

Geometric Stable Roommates*

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Abstract

We consider instances of the Stable Roommates problem that arise from geometric representation of participants' preferences: a participant is a point in a metric space, and his preference list is given by the sorted list of distances to the other participants. We show that contrary to the general case, the problem admits a polynomial-time solution even in the case when ties are present in the preference lists.

We define the notion of an α -stable matching: the participants are willing to switch partners only for a (multiplicative) improvement of at least α . We prove that, in general, finding α -stable matchings is not easier than finding matchings that are stable in the usual sense. We show that, unlike in the general case, in a three-dimensional geometric stable roommates problem, a 2-stable matching can be found in polynomial time.

Keywords: stable roommates with ties • consistent preferences • α -stable matching

1 Introduction

The Stable Marriage problem has multidisciplinary interest, as it is well-studied in economics, computer science, and combinatorics. The problem and its numerous extensions continue to receive considerable attention, both from the theoretical point of view and from the perspective of real-world applications [13].

The Stable Roommates problem (SR) is a non-bipartite version of the Stable Marriage problem. In an instance of SR each of $2n$ participants ranks all other participants in *strict* order of preference. A *matching* is a set of n disjoint pairs of participants. Two participants *block* a matching if each of them prefers the other to his partner in the matching. A matching is *stable* if there is no blocking pair. The objective in SR is to find a stable matching or show that none exists. We denote by $M(p)$ the participant to which participant p is matched in the matching M .

The Stable Roommates *with ties* problem (SRT) is a generalization of SR in which the preference lists may contain ties. In the presence of ties, there are three natural notions of blocking, giving

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rise to three notions of stability. Specifically, a pair of participants $\{x, y\} \notin M$ may be defined to block M in the following ways:

- 1) *Super-Blocking*: $\{x, y\}$ is a *super blocking* pair if x prefers y to $M(x)$, or considers them to be tied, and y prefers x to $M(y)$, or considers them to be tied.
- 2) *Strongly-Blocking*: $\{x, y\}$ is a *strongly-blocking* pair if $\{x, y\}$ is a super-blocking pair and, additionally, either x strictly prefers y to $M(x)$ or y strictly prefers x to $M(y)$.
- 3) *Weakly-Blocking*: $\{x, y\}$ is a *weakly-blocking* pair if x strictly prefers y to $M(x)$ and y strictly prefers x to $M(y)$.

Accordingly, *super-stable*, *strongly-stable*, and *weakly-stable* matchings are defined as those for which no super-blocking, strongly-blocking, and weakly-blocking pair exists, respectively.

The preference list of a participant may be just an ordered list of the others. Alternatively, a participant may associate a cost, or *regret*, of being matched with each of the other participants; the preference list is given then by the regrets sorted in increasing order.

In the *three-dimensional* version of SR (3D-SR), *three* people live in a room. There are $3n$ participants, and each of them ranks all pairs of the others. A matching M is a set of n disjoint triples of participants. Let $M(u)$ denote the pair of roommates assigned to participant u in M . A triple $\{u, v, w\} \notin M$ *blocks* M if u prefers $\{v, w\}$ to $M(u)$, v prefers $\{u, w\}$ to $M(v)$, and w prefers $\{v, u\}$ to $M(w)$. A matching is *stable* if no triple blocks it.

1.1 Previous Work

The book [7] is a comprehensive survey of the classical results; we provide only a brief summary here. It was shown [6] that in SR, unlike in the Stable Marriage problem, a stable matching does not always exist. It can be checked in $\Theta(n^2)$ if a matching exists; if it does, it can be found within the same time bounds [12, 7]. 3D-SR is NP-complete [19, 22]. For SRT, super-stable and strongly-stable matchings can be found in $O(n^2)$ and $O(n^4)$ time resp. [14, 21], while finding a weakly-stable matching is NP-complete [20, 14].

Consistent Preferences

In the general SR the preference lists of different participants are allowed to be completely uncorrelated. Several approaches introduce some form of consistency by means of preference lists. Irving et al. [11] considered “master lists” as a means of producing (possibly incomplete) preference lists from a centralized “source”; Bartholdi and Trick [2] studied a similar version, with a common “frame of reference” for the participants. Ng and Hirschberg [19] suggested studying three-dimensional matching problems in which for any participants y, z , and x the following holds: either x prefers the pair $\{w, y\}$ to $\{w, z\}$ for all w , or x prefers $\{w, z\}$ to $\{w, y\}$ for all w . Knuth [16] suggested considering 3D-SR with *cyclic* preferences: the participants are split into 3 groups, A, B, C , and people in A (resp., B, C) rank those in B (resp., C, A). In a recent work, Iwama, Miyazaki, and Okamoto [15] showed that finding a stable matching in 3D-SR remains NP-complete even if each participant ranks (as in the usual SR) *all* other participants (and not *pairs* of the others). Huang [10] showed that some of the 3D-SR problems with consistent preferences are still NP-complete; in particular, he showed hardness of the variant in which the participants submit lists ranking the others and the pairs are ranked by the sum of their ordinal numbers. Boros et al. [4] and Eriksson et al. [5] considered some generalizations of the matching problems with cyclic preferences.

Our geometric representation of the preferences is yet another natural way to produce consistent preference lists. In particular, for the three-dimensional matching, the geometric SR has preference lists, consistent in the sense of [19].

Our work may be viewed as an extension of Bartholdi and Trick’s paper [2], where a “1-dimensional” version of the geometric SR without ties was considered. In this paper we extend the geometric SR to “higher dimensions” and to SR with ties.

1.2 Problem Formulation

We consider points in \mathbb{R}^d , where d is some constant. The distances are measured according to an L_t metric for some $t > 0$; we denote \mathbb{R}^d , equipped with the L_t metric, by ℓ_t^d . For $x, y \in \ell_t^d$ we write $|xy|$ for $\|x - y\|_t$. All of our algorithms extend verbatim to points in an *arbitrary* metric space, but the specific running times are stated for ℓ_t^d .

An instance of the geometric SR is specified by a set of points $P = \{p_1, \dots, p_{2n}\} \subset \ell_t^d$. Regrets are given by distances from p_i to the other points: participant p_i prefers p_j to p_k if $|p_i p_j| < |p_i p_k|$. If $|p_i p_j| = |p_i p_k|$, then p_i is *indifferent* between p_j and p_k ; in this case, we say that there is a *tie* in the preferences. The problem is to find a stable matching in the instance or to conclude that none exists.

In the *three-dimensional* version of geometric SR (geometric 3D-SR) the number of points is $3n$. (Note that “three-dimensional” refers to the definition of a matching in terms of triples, not to the dimension, d , of the underlying geometric space.) The preference of a participant p_i over the *pairs* of the others is given by the sum of the distances to the points in the pair: p_i prefers $\{p_j, p_k\}$ to $\{p_l, p_m\}$ if $|p_i p_j| + |p_i p_k| < |p_i p_l| + |p_i p_m|$. The problem is to partition P into disjoint triples so that no blocking triple exists or to conclude that it is impossible.

α -Stable Matching Commonly, a participant in a blocking pair is ready to switch his match as soon as his regret after the switch decreases by any (arbitrarily small) amount. It seems more realistic that people would switch their partners only in favor of a significant improvement (the grass may be greener on the other side, but it takes effort to cross the fence). To quantify this observation, we introduce the notion of an α -stable matching. For $\alpha > 1$ we say that $p_i, p_j \in P$ form an α -blocking pair if p_i ’s (resp., p_j ’s) regret of being matched with p_j (resp., p_i) is α or more times less than that of being matched with his current partner. In particular, in a geometric 3D-SR, participants $p_i, p_j, p_k \in P$ are said to α -block a matching if $|p_i p_j| + |p_i p_k| < (|p_i p'_i| + |p_i p''_i|)/\alpha$, $|p_j p_i| + |p_j p_k| < (|p_j p'_j| + |p_j p''_j|)/\alpha$, and $|p_k p_i| + |p_k p_j| < (|p_k p'_k| + |p_k p''_k|)/\alpha$, where p', p'' are the partners of p in the matching. The α -stable roommates problem (α -SR) is to find an α -stable matching or to conclude that none exists.

1.3 Motivation

Geometric stable matching (its bi-partite version) may be used to match observations and objects in a classification task. It was noted that stable matching performs well in completing curve reconstructions, when pieces of the curve are available from other heuristics [17]. One may also imagine that when people are looking for partners (say, for a chess game), the only thing they care about is the distance to travel. Finally, in a real-world roommates-assigning task, the roommates-to-be may be represented by points in some space of “features”: the coordinate axes of the space may be, e.g., preferred time to go to bed, desired level of neatness of the room, number of parties/beers

per semester, etc. It is reasonable to assume that people would prefer to become roommates with those having similar features.

1.4 Our Contributions

We introduce the geometric Stable Roommates problem and give combinatorial and algorithmic results. Many of our results are in contrast with the general cases of SR and SRT; the most striking is a polynomial-time algorithm for finding a weakly-stable matching.

In Section 2, we observe that, unlike in the general SR, if there are no ties in the preference lists, any geometric SR instance admits a unique stable matching. The matching is given by a simple greedy $\Theta(n \log n)$ -time algorithm.

In Section 3, we consider geometric SR with ties. We show that it can be checked in $O(n \log n)$ time whether a super-stable matching exists, and in $O(n^{1.5})$ time if a strongly-stable matching exists (in $O(n^{1.19})$ time for $d = 2$). Our algorithms are constructive and find the matchings within the same time bounds. We also show that, in contrast with the general case, a weakly-stable matching always exists and can be found in $O(n \log n)$ time.

We introduce the notion of α -stable matching, in which a participant is willing to switch his match only if he improves his utility by more than a factor of α . To the best of our knowledge, this is a novel notion.

In Section 4, we show that for $\alpha > 1$ finding an α -stable matching is at least as hard as finding a regular stable matching. We consider geometric instances of 3D-SR, in which a participant ranks the pairs of the other participants by the sum of the distances to the participants in the pairs. We prove that, in contrast with the general (non-geometric) case, there always exists a 2-stable matching, and also that, in contrast with the general case, it can be found in polynomial time.

2 Geometric SR

We first consider the very basic version of geometric SR, in which the distances between different pairs of points in P are all distinct: $\forall \{i, j\} \neq \{k, l\}, |p_i p_j| \neq |p_k p_l|$.

Theorem 2.1. *Any instance of geometric SR without ties admits a unique stable matching, which can be found in optimal $\Theta(n \log n)$ time.*

Proof. Let $\{p_1, p_2\}$ be the *closest pair* in P : $|p_1 p_2| = \min_{p_i, p_j \in P, i \neq j} |p_i p_j|$. We claim that p_1 and p_2 must be roommates in any stable matching. Indeed, if it is not so, each of p_1, p_2 will be willing to switch to become roommates. Thus, we match p_1 and p_2 , remove them from P , and continue recursively with remaining points.

In order to achieve the claimed running time, we use the algorithm of Bespamyatnikh [3], which maintains, under changes caused by point insertions or deletions, closest pairs among a set of points, in any fixed dimension d (and any L_t metric), in time $O(\log n)$ per operation (insertion or deletion). In our case, there are $O(n)$ deletions, so the overall time is $O(n \log n)$.

It is easy to see that any algorithm for geometric SR requires $\Omega(n \log n)$ time. Indeed, since the closest pair must be in the stable matching, solving geometric SR is at least as hard as finding a closest pair, which has lower bound $\Omega(n \log n)$ from Element Uniqueness [1]. \square

3 Geometric SRT

In this section we consider the case in which there may be ties in the interpoint distances in P . We show that super-stable and strongly-stable matchings can be tested for existence (and found, if they exist) in near-linear time. We also show that a weakly-stable matching always exists and can be found in near-linear time, in contrast with the general, non-geometric SR, which is NP-complete.

Closest-Pairs Graph As in the previous section, all of our results are based on considering matchings in the *closest-pairs graph* $CP(P)$, defined as follows. Let $V \subseteq P$ be the points achieving the minimum interpoint distance in P . The vertices of $CP(P)$ are V , and the edges connect pairs of points that achieve the minimum distance (i.e., for every vertex $p \in V$ we connect p to all of its nearest neighbors).

Lemma 3.1. *The number of edges in $CP(P)$ is $O(|P|)$.*

Proof. We show that the maximum degree of $CP(P)$ is constant. Let $p \in CP(P)$, let $p_1 \dots p_k$ be p 's neighbors in $CP(P)$; suppose, without loss of generality, that the common length of edges of $CP(P)$ is 2. Then the unit (in L_t) ball, centered at p , touches each of the unit balls centered at $p_1 \dots p_k$. Also, the latter k balls are pairwise-disjoint, as otherwise 2 would not be the smallest interpoint distance in P . Thus, k is at most the number of non-overlapping translates of the unit ball that can be brought in contact with the unit ball. This number, known as the Hadwiger number [8], is at most $3^d - 1$, a constant for constant d . \square

Super-Stable Matching

Suppose that $CP(P)$ is a perfect matching, i.e., $CP(P)$ is a set of disjoint edges. By the argument of Theorem 2.1, we can match the points in $CP(P)$ according to the edges of $CP(P)$, remove them, and recurse.

On the other hand, suppose $CP(P)$ is *not* a perfect matching, i.e., the graph has a vertex p_1 of degree more than 1. Let M be a matching in P , and let p_2 be a vertex of $CP(P)$, adjacent to p_1 such that p_1 is not matched to p_2 . It is easy to see that $\{p_1, p_2\}$ is a super-blocking pair for M . Indeed, no matter which points (say, p'_1 and p'_2 , resp.) are matched with p_1 and p_2 (resp.), $|p_1p_2| \leq |p_1p'_1|$, $|p_1p_2| \leq |p_2p'_2|$, since p_1p_2 is an edge of $CP(P)$.

Theorem 3.2. *A super-stable matching in geometric SR can be tested for existence in $O(n \log n)$ time, and can be found (if it exists) in the same time bound.*

Proof. As in the proof Theorem 2.1, we use the structure of Bespamyatnikh [3] for maintaining closest pairs under deletions. After $O(|P| \log |P|)$ preprocessing time, the structure can report *all* closest pairs in time $O(\log |P| + k)$, where k is the number of the pairs. Thus, $CP(P)$ can be obtained in time $O(\log |P| + |CP(P)|)$, where $|CP(P)|$ is the number of edges of $CP(P)$. We check whether there is a vertex of degree greater than one in $CP(P)$. If any, we report that there is no super-stable matching. Otherwise, $CP(P)$ is a sub-matching of a super-stable matching. We delete the points involved in $CP(P)$ and repeat. By Lemma 3.1, at each stage the size of the closest-pair graph is proportional to the number of matched points. Since at least 2 points are matched at every stage, the number of stages is at most n . \square

Strongly-Stable Matching

Suppose that $CP(P)$ has a perfect matching. We can match the points in $CP(P)$ according to the edges of the perfect matching; none of the matched points will participate in a blocking pair since all matched points are matched to their top choices. Thus, we may remove the matched points and repeat.

On the other hand, suppose that $CP(P)$ does not contain a perfect matching. Let M be a matching in P , and let p_1 be a vertex of $CP(P)$ that is not matched in M to one of its neighbors in $CP(P)$; let $p_2 \in CP(P)$ be a neighbor of p_1 in $CP(P)$. Suppose that M matches p_1 with p'_1 and p_2 with p'_2 . Then, $|p_1p_2| < |p_1p'_1|$ and $|p_1p_2| \leq |p_2p'_2|$, since (p_1, p_2) is an edge in the *closest-pairs* graph, $CP(P)$. Thus, $\{p_1, p_2\}$ will block M .

Theorem 3.3. *A strongly-stable matching in geometric SR can be tested for existence in $O(n^{1.5})$ time, and can be found (if it exists) in the same time bound. If $d = 2$, the time bound is $O(n^{1.19})$.*

Proof. We recursively compute a maximum matching in $CP(P)$, and, if the matching is perfect, remove the matched pairs. (If there is no perfect matching, we report that no stable matching exists.) To find the matchings we use the the $O(\sqrt{nm})$ -time algorithm of Hopcroft and Karp [9] for maximum matching in a graph with n vertices and m edges. By Lemma 3.1, this is $O(|CP(P)|^{1.5})$ for the closest-pair graph. Using the structure of Bespamyatnikh [3], we find the graph at each stage of the recursion in $O(|CP(P)| \log |P| + |CP(P)|)$ time. Thus, at each stage, computing the graph and the maximum matching in it can be done in $O(|CP(P)| \log |P| + |CP(P)|^{1.5})$ time. The number of stages is at most n , and the total size of the closest-pair graphs at all stages is $O(n)$. Thus, the overall running time of the algorithm is $O(n \log n + n^{1.5}) = O(n^{1.5})$.

For $d = 2$, it is easy to see, using the triangle inequality, that $CP(P)$ is planar. Thus, instead of Hopcroft-Karp algorithm, we can use the $O(n^{1.19})$ -time algorithm of Mucha and Sankowski [18] for maximum matching in planar graphs. \square

Weakly-Stable Matching

Let M_{CP} be a *maximal* matching in $CP(P)$. We match the points in $CP(P)$ according to the edges of M_{CP} ; none of the matched points will participate in a blocking pair, since all matched points are matched to their top choices. Thus, we remove the matched points and repeat. Eventually, all points get matched in a weakly-stable matching for P .

Theorem 3.4. *A weakly-stable matching in geometric SR in \mathbb{R}^d always exists and can be found in $O(n \log n)$ time.*

Proof. A maximal matching can be obtained in a greedy way in linear time. Thus, maintaining the structure of closest pairs dominates the time complexity, as in Theorem 3.2. \square

4 Geometric 3D-SR

We now turn to the three-dimensional geometric SR (3D-SR). First we present an instance in which even a weakly-stable matching does not exist. We then consider α -stable matchings; we note that for $\alpha > 1$, finding an α -stable matching is no easier than finding a stable matching (which is a 1-stable matching). Finally, we show that in 3D-SR a 2-stable matching can be found in polynomial time.

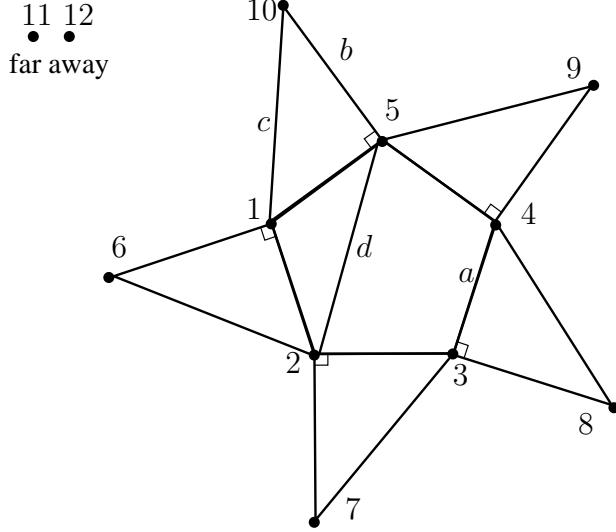


Figure 1: $(1,2,3,4,5)$ is a regular pentagon. $(1,5,10)$, $(1,2,6)$, $(2,3,7)$, $(3,4,8)$, and $(4,5,9)$ are congruent right triangles. The points 11 and 12 are far away from the others. $a < b < c < d$.

Theorem 4.1. *There are instances of geometric 3D-SR for which there is no weakly-stable 3D matching.*

Proof. Consider the instance in Fig. 1. We claim that there exists no weakly-stable 3D matching. Suppose first that none of the edges of the pentagon is in the matching. Then, for any participant the regret of the matching is at least $b + c$, and, for participant 10, the regret is strictly greater than $b + c$. On the other hand, if $\{1,5,10\}$ are matched, their regrets are $a + c$, $a + b$, and $b + c$, respectively; hence, they will be a blocking triple.

Thus, one of the edges of the pentagon, say, $(1,5)$, is in the matching. If none of 2, 4, or 6 is in the same triple with $\{1,5\}$, then $\{1,5,10\}$ is blocking. Now consider three cases:

$\{1,5,2\}$ is in the matching. Then participant 3 must be matched either to $\{4,7\}$ or to $\{4,8\}$, since, otherwise, $\{2,3,7\}$ blocks. In the former case $\{3,4,8\}$ blocks, in the latter case $\{4,5,9\}$ blocks.

$\{1,5,6\}$ is in the matching. Then, in order for $\{1,5,2\}$ not to block, participant 2 must be matched either to $\{3,4\}$ or to $\{3,7\}$. In both cases, $\{4,5,9\}$ blocks.

$\{1,5,4\}$ is in the matching. Then, in order for $\{1,2,6\}$ not to block, participant 2 must be matched to $\{3,7\}$. But then $\{3,4,8\}$ blocks.

□

α -stable Matchings

Let $\alpha > 1$, and consider any stable matching problem (SMP), any-dimensional, bipartite or not. Suppose that $f(n)$ is a lower bound on the time that is needed to find a stable matching in an instance of SMP of size $O(n)$. (If SMP is NP-complete, $f(n)$ may be superpolynomial.)

Theorem 4.2. $\Omega(f(n))$ is a lower bound on the running time for computing an α -stable matching in SMP.

Proof. Consider a person p , participating in SMP, and let $p(i)$ be the i th entry in p 's preference list. (Thus, $p(i)$ is a single person in the standard matching problem, a pair in a 3D matching problem, or another entity in another SMP.) Assign regret α^i to the match of p with $p(i)$. Now p prefers to be matched with $p(j)$ to being matched with $p(k)$ if and only if p 's regret of being matched with $p(j)$ is at least α times less the regret of being matched with $p(k)$. Thus, a matching is stable in SMP if and only if it is α -stable in the problem with the regrets as defined above. \square

Corollary 4.3. *There exist instances of 3D-SR in which no 2-stable matching exists. Finding a 2-stable matching in 3D-SR is NP-complete.*

Proof. It was shown in [19, 22] that finding a stable matching in 3D-SR is NP-complete. \square

2-Stable Matching in Geometric 3D-SR

The next result is in contrast with Corollary 4.3:

Theorem 4.4. *In a geometric instance of 3D-SR there always exists a 2-stable matching, and it can be found in polynomial time.*

Proof. Let $p_1p_2p_3$ be a minimum-perimeter triangle defined by a triple of distinct points of P : $|p_1p_2| + |p_2p_3| + |p_3p_1| = \min_{p_i, p_j, p_k \in P, i \neq j \neq k} (|p_ip_j| + |p_jp_k| + |p_kp_i|)$. Put p_1, p_2, p_3 into one room. We claim that none of them will participate in a 2-blocking triple.

Indeed, suppose that p_1 is willing to abandon p_2, p_3 for another pair, p_ip_j . Then $|p_1p_i| + |p_1p_j| < (1/2)(|p_1p_2| + |p_1p_3|)$. By the triangle inequality, $|p_1p_i| + |p_1p_j| + |p_ip_j| \leq 2(|p_1p_i| + |p_1p_j|) < |p_1p_2| + |p_1p_3| < |p_1p_2| + |p_2p_3| + |p_3p_1|$, which means that the perimeter of $p_1p_ip_j$ is less than that of $p_1p_2p_3$, a contradiction.

Thus, we can match p_1, p_2, p_3 , remove them, and repeat, eventually obtaining a 2-stable matching. \square

Remarks

It is possible to define an "additive" version of α -stability, in which a participant prefers to break partnership only when the new partner is at least α positions higher, in the participant's preference list, than the current partner. How hard is finding α -stable matchings in this setting? For a stable matching problem with no guaranteed solution, e.g., SR, what is the minimum α such that any instance of the problem is α -stable?

We left open the hardness of finding a stable matching in a geometric 3D-SR. Another open question, suggested by the reviewer, is to consider geometric 3D-SR where the preferences are defined as the maximum (or the minimum) of the distances to the participants in a pair.

It would also be interesting to investigate the geometric counterparts of other stable matching problems — for example, finding the minimum-regret stable matching.

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