Communications and Transport Systems
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TNSL20 - basic logistic algorithms

Homework Set 1, 2017

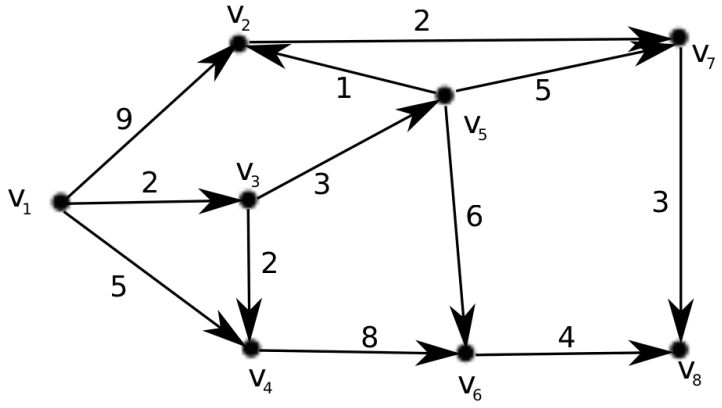
Solutions are due September 12, 2017.

Question 1 (Implementation: Mail Route Offers):
Mister Thorough is responsible to plan the collection of mail from several facilities of his company. Until now, FastMail collected the mail. Each day, FastMail served the route
ACBDAEDEFGBACBGHIBDCJKGHABLMEDECBA,
where each letter stands for a different facility. Recently, Mister Thourough thought that FastMail's service was rather expensive, and invited offers from other companies. Various companies gave an offer, amongst others, FasterMail. Before Mister Thorough can even start to compare the cost for different companies, he first has to figure out whether the offers are actually valid, that is, that they would serve all facilities as often every day as the old route form FastMail did. For example, FasterMail gave an offer for the route CAEDAFBGDABCEBGHILGJDKBCECHABBAMDE, EvenFasterMail for the route AGCAEDGELEHBFGBDACKBIBDBCAJDHMEBA.

(a) Can you help Mister Thorough to figure out whether FasterMail and EvenFasterMail gave offers for the correct facilities? Implement your approach, such that you can test offers of arbitrary companies for validity for Mister Thorough.

(b) Generalize your program to take an arbitrary input route that lets you check whether other routes serve the same facilities.

Question 2 (Application of Dijkstra's Algorithm):



Use the algorithm of Dijkstra from the lecture to determine the shortest path from v_1 to v_8. If, during an iteration, more than one vertex could be chosen, always choose the vertex with the smallest index. Indicate whenever distance label and predecessor change.

Question 3 (Paths and Reliability):

Assume we are given a communication network, but each link only establishes a connection with a certain reliability (larger than zero, and smaller or equal to 1). Now if vertex s wants to communicate with vertex t he would link to use a path of communication links that have the highest reliability (i.e., the links are least likely to fail). Hence, s and t have the highest possible chance to actually communicate successfully. Let us formulate that problem: Given a digraph G with $s, t \in V(G)$. For each edge $e \in E(G)$ we are given a number $r(e) \in [0, 1]$, its reliability. The reliability of a path is the product of the reliabilites of its edges. We are looking for the path from s to t with maximum reliability.

a) Show that you can reduce the problem to the Shortest Path Problem by using logarithms.

b) Solve the problem in polynomial time without using a logarithm.

Question 4 (Big-O Notation):
In the lecture we talked about the Big-O notation to bound the running time (or functions).

a) Find a constant c and a constant n_0 and show, with the help of these constants, that the function is in the given class.

$$\begin{aligned} f_1(n) &= \frac{n^{14}}{4^n} \in O(1) \\ f_2(n) &= 23n^4 + 17n^2 - 20n \in O(n^4) \\ f_3(n) &= \sum_{i=1}^{n} i^2 \in O(n^3) \end{aligned}$$

b) Mark the classes a function belongs to with a cross.

$f(n)$	$O(1)$	$O(n)$	$O(2^n)$
1735			
$23n - 75$			
$51n^2$			
$n \log n$			
n^{2n}			
$\sum_{i=0}^{n} q^i, \lvert q \rvert < 1$			