Communications and Transport Systems
Department of Science and Technology
Linköping University

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TNSL20 - basic logistic algorithms

Additional Problem Set October 18, 2017

Question 1 (Shortest Paths):

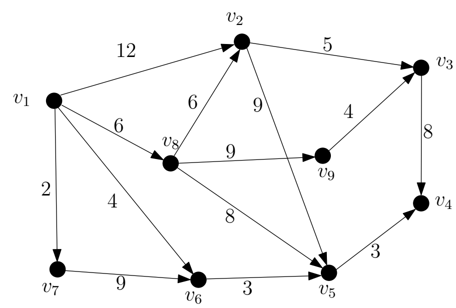

Figure 1: The graph G.

Use Dijkstra's algorithm to compute a shortest path from v_1 to v_4 in the graph G from Figure 1 For each iteration give the length and predecessors that change. If you can choose from more than one vertex, use the one with a smaller index.

Question 2 (FOR loops): Consider the following small algorithm:
$k = 0$
FOR $n = 1$ **TO** 20
    k=k+1

What is the value of k after running this algorithm?

Question 3 (FOR loops): Consider the following small algorithm:
$k = 0$
FOR $n = 1$ **TO** 5
    k=(k+1)*n

What is the value of k after running this algorithm?

Question 4 (Stable Marriage): Consider the following preferences for Anna, Bella, Carla, Daniel, Emil, and Frank:

Table 1: Women's preferences: leftmost is best, rightmost is least preferred.

Anna	Emil	Frank	Daniel
Bella	Frank	Daniel	Emil
Carla	Emil	Daniel	Frank

Table 2: Men's preferences: leftmost is best, rightmost is least preferred.

Daniel	Bella	Carla	Anna
Emil	Carla	Anna	Bella
Frank	Carla	Bella	Anna

(a) Use the appropriate algorithm to determine a stable matching for these eight people.

(b) Daniel suggests the following pairs: Anna+Frank, Bella+Emil, Carla+Daniel Is her suggestion a stable matching? Justify your claim.

Question 5 (Independent Sets and Vertex Coloring):

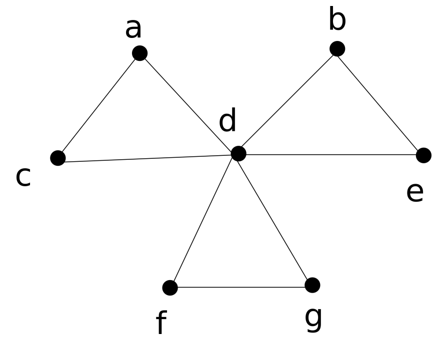

Figure 2: A graph G.

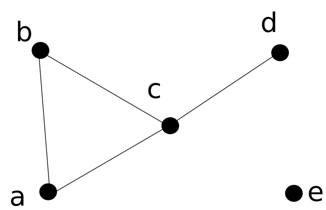

Figure 3: A second graph G.

Use algorithms 6.3 to find a maximal independent set in the graph from Figure 2. Use algorithm 6.5 to compute a maximum independent set for the graph from Figure 3. Do not only present the results, but the intermediate steps of the algorithms.

Question 6 (Scheduling Conflicting Jobs): At a small company 5 jobs need to be completed (j1, ... , j5), the company has two machines (M1, M2) that are needed for these jobs, and three workers (Frank, Jacob, and Anna) that are also needed for some of the jobs. The following table tells you exactly who and what is needed for which job. The execution of each

job	machine	workers needed
j1	M1	Frank
j2	M1	Frank
j3	M2	Frank
j4	M2	Jacob
j5	M2	Anna

single job takes exactly one working day.

Use the given information to construct the corresponding graph for scheduling conflicting jobs. Apply the appropriate algorithm from the lecture to tell the company after how many days they can complete all their jobs.

Question 7 (Mail Delivery): FastMail plans a new route for its postman Jacob. They are trying to work more efficiently, and ask you to help them replan the route. The streets Jacob needs to cover are given by the graph G_1 in Figure 4. Jacob must deliver mail along all these routes, that is, his route needs to pass each street at least once.

(a) Can Jacob cover all of his streets without using any street twice? Give an argument for your statement.

(b) Use the appropriate algorithm to find the best rotue for Jacob. What is the length of the resulting tour?

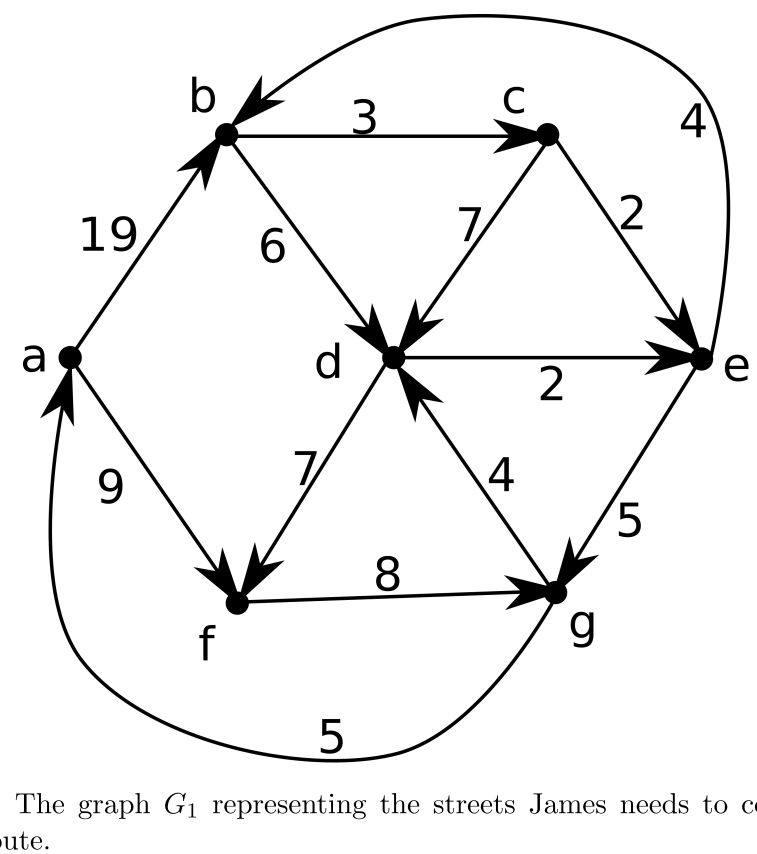

Figure 4: The graph G_1 representing the streets James needs to cover for his delivery route.

Question 8 (Maximum Flow I.):

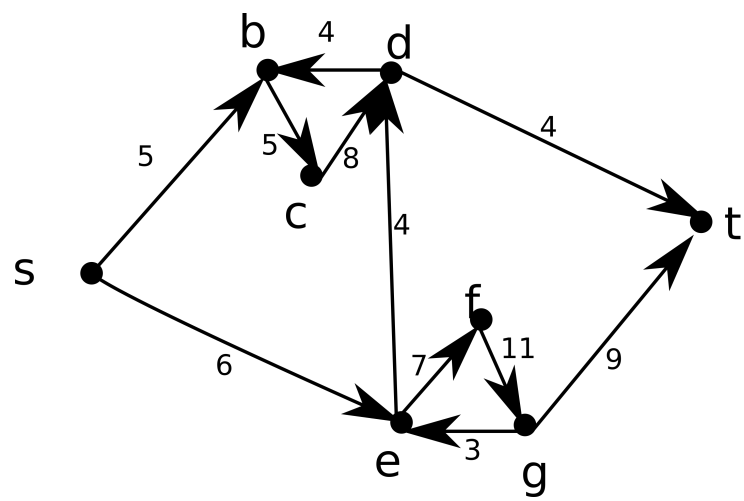

Figure 5: The network (G, c, s, t). The numbers at the edges denote the capacities c.

Use the algorithm from Edmonds-Karp to determine a maximum s-t-flow in the network (G, c, s, t) from Figure 5. Give the residual graph for each step.

Question 9 (Maximum Flow II):

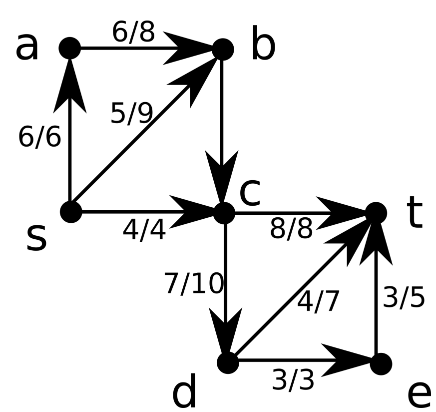

Figure 6: The network (G, c, s, t). The tupels at the edges have the form (flow/ capacity).

(a) Give the residual graph and the residual capacities for the network (G, c, s, t) from Figure 6.

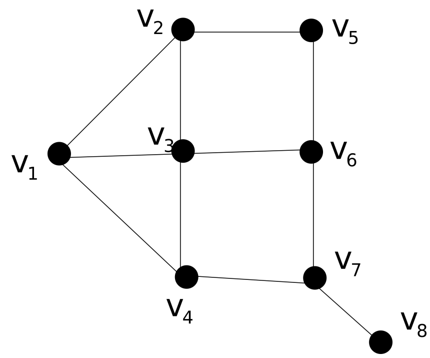

Figure 7: The graph H.

(b) Execute an iteration of the algorithm of Edmonds and Karp. Give the augmenting path and the network with the new flow values.

(c) Is the flow you found optimal? Justify your claim.

Question 10 (BFS and DFS):
Apply DFS and BFS to the graph H from Figure 7; start with vertex v_1.
If, at any time, you could choose more than one vertex for the next step, use the one with smalles index (that is, if you for example could choose v_2, v_4 or v_5 choose v_2).
Give the set Q every time it changes and draw the tree T you found.

Question 11 (Maximum Matching in Bipartite Graphs):

(a) Consider the bipartite graph G from Figure 8. Using the flow formulation we want to determine a maximum matching in G. Draw the network in which a maximum flow needs to be computed.

(b) Enter a flow with value 3 in the network from (a). (Hint: You do not need to use an algorithm to do so.) What does that tell you about a maximum matching?

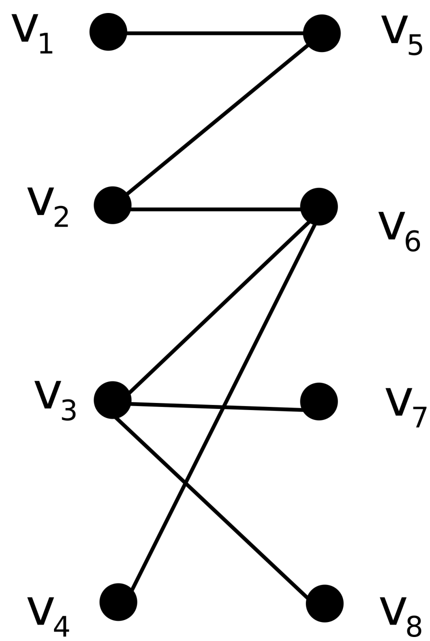

Figure 8: Graph G.