GP examples

- consider the problem

$$\begin{array}{ll} \text{maximize} & x/y \\ \text{subject to} & 2 \leq x \leq 3 \\ & x^2 + 3y/z \leq \sqrt{y} \\ & x/y = z^2 \end{array}$$

- it can be reformulated as a standard GP

$$\begin{array}{ll} \text{minimize} & x^{-1}y \\ \text{subject to} & 2x^{-1} \leq 1, \quad (1/3)x \leq 1 \\ & x^2 y^{-1/2} + 3y^{1/2} z^{-1} \leq 1 \\ & xy^{-1} z^{-2} = 1 \end{array}$$

GP examples

- consider the problem

$$\begin{array}{ll} \text{minimize} & p(x)/(r(x) - q(x)) \\ \text{subject to} & r(x) > q(x) \end{array}$$

where p, q are posynomials and r is monomial

- it can be reformulated as a standard GP

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & p(x) \leq t(r(x) - q(x)) \\ & (q(x)/r(x)) < 1 \end{array}$$

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & (p(x)/t + q(x))/r(x) \leq 1 \\ & (q(x)/r(x)) < 1 \end{array}$$

GP examples

- power control for S interfering links (MIMO-MU, Ad-hoc,etc.)
  - SINR for link i

$$\gamma_i = \frac{G_{ii} P_i}{\sigma_i^2 + \sum_{k=1, k \neq i}^{S} G_{ik} P_k} \qquad i = 1, \ldots, S$$

  - P_i power allocated to link i
  - G_{ik} power gain from TX of link k to RX of link i

- maximization of the min-SINR under sum-power constraint

$$\text{maximize} \quad \min_i \frac{G_{ii} P_i}{\sigma_i^2 + \sum_{k=1, k \neq i}^{S} G_{ik} P_k}$$

$$\text{subject to} \quad \sum_{i=1}^{S} P_i \leq P_{\max}$$

(other constraints can be added)

GP examples

$$\begin{aligned} \text{minimize} \quad & \max_i \frac{\sigma_i^2}{G_{ii}} P_i^{-1} + \sum_{\substack{k=1 \\ k\neq i}}^{S} \frac{G_{ik}}{G_{ii}} P_k P_i^{-1} \\ \text{subject to} \quad & \sum_{i=1}^{S} P_i \le P_{\max} \end{aligned}$$

$$\begin{aligned} \text{maximize} \quad & \min_i \frac{G_{ii} P_i}{\sigma_i^2 + \sum_{\substack{k=1 \\ k\neq i}}^{S} G_{ik} P_k} \\ \text{subject to} \quad & \sum_{i=1}^{S} P_i \le P_{\max} \end{aligned}$$

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & \max_i \frac{\sigma_i^2}{G_{ii}} P_i^{-1} + \sum_{\substack{k=1 \\ k\neq i}}^{S} \frac{G_{ik}}{G_{ii}} P_k P_i^{-1} \le t \\ & \sum_{i=1}^{S} P_i \le P_{\max} \end{aligned}$$

$$\begin{aligned} \text{minimize} \quad & t \\ \text{subject to} \quad & \frac{\sigma_i^2}{G_{ii}} P_i^{-1} + \sum_{\substack{k=1 \\ k\neq i}}^{S} \frac{G_{ik}}{G_{ii}} P_k P_i^{-1} \le t \,, \quad i = 1, \ldots, S \\ & \sum_{i=1}^{S} P_i \le P_{\max} \end{aligned}$$

GP examples

- It can be also cast as a generalized linear fractional program

$$\begin{aligned} \text{maximize} \quad & \min_i \frac{G_{ii} P_i}{\sigma_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^{S} G_{ik} P_k} \\ \text{subject to} \quad & \sum_{i=1}^{S} P_i \leq P_{\max} \end{aligned}$$

$$\begin{aligned} \text{minimize} \quad & \max_i \frac{\sigma_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^{S} G_{ik} P_k}{G_{ii} P_i} \\ \text{subject to} \quad & \sum_{i=1}^{S} P_i \leq P_{\max} \end{aligned}$$

- and solved via bisection

GP examples

- power control for S interfering links (MIMO-MU, Ad-hoc,etc.)
  - SINR for link i

$$\gamma_i = \frac{G_{ii} P_i}{\sigma_i^2 + \sum_{k=1, k \neq i}^{S} G_{ik} P_k} \qquad i = 1, \ldots, S$$

  - P_i power allocated to link i
  - G_{ik} power gain from TX of link k to RX of link i

- weighted sum-rate maximization in high SINR (low interference)

$$R_i = \log\left(1 + \gamma_i\right) \approx \log \gamma_i$$

$$\mathrm{WSR} = \sum_{i=1}^{S} w_i R_i \approx \sum_{i=1}^{S} w_i \log \gamma_i = \log \prod_{i=1}^{S} \gamma_i^{w_i}$$

GP examples

- the WSRM problem can be formulated as

$$\begin{aligned} \text{maximize} \quad & \log \prod_{i=1}^{S} \gamma_i^{w_i} \\ \text{subject to} \quad & \gamma_i = \frac{G_{ii} P_i}{\sigma_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^{S} G_{ik} P_k}, \ i = 1, \ldots, S \\ & \sum_{i=1}^{S} P_i \leq P_{\max} \end{aligned}$$

- which is equivalent to

$$\begin{aligned} \text{maximize} \quad & \prod_{i=1}^{S} \gamma_i^{w_i} \\ \text{subject to} \quad & \gamma_i \leq \frac{G_{ii} P_i}{\sigma_i^2 + \sum_{\substack{k=1 \\ k \neq i}}^{S} G_{ik} P_k}, \ i = 1, \ldots, S \\ & \sum_{i=1}^{S} P_i \leq P_{\max} \end{aligned}$$

GP examples

- which is further equivalent to

$$\begin{array}{ll} \text{minimize} & \prod_{i=1}^{S} \gamma_i^{-w_i} \\ \text{subject to} & \gamma_i \sigma_i^2 G_{ii}^{-1} P_i^{-1} + \sum_{\substack{k=1 \\ k\neq i}}^{S} \gamma_i G_{ik} P_k G_{ii}^{-1} P_i^{-1} \leq 1,\ i = 1,\ldots,S \\ & \sum_{i=1}^{S} P_i P_{\max}^{-1} \leq 1 \end{array}$$

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- Codreanu M, Tölli A, Juntti M & Latva-aho M (2007) Joint design of Tx-Rx beamformers in MIMO downlink channel. IEEE Transactions on Signal Processing 55(9): 4639–4655