Evaluation of Parallel Coordinates: Overview, Categorization and Guidelines for Future Research

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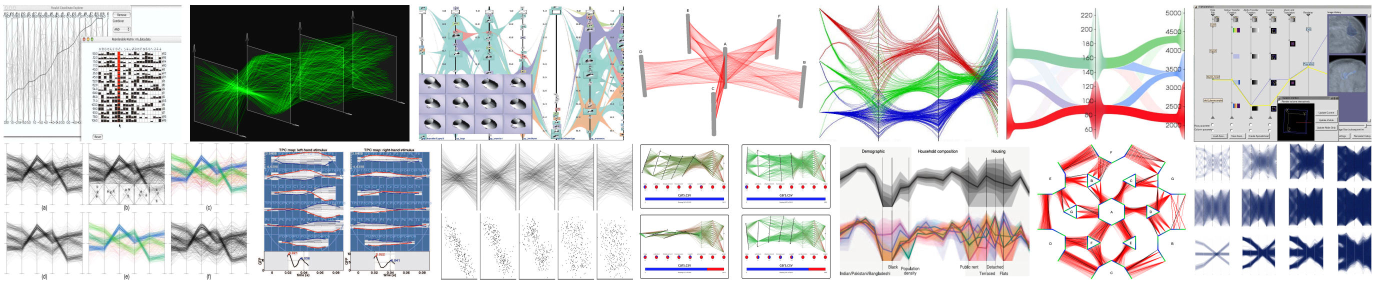

Fig. 1. Examples of extensions and modifications of parallel coordinates that have been evaluated. All images are reproduced with the permission of their respective copyright holder.

Abstract—The parallel coordinates technique is widely used for the analysis of multivariate data. During recent decades significant research efforts have been devoted to exploring the applicability of the technique and to expand upon it, resulting in a variety of extensions. Of these many research activities, a surprisingly small number concerns user-centred evaluations investigating actual use and usability issues for different tasks, data and domains. The result is a clear lack of convincing evidence to support and guide uptake by users as well as future research directions. To address these issues this paper contributes a thorough literature survey of what has been done in the area of user-centred evaluation of parallel coordinates. These evaluations are divided into four categories based on characterization of use, derived from the survey. Based on the data from the survey and the categorization combined with the authors' experience of working with parallel coordinates, a set of guidelines for future research directions is proposed.

Index Terms—Survey, evaluation, guidelines, parallel coordinates.

1 Introduction

Parallel coordinates [16] has, for many years, been a popular visualization technique for multivariate data. The design (Fig. 2), with polylines describing multivariate items that intersect with parallel axes representing variables, can be used for the analysis of many properties of a multivariate data set [50]. A few examples include identifying multivariate outliers, trends, and clusters. During the last 20 years the technique has been the focus of many research projects, primarily within the information visualization community although it has also been applied within scientific visualization and visual analytics, across numerous application areas such as mathematics, statistics, bioinformatics, medicine and climate science. The technique is also incorporated within several commercial applications and frameworks, making it available to a large number of users. An excellent compilation of current research on parallel coordinates can be found in the state-of-the-art report by Heinrich and Weiskopf [14].

Research on parallel coordinates has primarily focused on making the technique less sensitive to visual clutter by reducing the number of polylines or by reducing, or reordering, the parallel axes. This research has many facets and has included work on using the visual representation with algorithmic data mining techniques [7, 22, 8, 38], using different interaction techniques [11, 43, 9] and choosing various axis layouts, in both 2D and 3D [49, 23, 5].

With respect to the large number of publications proposing variations of parallel coordinates, only a limited number present results from user-centred evaluations, thus addressing usability aspects [17]. Evaluation is necessary in order to understand and communicate the potential and limitations of all proposed variations of parallel coordinates. In general, evaluation will advance information visualization research by promoting promising research ideas but also preventing less promising ideas from being accepted as valid ones to be expanded upon. This is also how we can present evidence of measurable benefits that will encourage more widespread adoption of successful variations of parallel coordinates by real users [25, 36].

The aim of this paper is to highlight the importance of, and need for, user-centred evaluation studies in order to investigate and validate presented research results. The reader of this paper will get an accessible overview and discussion of previous work (which will be especially helpful for researchers new to the area of parallel coordinates and/or evaluation studies), gain knowledge of important findings regarding the usefulness of parallel coordinates, and finally receive guidance in interesting and important directions for future work.

1.1 Contributions

The first contribution of this paper is a survey of the 23 existing papers that present user-centred evaluations of the standard 2D parallel coordinates technique and its variations, see Fig. 1. Each evaluation is described by a concise summary. Besides variations of parallel coordinates, many other visualization techniques are included in the evaluations. In addition to comparing different versions of parallel coordinates with each other it is common that a specific version of parallel coordinates is compared with other visualization techniques, for exam-

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Table 1. The 23 identified papers on evaluating parallel coordinates (PC) divided into the four constructed categories. Most evaluations have been performed within categories 1-Evaluating axis layouts of PC and 4-Comparing PC with other analysis techniques. The study presented in [47] is classified into both category 3 and 4.

	Number of studies	References
1. Evaluating axis layouts of PC	7	[42, 6, 21, 30, 5, 20, 48]
2. Comparing clutter reduction methods for PC	4	[15, 12, 33, 38]
3. Show practical applicability of PC	4	[45, 2, 47, 3]
4. Comparing PC with other analysis techniques	9	[28, 29, 10, 26, 43, 41, 35, 47, 44]

ple scatter plots, radar charts, pie charts, etc. In some cases a version of parallel coordinates is even compared with non-visual techniques such as tables or lists.

This paper has a strict focus on results from user-centred evaluations and does not consider papers with use cases or similar that attempt to demonstrate results by providing 'walkthroughs' by the authors. If performed correctly such demonstrations can, of course, present valuable support for a proposed idea but are different from evaluations based on user studies and will not be considered in this paper. In addition, it is important to note that this paper does not attempt to evaluate the quality of the evaluations but focuses on what aspects have been studied, and it is up to the reader to form their own opinion regarding the quality of the individual studies.

The second contribution is a categorization, derived from the literature survey, of the performed evaluations. The four categories are based on how parallel coordinates is used as a visual analysis tool: (1) evaluating axis layouts of parallel coordinates, (2) comparing clutter reduction methods for parallel coordinates, (3) showing practical applicability of parallel coordinates, and (4) comparing parallel coordinates with other analysis techniques.

Since the standard 2D version of parallel coordinates is by far the most used of all the parallel coordinates versions, this version has been given special attention. Besides summaries of all the evaluations this paper contributes a table which easily illustrates how standard 2D parallel coordinates performs, compared with 26 other techniques, across 7 tasks. In the table, only combinations of techniques and tasks that produce evaluation results specific enough to replicate are included. For example, results such as 'obtain an overview', 'understanding parameters', 'is more appropriate', etc. are hard to measure, making them difficult to replicate.

The final contribution is a set of guidelines for future research. These guidelines are based on the data collected from the evaluations as well as the authors' experience of working with parallel coordinates. These guidelines will hopefully advance the state-of-the-art of parallel coordinates. To summarize, the major contributions of this paper are:

- a survey of existing user-centred evaluations of parallel coordinates,
- a categorization of the identified user-centred evaluations based on characterization of use,
- a compilation and description of the performance of standard 2D parallel coordinates versus 26 other data analysis techniques for 7 identified tasks,
- guidelines for future research on user-centred evaluations of parallel coordinates.

Although the authors have been thorough in the literature review it can not be guaranteed that research is not missing. The contributions and the general conclusions do not, however, depend on any single work.

1.2 Structure of the Paper

The remainder of this paper is organized as follows. Section 2 presents and discusses the categorization of previous evaluations. Sections 3–6 each presents and discusses one category and can be read individually, ending with a short summary and discussion. How 2D parallel coordinates performs in relation to other techniques is discussed in Section 7. More detailed discussions and guidelines for future research are presented in Section 8 and concluded in Section 9.

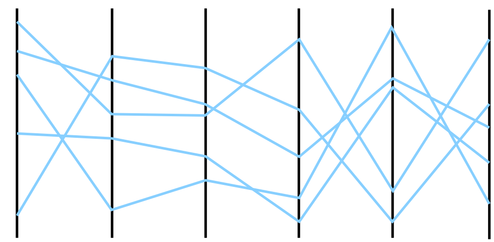

Fig. 2. Standard 2D parallel coordinates [16] with variables represented as parallel axes and multivariate items represented as polylines intersecting the axes at their corresponding values.

2 Categorization of Parallel Coordinates Evaluations

From a comprehensive literature review of all major journals and conferences on visualization, 23 papers were found that present user-centred evaluations of parallel coordinates.

That only 23 papers were found that deal with evaluation was, unfortunately, not surprising but far from satisfying when the goal of information visualization is to develop useful visualization techniques. Among the journals and conferences included in the review there was no apparent difference in the ratio between evaluation papers and the total number of papers on parallel coordinates. Looking at, for example, IEEE Infovis from the ten years between 2005 and 2014 there were in total 28 papers where parallel coordinates was the main topic and of these only four presented some type of user-centred evaluation. The 23 identified papers focus on a wide variety of topics so a categorization was not straightforward. Existing work on categorizing evaluations, such as [36, 27, 18], could not be directly incorporated into this work but provided much inspiration. The categorization is derived from the literature survey. The four categories have been constructed so that they, together, cover all major aspects of previous work on parallel coordinates and, at the same time, do not significantly overlap. The categories are as follows:

1. evaluating axis layouts of parallel coordinates,
2. comparing clutter reduction methods for parallel coordinates,
3. showing practical applicability of parallel coordinates,
4. comparing parallel coordinates with other data analysis techniques.

An overview of the publications and categories is presented in Table 1. The four categories with the assigned publications are described in detail below.

3 Evaluating Axis Layouts of Parallel Coordinates

The reason for studying alternative axis layouts is mainly because the standard 2D parallel coordinates technique only allows the identification of relationships between adjacent axes. This topic has received

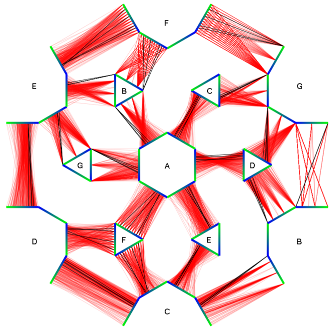

Fig. 3. Many-to-many relational parallel coordinates [30].

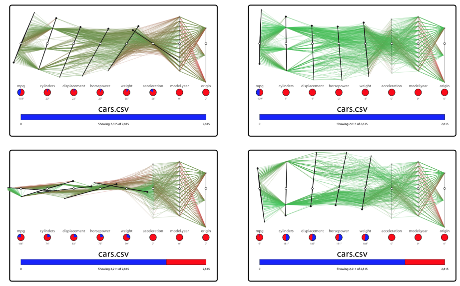

Fig. 4. Force-directed parallel coordinates [48].

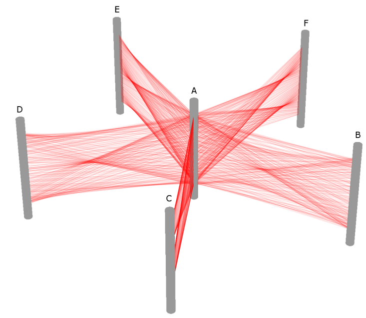

Fig. 5. 3D multi-relational parallel coordinates [19] was evaluated with respect to its efficiency in conveying complex patterns [6] and noisy data [21].

much attention during recent years and many different approaches have been proposed. These can mainly be categorized into:

- techniques for arranging axes in 2D parallel coordinates in order to highlight specific types of relationships, or for reducing clutter,
- techniques for extending parallel coordinates from 2D to 3D with the aim of allowing more relationships to be simultaneously displayed and analysed.

3.1 Evaluations

The following work addresses the usability of different axis layouts of standard 2D parallel coordinates and its extensions into 3D.

Focusing on 2D parallel coordinates, Siirtola et al. [42] based their evaluation on eye-tracking. Nine participants took part in a quantitative study where each performed nine different tasks related to a cars data set. The tasks included: searching for cars with four or six cylinders, finding the average mileage for cars having six cylinders, and finding which Japanese cars that have the best acceleration. Eye-tracking was used to capture the participants' gaze data, in terms of fixation target and fixation length. The results confirmed a previous study presented in [43] (see section 6) that the parallel coordinates technique is quite easy to learn and that its immediate usability is good.

Lind et al. [30] developed the 2D many-to-many relational parallel coordinates technique (see Fig. 3) and compared it with standard parallel coordinates. The aim of their quantitative study was to investigate whether the many-to-many relational display was more effective and efficient than standard parallel coordinates for finding interesting relationships between variables. For both standard parallel coordinates and many-to-many relational parallel coordinates, 12 participants were asked to identify negative correlations between adjacent axes. The results indicate that the participants in the study performed the task about 20% faster using the many-to-many relational configuration.

Claessen and van Wijk [5] further developed the concept of many-to-many relational parallel coordinates into the system FLINA (Flexible LINked Axes). They performed a qualitative evaluation in which the think-aloud method was used together with a follow up survey in which the participants rated a number of statements on a 5-point Likert scale. In one part of the study, the participants used the FLINA system to construct simple visual representations, such as scatter plots, parallel coordinates and radar charts, of the Iris data set. The participants performed a number of tasks such as finding the range of values of attributes, identifying average values for attributes, and studying correlations between attributes. In the next step the participants created their own combinations of axes while exploring their own data sets. The overall results of the study were positive with respect to the flexibility of the system and the possibility of interactively adding and removing axes.

Walker et al. [48] introduced force-directed parallel coordinates based on physical metaphors, see Fig. 4, which together with a number of interaction methods can be used to explore multivariate data. Some examples include cutting, axis swinging and axis pinning. A qualitative user study, using the same methodology as in [5], was performed with 13 participants. After a test session the think-aloud method was used while the participants explored a data set containing world food prices since 1990. After the experiment the participants filled in a survey in which they had to rank a number of statements on a 5-point Likert scale. The results indicate that the strengths of the force-directed approach is in its identification of correlations and trends. The main weaknesses concerned the possibilities to undo interactions and that the interactions required very precise mouse inputs.

Instead of only focusing on axis layouts of 2D parallel coordinates, Forsell and Johansson [6] investigated the performance of 2D parallel coordinates compared with 3D multi-relational parallel coordinates (Fig. 5). The evaluation was conducted as a quantitative experiment in which 30 participants performed a simple and a complex task. The simple task concerned finding one specific pattern from a set of five. In the complex task, the participants were asked to search for all five patterns and identify the one pattern that was missing. In a post-test questionnaire the participants rated a number of statements on a 5-point Likert scale. The questions were related to how easy or difficult it was to use the different parallel coordinates techniques for the different tasks. The results showed that when manually exploring a complex interrelated multivariate data set, the user performance with 3D multi-relational parallel coordinates is significantly faster. For simpler tasks, such that finding a relationship between two adjacent axes, the difference is negligible.

Standard 2D parallel coordinates and 3D multi-relational parallel coordinates were further evaluated by Johansson et al. [21]. In a first quantitative study with 13 participants the aim was to investigate acceptable distortions (noise) in 2D parallel coordinates and investigate users' ability to recognize different relationships in data when these were shown with noise (Fig. 6). 13 participants were asked to identify which of three noisy patterns was the target pattern. The results indicated that a noise level of 13% is the threshold for efficiently discriminating between patterns. In a second quantitative study, 13 participants were asked to identify patterns at different angles in 3D in order to investigate the maximum number of variables that can be efficiently displayed in 3D multi-relational parallel coordinates. The result of this study showed that a 3D multi-relational parallel coordinates display having 11 axes (variables) is as efficient as 2D parallel coordinates. For larger number of variables, the efficiency is reduced since a greater amount of manipulation, in terms of interaction, is required.

In a recent publication, Johansson et al. [20] evaluated the 3D parallel coordinates technique developed by Wegenkittl et al [49], see Fig. 7. This technique yields a 3D representation by using parallel planes instead of parallel lines as axes. The 3D parallel coordinates representation has the possibility to simultaneously display more variables compared with standard parallel coordinates. A quantitative study was performed that compared 2D parallel coordinates with 3D

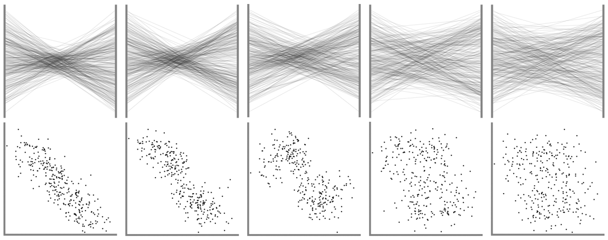

Fig. 6. Noisy data in parallel coordinates was studied in [21].

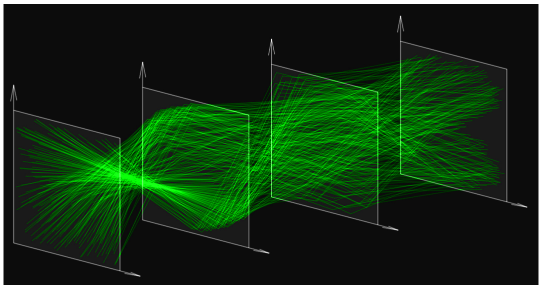

Fig. 7. 3D parallel coordinates was compared with standard 2D parallel coordinates for the identification of 2D patterns [20].

parallel coordinates for the identification of patterns. More specifically, 24 users searched for five patterns displayed in both representations. Both response times and accuracy were measured. The results strongly indicate that 2D parallel coordinates outperforms 3D parallel coordinates, for the specific tasks and users. Users took on average seven times longer when using 3D parallel coordinates, as compared with 2D parallel coordinates when identifying the patterns in the study. A follow-up questionnaire detailing the participants' subjective ratings on a Likert scale agreed with the results.

3.2 Summary and Discussion

The evaluations discussed in this section have shown that the 2D parallel coordinates axis layout is both effective and efficient for tasks involving comparing relationships between variables. The standard 2D axis layout is also regarded as intuitive and novice users learn it without effort. One study has also shown that parallel coordinates is quite robust to noisy data. For tasks involving examining multiple relationships, extensions into 3D have been shown to be advantageous. It should be noted, however, that the existing studies are in strong favour of extensions showing co-planar structures whereas extensions producing 3D structures distort the data and are difficult to interpret.

There are several aspects within this area that remain unexplored and if studied could provide a better understanding of possible axis configurations in parallel coordinates. Automatic ordering of axes based on various criteria is a well studied approach. What is missing is knowledge about when and how these should be used. Differences between axis layouts need to be further investigated and systematically studied or different tasks and users.

4 Comparing Clutter Reduction Methods for Parallel coordinates

The parallel coordinates technique is well-known to be sensitive to visual clutter and even a moderate data size can produce cluttered displays. A large number of methods have been proposed to overcome this limitation by, in some form, visualizing an abstraction of the data. The proposed techniques can, broadly, be categorized into:

- reduction techniques that result in the appearance of fewer rendered polylines with the aim to better highlight structures in the data,
- techniques that use rendering of semi-transparent polylines to enhance underlying structures in the data.

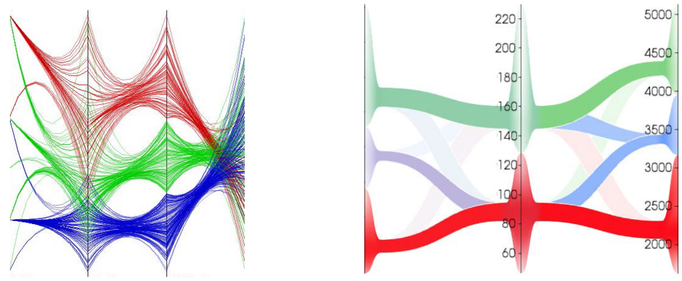

Fig. 8. Bundled [31] (left) and edge-bundled [33] (right) parallel coordinates.

4.1 Evaluations

Different ways of using curves in parallel coordinates are common, see for example [46, 8, 31]. Heinrich et al. [12] focused on a bundling technique for parallel coordinates. Although the actual number of rendered polylines is the same as in standard parallel coordinates, the visual appearance is a reduction that aims to reveal structures. (Fig. 8, left). The aim of their quantitative study was to investigate the efficiency of bundled parallel coordinates compared with standard parallel coordinates for visual identification of linear correlations and clusters. 14 participants took part in the experiment. For the part on correlations, each participant was asked to categorize the strength of the correlation using five different categories ranging from 'strong negative' to 'strong positive'. For the clustering task, each participant viewed clusters of different dimensionality and were asked to identify the number of clusters. The results of the study showed that visual identification of linear correlations was the same for both techniques and that bundled parallel coordinates is efficient for identifying clusters.

In a recent publication, Palmas et al. [33] introduced and evaluated an edge-bundling technique that is based on density clustering, see Fig. 8, right. The two tasks included in the quantitative evaluation were to judge the strength of a correlation (55 participants) and to trace a subset of the data over several variables (82 participants). The first result indicates that the participants were more accurate when judging correlations in edge-bundled parallel coordinates compared with standard 2D parallel coordinates. The second result shows that bundled parallel coordinates is also more accurate when tracing a subset of the data over several variables.

Another proposed method for clutter reduction is progressive parallel coordinates [38] which is based on progressive refinements of the display. This technique is used to reduce the number of data items needed to be rendered as polylines, making it possible to visualize larger data sets (see Fig. 9 for examples). In [38] the authors performed a quantitative evaluation with 43 participants of various levels of expertise to compare the effectiveness of progressive parallel coordinates with standard parallel coordinates. The participants were asked to perform a number of tasks such as identifying example patterns and searching for patterns in different refinement levels. The results indicate that in terms of correctness there was no significant difference between the two techniques. For pattern detection, however, progressive parallel coordinates was substantially faster and, on average, only 37% of the data was needed in order to detect the patterns.

Holten and van Wijk [15] evaluated parallel coordinates for cluster identification, which is a commonly used technique for reducing visual clutter. Nine types of parallel coordinates (eight variations, both static and animated, as well as standard 2D parallel coordinates) were evaluated. The six static variations were: scatter plots embedded in parallel coordinates, coloured polylines, blended polylines, coloured plus blended polylines and curved polylines, see Fig. 10. The animated versions were: random tour animation, permutation tour animation and wobble animation. In a quantitative experiment, 20 participants used all the different visual representations to identify the number of clusters present. The performance was measured in terms of response time and accuracy. The main findings of the study were that, except

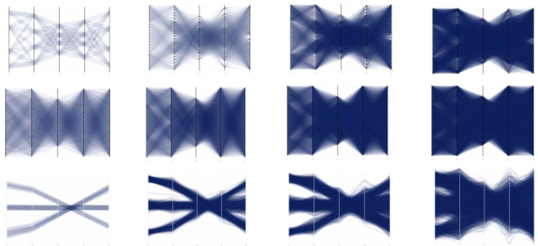

Fig. 9. The technique of progressive parallel coordinates [38].

for embedded scatter plots, none of the previously proposed variations significantly improved the user performance. The results of the experiment agreed with the subjective findings stated by the participants.

4.2 Summary and Discussion

Much work has been dedicated to addressing issues related to cluttered parallel coordinates displays. Techniques based on colour, blending and curved lines are commonly seen in the literature as suggestions for improving the visual quality. For the task of cluster identification no benefit in terms of improved performance has been found for any of these variations. Only the combination of standard 2D parallel coordinates and scatter plots has been shown to be advantageous. Compared to standard parallel coordinates, the bundled technique makes visual identification of clusters in data easier and is advantageous for judging correlation and for tracing subsets of the data across multiple variables.

The above described evaluations provide a basis for understanding how visual clutter effects pattern identification. There are, however, important issues that have not been addressed. Knowledge about the relationships between display size, data size and pattern identification needs to be increased, which can only be achieved through user studies. Another area that is becoming more important as the size of the data increases is progressive parallel coordinates. An interesting idea would be to extend the method to other variants of parallel coordinates.

5 Showing Practical Applicability of Parallel Coordinates

To show practical applicability of parallel coordinates means, for example, to:

- introduce parallel coordinates in a new application domain,
- make novel use of parallel coordinates for solving tasks specific to an application.

When using parallel coordinates for a new application, the technique is typically compared with what is state-of-the-art in the area.

5.1 Evaluations

Tory et al. [47] introduced parallel coordinates as an interface for exploratory volume visualization (Fig. 11). They also performed a qualitative evaluation in which they compared the parallel coordinates interface to table based and traditional interfaces. A traditional volume visualization interface typically consists of a rendering view, a transfer function editor and some widgets for zoom and rotation. Their evaluation was based on five expert users. The experts were asked to perform two tasks: explore the data set, and search for a specific identifiable object in the data. Thereafter, they rated the visual representations using a set of heuristics as statements. Their main findings showed that the table and parallel coordinates interfaces were mostly rated higher than the traditional interface. Furthermore, parallel coordinates and tables had different strengths, with tables considered good for investigating relationships while parallel coordinates were rated higher for understanding parameters and changing parameter values.

Another example of introducing parallel coordinates into a new domain is presented by ten Caat et al. [45] who developed tiled parallel coordinates for analysis of multi-channel EEG data (Fig. 12). More

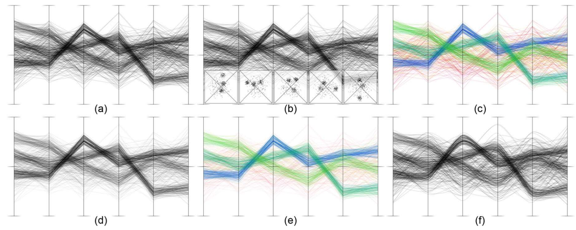

Fig. 10. The six non-animated variations that were evaluated in [15]: (a) standard parallel coordinates, (b) embedded scatter plots, (c) coloured polylines, (d) blended polylines, (e) coloured and blended polylines, and (f) curves instead of polylines.

specifically, they used a data set containing somatosensory evoked potential (SEP) which is obtained by electrical stimulation of a nerve. Traditional analysis tools for this type of data consist of simple graphs showing time and voltage. The authors performed a qualitative evaluation of these tools based on four criteria: number of time steps that can be visualized, clarity of the temporal dimension, number of channels that can be properly analysed, and preservation of the spatial order. Tiled parallel coordinates was the technique that enabled analysis of most time steps but was less good at preserving an explicit time ordering. After the initial qualitative study, the authors performed a quantitative user study with 12 participants analysing 16 different SEP data sets. Tasks specifically designed for SEP data sets, such as identifying latencies and symmetry values, were used. The results of this study indicated that visual analysis using tiled parallel coordinates was, on average, 40% faster than using standard clinical EEG visualization methods, with accuracy being maintained.

Azhar and Rissanen [2] used parallel coordinates as a tool for interactive alarm filtering in the application area of process control. When an industrial process (electricity, gas, heating, etc.) runs into an abnormal status, an alarm is generated. These alarms are typically stored in long alarm lists which are tedious to search through since it is often difficult to filter out alarms of particular interest. A prototype application was developed, including both a traditional alarm list and parallel coordinates. To evaluate the effectiveness and efficiency of parallel coordinates compared with traditional alarm lists, the authors performed a quantitative evaluation engaging 12 participants. The evaluation aimed at identifying whether there would be an efficiency gain through using parallel coordinates as well as getting insights into what the participants thought of this new way of analysing alarms. The participants performed a total of 11 tasks that were categorized into tasks related to selection and filtering. The evaluation showed that, for simpler tasks (involving a single or few parameters), there was no significant difference. For more complex tasks (involving several parameters) there was a significant reduction in time of 40%–80% in favour of parallel coordinates. In a post-test questionnaire, ratings from the participants indicated that they found parallel coordinates faster, more intuitive, more accurate, easier to learn and remember, as well as more supportive of pattern identification as compared with traditional alarm lists.

Beham et al. [3] used parallel coordinates as a tool for exploring the results of geometry generators, see Fig. 13. A geometry generator creates a polygonal mesh that approximates a geometric shape. Such a generator has many parameters that describe features of the geometry. Parallel coordinates was used to explore the parameters to find similarities and errors in the geometric shapes and their corresponding parameters. This use of parallel coordinates shares many similarities with the work by Tory et al. [47]. The authors conducted a quantitative evaluation with three domain experts that performed the following three tasks: (1) evaluation of the parameter space and geometric result, (2) finding implausible shapes, and (3) determining the influence and sensitivity of the parameters. The results from the study were positive and there was a clear demand for similar tools. The experts could solve the two first tasks but had difficulties in the sensitivity task due to missing interaction features in the parallel coordinates implementation.

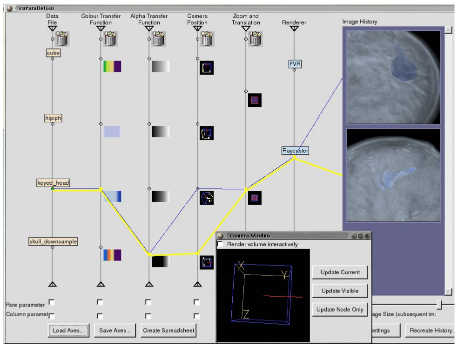

Fig. 11. A parallel coordinates style interface for exploratory volume visualization [47].

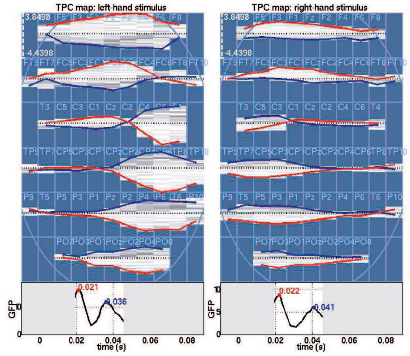

Fig. 12. Tiled parallel coordinates for exploration of multi-channel EEG data [45].

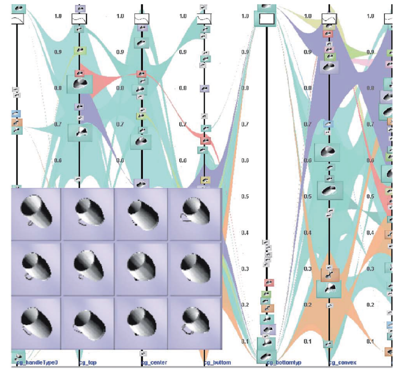

Fig. 13. Parallel coordinates used as a tool for exploring parameters of geometry generators [3].

In a recent publication, Slingsby et al. [44] used an interactive system for exploration of uncertainty in geodemographics, in which parallel coordinates was an important component for showing uncertainties. This was done by using faded bands, as illustrated in Fig. 14. The authors focused on geodemographics classifiers which are used to characterize a population by categorizing different geographical areas with respect to the demographic and lifestyle characteristics of the people living within them. This classification is typically of an hierarchical nature, where smaller areas are combined into larger ones. This generalization process inherently introduces uncertainties which might have large impacts on the analysis. To test their developed interactive system, and how it could be used for analysing the uncertainty in the data, the authors performed a qualitative evaluation with six expert users. After the participants had used the system to explore the data they answered a number of questions regarding what they learned about the classifications and the population and which functionality of the system they found useful. The results related to uncertainty representation in parallel coordinates indicate that the summarized variation in the demographic variables were appreciated by all users and that depicting variation as shaded areas around the median was intuitive.

5.2 Summary and Discussion

Parallel coordinates has often been found to be advantageous to state-of-the-art techniques when introduced in a new application area. Although users tend to be confused at the beginning they quickly learn how to use parallel coordinates and tend to appreciate the way they can interact with their data. From the authors point of view this is one of the most interesting areas for future research on parallel coordinates and there are several interesting aspects to study.

Studies in new application areas should be encouraged. Particularly using longitudinal studies as well as using real data. This would provide valuable insight into how the technique is used over time and could give information on aspects such as learnability and the need for new, or customized, features.

6 Comparing Parallel Coordinates with Other Data Analysis Techniques

As discussed earlier in this paper, parallel coordinates is a popular technique for exploring multivariate data although many other multivariate visualization techniques can be used for the same tasks. It is, thus, important to investigate which techniques that work best for different conditions. This part of evaluation research focuses solely on comparing parallel coordinates with other visualization techniques. Typical reasons for conducting such studies are:

- to determine if parallel coordinates is ‘better’ than traditionally used techniques for identifying patterns in data,
- to investigate advantages and disadvantages of parallel coordinates compared with other techniques for different user groups.

6.1 Evaluations

Lanzenberger et al. [28] compared standard parallel coordinates with stardinates in an evaluation with 22 participants. The aim was to investigate: the time participants were engaged in testing, the number of correct answers, subjective statements, and key statements in comparison with those defined by an expert. The evaluation was performed on two different data sets: the position data of aeroplanes and a psychotherapeutical data set of five patients. For the first data set, the participants performed tasks such as identifying collisions between aeroplanes and answered questions regarding which problems occurred during the analysis. For the second data set, tasks included identifying outstanding characteristics of the data and, similarly to the first data set, specifying problems that occurred during the analysis. The main results were that the stardinates technique was found to be more appropriate for analysing details in structured data while the parallel coordinates technique was found to better provide information quickly. The authors suggested the use of a combination of the two techniques for best results.

Li et al. [29] compared standard parallel coordinates with scatter plots, focusing on correlation patterns. 25 participants were asked to judge the strength of linear correlations in both scatter plots and parallel coordinates, much like the illustrations in Fig. 6. The results showed that the visual resolution of scatter plots is superior to parallel coordinates and users can distinguish twice as many correlation levels when using scatter plots as compared with parallel coordinates. They also found that the participants often overestimated a negative correlation when using parallel coordinates.

Kuang et al. [26] also compared parallel coordinates and scatter plots, but for value retrieval. In two quantitative experiments, they compared standard parallel coordinates with three versions of scatter plots: standard, rotated and staircase. In the first experiment 12 participants used all four visualization techniques to read numerical values. The result clearly showed that the two winning techniques are standard scatter plots and standard parallel coordinates. In the second experiment, these two techniques were studied further. Here, 18 participants tried to read numerical values in the representations. The main results from the second study are that the error when reading values in scatter plots is stable as dimensionality and density change, but in parallel coordinates the error dramatically increases as the dimensionality and density increases.

In a recent study Harrison et al. [10] expanded on previous work by studying the perception of correlations in parallel coordinates compared with eight other visualization techniques: scatter plots, stacked areas, stacked lines, stacked bars, donuts, radar charts, line plots, and ordered line plots. 1687 participants took part in the test, using a crowdsourcing platform. The task was, again, to judge the strengths of different correlations. Their results are in agreement with the work from Li et al. [29], in that scatter plots depict correlations better overall than parallel coordinates.

Fig. 14. Parallel coordinates showing 41 census variables used in the Output Area Classification [44].

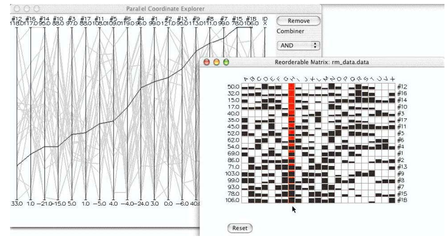

Fig. 15. Parallel coordinates combined with a reorderable matrix [41].

Siirtola [41] combined parallel coordinates with a reorderable matrix so that all interactions performed on either technique were propagated to the other in a coordinated and linked views setup (Fig. 15). A number of interaction techniques, such as selecting a single data item or subsets of items and selecting and permuting dimensions, were implemented. An evaluation was carried out with 20 participants. They performed ten common data analysis tasks on a synthetically generated data set using both techniques; either as two standalone techniques, or linked together. Examples of tasks were to identify small or large values of variables, cases that resemble each other, and cases which have the smallest/largest values on several variables. The main findings were that the difference in completion times between the non-linked and the linked versions was about 15% in favour of the linked version. Subjective ratings obtained from a subsequent questionnaire indicated, for example, that the view linking made it easier to solve the tasks as well as improving the execution times.

The work presented by Tory et al. [47], previously discussed in section 5, in which they introduced parallel coordinates as an interface for volume visualization (Fig. 11) is also applicable in this category since, besides introducing parallel coordinates in a new area, they also compare them with other visualization techniques.

Pillat et al. [35] performed an evaluation using the think-aloud method with five participants with the aim of identifying usability problems with parallel coordinates and RadViz. The participants used both techniques to analyse a data set with cars and were instructed to answer four specific questions about relationships in the data set. In which period was the largest number of Japanese cars produced? Analyse the data and describe the main features of American cars. Are Japanese 4-cylinder cars generally heavier than American 6-cylinder cars? What is the tendency of present European cars along the years in relation to their features? The main results of their study showed that the parallel coordinates technique was efficient for identifying outliers and relationships in subsets of the data. On the other hand, RadViz was efficiently used for identifying clusters in the data as well as for gaining an overview of general structures.

In [43] Siirtola and Räihä tested the usability of parallel coordinates compared to using the SQL query language. 16 SQL programmers were given a number of tasks to perform on a data set with cars. The tasks concerned, for example, to identify the cars with four or six cylinders, finding the origin of cars with six cylinders manufactured in a specific year, or to estimate the weight of a specific car make. The tasks should be carried out using both SQL and parallel coordinates. The participants were all experts in the SQL query language but had not previously used parallel coordinates. The results of this quantitative study showed that parallel coordinates is not as difficult to use as many novice users first expect. The participants completed their tasks on average 43% faster using parallel coordinates than with the SQL query language. The participants also answered a questionnaire with a set of Likert-scale items. The main subjective opinions of the participants were that although parallel coordinates initially was considered complex the technique was easy to learn.

6.2 Summary and Discussion

When compared with other visualization techniques a general comment about parallel coordinates is that the technique gives a good overview of the data. For analysis of specific relationships such as linear relationships, however, parallel coordinates has been found to be outperformed by scatter plots. One study has shown that outliers are easily spotted in parallel coordinates as well as relationships in subsets of the data.

The overall understanding of when to use a specific visualization technique for a specific pattern is still low. Further studies in this area is therefore of high importance. It is also necessary to study more advanced relationships and not only focus on clusters, correlations and outliers. A good understanding of how non-linear relationships are interpreted in parallel coordinates would be valuable and could potentially increase the use of parallel coordinates.

7 Performance of Standard 2D Parallel Coordinates

The previous sections have focused on how the different versions of parallel coordinates perform compared with a variety of other data analysis techniques. Although some of the variations of standard 2D parallel coordinates are commonly used it is still the original version that is by far the most known and used. Therefore it is of interest to further investigate how all the other described data analysis techniques perform compared with standard 2D parallel coordinates. Performance is, in this section, defined as any aspect that has been measured or collected from users and which can be used to describe the effectiveness or efficiency of a technique. In the identified 23 publications there are 27 techniques (including standard 2D parallel coordinates) and 7 tasks that together produce evaluation results specific enough to replicate in future studies. This means that tasks and dependent measures are clearly defined and thus possible to repeat and compare between studies. On the other hand, tasks such as 'obtain an overview', 'good for investigating relationships', 'understanding parameters', etc. are more vague and may be ambiguous and are therefore not included in the table. From this data, a compilation has been performed that shows the performance of standard 2D parallel coordinates compared with the 26 techniques, for each of the 7 tasks. The result is presented in Table 2. The performance is categorized into equal performance (yellow colour), performs better than standard 2D parallel coordinates (green colour) and performs worse than standard 2D parallel coordinates (red colour). A light blue colour means that no evaluation for the specific combination of technique and task has been made. As can be seen in the table, the majority of combinations of techniques and tasks are not evaluated, which highlights the need for further evaluation in the area. For example, tasks such as analysing clusters and correlations have been studied in several papers while information on tasks such as identifying outliers and tracing lines are missing for most visualization techniques.

8 Discussion and General Guidelines for Future Research

The categorization of evaluations of parallel coordinates made in this paper is a step in highlighting the need for user-centred evaluation studies in order to investigate and validate proposed research results. The compilation shown in Table 2 clearly indicates that more research is called for. The presented review is also aimed at helping new researchers within this area and could, hopefully, be used as an easily accessible overview of what has previously been done within this area.

This section presents guidelines for future research for each of the four categories (Sections 3–6). Finally some general guidelines related to design and aesthetics, and evaluation methodologies are discussed.

Table 2. This table illustrates how 26 identified techniques perform *in relation to standard 2D parallel coordinates* (2DPC) for 7 tasks. A yellow colour indicates no significant difference in performance. A green colour means that the technique outperforms 2DPC for the specific task. A red colour means that the technique performs worse than 2DPC. A light blue colour shows that no evaluation has been found in the literature. ∇ denotes that the technique is based on animation.

	Analyse Clusters	Analyse Correlations	Find Outliers	Value Retrieval	Detect 1 pattern	Detect *N* patterns	Trace Lines
2DPC colour [15]	Equal to 2DPC						
2DPC blending [15]	Equal to 2DPC						
2DPC colour+blending [15]	Equal to 2DPC						
2DPC curves [15]	Equal to 2DPC						
2DPC + Scatter plots [15]	Better than 2DPC						
2DPC random ∇ [15]	Worse than 2DPC						
2DPC permutation ∇ [15]	Worse than 2DPC						
2DPC wobble[15] ∇	Equal to 2DPC						
3D Multi-relational PC [6, 21]					Equal to 2DPC	Better than 2DPC	
Bundled PC [12]	Better than 2DPC	Equal to 2DPC					
Edge-bundled PC [33]	Better than 2DPC	Better than 2DPC					Better than 2DPC
Many-to-many PC [30, 5]					Better than 2DPC		
3D PC [20]					Worse than 2DPC	Worse than 2DPC	
Progressive PC [38]					Better than 2DPC	Better than 2DPC	
Scatter plots [29, 26, 10]		Better than 2DPC		Better than 2DPC			
Scatter plots, rotated [26]				Worse than 2DPC			
Scatter plots, staircase [26]				Worse than 2DPC			
Stacked areas [10]		Worse than 2DPC					
Stacked lines [10]		Worse than 2DPC					
Stacked bars [10]		Worse than 2DPC					
Donuts [10]		Worse than 2DPC					
Radar Charts [10]		Worse than 2DPC					
Line plots [10]		Worse than 2DPC					
Ordered line plots [10]		Worse than 2DPC					
Tables/lists [2]					Equal to 2DPC	Worse than 2DPC	
Radviz [35]	Better than 2DPC		Worse than 2DPC				

Equal to 2DPC | Better than 2DPC | Worse than 2DPC | Not evaluated

8.1 Evaluating Axis Layouts of Parallel Coordinates

The existing axis configurations in parallel coordinates have not been thoroughly studied and 3D configurations have received conflicting performance results. Some have been found to efficiently convey complex patterns while others have been found to be more or less useless. In addition, it has not been concluded for which specific tasks and types of data the different axis layouts are most efficient. An area of research is thus to further develop and evaluate existing layouts for different types of data, user scenarios and tasks.

The need for added interaction in 3D layouts, mainly rotation, to produce the 'best' 3D view of the sought pattern is often time-consuming. Developing and evaluating automatic approaches for such analyses could hopefully improve the efficiency of 3D parallel coordinates displays. Combinations of 2D and 3D parallel coordinates displays is another area in which more investigation is needed.

Stereopsis, used to create depth perception in 3D displays, has not been widely used in information visualization and only one example of stereoscopic parallel coordinates has been found. In the work by Nunnally et al. [32] stereoscopic 3D parallel coordinates is used for the analysis of network scans. Since stereopsis has not been thoroughly studied in the field of information visualization, little is known about its advantages and disadvantages for representations of abstract data.

Automatic ordering of axes based on various criteria, such as minimizing outliers [34] or enhancing cluster structures [24], is a well studied approach. At the same time, providing the ability for users to interactively change axes positions is considered a standard way of interacting with parallel coordinates. What is missing is knowledge about when and how automatic approaches and user defined approaches should be used, how they might be combined, etc. This is a gap in the literature that can only be filled via evaluation. For example, it would be interesting to investigate the speed and accuracy of automatic approaches versus user interactions for finding typical patterns such as clusters, correlations and outliers.

The benefits of different axis layouts need to be further investigated since, among the many proposed axis layouts, only a few have been evaluated. The different configurations need to be systematically studied to find their strengths and weaknesses for different tasks and users. One example would be to use eye-tracking in order to quantitatively measure and analyse users' scan paths in order to gain a deeper understanding of how users read the display when looking for patterns.

8.2 Comparing Clutter Reduction Methods for Parallel Coordinates

Visual clutter is still a major limitation of parallel coordinates, regardless of whether the technique is implemented as a 2D or 3D display. The evaluations performed present no improved general performance of the different variations. In order to advance this area of research, existing approaches need to be further evaluated in order to find out which techniques have potential and which should be avoided. The knowledge gained from these studies should then be fed into the development of new approaches for clutter reduction. It is not believed that a single technique can be used to completely overcome this limitation. Hence, being able to develop new techniques in close collaboration with expert users that test the new techniques on real tasks in real working environments would most certainly drive the research forward and, hopefully, result in a deeper understanding of how to visually emphasize complex patterns in the data.

The effect of display size and data size for pattern identification is an issue that has not been thoroughly investigated. Small displays on smart phones and large displays in, for example, control rooms have very different properties and require different methods for rendering, interaction etc. in order to achieve efficient visualization.

Progressive parallel coordinates seems to be a promising technique for reducing visual clutter while making patterns visible and should be further enhanced and studied. Techniques that give the possibility to progressively refine patterns in the display become increasingly important as the size of the data continues to increase.

8.3 Showing Practical Applicability of Parallel Coordinates

Practical applicability of parallel coordinates seems to be high. All evaluations found reported advantages in using parallel coordinates compared with existing techniques used in the various application areas. The large amount of data produced today suggests an increasing use of more advanced analysis tools such as parallel coordinates. Collaborations between information visualization researchers and domain experts in a specific application area are likely to result in versions of parallel coordinates optimized for specific analysis tasks. This might also result in new enhancements of parallel coordinates that would be of interest for a wider audience in the visualization community. Studies in new application areas are something that the information visualization community really cannot get too many of. Knowing the advantageous and disadvantages from many different types of use and from a broad range of users would be extremely valuable. This would be particularly helpful for new users of parallel coordinates.

Longitudinal studies are scarce within the information visualization community [40]. Concerning parallel coordinates there are none. Although time consuming and expensive to perform, such studies are excellent at providing insight into how parallel coordinates is perceived and used in reality and over time. A state-of-the-art on "practical use of parallel coordinates" based on results from numerous evaluations would be useful for everyone working with parallel coordinates.

8.4 Comparing Parallel Coordinates with Other Data Analysis Techniques

A general strength of parallel coordinates that has been reported in several studies is its ability to quickly provide an overview of the data. In several applications it is also the visualization technique with which users prefer to interact. In a coordinated and multiple views setup, parallel coordinates is often used to perform brushing and filtering of the data. When it comes to representation of specific patterns, parallel coordinates has been outperformed by scatter plots for linear correlation (see Table 2). For cluster analysis parallel coordinates performs better than most other techniques that have been studied. For outliers there is not yet sufficient evidence to draw any conclusions (see Table 2).

The knowledge of when to use a specific technique is still low and substantial research is needed to fill the gaps in Table 2. It will also be necessary to go beyond linear relationships and study more complex, non-linear relationships in order to fully utilize parallel coordinates.

Another area of interest is to investigate temporal trends in the existing 3D versions of parallel coordinates. Temporal data analysis puts specific demands on the visualization technique. Although this is a major research area within the information visualization community, almost no work has been dedicated to evaluation of any of the many 3D extensions to parallel coordinates that could potentially be efficiently used for analysis of temporal trends.

8.5 Design and Aesthetics

The design and aesthetics of parallel coordinates have not received much attention in previous work. This is an important aspect to consider when designing a parallel coordinates interface [1, 4]. Whether standard 2D parallel coordinates or any of its 3D extensions appeal to users is unknown. The only work found on creating appealing images of parallel coordinates is by Heinrich and Weiskopf [13] who used density footprints and alpha-blending to obtain visually pleasing displays of high dimensional data. Similarly, when it comes to interaction we prefer things that are intuitive and easy to learn to use [1]. This is another important area that has not received much attention in parallel coordinates research. What has been shown is that people are less prone to use parallel coordinates that have bad usability/interaction mechanisms [39]. It can be expected that a parallel coordinates display that is visually appealing with intuitive interactions would attract the attention of more users and stimulate uptake and usage. To reach such results would require qualitative evaluation that investigates users' subjective actions, opinions and attitudes in depth.

8.6 Evaluation Methodology

One important direction for future research is more qualitative studies and studies executed in the field rather than in controlled lab settings. Traditional metrics such as error rates and response times are important but narrow. The community needs in-depth knowledge about the causes behind such figures and users' expectations, experiences and attitudes. regarding the use and applicability of the parallel coordinates technique. As mentioned in Section 5.2, longitudinal studies would provide insight into how parallel coordinates is perceived and used in reality and over time. For example, it would allow users to experience varied use, allow them to think of improvements, customization and new ways of interaction that will, probably, not come to mind during short evaluation sessions in a lab setting.

Finally, as stated earlier, this paper does not consider the quality of the reported evaluation studies. It is clear, however, that the quality of some of the studies could have been improved if they had applied a better methodology and/or a more clear and informative reporting of execution, data analysis and results. For example, the outcome from some studies is based on a low number of participants, they fail to clearly define tasks, dependent measures, set-up or procedure, and the reporting of data and statistical analysis is, at times, weak. Soundness in methodology and clarity in reporting have a great impact on the reliability, validity, generalizability and the overall trustworthiness of a study. It is common that calls for papers encourage authors to collaborate with experts in evaluation and data analysis. However, in reality this is hard to achieve and to improve knowledge and skill in conducting sound studies the authors refer to [37] for an excellent guide.

9 Conclusions

This paper reports a survey and categorization of research on evaluating the parallel coordinates technique. A comprehensive review of previous work identified 23 papers that presented user-centred evaluations to report on usability aspects of standard 2D parallel coordinates and its many extensions. The previous research was categorized into four identified categories: axis layouts, clutter reduction, practical applicability, and comparison with other techniques. All but one of the papers could be uniquely assigned to a single category. The presented categorization is hoped to help researchers by providing an accessible summary of previous work making it easy to get familiar with what has and has not been done. In addition, the paper compiles evaluation studies on standard 2D parallel coordinates and illustrates, in a table, how the technique performs in comparison with 26 other techniques for a number of tasks. The table gives an overview of previous work, exposes gaps and thus provides an aid for future work. Finally, the paper contributes directions and guidelines for future research within each category as well as more general guidelines.

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References

[1] A. Andrea and C. Dickens. When users want what's not best for them. *The Quarterly of Human Factors Applications*, 3(4):10–14, 1995.

[2] S. Azhar and M. Rissanen. Evaluation of parallel coordinates for interactive alarm filtering. In *15th International Conference on Information Visualisation*, pages 102–109, 2011.

[3] M. Beham, W. Herzner, E. Gröller, and J. Kehrer. Cupid: Cluster-based exploration of geometry generators with parallel coordinates and radial trees. *IEEE Transactions on Visualization and Computer Graphics*, 20(12):1693–1702, 2014.

[4] N. Cawthon and A. Moere. The effect of aesthetic on the usability of data visualization. In *11th International Conference Information Visualization (IV07)*, pages 637–648, 2007.

[5] J. Claessen and J. van Wijk. Flexible linked axes for multivariate data visualization. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2310–2316, 2011.

[6] C. Forsell and J. Johansson. Task-based evaluation of multi-relational 3D and standard 2D parallel coordinates. In *Proceedings of SPIE-IS&T Electronic Imaging*, volume 6495, pages 1–12, 2007.

[7] Y.-H. Fua, M. O. Ward, and E. A. Rundensteiner. Hierarchical parallel coordinates for exploration of large datasets. In *Proceedings IEEE Visualization 1999*, pages 43–50, 1999.

[8] M. Graham and J. Kennedy. Using curves to enhance parallel coordinate visualisations. In *Seventh International Conference on Information Visualization*, pages 10–16, 2003.

[9] P. Guo, H. Xiao, Z. Wang, and X. Yuan. Interactive local clustering operations for high dimensional data in parallel coordinates. In *Pacific Visualization Symposium (PacificVis)*, pages 97–104, 2010.

[10] L. Harrison, F. Yang, S. Franconeri, and R. Chang. Ranking visualizations of correlation using weber's law. *IEEE Transactions on Visualization and Computer Graphics*, 20(12):1943–1952.

[11] H. Hauser, F. Ledermann, and H. Doleisch. Angular brushing of extended parallel coordinates. In *Proceedings IEEE Symposium on Information Visualization 2002*, pages 127–130, 2002.

[12] J. Heinrich, Y. Luo, A. E. Kirkpatrick, and D. Weiskopf. Evaluation of a bundling technique for parallel coordinates. In *Proceedings of the International Conference on Computer Graphics Theory and Applications and International Conference on Information Visualization Theory and Applications*, pages 594–602, 2011.

[13] J. Heinrich and D. Weiskopf. Parallel-coordinates art. In *Proceedings of the IEEE VIS Arts Program (VISAP)*, 2013.

[14] J. Heinrich and D. Weiskopf. State of the Art of Parallel Coordinates. In *Eurographics 2013 - State of the Art Reports*, 2013.

[15] D. Holten and J. J. Van Wijk. Evaluation of cluster identification performance for different PCP variants. *Computer Graphics Forum*, 29(3):793–802, 2010.

[16] A. Inselberg. The plane with parallel coordinates. *The Visual Computer*, 1(4):69–91, 1985.

[17] ISO 9241-11. Ergonomic requirements for office work with visual display terminals, part 11: Guidance on usability, 1998. http://www.iso.ch/iso/en.

[18] T. Isenberg, P. Isenberg, J. Chen, M. Sedlmair, and T. Möller. A systematic review on the practice of evaluating visualization. *IEEE Transactions on Visualization and Computer Graphics*, 19(12):2818–2827, 2013.

[19] J. Johansson, M. Cooper, and M. Jern. 3-dimensional display for clustered multi-relational parallel coordinates. In *Proceedings International Conference on Information Visualization, IV05*, pages 188–193, 2005.

[20] J. Johansson, C. Forsell, and M. Cooper. On the usability of 3D display in parallel coordinates: Evaluating the efficiency of identifying 2D relationships. *Information Visualization Journal*, 13(1), 2014.

[21] J. Johansson, C. Forsell, M. Lind, and M. Cooper. Perceiving patterns in parallel coordinates: Determining thresholds for identification of relationships. *Information Visualization*, 7(2):152–162, 2008.

[22] J. Johansson, P. Ljung, M. Jern, and M. Cooper. Revealing structure within clustered parallel coordinates displays. In *Proceedings IEEE Symposium on Information Visualization 2005*, pages 125–132, 2005.

[23] J. Johansson, P. Ljung, M. Jern, and M. Cooper. Revealing structure in visualizations of dense 2D and 3D parallel coordinates. *Information Visualization*, 5(2):125–136, 2006.

[24] S. Johansson and J. Johansson. Interactive dimensionality reduction through user-defined combinations of quality metrics. *IEEE Transactions on Visualization and Computer Graphics*, 15(6):993–1000, 2009.

[25] R. Kosara, C. G. Healey, V. Interrante, D. H. Laidlaw, and C. Ware. User studies: Why, how, and when? *IEEE Computer Graphics and Applications*, 23(4):20–25, 2003.

[26] X. Kuang, H. Zhang, S. Z. S., and M. McGuffin. Tracing tuples across dimensions: A comparison of scatterplots and parallel coordinate plots. *Computer Graphics Forum*, 31(3):1365–1374, 2012.

[27] H. Lam, E. Bertini, P. Isenberg, C. Plaisant, and S. Carpendale. Empirical studies in information visualization: Seven scenarios. *Visualization and Computer Graphics, IEEE Transactions on*, 18(9):1520–1536, 2012.

[28] M. Lanzenberger, S. Miksch, and M. Pohl. Exploring highly structured data: a comparative study of stardinates and parallel coordinates. In *Proceedings IEEE International Conference on Information Visualisation, IV05*, pages 312–320, 2005.

[29] J. Li, J.-B. Martens, and J. J. van Wijk. Judging correlation from scatterplots and parallel coordinate plots. *Information Visualization*, 9(1):13–30, 2010.

[30] M. Lind, J. Johansson, and M. Cooper. Many-to-many relational parallel coordinates displays. In *13th International Conference on Information Visualisation*, pages 25–31, 2009.

[31] K. T. McDonnell and K. Mueller. Illustrative parallel coordinates. *Computer Graphics Forum*, 27(3):1031–1038, 2008.

[32] T. Nunnally, P. Chi, K. Abdullah, A. Uluagac, J. Copeland, and R. Beyah. P3D: A parallel 3D coordinate visualization for advanced network scans. In *International Conference on Communications*, pages 2052–2057, 2013.

[33] G. Palmas, M. Bachynskyi, A. Oulasvirta, H. Seidel, and T. Weinkauf. An edge-bundling layout for interactive parallel coordinates. In *IEEE Pacific Visualization Symposium (PacificVis)*, pages 57–64, 2014.

[34] W. Peng, M. O. Ward, and E. A. Rundensteiner. Clutter reduction in multi-dimensional data visualization using dimension reordering. In *Proceedings IEEE Symposium on Information Visualization 2004*, pages 89–96, 2004.

[35] R. M. Pillat, E. R. A. Valiati, and C. M. D. S. Freitas. Experimental study on evaluation of multidimensional information visualization techniques. In *Proceedings of the 2005 Latin American conference on Human-computer interaction*, pages 20–30, 2005.

[36] C. Plaisant. The challenge of information visualization evaluation. In *Advanced Visual Interfaces*, pages 109–116, 2004.

[37] H. C. Purchase. *Experimental Human-Computer Interaction: A Practical Guide with Visual Examples*. Cambridge University Press, 2009.

[38] R. Rosenbaum, J. Zhi, and B. Hamann. Progressive parallel coordinates. In *Pacific Visualization Symposium (PacificVis)*, pages 25–32, 2012.

[39] P. Saraiya, C. North, and K. Duca. An evaluation of microarray visualization tools for biological insight. In *IEEE Symposium on Information Visualization*, pages 1–8, 2004.

[40] B. Shneiderman and C. Plaisant. Strategies for evaluating information visualization tools: Multi-dimensional in-depth long-term case studies. In *Proceedings of the 2006 AVI Workshop on Beyond Time and Errors: Novel Evaluation Methods for Information Visualization*, pages 1–7, 2006.

[41] H. Siirtola. Combining parallel coordinates with the reorderable matrix. In *Coordinated and Multiple Views in Exploratory Visualization*, pages 63–74, 2003.

[42] H. Siirtola, T. Laivo, T. Heimonen, and K.-J. Raiha. Visual perception of parallel coordinate visualizations. In *13th International Conference on Information Visualisation*, pages 3–9, 2009.

[43] H. Siirtola and K.-J. Räihä. Discussion: Interacting with parallel coordinates. *Interacting with Computers*, 18(6):1278–1309, 2006.

[44] A. Slingsby, J. Dykes, and J. Wood. Exploring uncertainty in geodemographics with interactive graphics. *IEEE Transactions on Visualization and Computer Graphics,*, 17(12):2545–2554, 2011.

[45] M. ten Caat, N. M. Maurits, and J. B. T. M. Roerdink. Tiled parallel coordinates for the visualization of time-varying multichannel EEG data. In *Proceedings Eurographics/IEEE VGTC Symposium on Visualization*, pages 61–68, 2005.

[46] H. Theisel. Higher order parallel coordinates. In *Proceedings of the Conference on Vision Modeling and Visualization*, pages 415–420, 2000.

[47] M. Tory, S. Potts, and T. Möller. A parallel coordinates style interface for exploratory volume visualization. *IEEE Transactions on Visualization and Computer Graphics*, 11(1):71–80, 2005.

[48] R. Walke, P. Legg, S. Pop, Z. Geng, R. Laramee, and J. C. Roberts. Force-directed parallel coordinates. In *Proceedings IEEE International Conference on Information Visualisation, IV13*, pages 36–44, 2013.

[49] R. Wegenkittl, H. Löffelmann, and E. Gröller. Visualizing the behavior of higher dimensional dynamical systems. In *Proceedings IEEE Visualization 1997*, pages 119–125, 533, 1997.

[50] E. J. Wegman. Hyperdimensional data analysis using parallel coordinates. *Journal of the American Staistical Association*, 85(411):664–675, 1990.