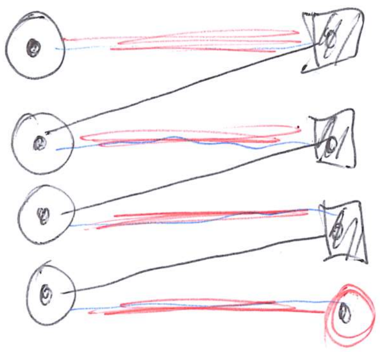
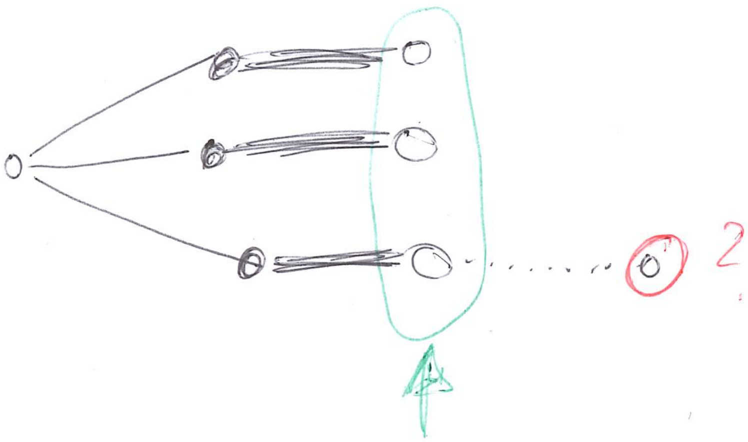
ad Matching

slide 206:





# Matching

I

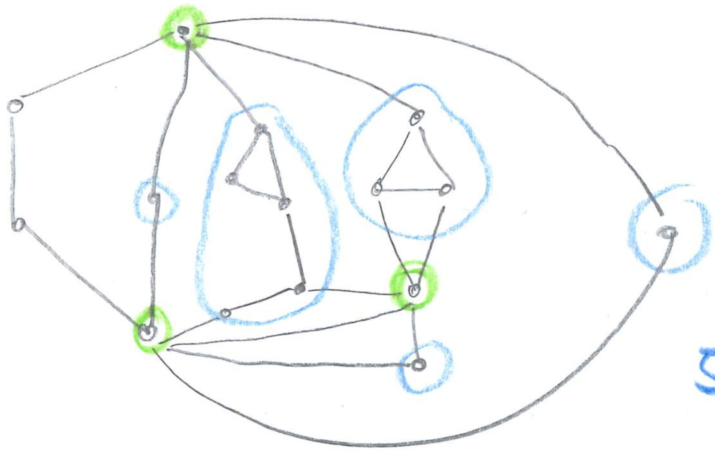
Consider: searching for perfect matching in general graphs?

Given: $G=(V,E)$

Output: Exists a perfect matching in G?

YES $\Rightarrow$ give matching

NO $\xrightarrow[\text{Th. 9.16}]{}$ set of vertices splitting graph in more odd components.



$5 > 3$

**Theorem 9.16 (Tutte 1947)**

A graph $G=(V,E)$ has a perfect matching
$\Leftrightarrow$ For each set $A \subseteq V$: $oc(G \setminus A) \leq |A|$

More general:

**Theorem 9.17 (Tutte-Berge formula 1958)**

For $G=(V,E)$

$$\max\{|M| \mid M \text{ is matching}\} = \min\left\{\tfrac{1}{2}\left(|V| - oc(G\setminus A) + |A|\right) \mid A \subseteq V\right\}$$

$\Rightarrow$ set A gives upper bound for card. of a matching

In the algorithm for bipartite matching?

**Theorem 9.18:** Let G be bipartite, M a matching, T an alternating tree, for which no edge of G connects a vertex in W(T) with a vertex not in V(T).
Then G does not have a perf. matching

proof: Consider $B(T) := V(T) \setminus W(T)$ - the set of black vertices in $T$ (II)

By construction: $|B(T)| = |W(T)| - 1$

G bipartite $\Rightarrow$ no two white vertices adjacent

$\Rightarrow$ Each vertex in $W(T)$ is an odd component of $G \setminus B(T)$

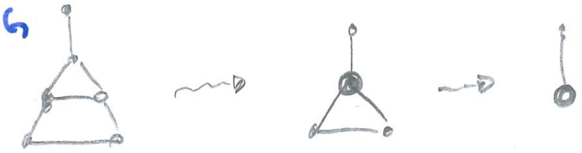
$\overset{|B(T)|<|W(T)|}{\underset{9.16}{\Longrightarrow}}$ statement holds. $\square$

For general graphs?

start with graph G

new graph G' (obtained from G by a serious of shrinkings of odd cycles)

Vertices in G':
- „original vertices"
- „pseudo-vertices": represent shrunk odd cycles




$\Rightarrow$ Each vertex in $V(G')$ represents an odd set of vertices $S(v) \subseteq V(G)$

↑ union of odd sets

**Theorem 9.19:** Let G' be derived graph from G, M' matching in G', T' ~~alternating tree~~ M'-alternating tree of G', such that no element of $B(T)$ (black) is a pseudo vertex.

If each edge ~~has~~ with one end in $W(T)$ has other end in $B(T)$, then G has no perfect matching.

T is „frustrated"

proof: Delete $B(T)$ from G. In G each $v \in W(T)$ is an odd component in $S(v)$

$\Rightarrow oc(G \setminus B(T)) > |B(T)| \Rightarrow$ no perfect matching $\square$

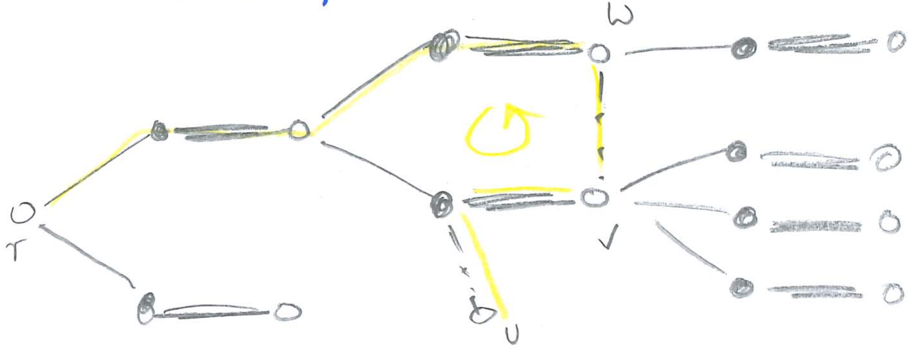
$\Rightarrow$ ~~slides~~

$\Rightarrow$ • derived graph has perf. matching $\Rightarrow$ obtain perf. matching in G (III)

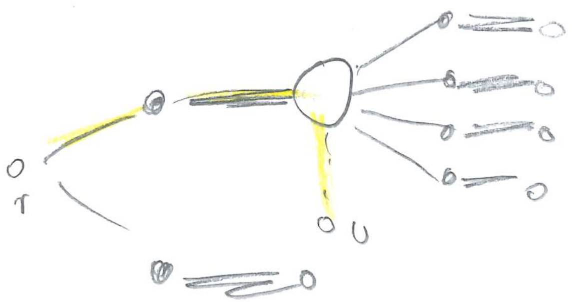
• —"— gives contradiction $\Rightarrow$ no —"— —"—

How to find cycles to shrink?

- edges $W(T)$ to $W(T)$




after shrinking




$\Rightarrow$ slides

End of Alg. 6.20:
- perf. matching
- a frustrated tree T: - ~~[illegible]~~ root unmatched
  - deleting black vertices shows that no matching partner exists for all white vertices

Done?

No, can have unmatched vertices elsewhere
↳ continue search there

⟹ slides (211)

proof 9.23: