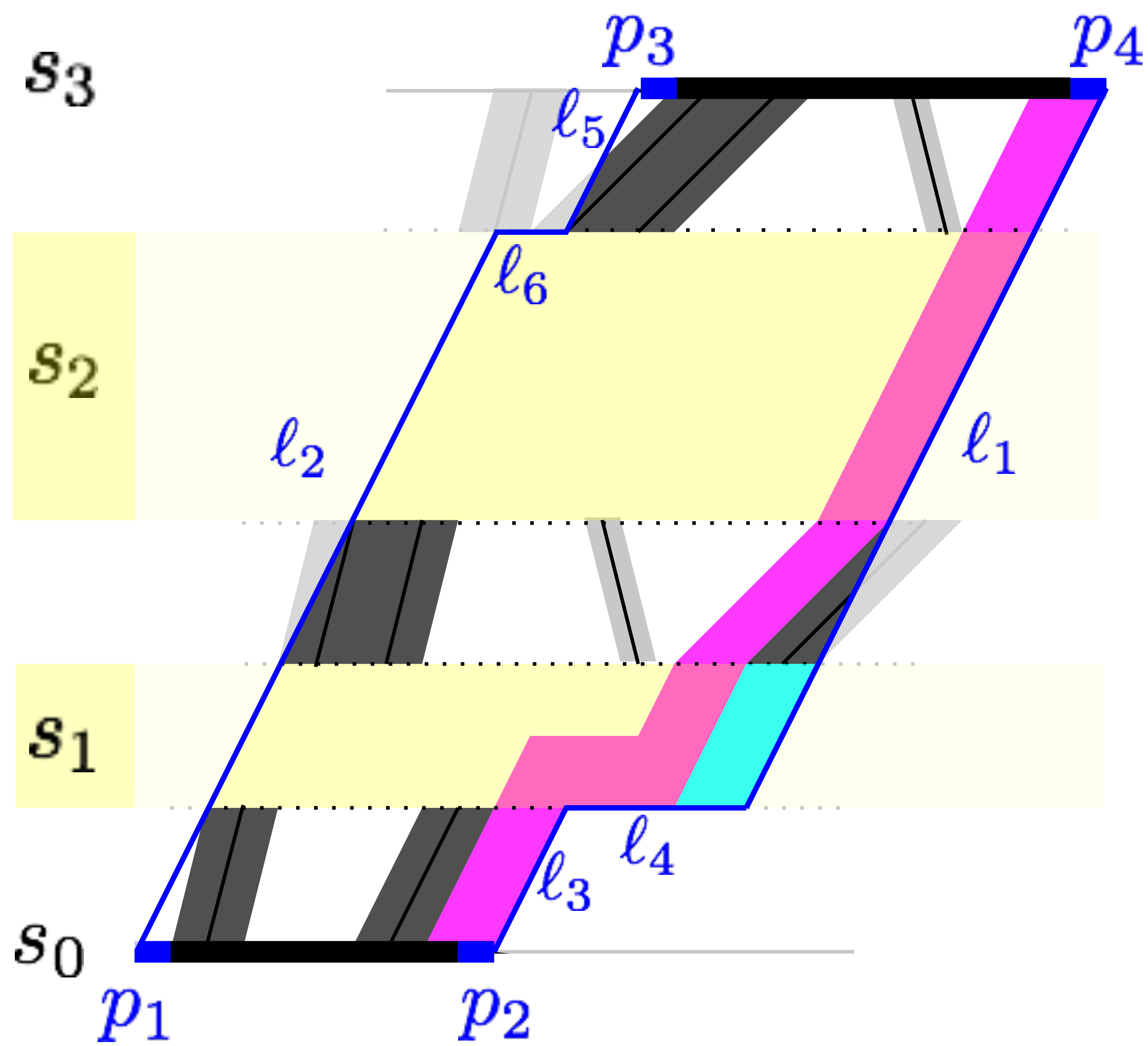
  ⇨ ℓ_1
- Some further boundary parts
- Intersect holes with boundary



Example

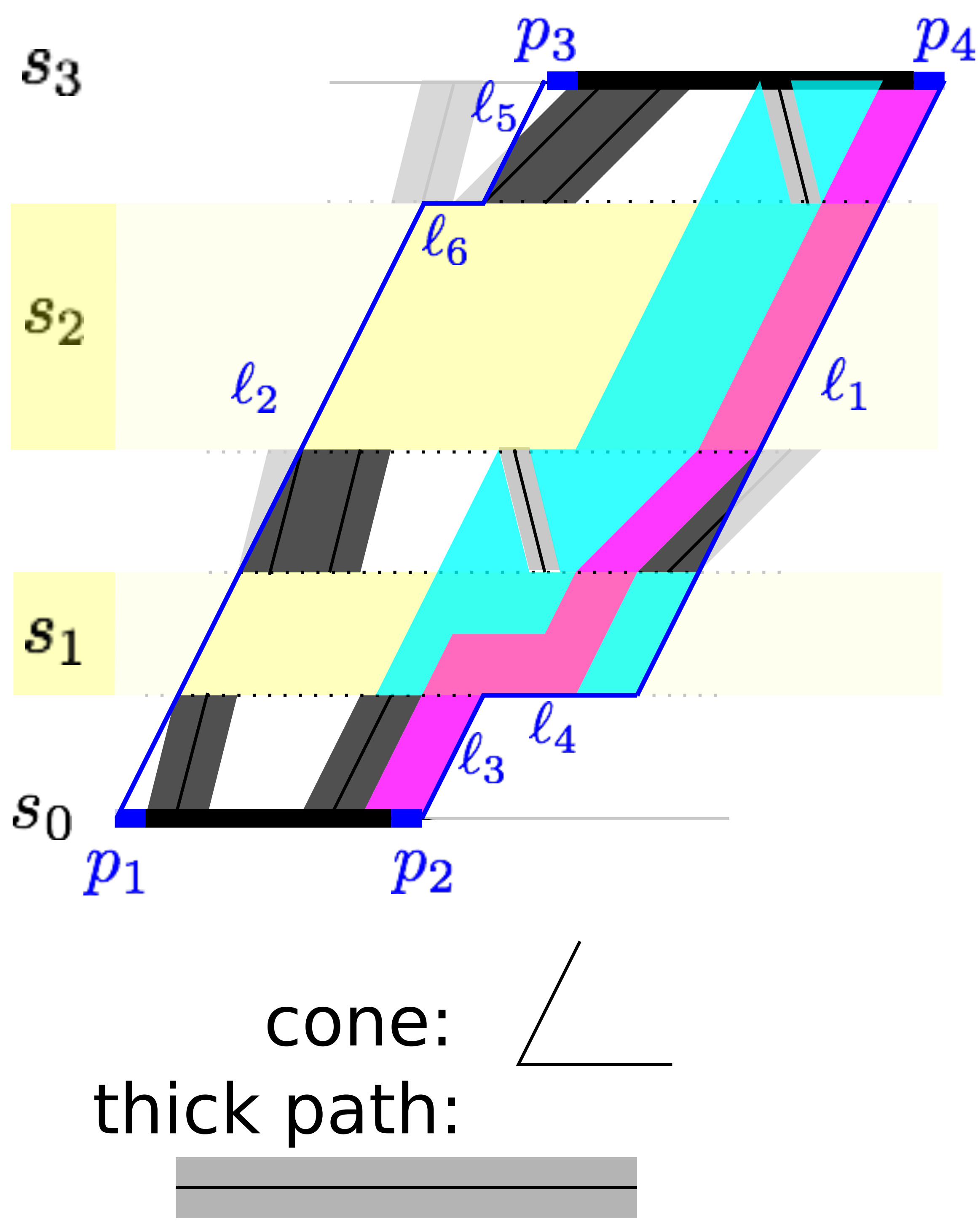


s_3
p_3
p_4
ℓ_5
ℓ_6
s_2
ℓ_2
ℓ_1
s_1
ℓ_4
ℓ_3
s_0
p_1
p_2
cone:
thick path:

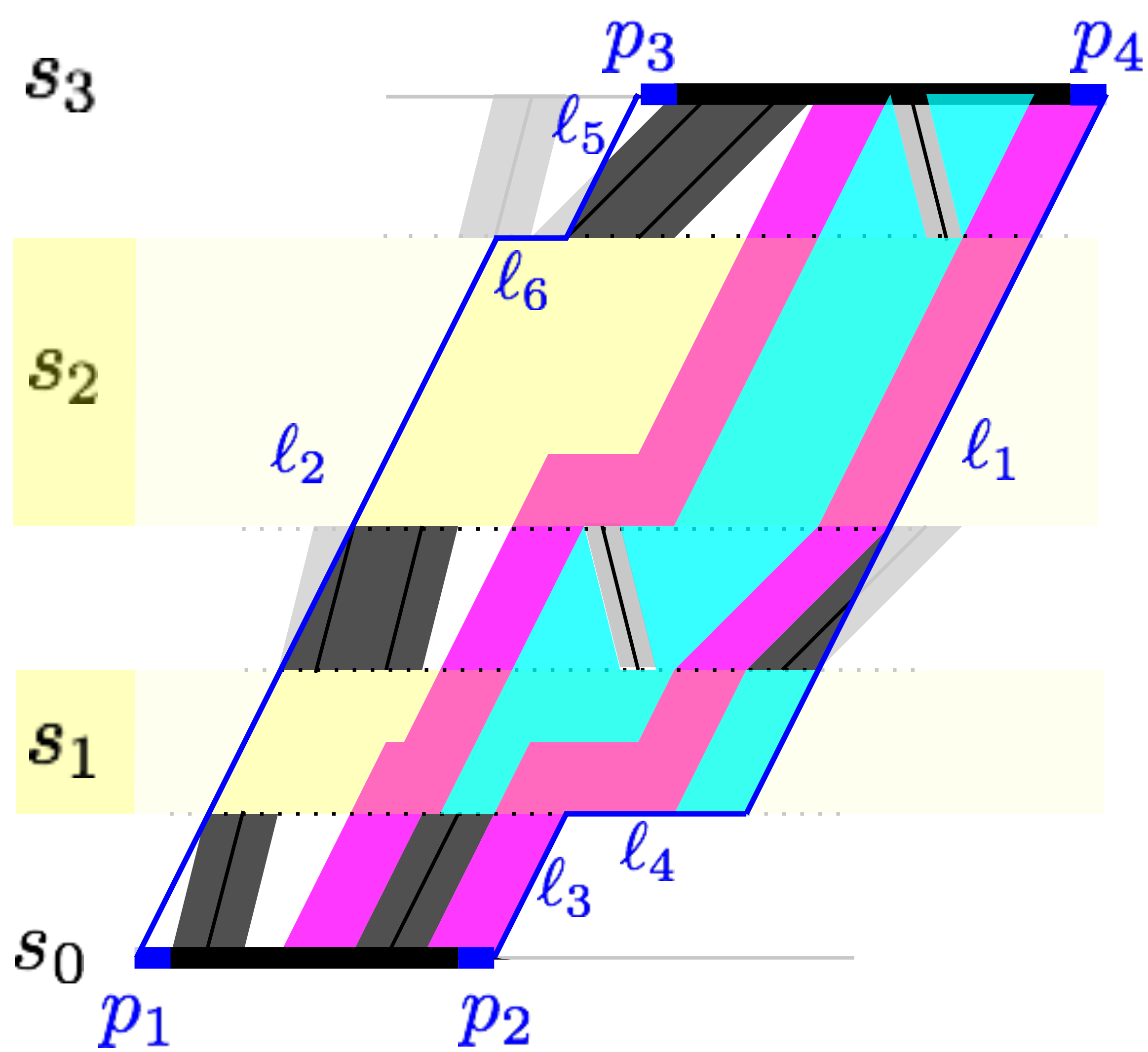


cone:

thick path:

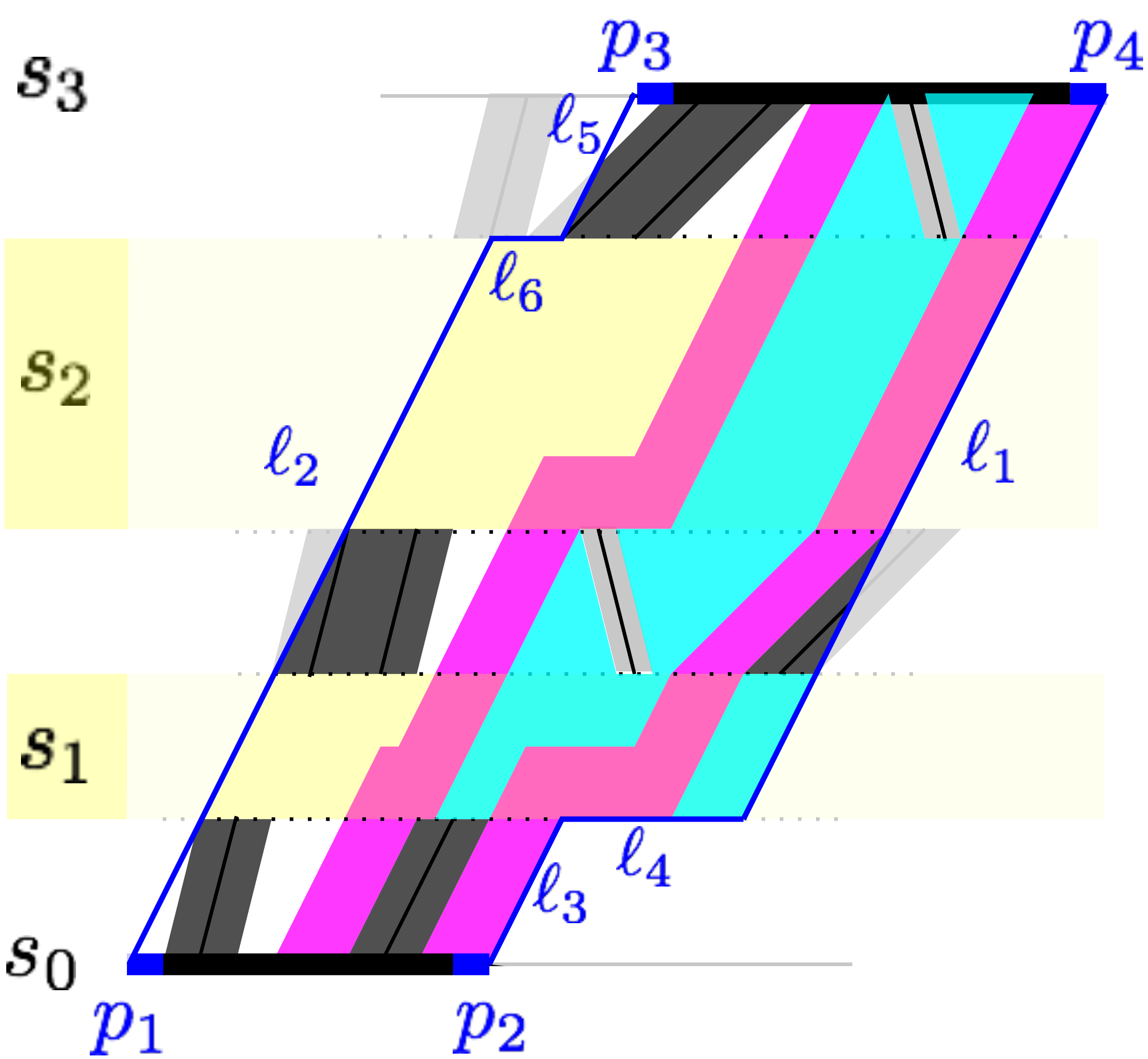


s_3
p_3
p_4
ℓ_5
ℓ_6
s_2
ℓ_2
ℓ_1
s_1
ℓ_4
ℓ_3
s_0
p_1
p_2
cone:
thick path:



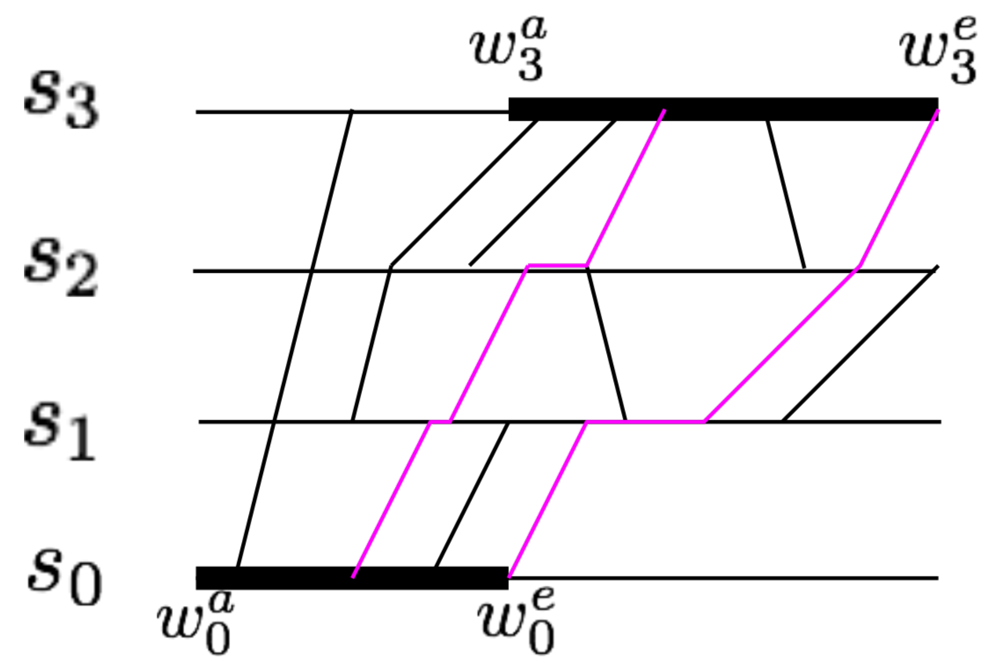
cone:

thick path:



cone:

thick path:



cone:

thick path:

Conclusion and Outlook

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Outlook

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Outlook

- Application to real-world example

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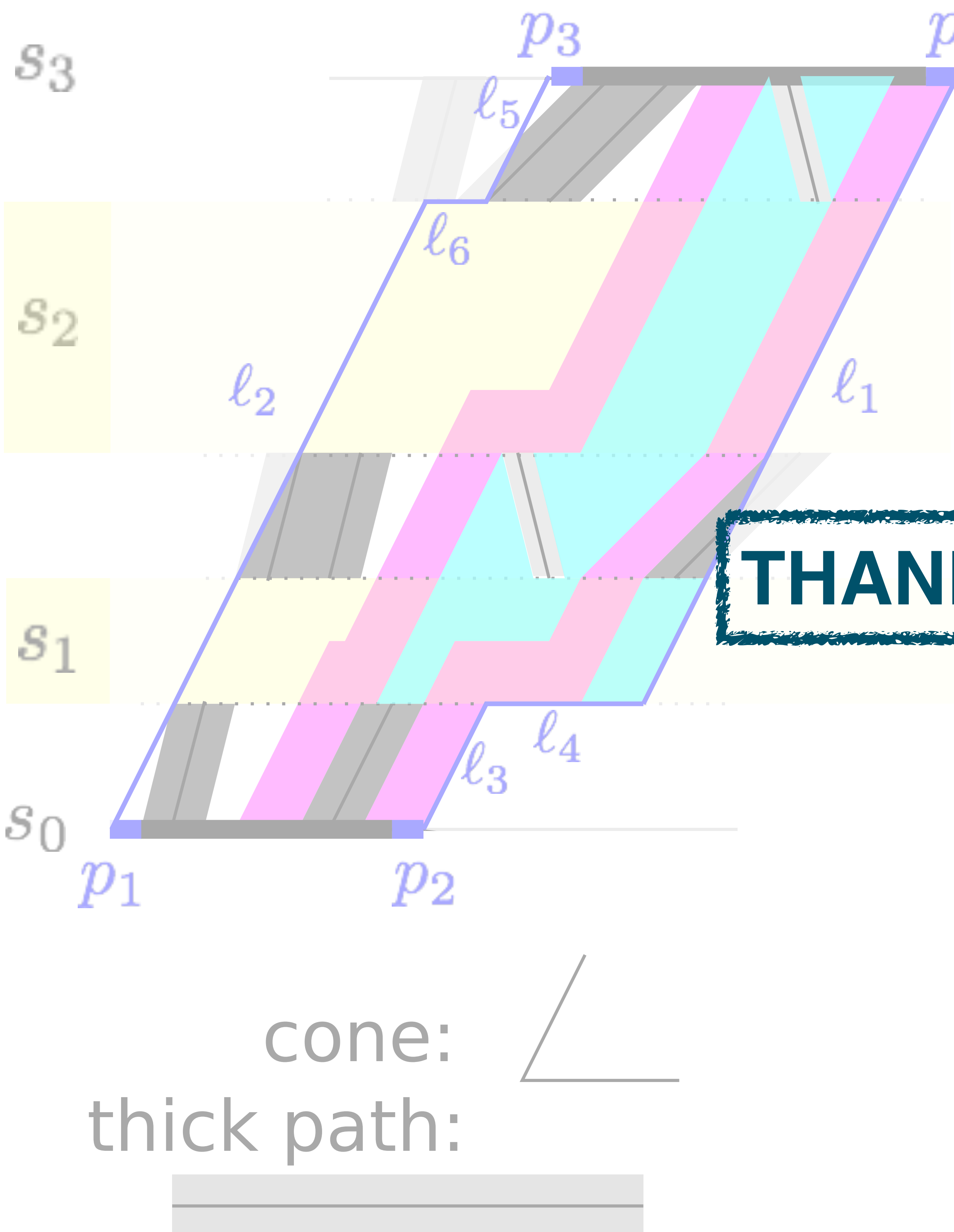
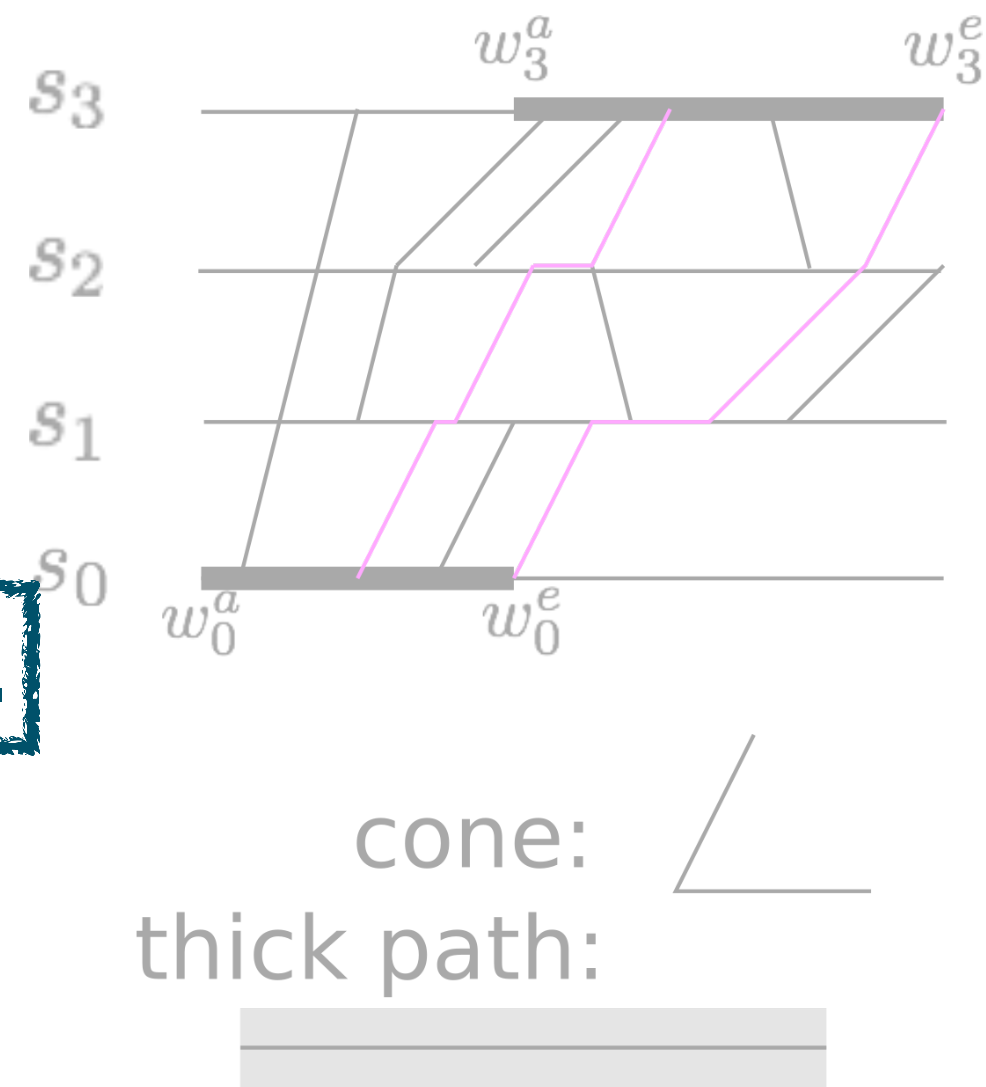
Outlook

- Application to real-world example
- What other geometric concepts can be used?



s_3
p_3
p_4
ℓ_5
ℓ_6
s_2
ℓ_2
ℓ_1
s_1
ℓ_3
ℓ_4
s_0
p_1
p_2
cone:
thick path:

THANKS.



w_3^a
w_3^e
s_3
s_2
s_1
s_0
w_0^a
w_0^e
cone:
thick path: