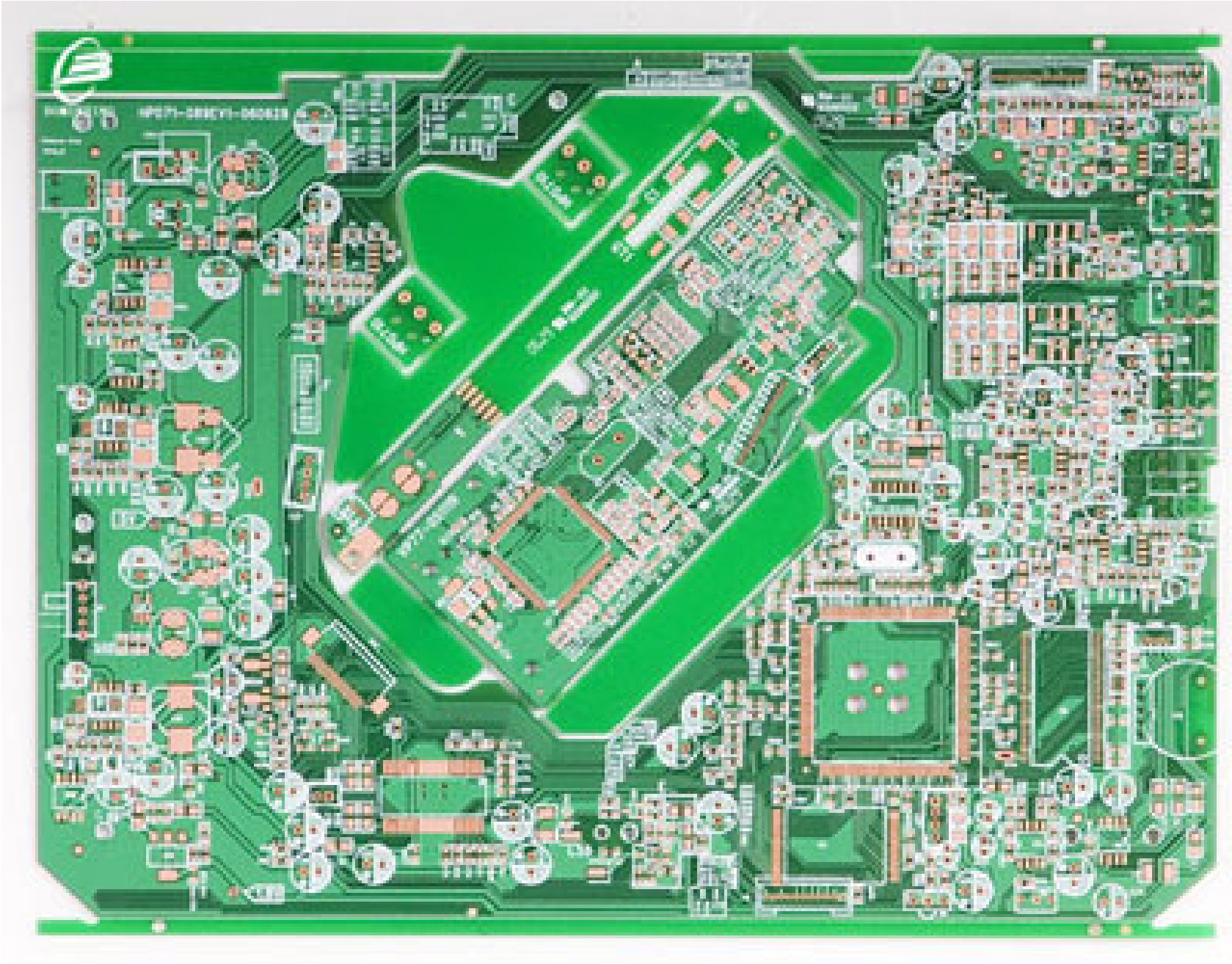
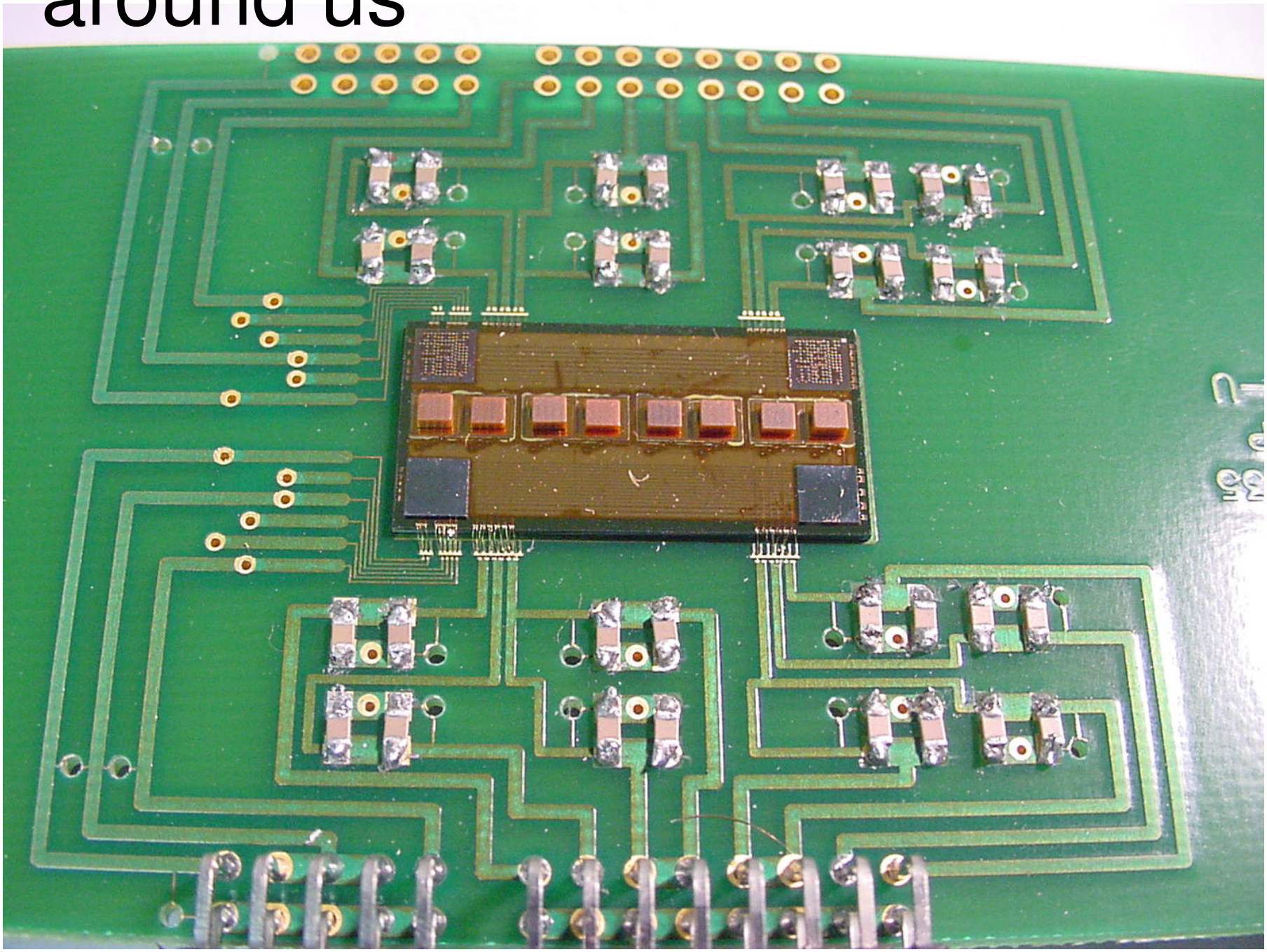
Simple Wriggling is Hard unless you are a Fat Hippo

Irina Kostitsyna — *Stony Brook University*
Valentin Polishchuk — *Helsinki Institute for Information Technology*

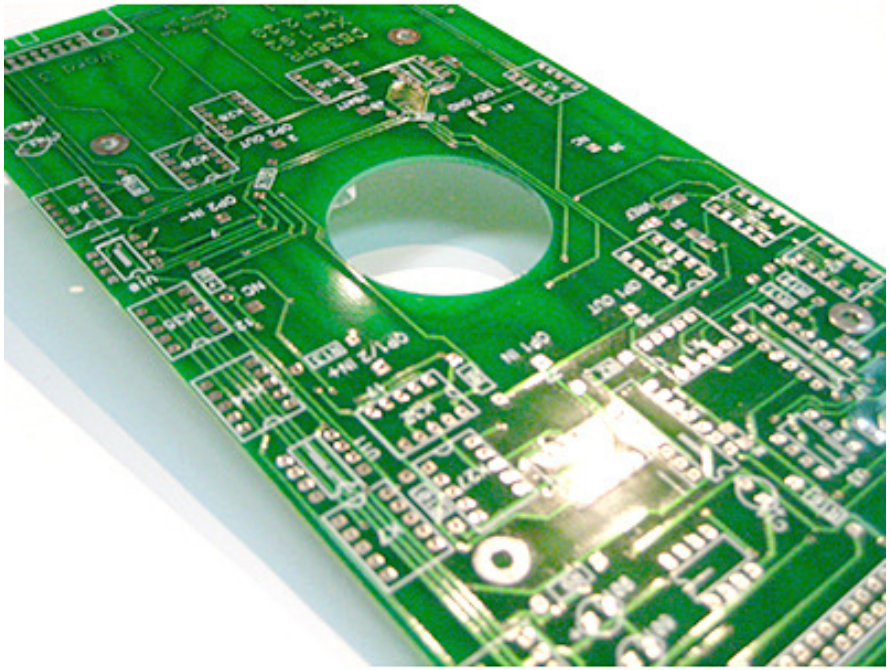
Things around us



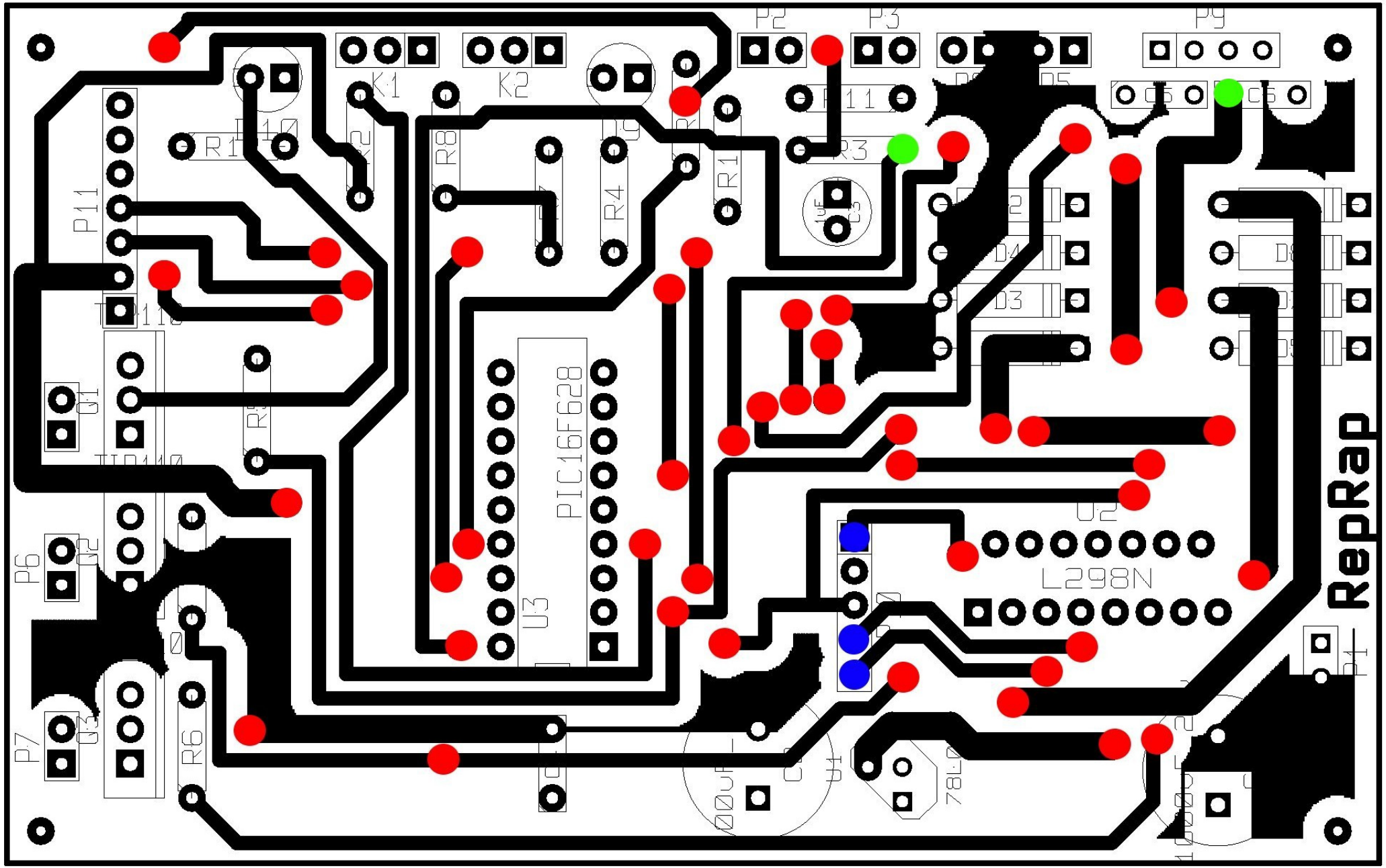
Things inside things around us



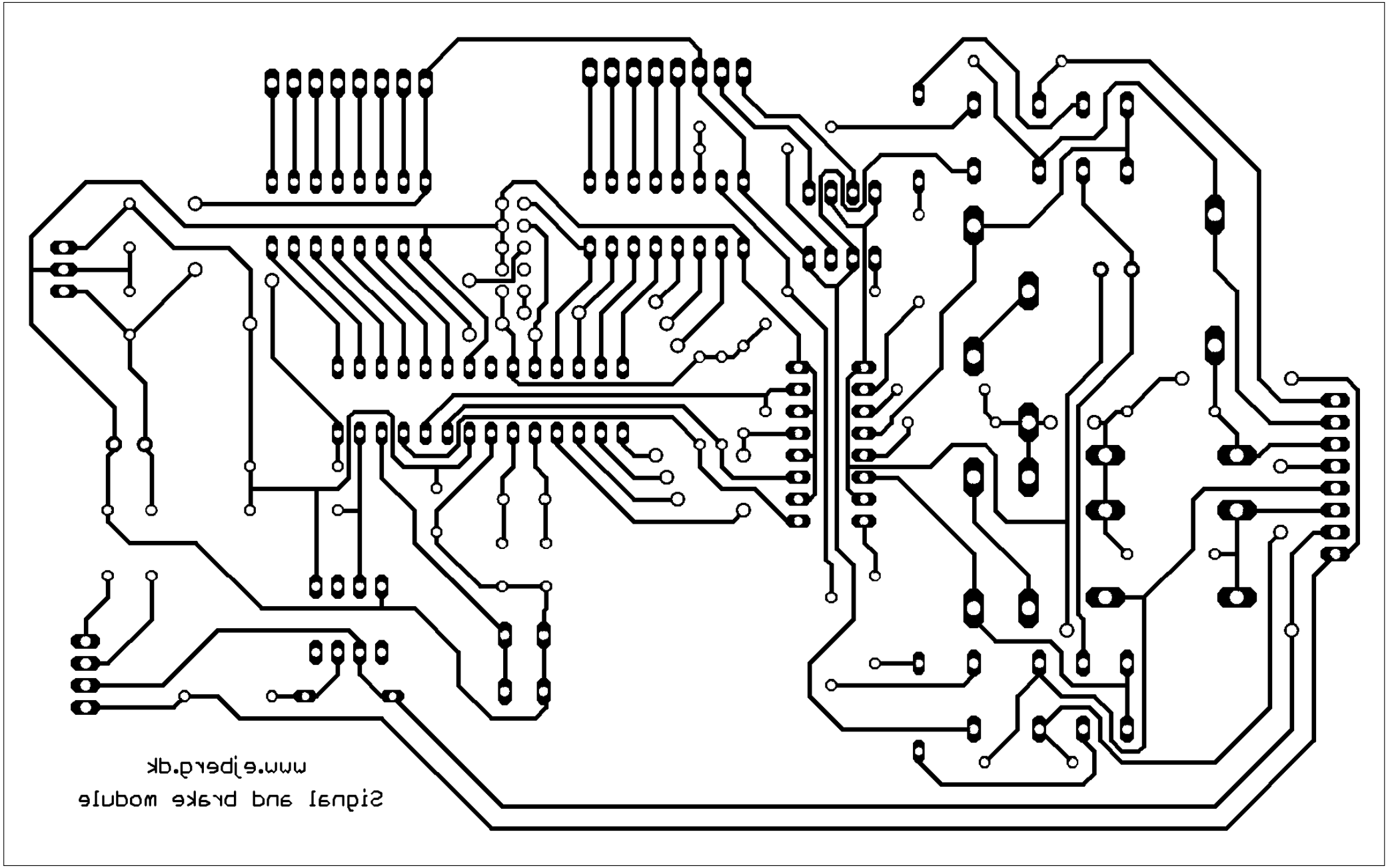
Polygon with holes



Connect terminals pairs



… by multiple (thick) paths

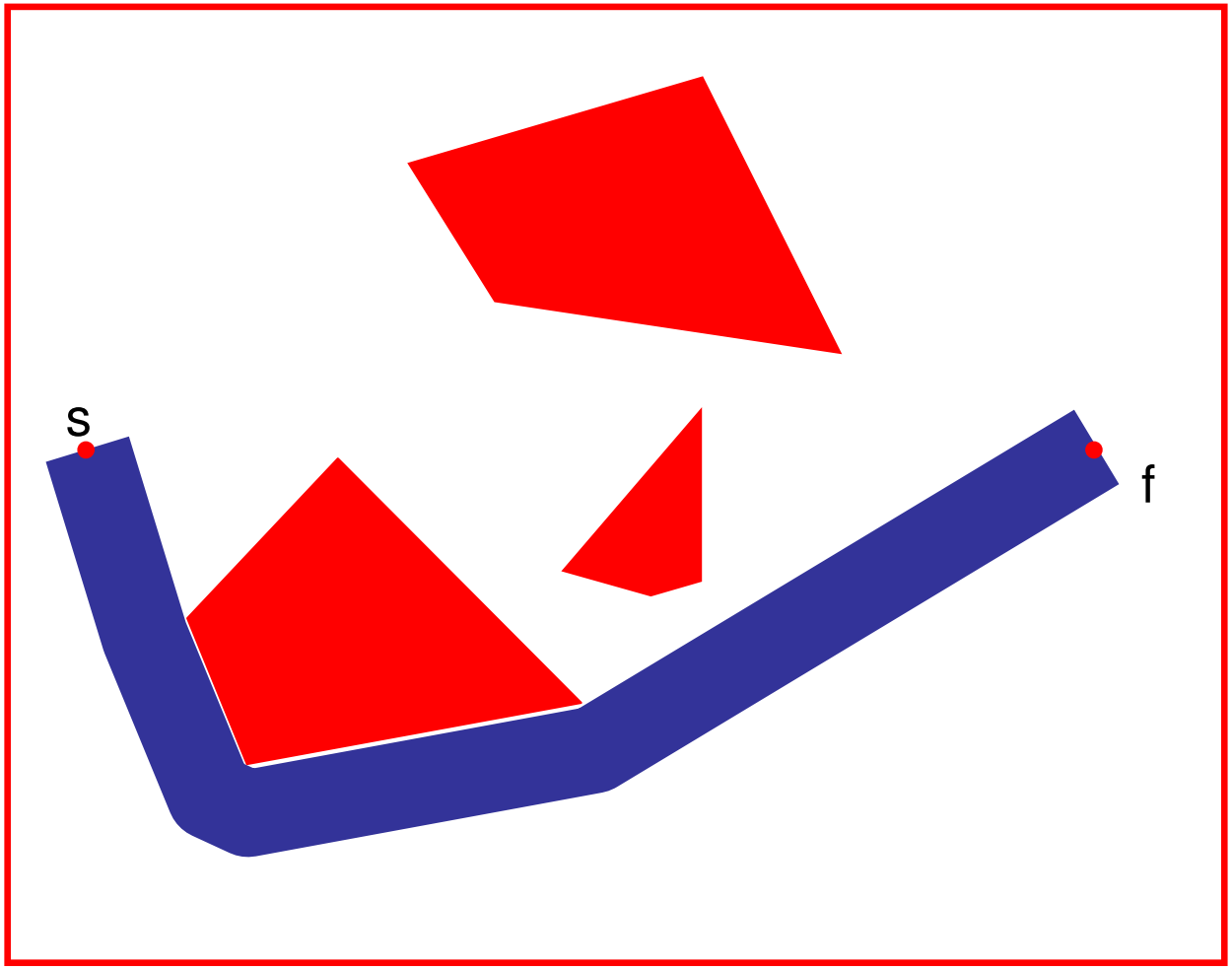


VLSI, Printed circuit board design

- 30 years of research
- Zillions of papers; theses; books
- Advanced algorithms
  - routing wires on different levels
  - taking into account multiple objectives
  - ...
- Commercial success

VLSI very very very basic problem

- Given:
  - domain
  - points s,f
- Find
  - thick s-f wire



NP-hard

already the feasibility problem

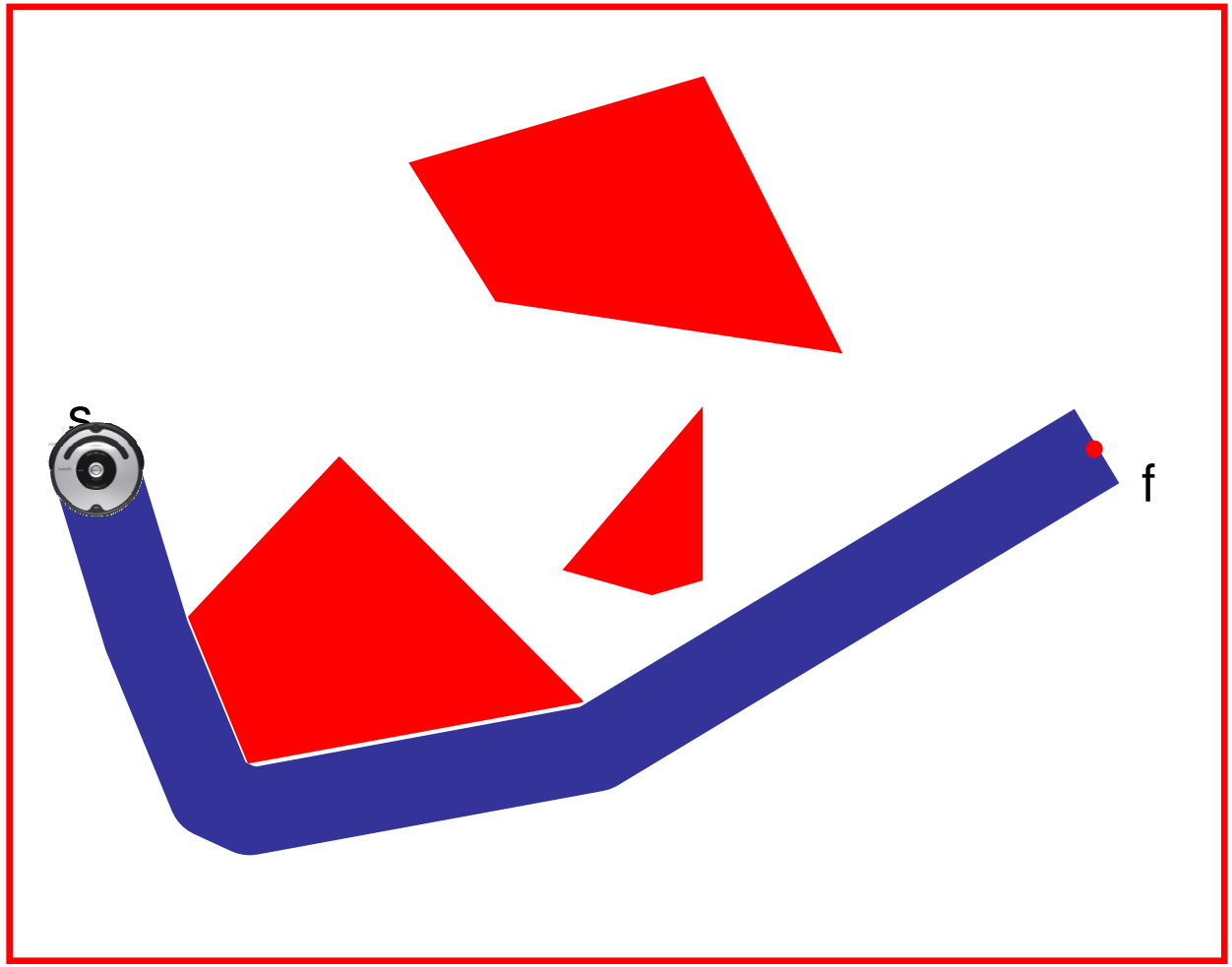
NP-hard to find *any* approximation to shortest wire

No attention for 30 years???

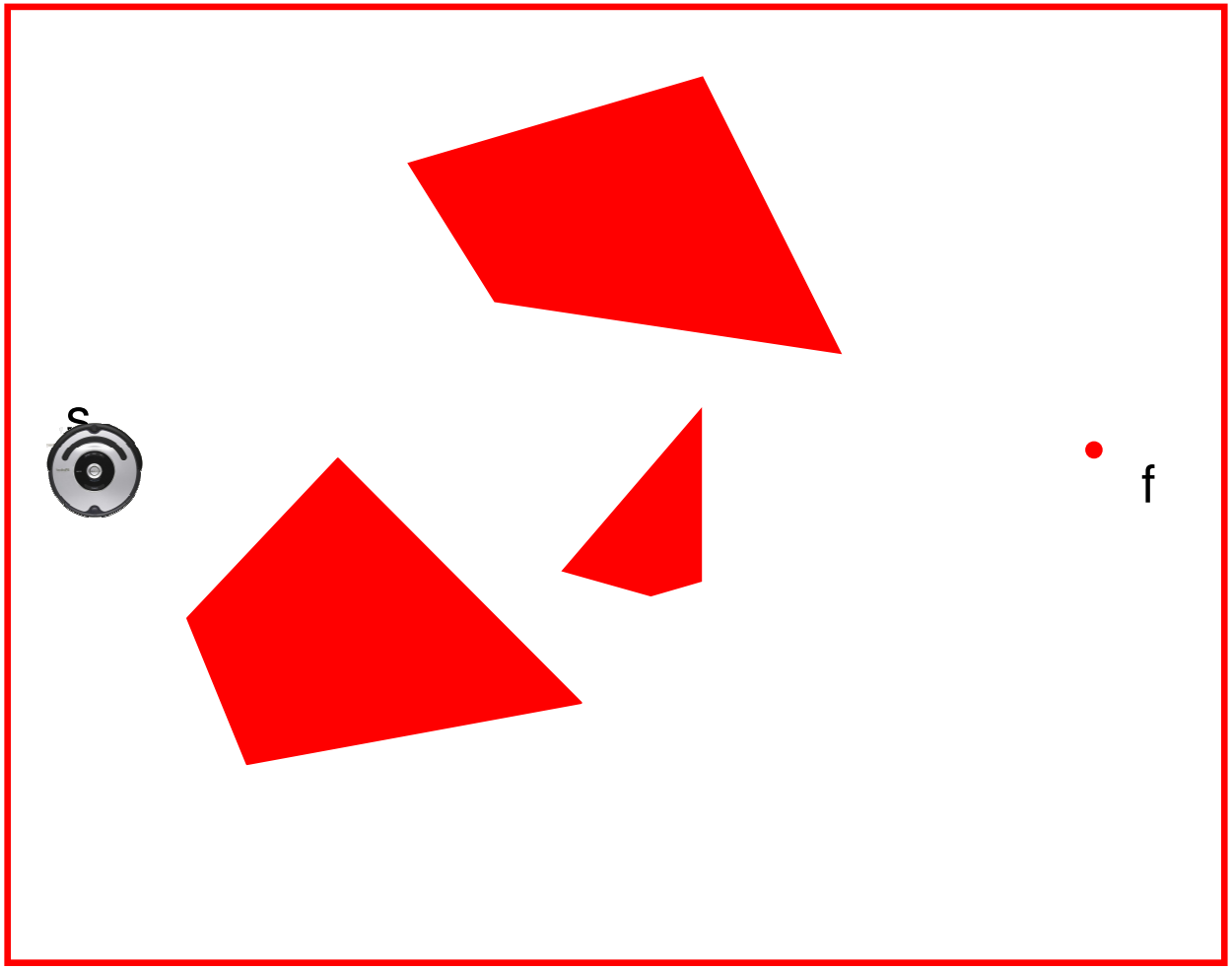
- VLSI,
  printed circuit
  board design

- Robotics

  paths for disks

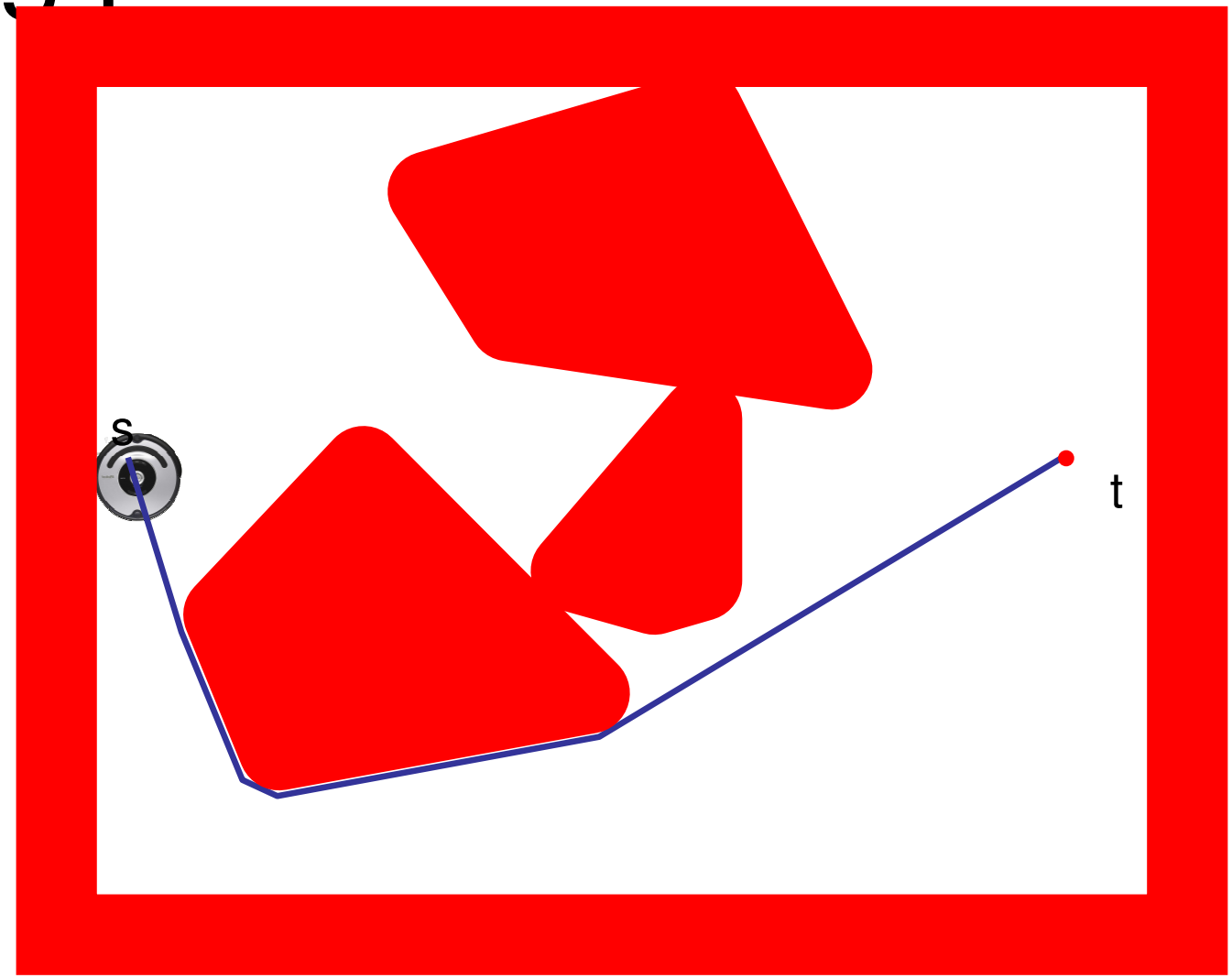


Finding path for a disk



Finding path for a disk

- Inflate the obstacles
- Find path avoiding inflated obstacles



Finding path for a disk

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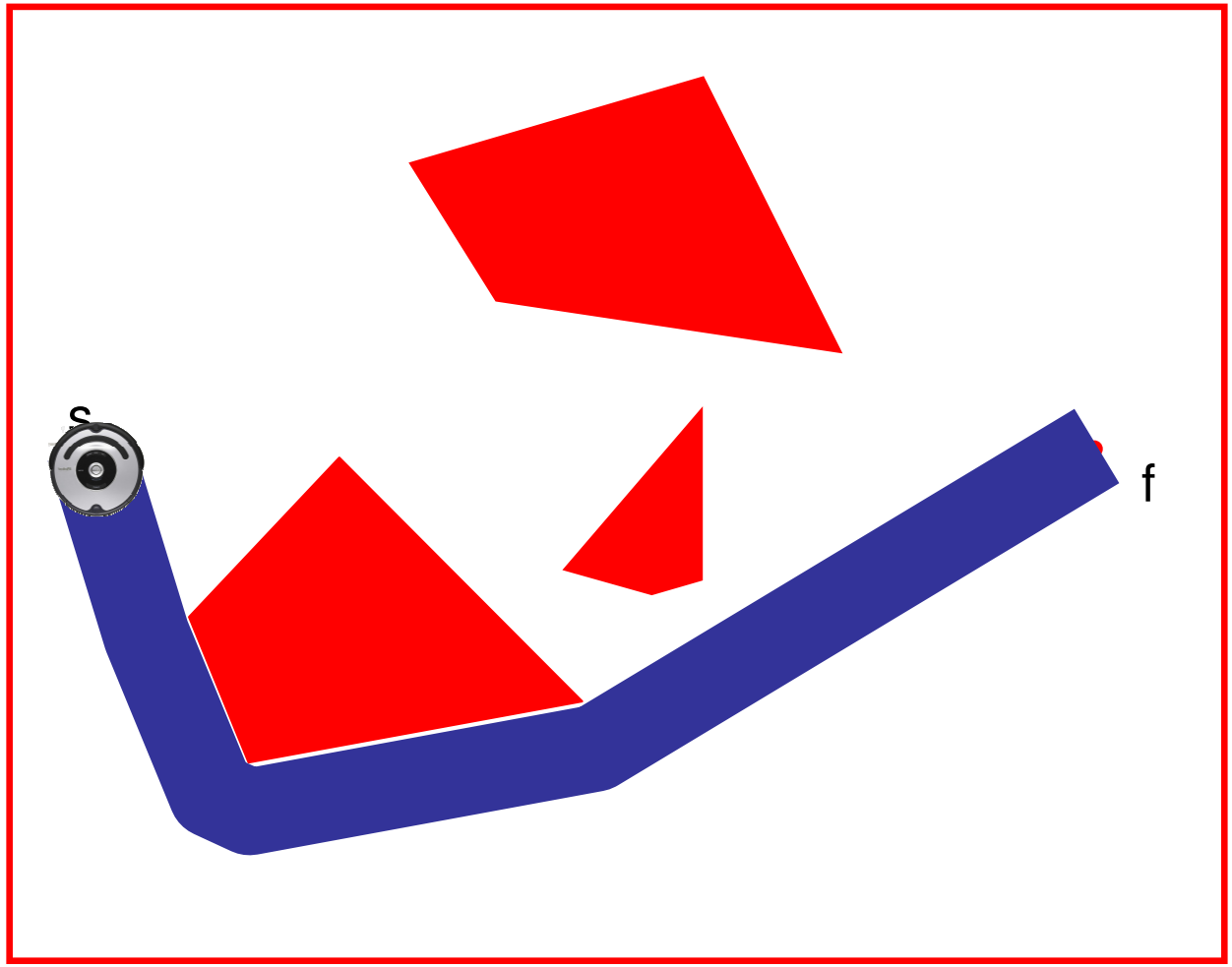
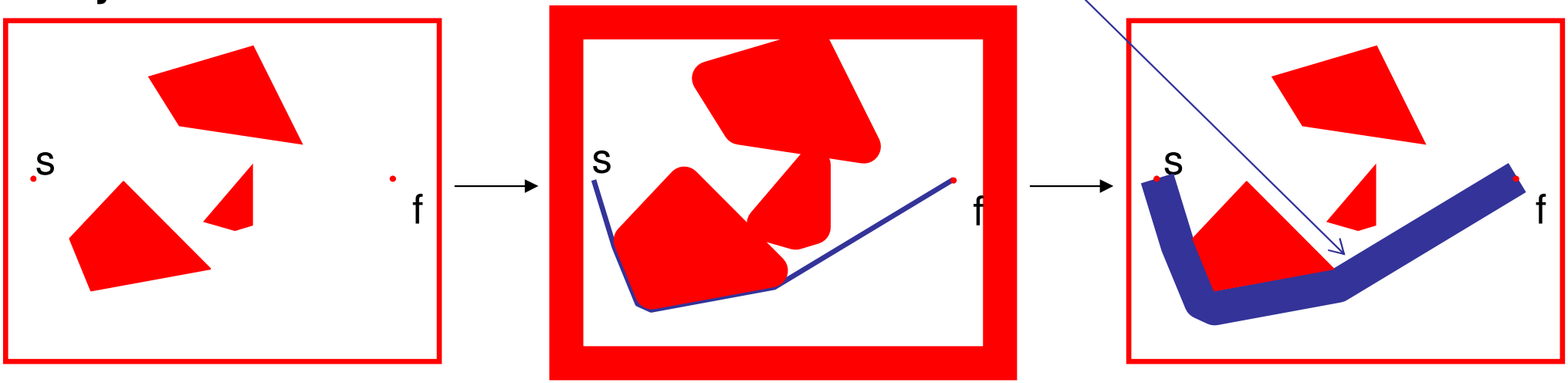


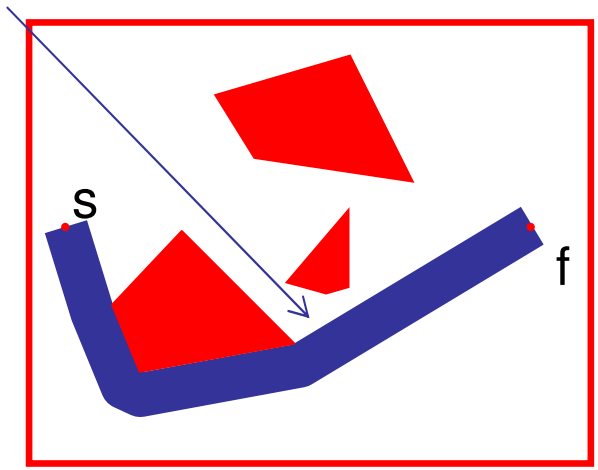
Finding thick path

Poly-time:

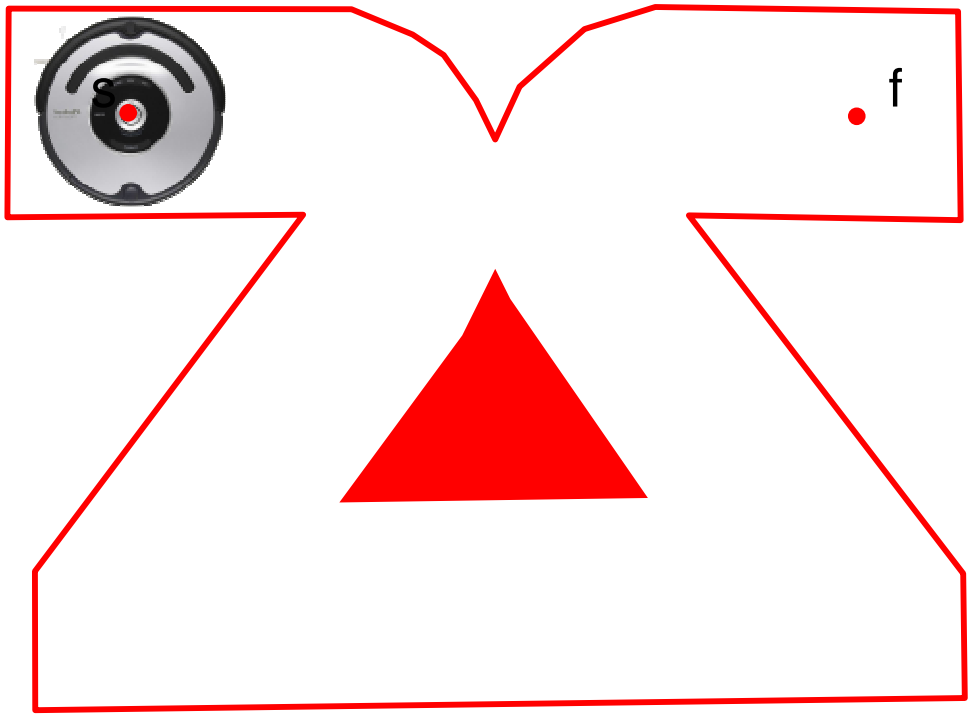


Finding thick wire

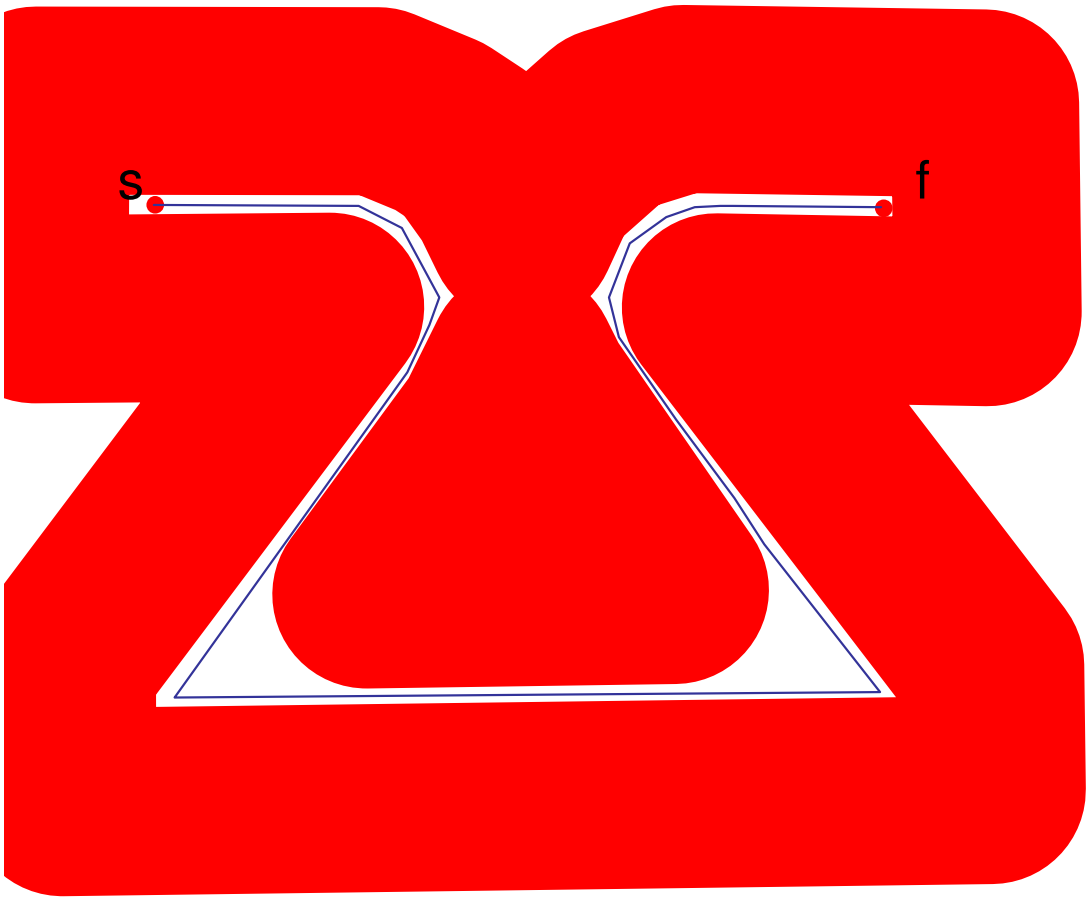
NP-hard



Path ≠ Wire

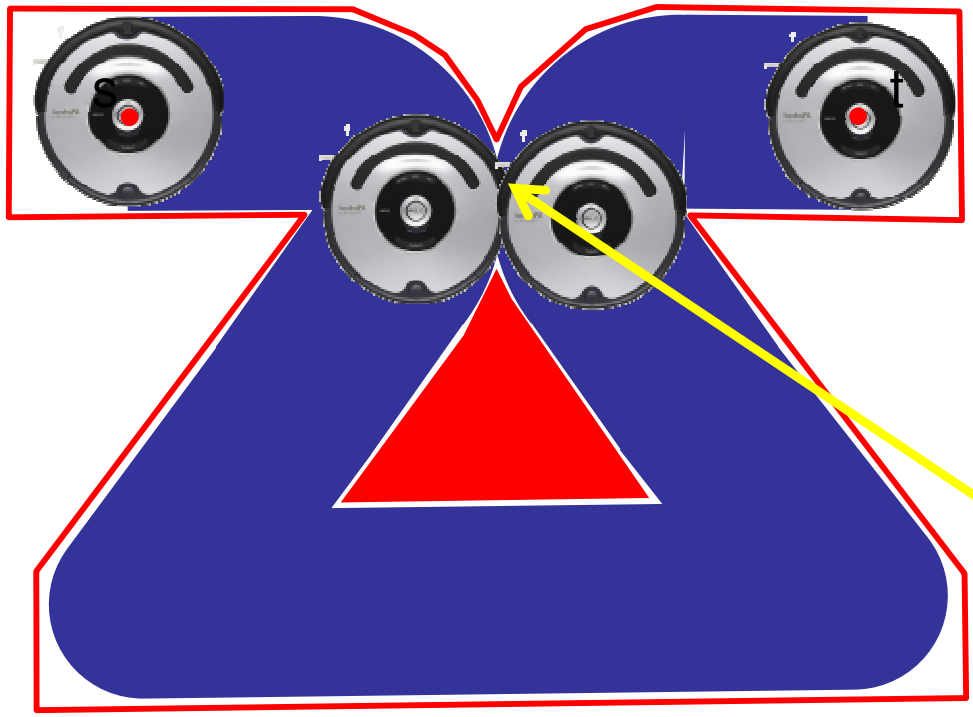


Path ≠ Wire



- Inflate obstacles
- Find (thin) path

Path ≠ Wire

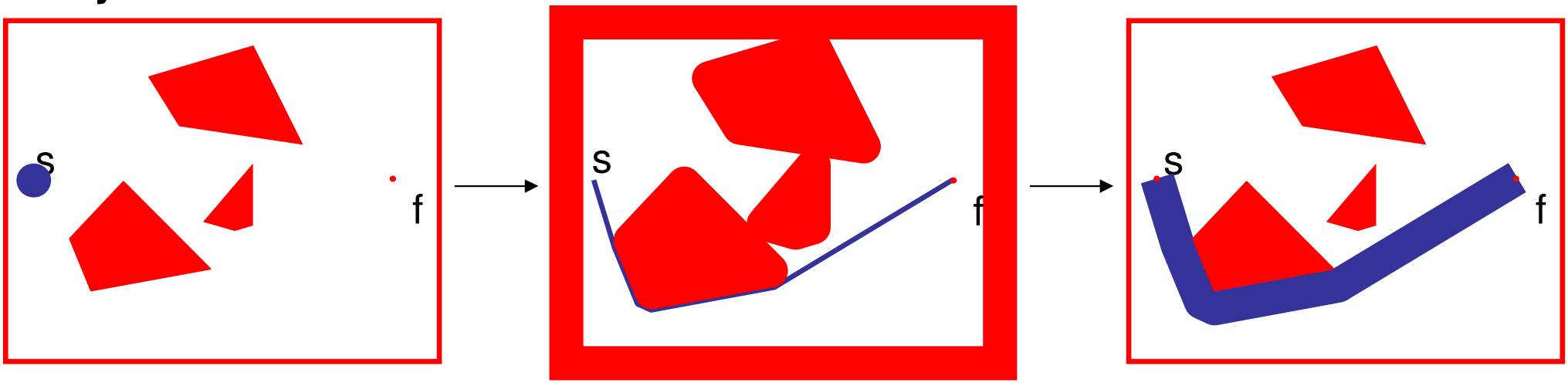


- Inflate obstacles
- Find (thin) path
- Inflate the path

~~Wire~~
self-overlap

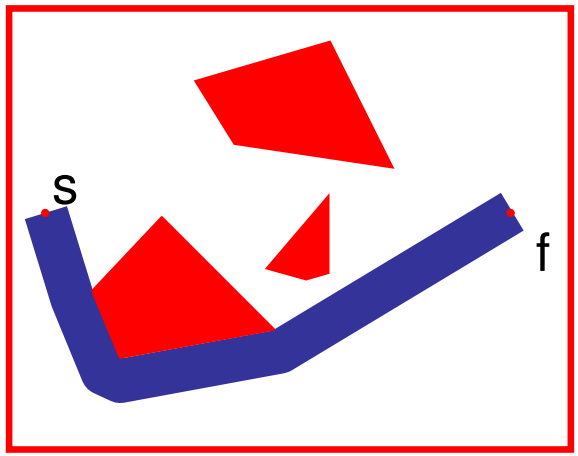
Finding thick path

Poly-time:



Finding thick wire

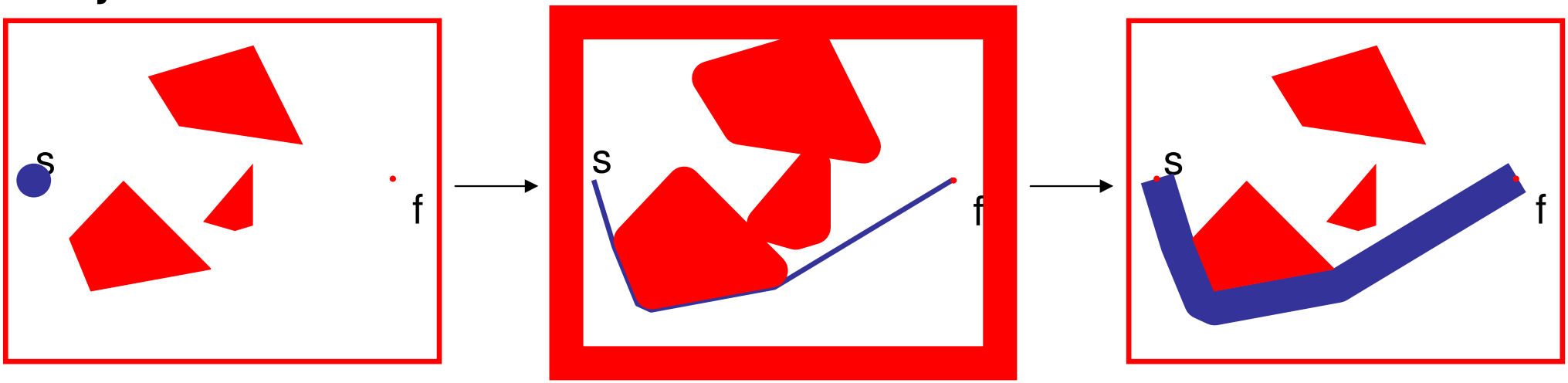
NP-hard



Proof: Reduction from planar 3SAT (time permitting)

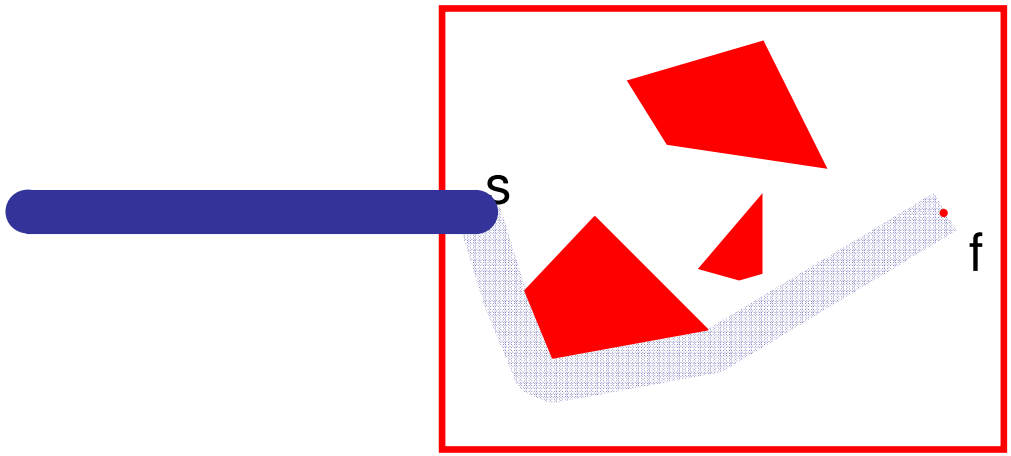
Finding path for a disk

Poly-time:



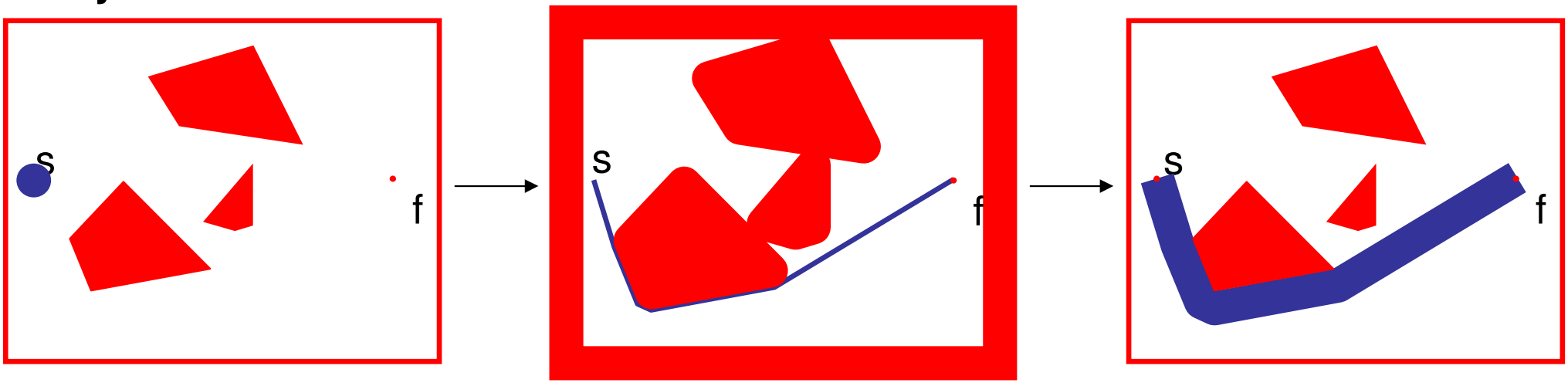
Laying out a snake

NP-hard



Shortest path for disk

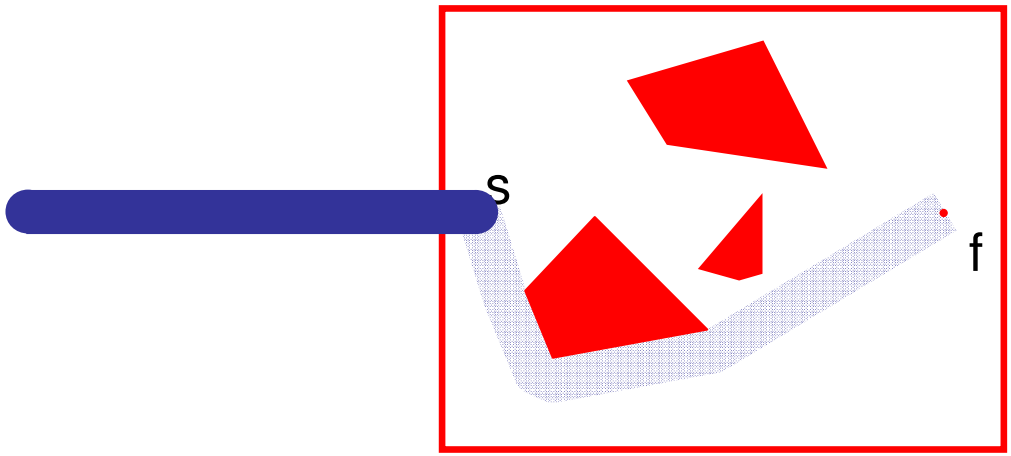
Poly-time:



Shortest path for snake

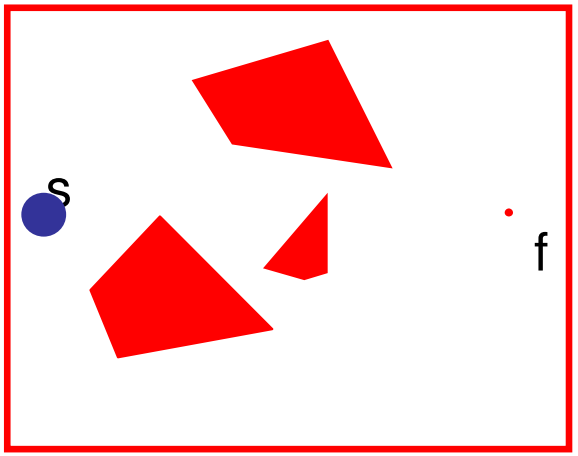
NP-hard

Even to approximate



Shortest path for short snake

Poly-time:

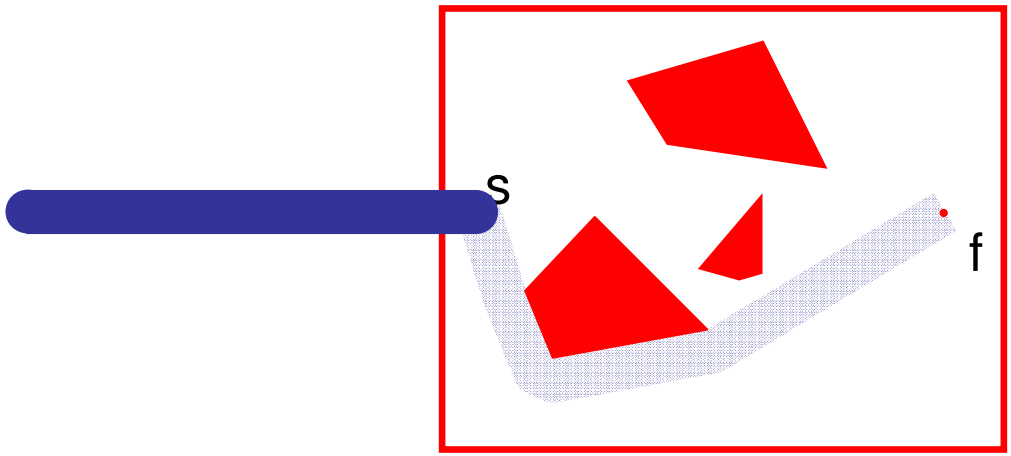


What is a snake (or a thick path), after all?

Shortest path for LOOONG snake

NP-hard

Even to approximate



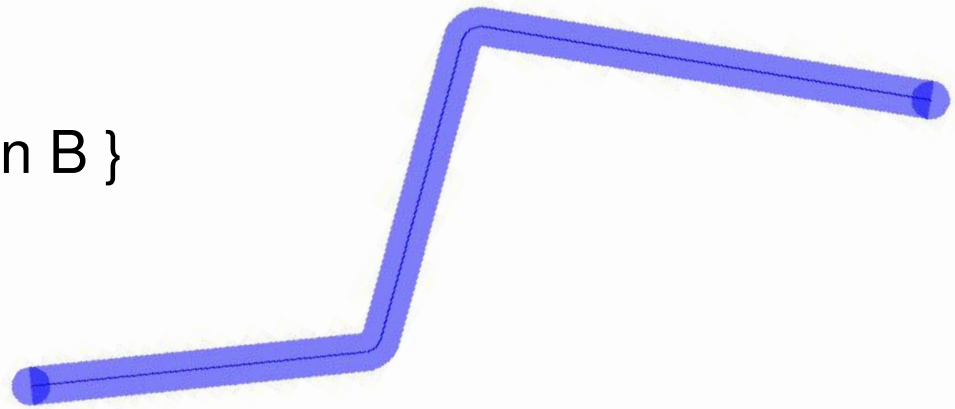
Formally…

Thick path =

reference_path + unit_disk

Minkowski sum

A + B = { a + b | a in A , b in B }

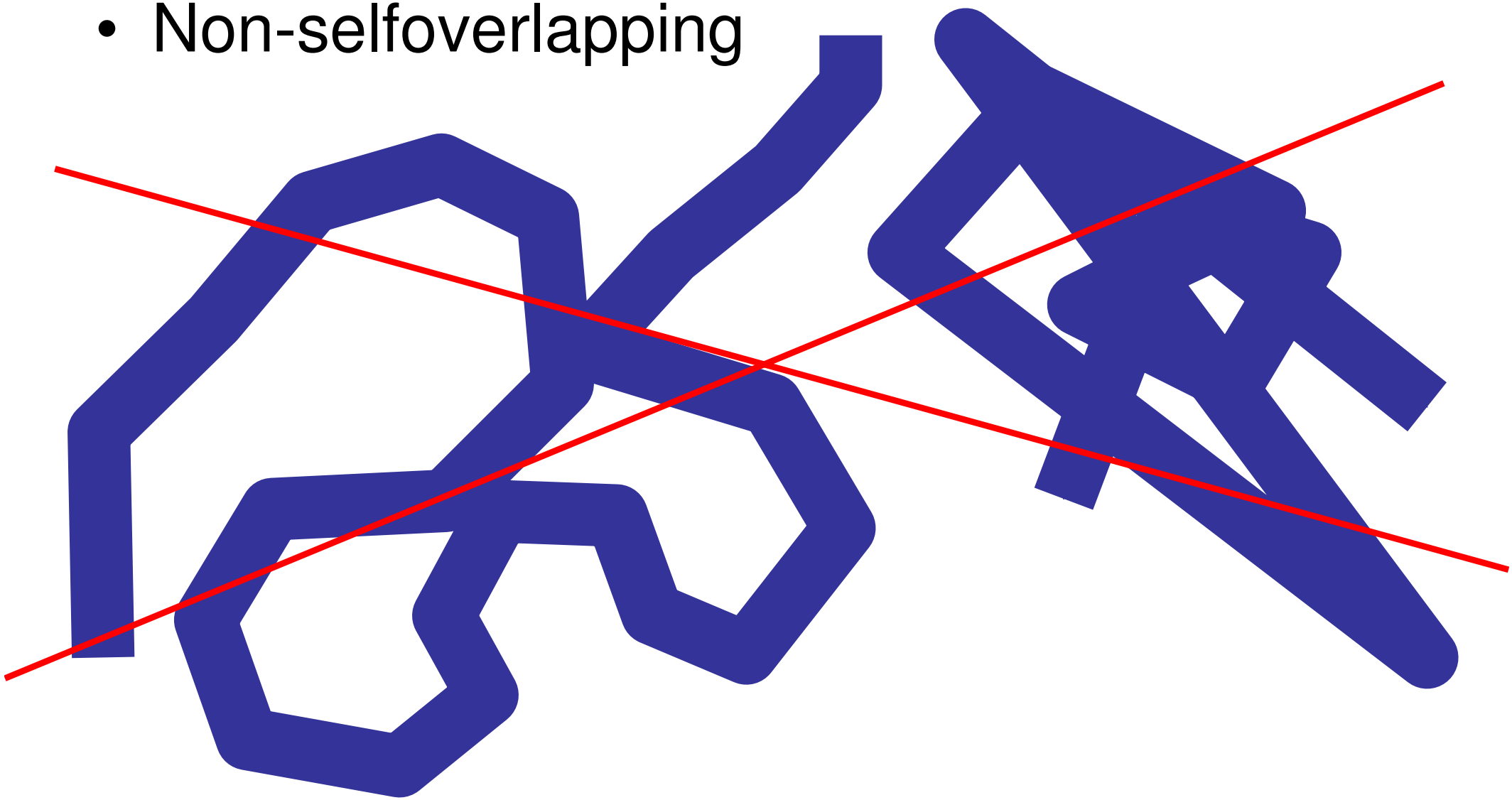


Length =

length(reference_path)

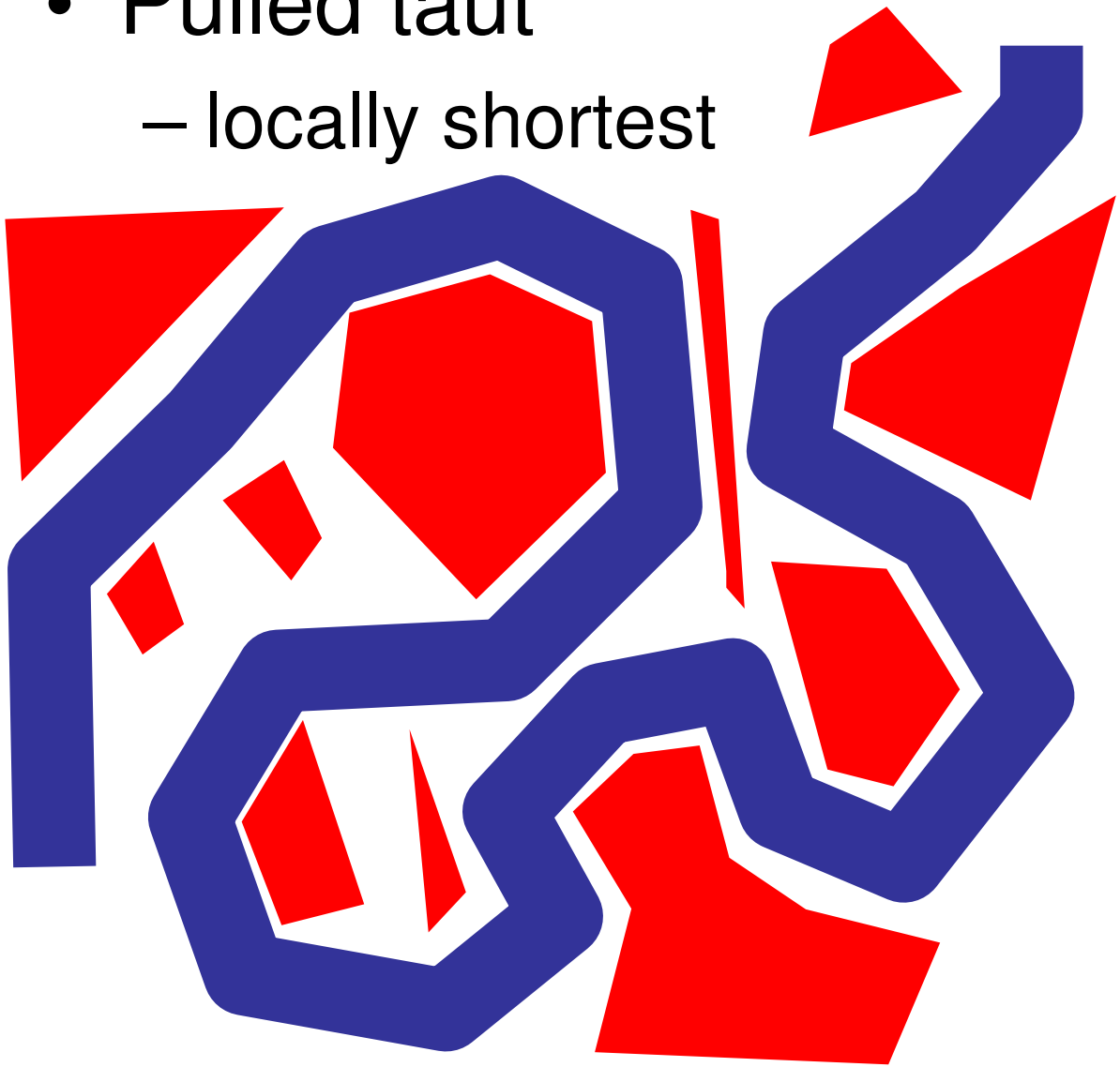
Snake – “nice” thick path

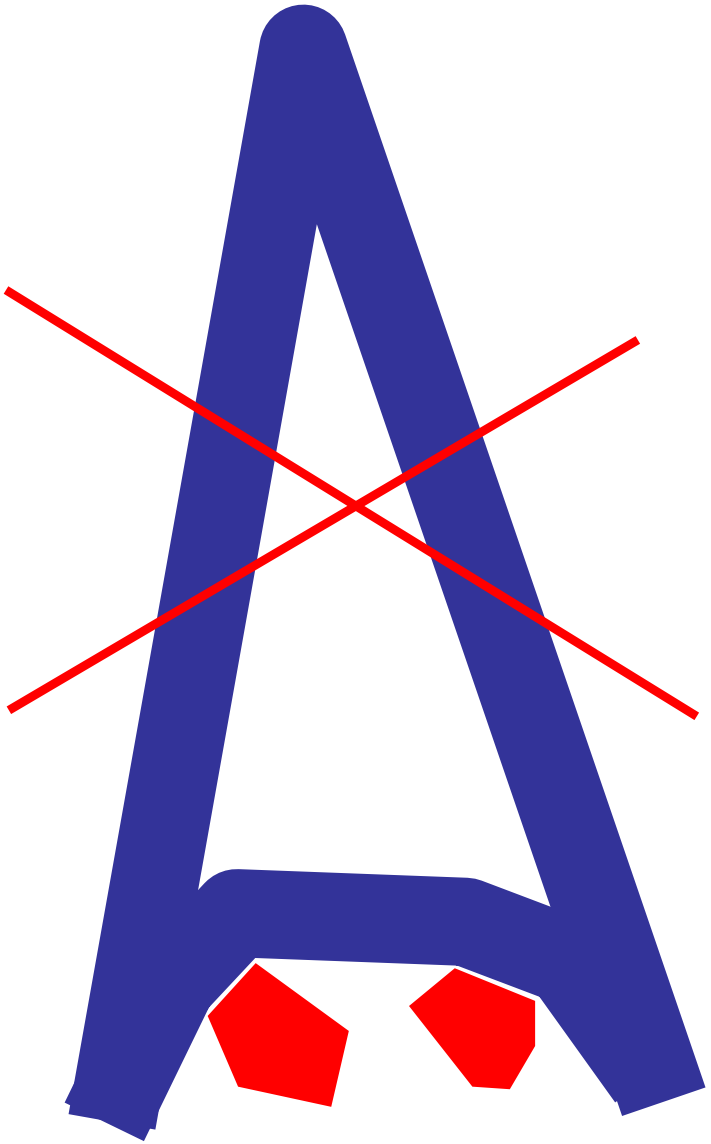
- Non-selfoverlapping



Snake – “nice” thick path

- Pulled taut
  - locally shortest



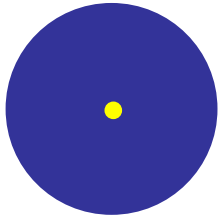


Definition

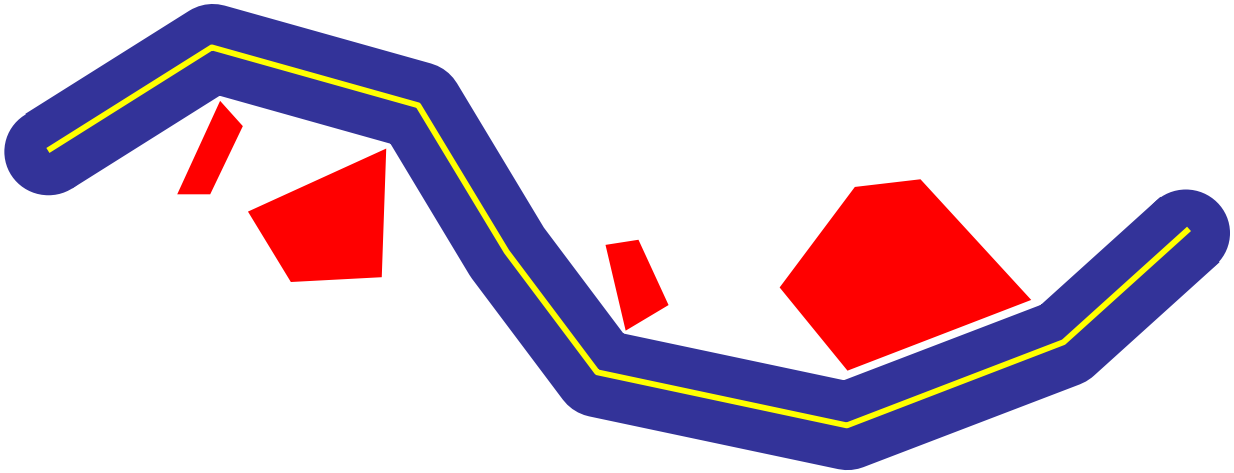
Snake =

non-selfoverlapping (simple)

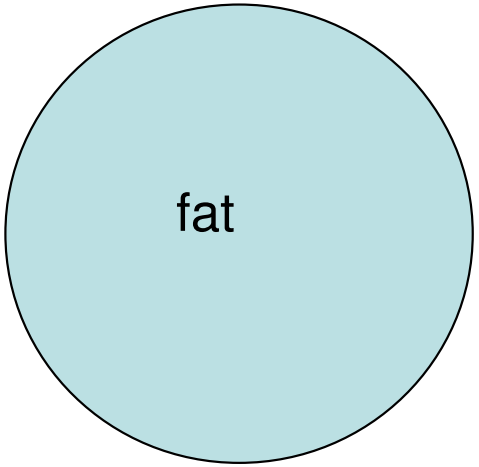
locally shortest (pulled taut)



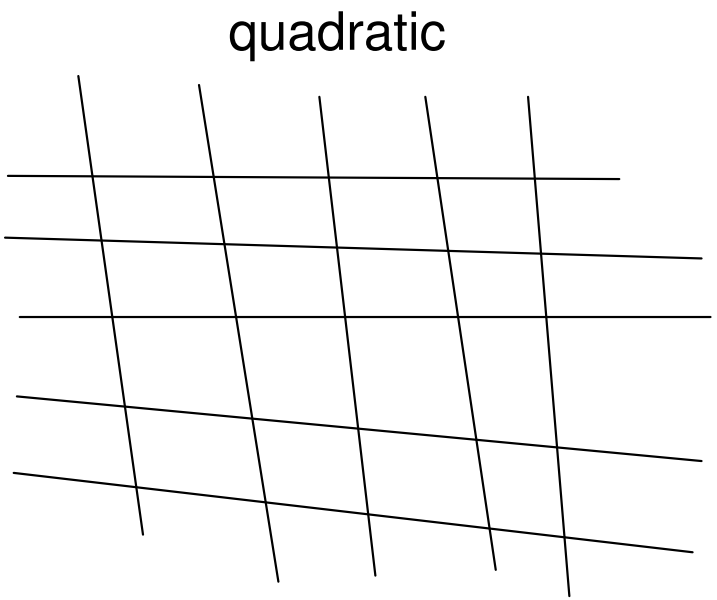
thick path

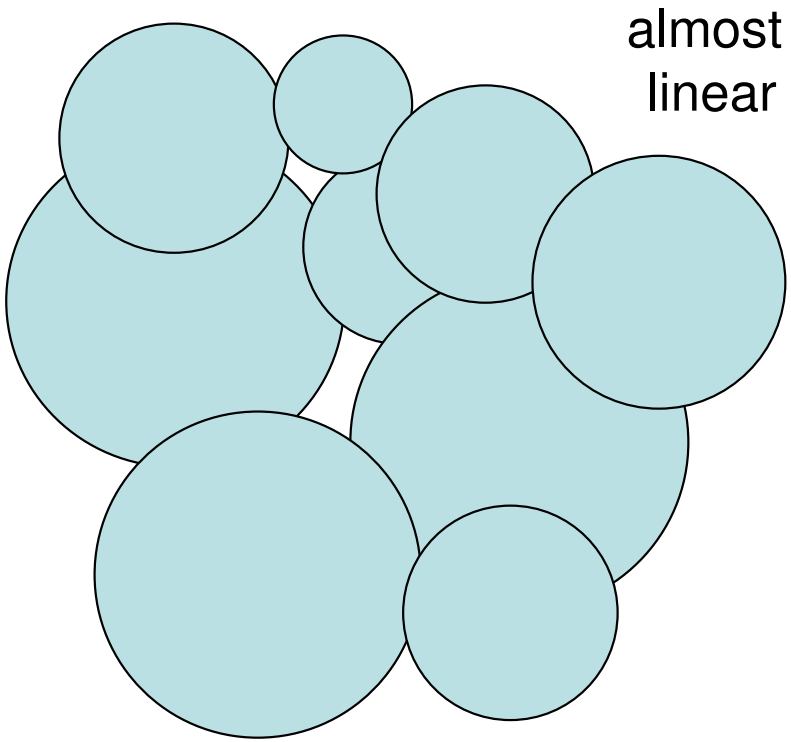


(Geometric) Fatness



- Union complexity





Snake fatness

- Snake – thick path

  Mink sum: path + unit disk

- Snake is stretched

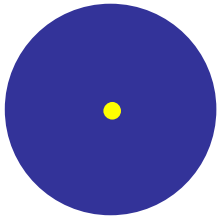
  Mink sum: segment + disk

  =

  hippodrome or hippo

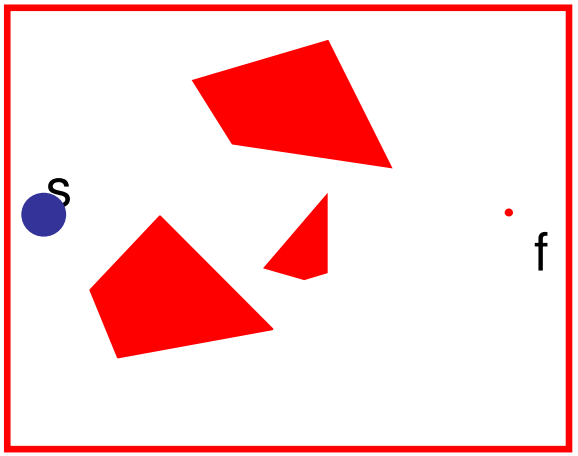
- Fatness = width / length
- Fat hippo

  L = O(1) (w = 2)



Shortest path for short snake

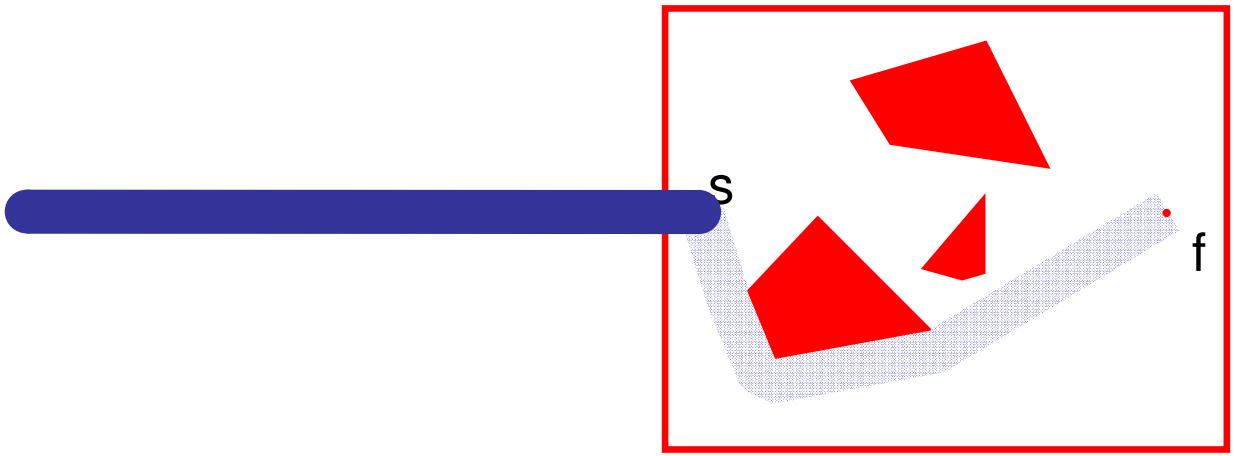
Poly-time:



Shortest path for LOOONG snake

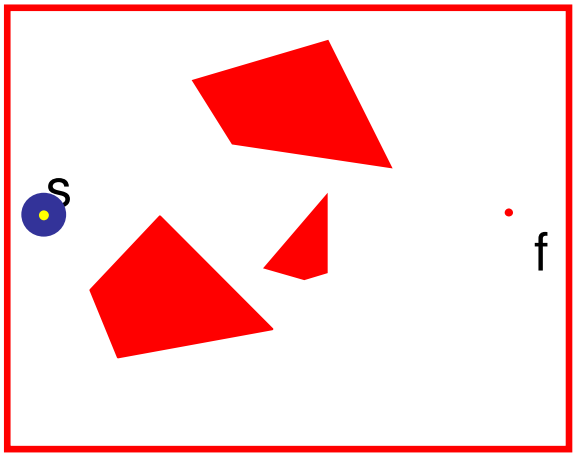
NP-hard

Even to approximate



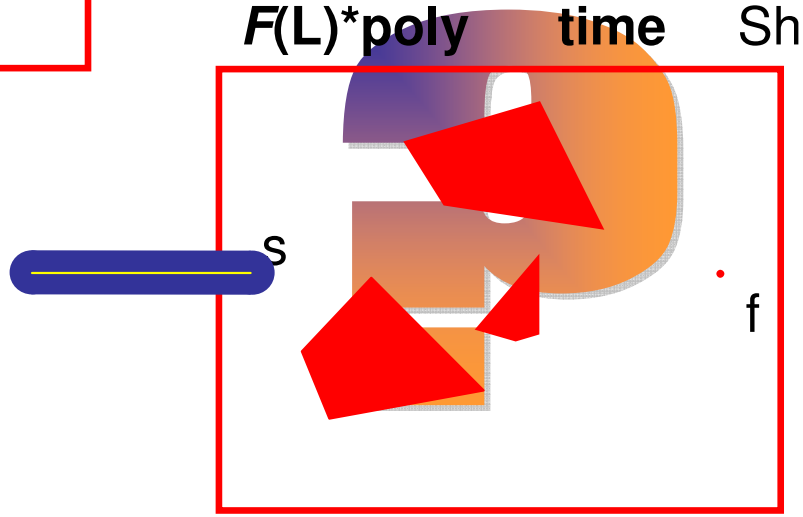
Shortest path for hippos

Poly-time:
∞-fat



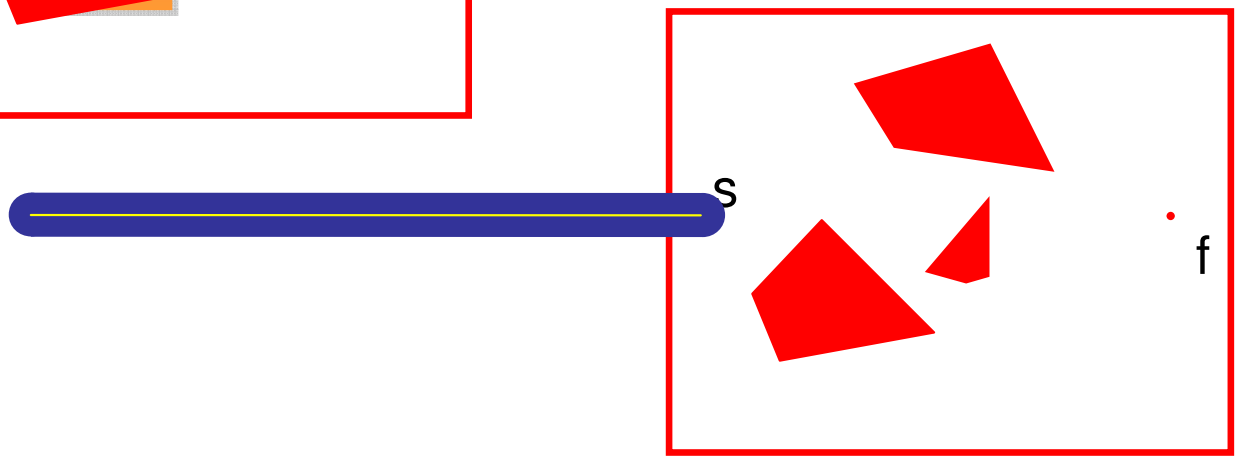
Fixed-parameter tractability:
NP-hard problem is FPT
F(k) * poly alg

***F*(L)*poly time**



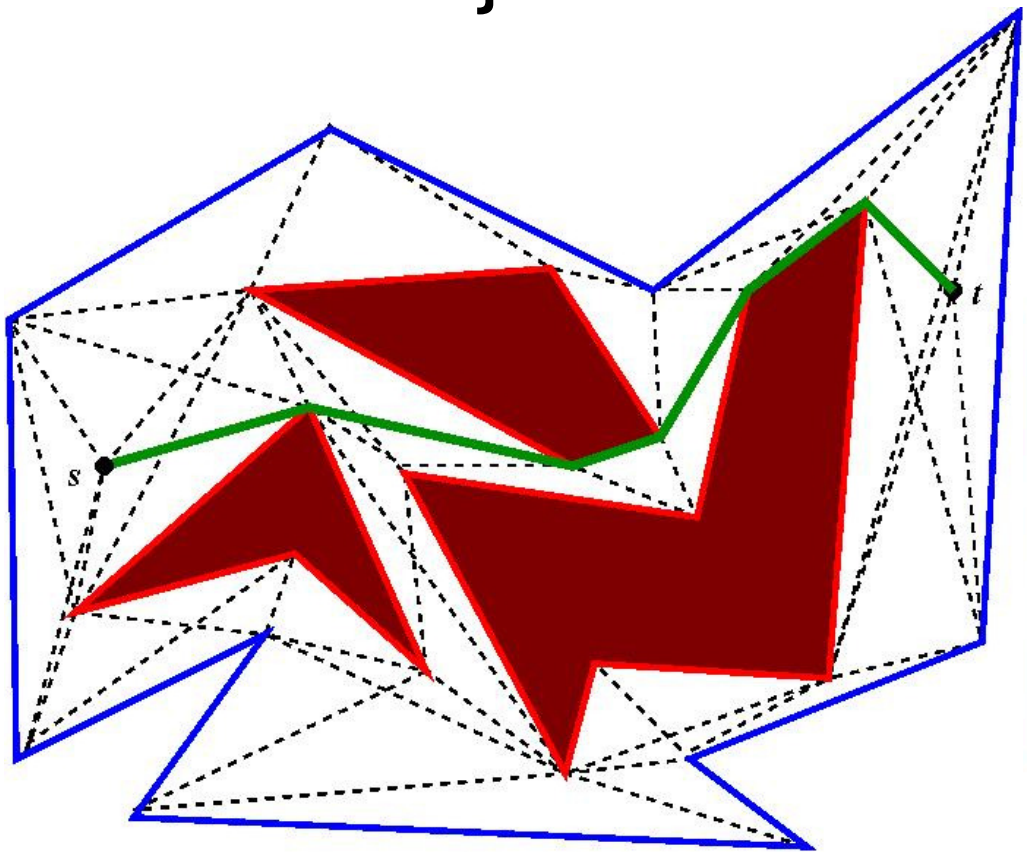
Shortest path for a snake
length-tractable
poly-time if L=O(1)

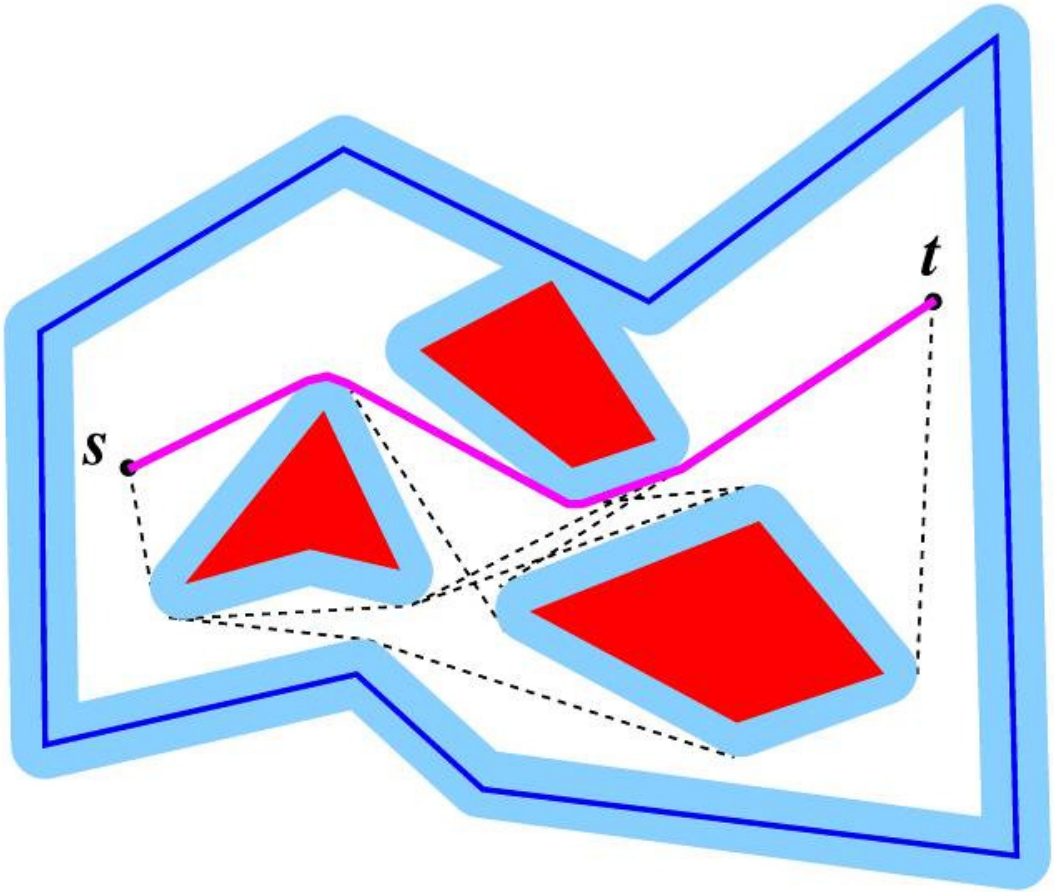
NP-hard:
no lower bound on fatness



Shortest path for a point/disk

- Build Visibility graph
- Run Dijkstra





Shortest path for a snake

- Build Visibility graph
- Run Dijkstra

v – node with smallest label
"propagate" labels along edges incident to v
update labels of reached nodes

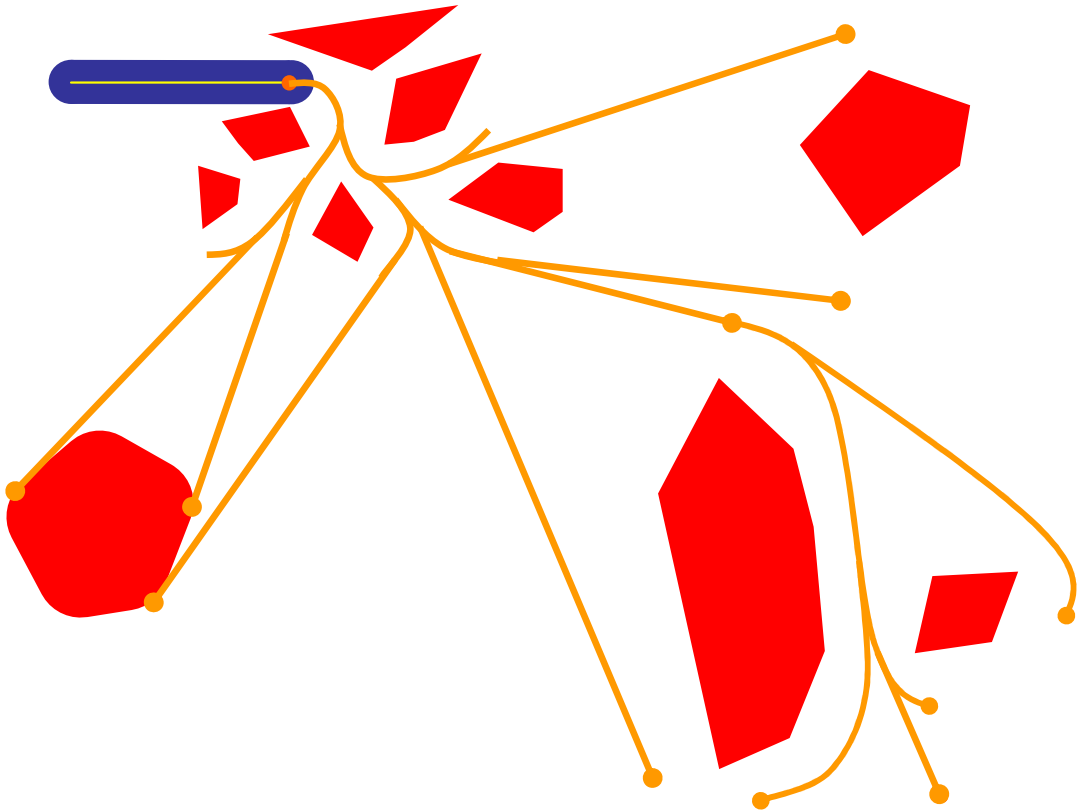
No need to know the nodes before they're reached
(Dijsktra on-demand)

Pull the snake along
"every possible" length-L path

Draw bitangents
endpoints – nodes

?

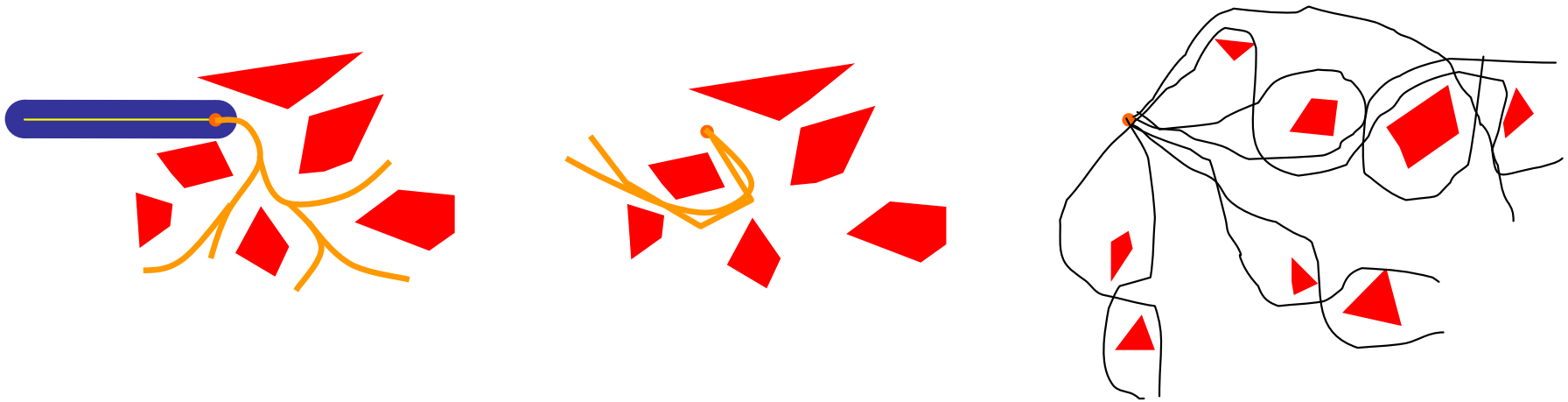
Runtime



f

of length-L paths

- Pulled taut
- Different combinatorial type ("wiggling amidst holes")

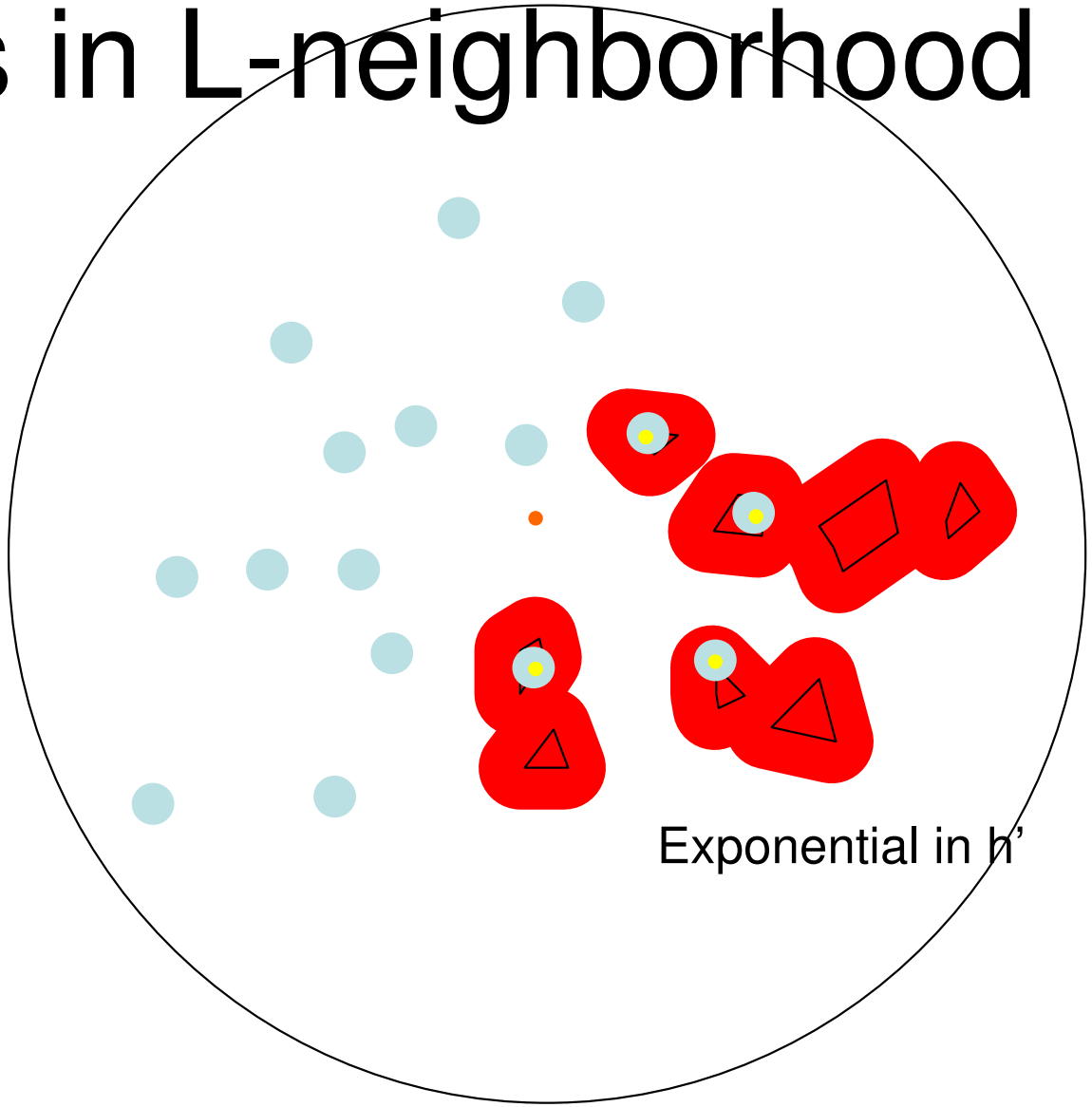


Exponential in h

- But!:
- Paths for the snake's centerline
- Stay away from obstacles
- Paths amidst inflated holes

Inflated holes in L-neighborhood

- $h' = O(1)$



- $h' \leq$ # of disjoint unit disks in radius-L disk $\leq L^2 = O(1)$

Shortest path for a snake

- Run Dijkstra

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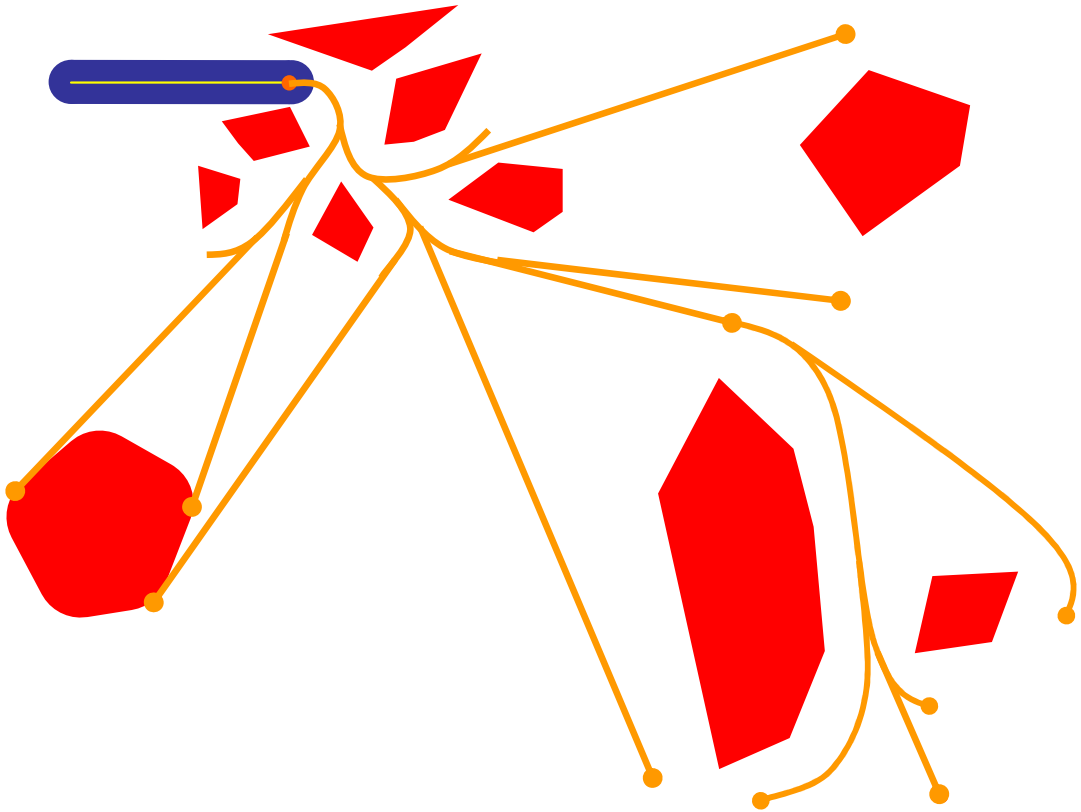
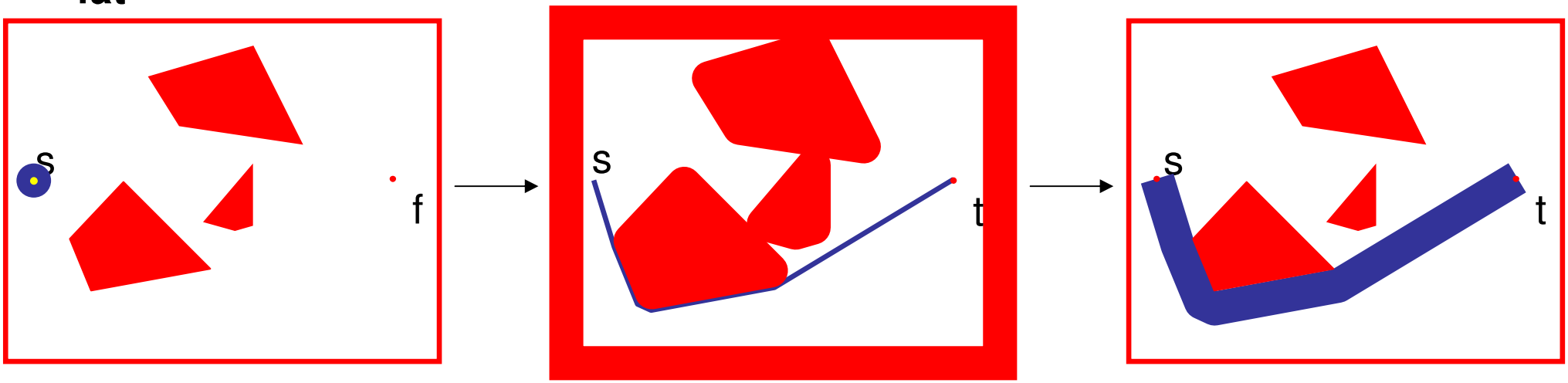


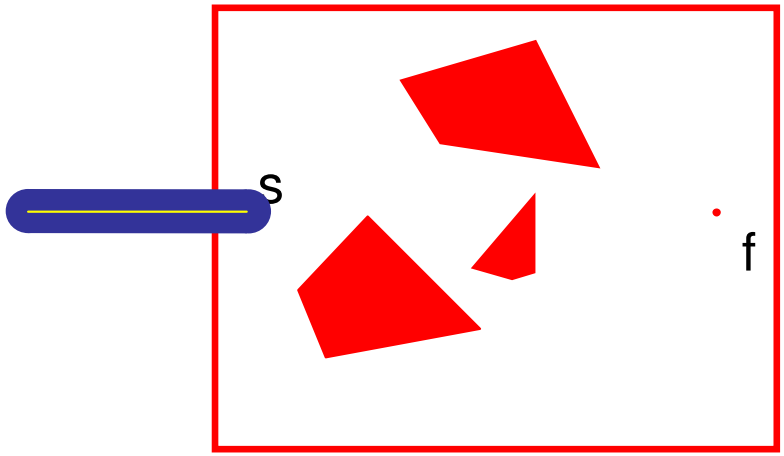
f

Poly-time:
∞-fat

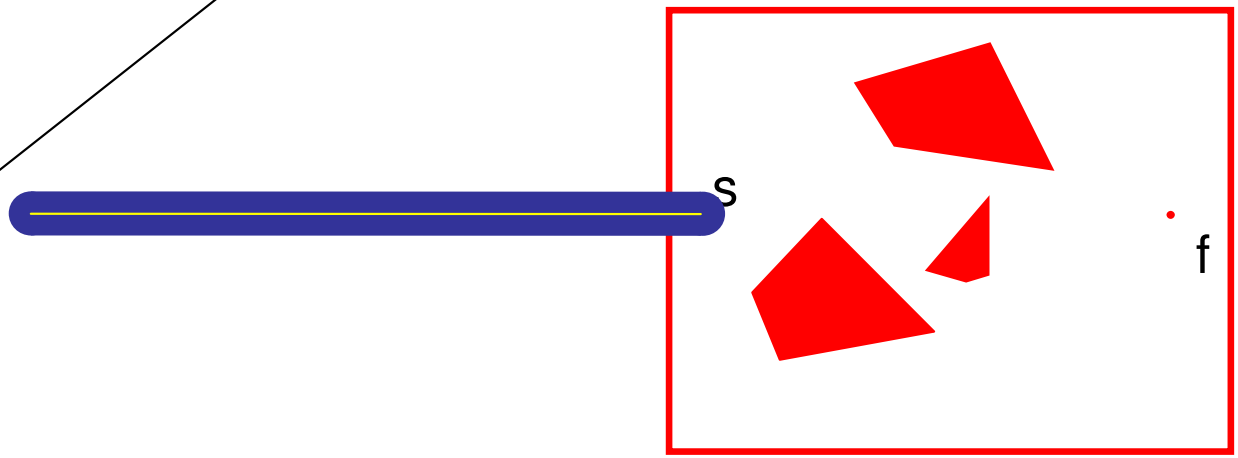


***F*(L)*poly time**

length-tractable
poly-time if L=O(1)

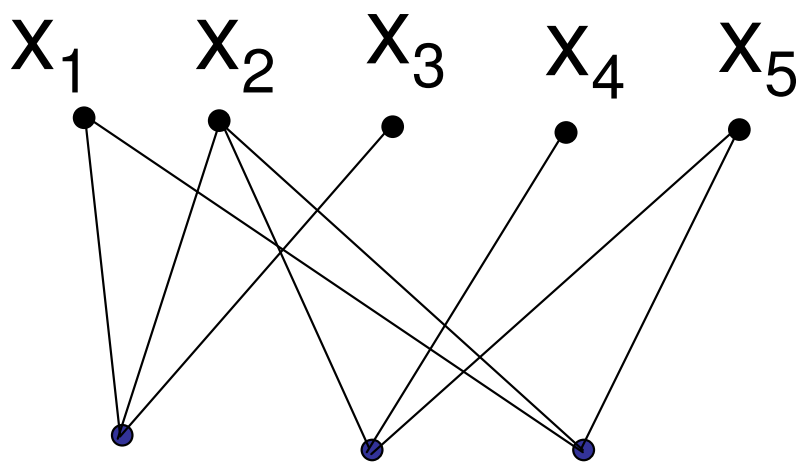


NP-hard:
no bound on fatness

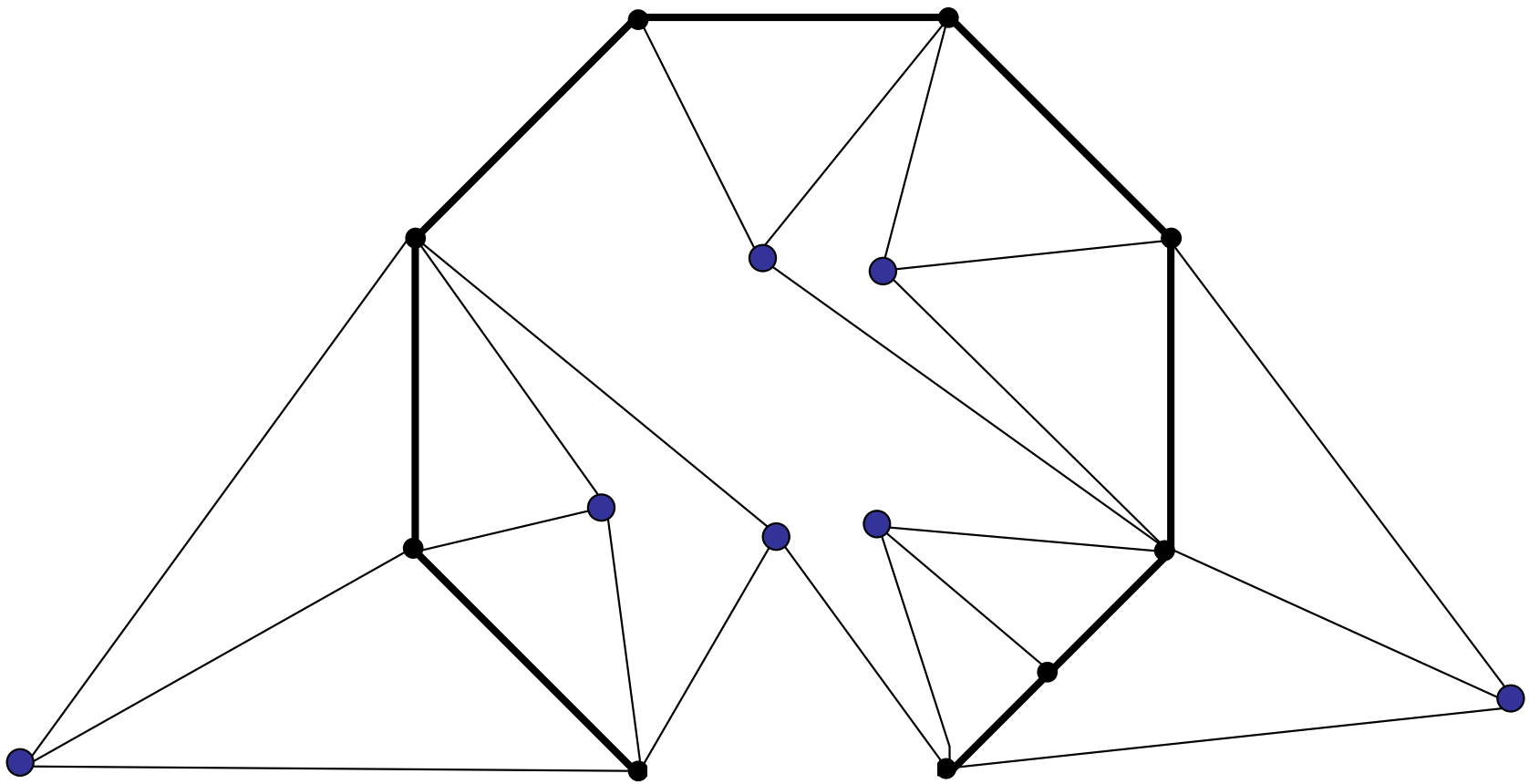


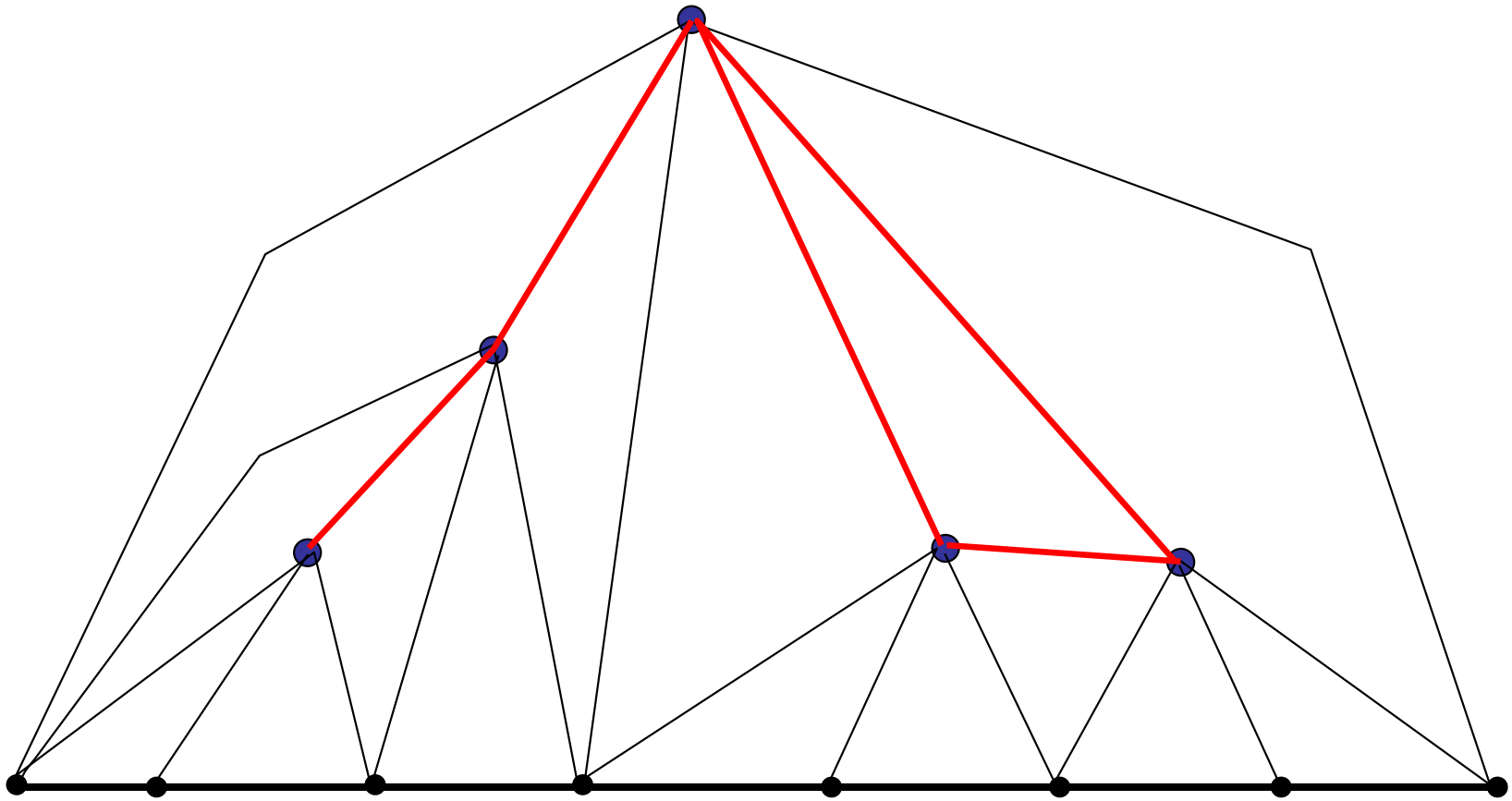
Reduction from 3SAT

- $(x_1, \neg x_2, x_3)$, $(\neg x_2, \neg x_4, x_5)$, $(x_5, \neg x_2, x_1)$
- Graph
  - nodes: vars and clauses
  - edge xC if var x is in clause C

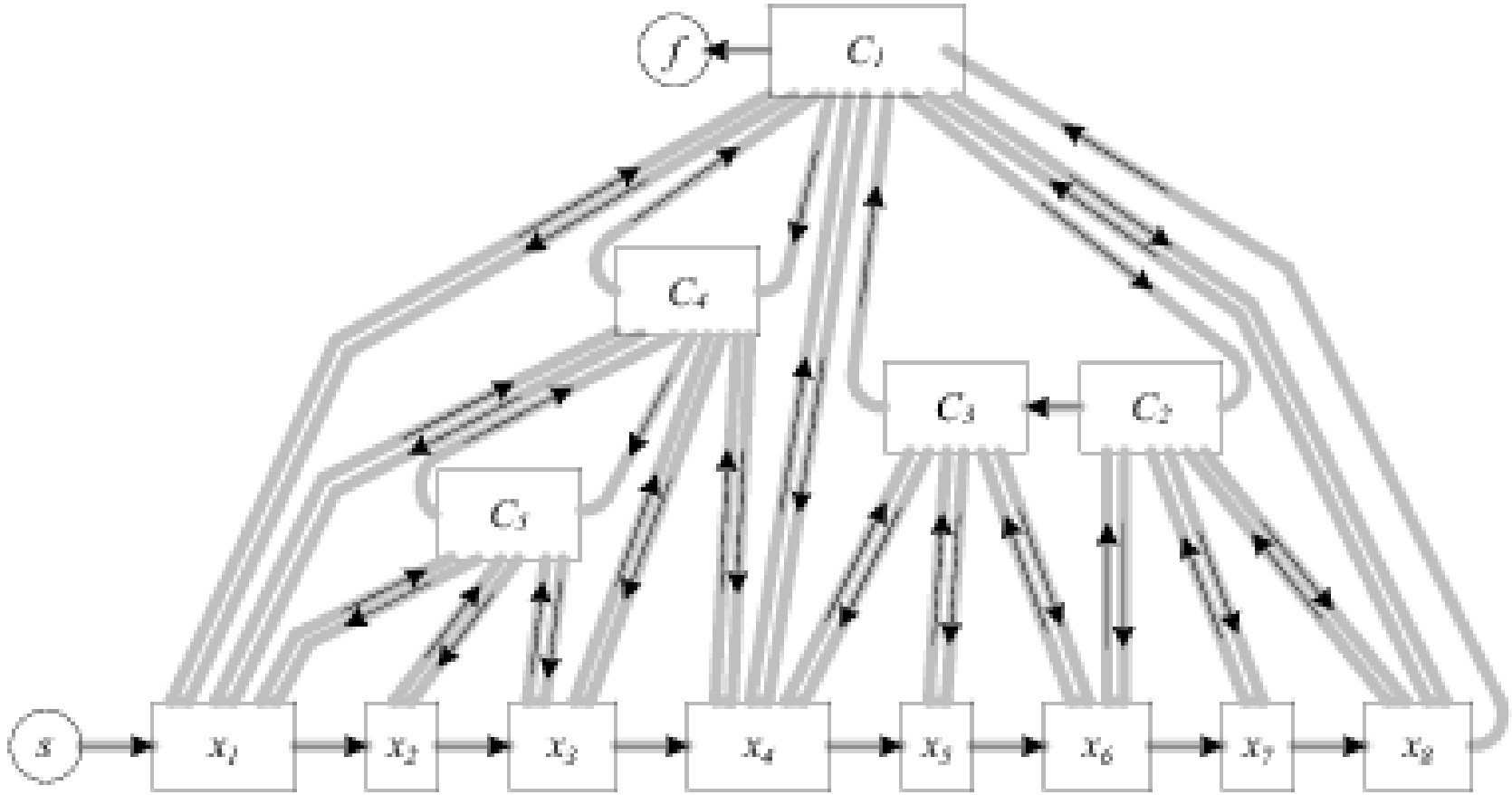


Planar 3SAT

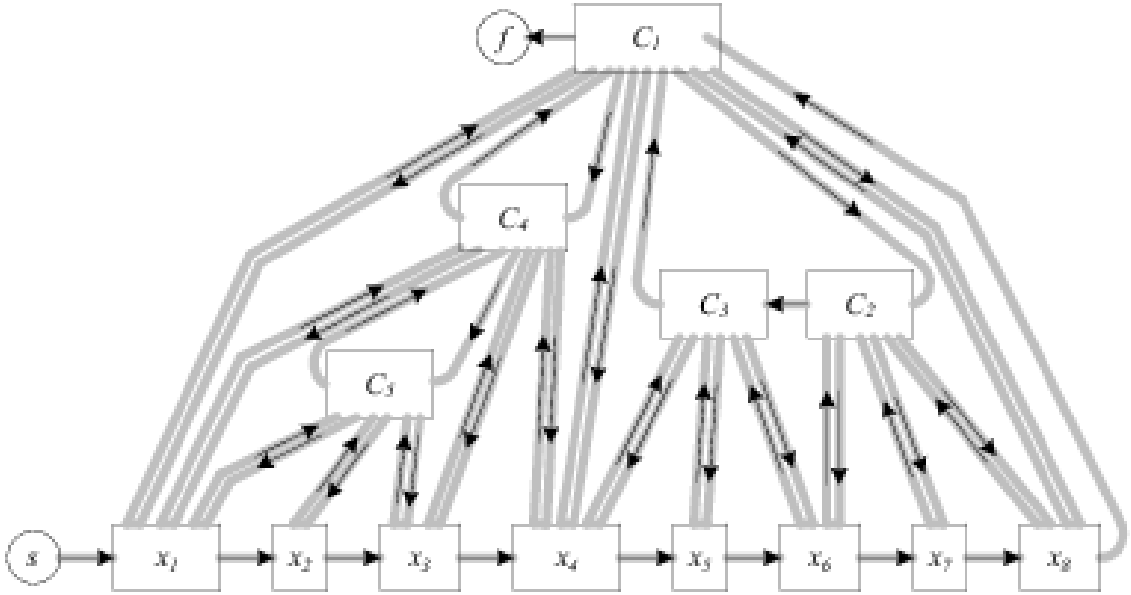


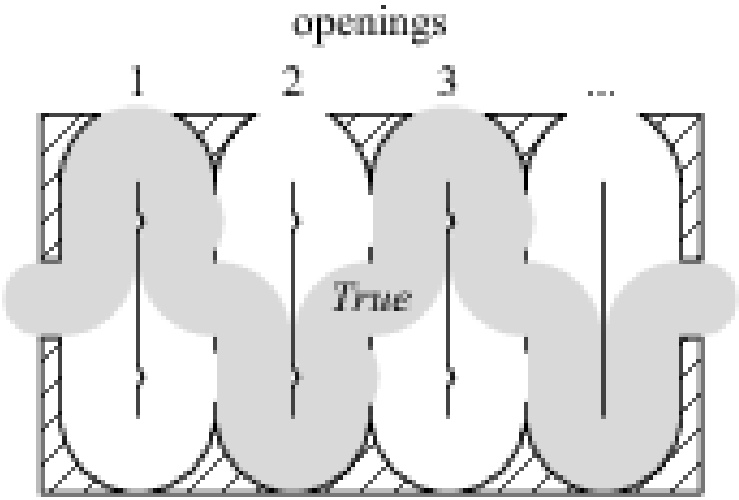


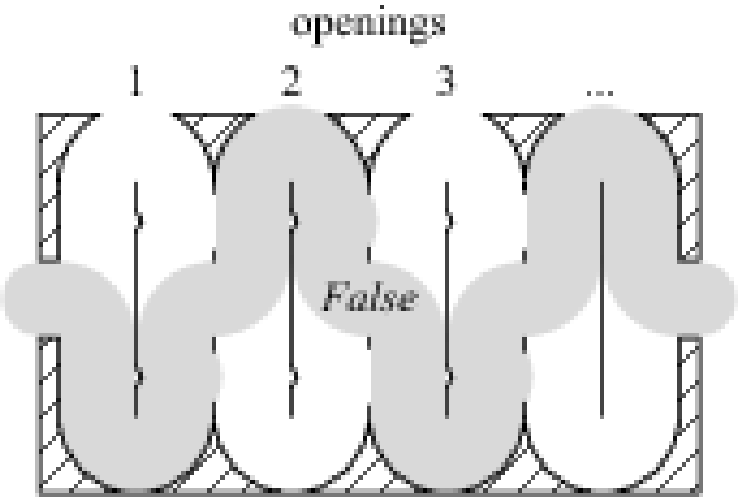
DFS-type path



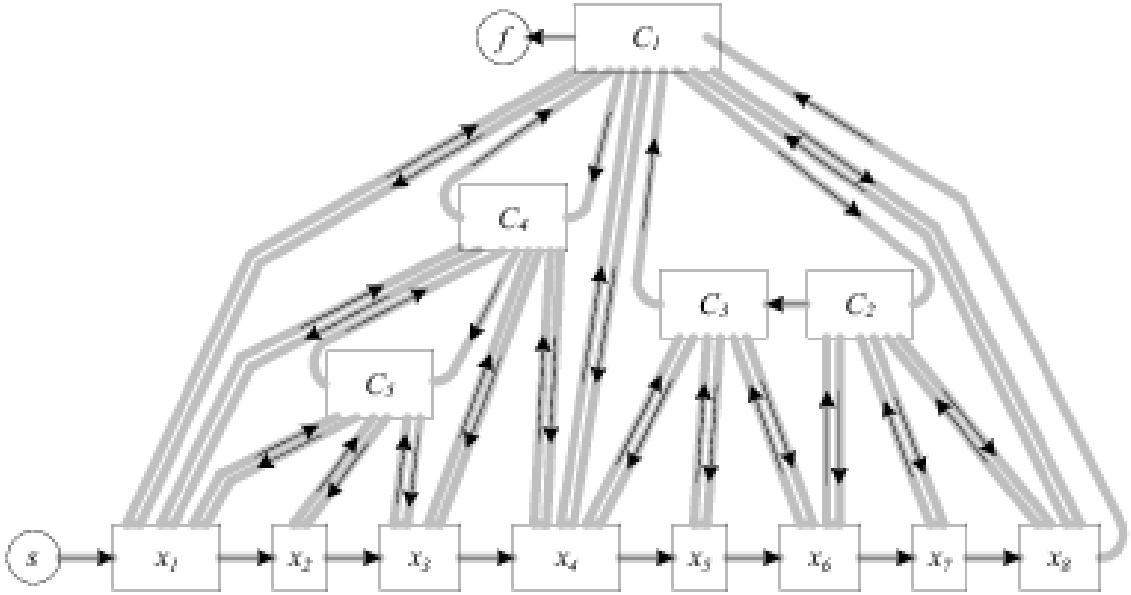
Variable gadget

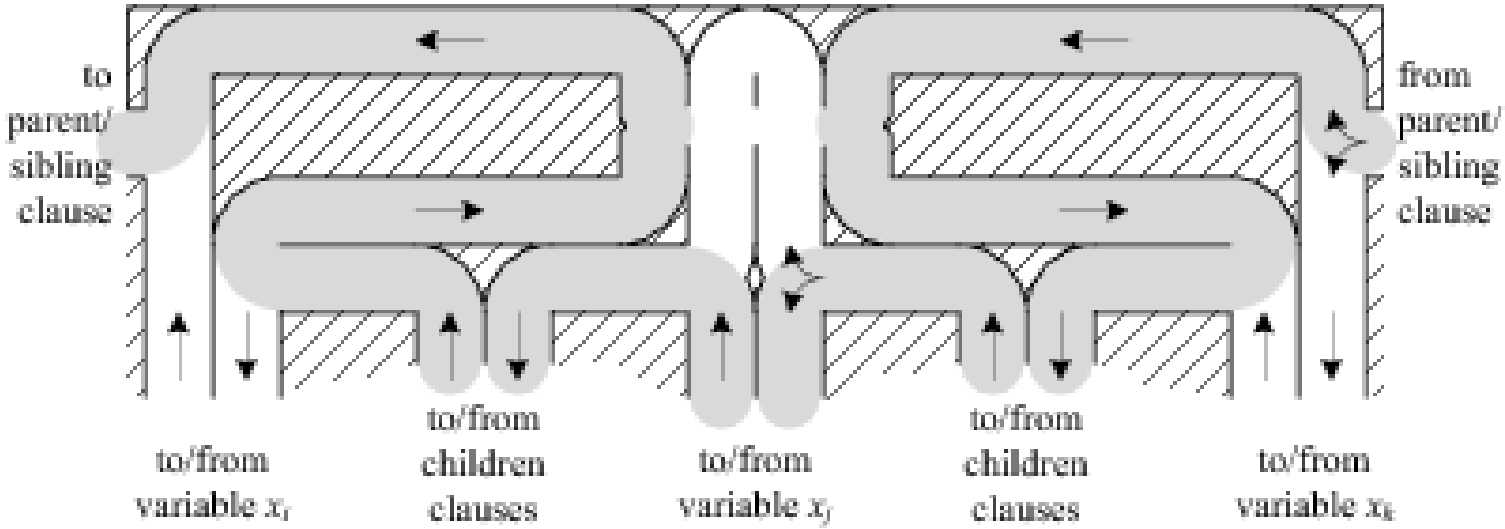




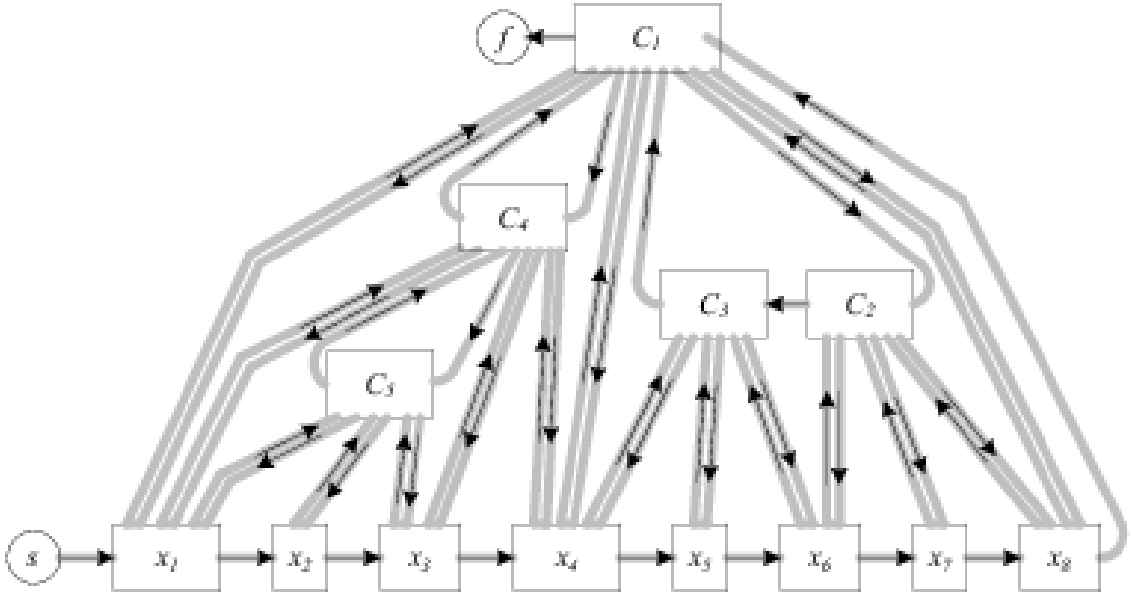


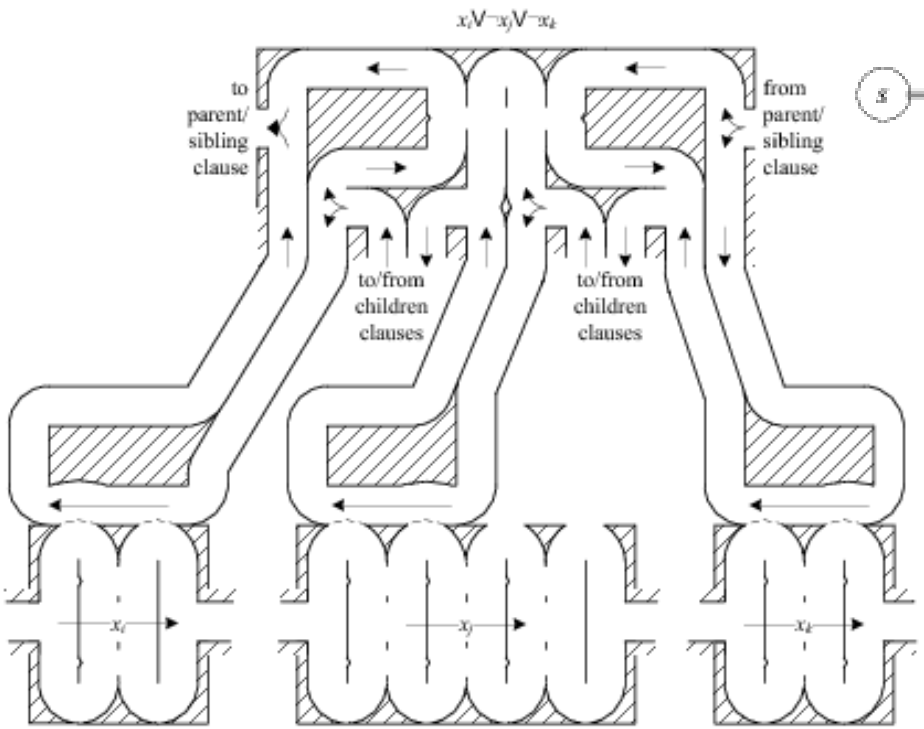
Clause gadget



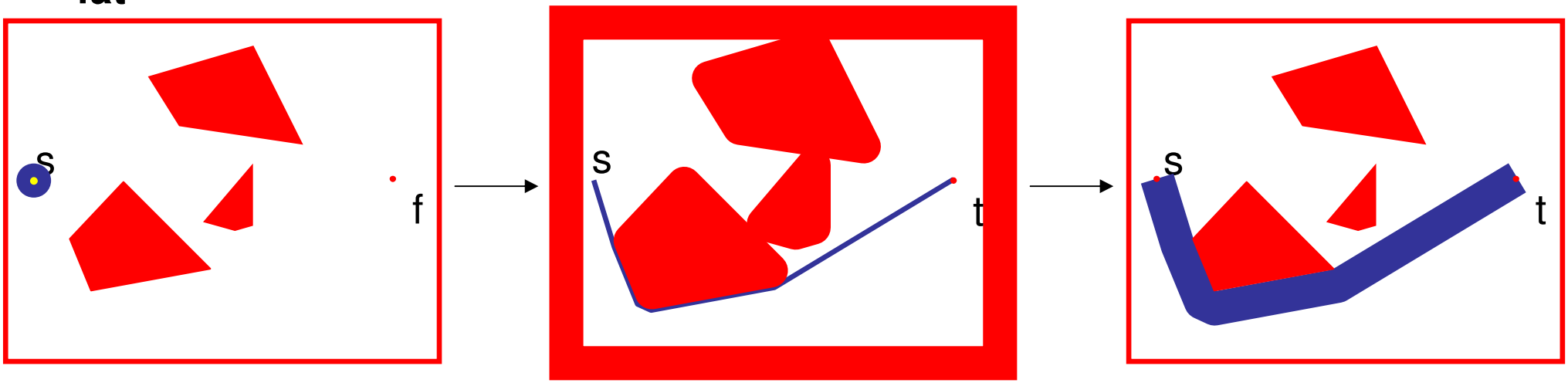


Edges – channels



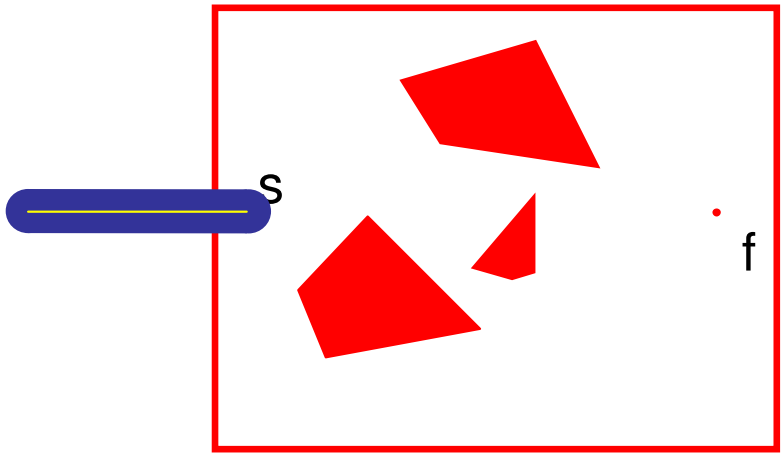


Poly-time:
∞-fat

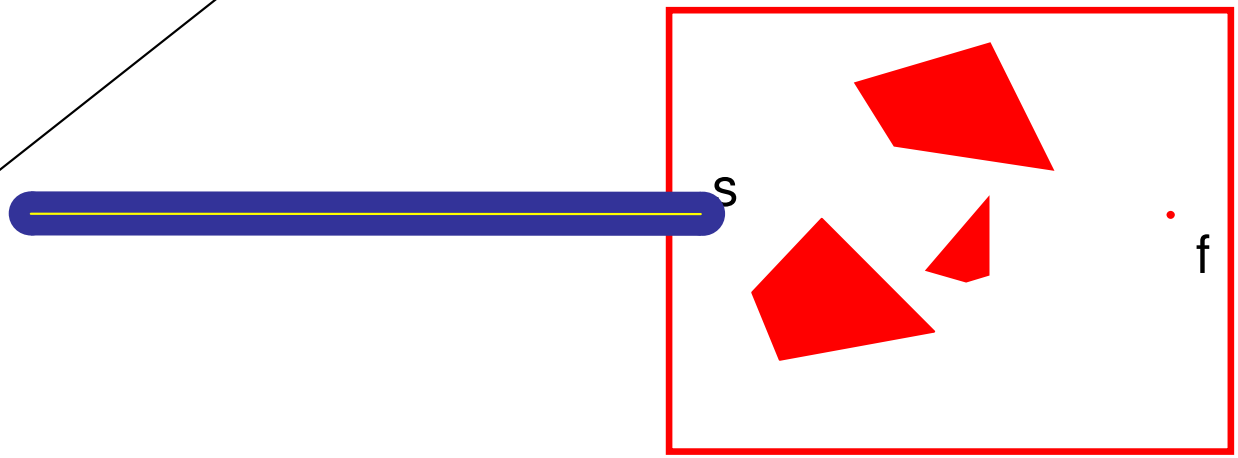


***F*(L)*poly time**

length-tractable
poly-time if L=O(1)

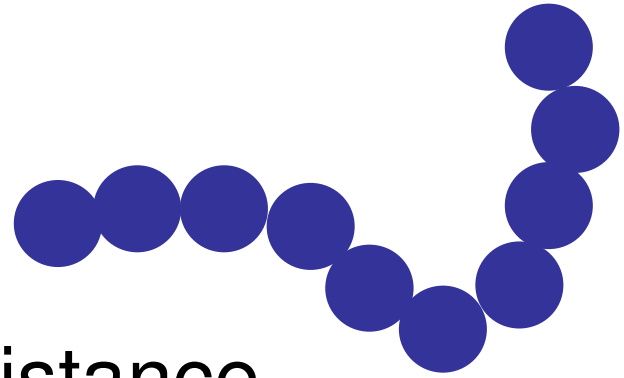


NP-hard:
no upper bound on length



Physical analogies

- Rope being pulled by endpoint
  - high friction with the surface
- Chain of robots
  - leader move
  - the others: follow keeping the distance
- Locomotion dance



Non-selfoverlaping
Pulled taut

Movie time!

http://www.youtube.com/watch?v=hrCBRVE_ctQ

- Images found via Google Images™ search at

  http://www.semissourian.com/gallery/3823/

  http://www.electrical-res.com/printed-circuit-manufacturing/

  http://www.allproducts.com/ee/pcb-togo/pcb_prototype_print.html \

  http://amarkonmywall.files.wordpress.com/2008/03/irobot-roomba-560.jpg

- Movie excerpt from

  Den vyborov (The Elections Day), 2007

  http://www.unescofound.com/denviborov/ http://www.imdb.com/title/tt1198196/