

# LEARNING AND TEACHING ELECTRICAL CIRCUIT THEORY

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Student's conceptions in circuit theory and electricity are not as well investigated as those in mechanics. Most studies have been on pre-university students understanding of simple circuits and little research has been done on university level students understanding. Some research indicates that the similar inappropriate understandings (For example: Confusion between voltage and current and between energy and current. Problems with sign and an inappropriate ability to reason globally) which can be found among younger students do exist even after that students have followed university level courses in physics or electrical engineering. Students understanding of more advanced topics in DC-theory, AC-theory and in applying transform (phasor, Fourier and Laplace) methods to circuits are, to our knowledge, not investigated at all. We are, by means of videotaping labs and with semistructured interviews, investigating engineering students understanding of electrical circuits including their understanding of AC-electricity (with complex phasor representation), periodic signals (Fourier series) and transients (Laplace transforms). We will report on some of our preliminary findings and also report on some learning approaches we have developed using for example conceptual labs to enhance student understanding.

## INTRODUCTION

“What do you *really* do using these complex numbers [in alternating current problems]?” This exclamation one of us (J. B.) heard from his co-instructor when preparing an alternating current lab for a university-level electricity course about 15 years ago. The person behind this statement had, at that time, recently got a Ph D in physics.

This experience points to two conjectures from a rich body of research in physics education [1–5]:

- A functional understanding (in this case an understanding why complex representation and phasors are used in theory of AC-electricity) is not typically an outcome of traditional instruction. Qualitative reasoning and the ability to make verbal explanations must specifically be addressed in teaching.
- Even faculty members, graduate students and students at high ranking institutions have problems with their conceptual understanding.

Learning electric circuit theory is important in engineering education. For an engineer it's

important to know not only DC-circuit theory but also AC-circuit theory since AC-electricity is much more common in technological practise. Students specialising in electrical engineering or engineering physics typically need to study not only AC-circuits but methods for handling more complex circuits and are usually requested to learn to apply various transform methods (phasor, Fourier and Laplace) and Fourier-series in circuit analysis. Understanding of concepts from circuit theory, and specially AC-electricity, periodic signals and transients, is important for understanding of for example electronics, telecommunication and system theory.

However research on student learning and understanding of electric circuit theory is still in its infancy. Student's conceptions in circuit theory and electricity are not as well investigated as those in mechanics. To our knowledge very little research has been done on student understanding of more advanced topics in DC-theory such as superposition, source transformation, mesh-current and node-voltage methods or on students' understanding of AC-electricity, periodic signals and on transients.

Most studies have dealt with pre-university students understanding of simple resistive DC-circuits or simple circuits with a few bulbs and a battery. Little research has been done on university level students' understanding.

## METHOD AND SAMPLES

This work has been done in the context of an introductory course in electric circuit theory for students in an electrical engineering program at a major Swedish university. In the 2001/2002 academic year 56 students were enrolled in the course. The students were studying towards the higher Swedish engineering diploma (equivalent to an M Sc.)<sup>1</sup> and the course was taken in the second semester of their first year. They had previously studied a first course in calculus and a course in linear algebra.

The electric circuits course included topics such as node-voltage method and mesh-current method (applied to DC-circuits, AC-circuits and with Laplace-transforms), superposition, source transformation, magnetic circuits, AC-electricity (including complex representation and phasors), periodic signals and Fourier series in circuit analysis and the use of transform methods (Fourier- and Laplace-transforms) in circuit analysis. Nilsson and Riedel [6] was used as textbook.

The course was taught using lectures, classes (the course was split into two sections) and labs (the group was further split into subsections totalling 4 subsections for the whole course). Labs were performed in groups of 2–3 students.

Student's communications and actions during labs were videotaped. Typically two lab-groups were videotaped each time. Thus about one-third of the total number of students were videotaped each time. On two occasions, for technical reasons, only one camera was used. We are planning to supplement the videotaping of labs with interviewing selected students inside and outside this course.

It should be pointed out that at the time of writing this paper the analysis of the videotaped lab-sessions had just begun. One lab-session remains in the course. Thus the findings we have put forward below should be treated as

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<sup>1</sup> In the present Swedish engineering education system we have two parallel tracks: One leading to a lower engineering diploma (~B Sc) and one leading to a higher diploma (~M Sc).

preliminary results and hence this paper should be regarded as a report of "work in progress".

## OVERVIEW OF EARLIER RESEARCH

### Learning and teaching of physics

Research into the learning and teaching of physics in general<sup>2</sup> have been summarised<sup>3</sup> into the following points [1-3]:

- Facility in solving standard quantitative problems is not an adequate criterion for functional understanding. *Questions that require qualitative reasoning and verbal explanation are essential.*
- A coherent conceptual framework is not typically an outcome of traditional instruction. Rote use of formulas is common. *Students need to participate in the process of constructing qualitative models that can help them understand relationships and differences among concepts.*
- Certain conceptual difficulties are not overcome by traditional instruction. *Persistent conceptual difficulties must be explicitly addressed by multiple challenges in different context.*
- Growth in reasoning ability does not usually result from traditional instruction. *Scientific reasoning skills must be expressly cultivated.*
- Connections among concepts, formal representations, and the real world are often lacking after traditional instruction. *Students need repeated practice in interpreting physics formalism and relating it to the real world.*
- Teaching by telling is an ineffective mode of instruction for most students. *Students must be intellectually active to develop a functional understanding.*

The late Arnold Arons has stressed [4] "The pre- and mis-conceptions found to be widely prevalent among students in introductory physics

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<sup>2</sup> In this paper we will not venture further into a discussion of learning theory in general.

<sup>3</sup> This summary summarises the consensus of physics education researchers at a meeting at Tufts University some years ago [2-3].

courses extend to students in upper division courses, to secondary school teachers, to graduate students, and even to some university faculty members. The *proportion* of individuals exhibiting such difficulties decreases significantly but does not drop to zero discontinuously beyond introductory level.” [Emphasis in original text]

## Understanding of electric circuits

As mentioned in the introduction most of the research done on electric circuits are in the domain of pre-university students understanding of DC-circuits.

According to this body of research [7-12] students tend to “cluster” together concepts such as voltage, current, power and energy. This means that students do not clearly distinguish between these concepts and from this “clustering” view follows conceptions such as:

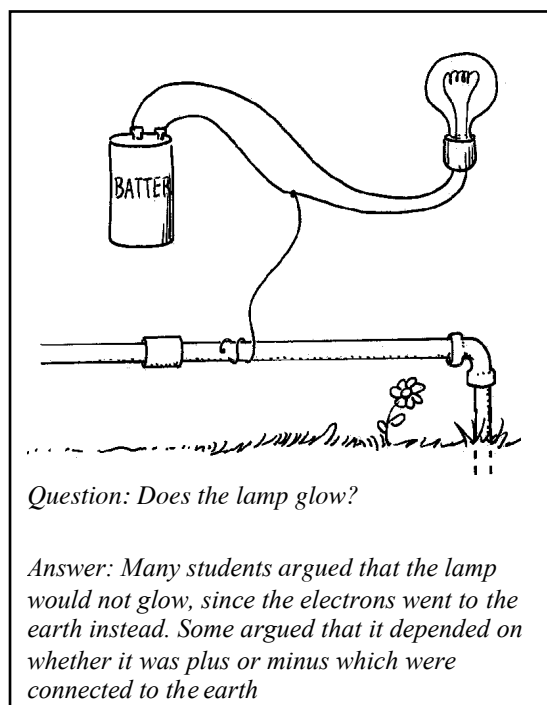
- *Current consumption.*
- *Battery as constant **current** supply.*
- *No current – no voltage.*

- *Voltage is a part or a property of current.*

Research has also shown that it is very difficult for students to see a circuit as a system and to understand that local changes in a circuit results in global changes and that all voltages and currents in a circuit are affected. One can see both:

- *Local reasoning.* Students focus their attention upon one point in the circuit. A change in the circuit is thought on as only affecting current and/or voltages in the circuit there the change is made.
- *Sequential reasoning.* If something is changed in the circuit this is thought on as only affecting current and/or voltages in elements coming after the place there the change was made, not before.

The research which has targeted university students (even electrical engineering students) or secondary school teachers understanding of electrical circuits indicates that these groups reveals very much the same difficulties as found among younger students.



*Question: A 24 V Direct Current source is connected to a transformer with 100 turns on the primary side and 50 turns on the secondary side. On the secondary side is a 12  $\Omega$  load connected. Calculate the voltage  $V_2$  on the secondary side and the current  $I_2$  through the load. The transformer can be treated as an ideal transformer.*

*Answer: Most students used  $V_2 = N_2/N_1 \cdot V_1$  uncritically got  $V_2 = 12$  V and  $I_2 = 1$  A as an answer instead of  $V_2 = 0$  V and  $I_2 = 0$  A. They overlooked the necessity of having an alternating current to have a varying magnetic field and to obtain induction.*

**FIGURE 1.** Examples of some of the ‘simple’ ‘conceptual’ questions one of us (J. B.) started to ask students on exams some years ago. These turned out to be the most difficult questions on exams. The question to the left is from Epstein: Thinking Physics [13].

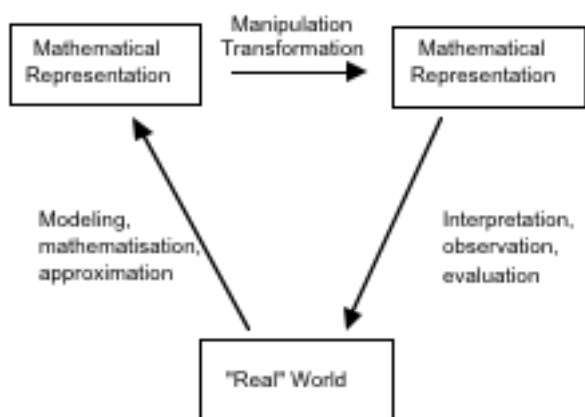


FIGURE 2. Steps involved in modelling or in the use of mathematics in for example problem solving.

## RESULTS AND DISCUSSION

We will once again stress the point that we report “work in progress” and that we are not finished with analysis of our data.

We however see in the videorecordings from the lab, and from in-class observations, very much the same problems with conceptual understanding of AC-electricity as have been reported before with DC-circuits. A typical example is from different tasks in the lab there students are asked to measure AC-voltages and AC-currents in a circuit using voltage and current sensors<sup>4</sup>. Students who have not conceptualised the difference between voltage and current struggle very much with how to connect these sensors. Typically some students would try to connect the current sensor to the circuit in the same way as a voltage sensors. This means that they are connecting the current sensor parallel to the circuit elements instead of connecting it in series.

We also see that many students struggle with the interpretation of mathematics in this context. Although some students have problems with their understanding of “pure” mathematics this is not our main point. Our conjecture is that the students have problems with the translation back and forth between the “real” world and the mathematical representation of the observed data. This means that we focus on first arrow (Real world → Mathematical Representation) and third arrow (Mathematical Representation → Real world) in figure 2. When mathematics is re-

<sup>4</sup> A computerised data acquisition system specially made for educational purposes from PASCO were used.

interpreted in a physics or in an engineering context many things change: The role of symbols, the conventions for interpreting the symbols and the way equations are interpreted. Physics is not just applied mathematics. The way one thinks about mathematics differs from what is taught in the subject of mathematics. This process is not transparent for students<sup>5</sup>.

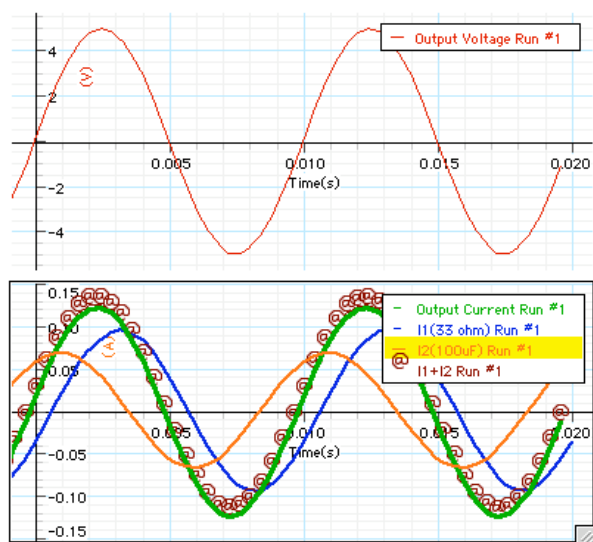


FIGURE 3. Measurement of AC-currents in one of the task in the laboratory aiming to give students an understanding of Kirchoffs’ current law in the context of AC-electricity. The circuit is arranged in such that “output current” should equal the sum of the currents I1 and I2. Included is also this summation (I1+I2) made by the software.

One task in the laboratory was to measure the AC-current in different circuit configurations (Figure 3). Our aim was to enhance student understanding of Kirchoffs’ current law. When students were asked to also represent the addition of currents  $i_1(t) + i_2(t)$  with the corresponding complex phasor representation many students were lost. Although they had learned complex numbers in mathematics and although the complex representation had been discussed in lectures and in the textbook students struggled with the translations of the measurements similar to the ones displayed in figure 3 to the corresponding complex mathematical representation.

The observations we have made are very similar to the observations of Roth and Bowen [14] in the context of graph interpretation: “Our

<sup>5</sup> This issue is also discussed in the other paper [15] written by us.

research shows that competent readings are related to understanding of both the phenomena signified and the structure of the signifying domain, familiarity with the conventions relating the two domains, and familiarity with the translating between the two domains. Graphs are not significant (signifying!) signs on their own. /.../ Finally, only through the continuous movement between the experiential and expressive domains do we expect students to begin to dissociate the features of the two, which lead, without familiarity in translating, to iconic errors.

To deal with all these issues will require much more than traditional instruction in graphing has allowed for. To read a graph competently, one needs more than instruction on the mechanical aspects of producing graphs. *One's extensive interaction with the phenomena and representational means seems to be prerequisite for competent graphing practises.* [Our emphasis]

The results of the preliminary analysis of our data is very much in agreement with the results and conclusion of Roth and Bowen [14] mentioned above. Our analysis also suggest that the competent use of mathematical representations and competent translations back forth between the experiential ("real" world observations) and expressive (mathematical representation, graphing, talking ...) domains is very similar to that required in graphing as discussed by Roth and Bowen.

One of us (J. B.) has previously successfully developed an innovative course in engineering mechanics [16] using conceptual labs. We are now developing conceptual labs in electric circuit theory (including some more advanced topics as discussed above) using many of the ideas used in the previous project and using findings expressed in other studies [1-5].

Our aim is to help students relate electric circuit phenomena to their representational means (mathematical and graphical). We are convinced, and the results of our research and other researches support this claim, that this must expressively and extensively be cultivated to make the process transparent to students.

$$\begin{aligned}
 y(t) &= y_1(t) + y_2(t) = \\
 &Y_{1m}\sin(\omega t + \phi_1) + Y_{2m}\sin(\omega t + \phi_2) = \\
 &\text{Im}(Y_{1m} \cdot e^{j(\omega t + \phi_1)}) + \text{Im}(Y_{2m} \cdot e^{j(\omega t + \phi_2)}) = \\
 &\text{Im}(Y_{1m} \cdot e^{j\phi_1} \cdot e^{j\omega t}) + \text{Im}(Y_{2m} \cdot e^{j\phi_2} \cdot e^{j\omega t}) = \\
 &\text{Im}(Y_1 \cdot e^{j\omega t}) + \text{Im}(Y_2 \cdot e^{j\omega t}) = \\
 &\text{Im}(Y_1 \cdot e^{j\omega t} + Y_2 \cdot e^{j\omega t}) = \text{Im}((Y_1 + Y_2) \cdot e^{j\omega t}) = \\
 &\text{Im}(Y \cdot e^{j\omega t}) = Y \cdot \sin(\omega t + \arg(Y))
 \end{aligned}$$

There

$$Y = Y_1 + Y_2 =$$

$$\begin{aligned}
 &Y_{1m} \cdot (\cos\phi_1 + j \cdot \sin\phi_1) + Y_{2m} \cdot (\cos\phi_2 + j \cdot \sin\phi_2) = \\
 &= Y_{1m} \cdot \cos\phi_1 + Y_{2m} \cdot \cos\phi_2 \\
 &+ j \cdot (Y_{1m} \cdot \sin\phi_1 + Y_{2m} \cdot \sin\phi_2) = \\
 &= Y_{1x} + Y_{2x} + j \cdot (Y_{1y} + Y_{2y})
 \end{aligned}$$

$$Y = |Y| = \sqrt{(Y_{1x} + Y_{2x})^2 + (Y_{1y} + Y_{2y})^2}$$

$$\arg(Y) = \arctan\left(\frac{Y_{1y} + Y_{2y}}{Y_{1x} + Y_{2x}}\right)$$

$$Y_{1x} = Y_{1m} \cdot \cos\phi_1$$

$$Y_{1y} = Y_{1m} \cdot \sin\phi_1$$

$$Y_{2x} = Y_{2m} \cdot \cos\phi_2$$

$$Y_{2y} = Y_{2m} \cdot \sin\phi_2$$

**FIGURE 4.** Some mathematics behind complex representation: Addition of two sines (such as in figure 3) with the *same* frequency. When relating to observations several translations back and forth, as discussed in figure 2 and in the text, are needed. Note that in this context the equality signs have different meaning in different lines above. Sometimes it signifies a mathematical transformation (second arrow in figure 2) and sometimes it represents a modelling or an interpretation (first and third arrows in figure 2).

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