Communications and Transport Systems
Department of Science and Technology **2022/2023**
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Design and Analysis of Algorithms Part 2 - Approximation and Online Algorithms Homework 2, January 13, 2023

Problem 1 (Planar 3SAT):
Read the paper "PLANAR FORMULAE AND THEIR USES" bei David Lichtenstein[1], and prepare a short presentation on the reduction presented.

Problem 2 (Hamiltonian Paths):
A Hamiltonian path, also called a Hamilton path, is a graph path between two vertices of a graph that visits each vertex exactly once. If a Hamiltonian path exists whose endpoints are adjacent, then the resulting graph cycle is called a Hamiltonian cycle , or Hamiltonian circuit. The following problems are NP-complete:
Hamiltonian Path Problem
Given: graph $G = (V, E)$
Find: A Hamiltonian path, or determine that none exists.

Hamiltonian Circuit Problem
Given: graph $G = (V, E)$
Find: A Hamiltonian circuit, or determine that none exists.

In the paper '’Hamilton Paths in Grid Graphs"[2] Itai et al. showed that the problems remain NP-complete in grid graphs. Read the proof by Itai et al. and use the construction to show the following problems to be NP-hard:

(a) **Traveling Tourist Problem in Grid Graphs**
Given: a grid graph G.
Find: a shortest tour through the graph such that every vertex is either on the tour or is adjacent to a vertex on the tour.

Show: The Traveling Tourist Problem is NP-hard in Grid Graphs.

(b) **Lawn Mowing and Milling Problems.** We are given a planar region, R, that describes the grass to be mowed or the pocket to be machined. We are also given a cutter, χ. We assume that χ is either a circle or an axisaligned square. Without loss of generality, we scale our problem instance so that χ is a unit circle (radius

[1] https://epubs.siam.org/doi/pdf/10.1137/0211025
[2] https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=4d2abfc483c9ffa4187d43269a53ec37e64e1c5d

1) or a unit square (side length 1). The reference point for the cutter χ is its centerpoint. We let $\chi(p)$ denote the placement of χ at the point $p \in \mathbb{R}^2$ (i.e., the unit circle/square with centerpoint at p). A *lawn mower path/tour* π is a path/tour such that every point of the region R is covered by some placement of χ along π; i.e., $R \subseteq \cup_{p \in \pi} \chi(p)$. A *milling path/tour* π is a path/tour such that every point of R is covered by some placement of χ along π, and no placement of χ along π ever hits a point outside of R; i.e., $R = \cup_{p \in \pi} \chi(p)$.

We consider two cases of allowed motions (translations) of the cutter: rectilinear (axis-parallel) and unrestricted (arbitrary translation). We measure the length of a path/tour of the cutter as its Euclidean (L_2) length. In the case of rectilinear motion, measuring the Euclidean length amounts to the same thing as measuring the L_1 length of the path/tour.

It is easy to see that, for any region R, there always exists a lawn mower path/tour; however, it may be that there exists no milling path/tour for a (connected) region R, as the cutter may not be able to fit into the ''corners" of R or pass through the ''bottlenecks" of R.

(b.1) Show: The lawn mowing problem for a connected polygonal region is NP-hard for the case of an aligned unit square cutter χ.

(b.2) Show: The lawn mowing problem is NP-hard even for simple polygonal regions R.

(b.3) Show: The milling problem is NP-hard for the case of an aligned unit square cutter χ and a multiply-connected polygonal region R (with holes).