2. An FPTAS for knapsack

idea ~~behind~~ for FPTAS: use (*)

- ignore a certain number of least significant bits of profits of objects (depending on error parameter ε)

  ↳ modified profits: numbers bounded by a polynomial in n and $1/\varepsilon$

⟹ find solution whose profit is at least $(1-\varepsilon)\cdot OPT$ in time bounded by a polynomial in n and $1/\varepsilon$

Algorithm

1. Given $\varepsilon > 0$, let $K = \frac{\varepsilon P}{n}$
2. For each object a_i, define $\text{profit}'(a_i) = \left\lfloor \frac{\text{profit}(a_i)}{K} \right\rfloor$
3. With these profits of objects use the dynamic progr. algorithm – find most profitable set: S'
4. Output S'

Lemma 1: Let A denote the set output by the algorithm. Then

$$\text{profit}(A) \geq (1-\varepsilon)\cdot OPT$$

Proof: O: optimal set

for any object a: $K\cdot \text{profit}'(a)$ can be smaller than $\text{profit}(a)$ (rounding down!!), but: by not more than K

$$\Rightarrow \text{profit}(O) - K \cdot \text{profit}'(O) \leq n \cdot K \qquad (**)$$

Dynamic Programing step: return set at least as good as O under new profits

$$\Rightarrow \text{profit}(S') \geq K \cdot \text{profit}'(O)$$

$$\overset{(**)}{\geq} \text{profit}(O) - n \cdot k$$

$$\overset{K = \frac{\varepsilon P}{n}}{=} OPT - \varepsilon P$$

$$\overset{OPT \geq P}{\geq} (1-\varepsilon)\, OPT$$

□

<u>Theorem 2</u>: The Algorithm is a FPTAS for knapsack.

<u>Proof:</u> Lemma 1 $\rightarrow$ solution found within $(1-\varepsilon)$ factor of OPT.

running time : $O(n^2 \lfloor \frac{P}{K} \rfloor) = O(n^2 \lfloor \frac{n}{\varepsilon} \rfloor)$, which is a polynomial in n and $1/\varepsilon$.

□