Communications and Transport Systems
Department of Science and Technology
Linköping University

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# TNK051: Planning of Air Traffic
# Homework Set 2, 2019

Solutions are due October 1, 2019, 12:00. **Please put your name on all pages!**

Many problems in this homework are about mathematical programming formulation for Fleet Assignment. To solve the problems, you may use the program we had in class:

$$\text{minimize } \sum c_{lf}\, x_{lf} \text{ subject to}$$

$$\sum_{lf:O(lf)=s} x_{lf} = \sum_{lf:D(lf)=s} x_{lf} \qquad \text{for each airport s and fleet type f}$$

$$\sum_{lf:O(lf)=s,d(lf)\leq t} x_{lf} \leq \sum_{lf:D(lf)=s,r(lf)\leq t} x_{lf} + x_{sf} \qquad \text{for each departure time t and fleet type f}$$

$$\sum_{s} x_{sf} \leq N_f \qquad \text{for each fleet type f}$$

$$\sum_{f} x_{lf} = 1 \qquad \text{for each link l}$$

$$x_{lf} = 0 \text{ or } 1 \;, \quad x_{sf} \geq 0$$

F: fleet types S: airports
lf: link l in the network for type f from F
sf: RON arc for airport s from S in the network for f

You may also use the program from the textbook, or the program from the MIT course; in these cases, please indicate which program exactly you are using.

To get credit for a problem, you must adhere to the following:

- If you introduce new notation, please describe it. In particular, describe any new variables you introduce. Remember to state how many you add: it can be either an exact number (1,2,3,etc.), or a description ("one new variable per fleet type", "one new variable per airport", etc.). Also remember to indicate how your new variables are related to the standard ones, i.e., write equations or inequalities that connect the new variables to the standard ones.
- If you add new constraints, remember to state how many you add. It can be either an exact number (1,2,3,etc.), or a description ("one new constraint per fleet type", "one new constraint per link", etc.).

- If the problem has a table, fill in each cell with Y (Yes) or N (No) as it applies to your solution. No credit will be given for problems with empty cells.
- Remember that a constraint may be only an equality (=) or an inequality (≥ or ≤), but not a "non-equality (≠).

1. P2P airline has 2 flights (one flight in each direction) between every pair of 5 airports that the airline serves. P2P has 3 fleet types, and any fleet type may fly any link between any pair of the airports. How many variables are there in the math program for the airline's Fleet Assignment?

2. FlyHigh's fleet consists of 3 Airbus-310, 2 Airbus-319, 2 Airbus-320 and 1 Boeing-777. FlyHigh's management wants to keep only 5 Airbuses (and sell the others). Model this by modifying the mathematical program for the fleet assignment problem? Describe the relevant variables (you may omit description of the other variables), the relevant constraints (you may omit description of the other constraints), and the objective function.

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|---|---|
| I added new variables | |
| I added new constraints | |
| I modified the standard constraints | |
| I modified the objective function | |

3. **a.** RonAir flies to several airports in the Ron province, recently being torn by riots. Because of the riots, every airplane staying overnight in Ron must be guarded by a policeman. Ron's government can delegate only 3 policemen to guard RonAir's airplanes; thus, the total number of RonAir's planes staying overnight in all airports in Ron must not exceed 3.
Model this situation by modifying the mathematical program for the Fleet Assignment problem. Describe the relevant variables (you may omit description of the other variables), the relevant constraints (you may omit description of the other constraints), and the objective function.

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**b.** As the riots subside, more police force becomes available, and Ron's government is able to rent policemen to airlines. Guarding one narrow-body jet costs 1000 Ronors (the local currency) per night; guarding a wide-body costs 2000 Ronors.
Model this by modifying the mathematical program for the Fleet Assignment problem. Describe the relevant variables (you may omit description of the other variables), the relevant constraints (you may omit description of the other constraints), and the objective function.

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4. The OD airline operates 1 Airbus-310 and 1 Boeing-777. OD has 4 flights per day between airports O and D: the morning flight from O to D, the early afternoon flight from D to O, the late afternoon flight from O to D, and the late evening flight from D to O. To simplify maintenance and crew scheduling, the airline management requests that the morning and the afternoon O->D flights are operated by the same plane.
**a.** Model this requirement by modifying the math program for the Fleet Assignment. Describe the relevant variables (you may omit description of the other variables), the relevant constraints (you may omit description of the other constraints), and the objective function.

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**b.** OD just purchased two more planes -- 1 Airbus-319 and 1 Boeing-787 -- so now its fleet consists of 2 Airbuses (-310 and -319) and 2 Boeings (-777 and -787). It still has the same 4 flights per day between O and D. The airline now wants to operate the same-make aircraft on both O->D flights. That is, either each of the 2 flights must be served by an Airbus (it’s OK to have one flight served by Airbus-319 and the other -- by Airbus-310), or each of the 2 flights must be served by a Boeing (it’s OK to have on flight served by Boeing-777 and the other -- by Boeing-787). Model this requirement by modifying the math program for the Fleet Assignment.

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**c.** Now the management again wants to have both O->D flights operated by the same plane. How to model this?

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5. **a.** Can you find an airline that never has all its planes on the ground in airports? That is, at any time of the day, the airline should have at least one of its planes in the air.
   **b.** The counting constraint in the mathematical program for the Fleet Assignment counts, for each type, the total number of planes that remain on the ground overnight. However for a major long-distance airline (especially one flying West-to-East) there may exist no time when all planes are on the ground. How would you formulate the counting constraint in this case?

6. **a.** The Little airline has a single plane. The demand for the plane is huge, and Little will not be able to satisfy all flight requests for the plane. Therefore the manager has to decide which flights to fly; the objective is to maximize the total number of flights served. Manager's friend, Professor May B. Wright, suggests that there is no need to waste Little's low budget on buying software for solving the math program for Fleet Assignment; instead, the professor claims, it is enough to work just with the time-line network, and assign each ready flight to the earliest available departure (recall from class that we used the Earliest-Departure-First strategy to test the feasibility of a schedule). Prove that professor is not right, by giving an example of a network where the Earliest-Departure-First strategy fails to find the maximum possible number of flights that can be flown by Little's plane.

**b.** Different potential flights generate different revenues for Little; let $p_l$ denote the revenue obtained if flying link l. The manager wants to decide which flights to fly, and the objective now is to maximize the total revenue. Model this by modifying the mathematical program for the Fleet Assignment problem. Describe the relevant variables (you may omit description of the other variables), the relevant constraints (you may omit description of the other constraints), and the objective function.

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|---|---|
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7. Discuss advantages and drawbacks of splitting the Fleet Management problem into Fleet Assignment and Aircraft Routing.

8. You work for an airline, and you have to convince the management that knowing the math programming formulation for Fleet Assignment is worthwhile. Come up with a modification of the standard Fleet Assignment problem, which takes into account some additional aspect of airline management (it can be a real aspect, or a fictitious one, as in problems above). Solve your modification.

9. How would you model the following constraints?

A. *Station exclusion*: Due to restrictions, fleet type f cannot fly into airport j.
B. *Noise restriction*: Due to municipal laws, no more than 30% of departures out of airport j can be of fleet type f or g.
C. *Crew*: The number of flying hours of fleet f cannot exceed the maximum number of available crew hours, C(f).