Exploring and Triangulating a Region by a Swarm of Robots

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Abstract. We consider online and offline problems related to exploring and surveying a region by a swarm of robots with limited communication range. The minimum relay triangulation problem (MRTP) asks for placing a minimum number of robots, such that their communication graph is a triangulated cover of the region. The maximum area triangulation problem (MATP) aims at finding a placement of n robots such that their communication graph contains a root and forms a triangulated cover of a maximum possible amount of area. Both problems are geometric versions of natural graph optimization problems.

The offline version of both problems share a decision problem, which we prove to be NP-hard. For the online version of the MRTP, we give a lower bound of 6/5 for the competitive ratio, and a strategy that achieves a ratio of 3; for different offline versions, we describe polynomial-time approximation schemes. For the MATP we show that no competitive ratio exists for the online problem, and give polynomial-time approximation schemes for offline versions.

1 Introduction

Exploration and Guarding. Many geometric problems of searching, exploring or guarding are motivated by questions from robot navigation. What strategies should be used for an autonomous robot when dealing with known or unknown environments?

A typical scenario considers an unknown polygonal region P that needs to be fully inspected by one or several robots; in a guarding problem, a (typically known) region needs to be fully covered from a set of guarding positions.

Triangulation is another canonical geometric problem that plays an important role in many contexts. It is an underlying task for many computer graphics approaches and the basis for a huge variety of problems in polygons, e.g., the

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computation of shortest watchman routes in simple polygons. Other contexts include mesh generation, divide-and-conquer algorithms, and even guarding problems: as Fisk [12] showed, a simple argument based on triangulations can be used to show that $\lfloor \frac{n}{3} \rfloor$ guards always suffice to guard any simple polygon with n vertices. Hence, triangulation has been intensively studied. Moreover, classical surveying relies on triangulation, which makes it possible to compute geographic coordinates with high precision by simple trigonometry.

In this paper, we present a number of geometric problems and solutions motivated by exploration and guarding of a region by a large swarm of robots. We consider a static sensor network that needs to react to different scenarios by adding further mobile sensors, e.g. sensor nodes attached to mobile robots. We have experimented with actual robot swarms, consisting of Roombas controlled by iSense sensor nodes, which in turn communicate using IEEE 802.15.4 radios. Localization based on signal strength turned out to be completely infeasible indoors, due to ranging errors well exceeding a factor of 10. However, we are convinced that we can steer a robot through a triangle, making it leave through a designated side. This is done by a simple strategy that tries to increase the signal strength to exactly two vertices. If we have a triangulated environment, we can use the connectivity information as a rough localization map, and navigate robots by switching from triangle to triangle.

We consider online problems (in which the region is unknown) and offline problems (in which the region is known). Another distinction arises from minimizing the number of relays necessary to completely cover and triangulate a region (Minimum Relay Triangulation Problem (MRTP)), or by maximizing the covered subregion for a given number of robots (Maximum Area Triangulation Problem (MATP)). We use the terms *robots* and *relays* synonymously; see Section 2 for more precise details.

For the MRTP we ask for complete coverage of the polygon. Hence, relays must be located at all vertices and the polygon has to be fully triangulated. The knowledge of necessary positions makes an actual placement easier. On the other hand, in combination with the edge lengths restriction, it complicates an NP-hardness proof.

Related Work. Hoffmann et al. [13] presented a 26.5-competitive strategy for the online exploration of simple polygons with unlimited vision. Icking et al. [15] and Fekete et al. [11] considered exploration with limited and time-discrete vision, respectively. Exploration with both limited and time-discrete vision is presented by Fekete et al. [10]. Placing stationary guards was first considered by Chvátal [6], see also O'Rourke [19].

Classical triangulation problems (see, e.g., [19]) ask for a triangulation of all vertices of a polygon, but allow arbitrary length of the edges in the triangulation. This differs from our problem, in which a edge lengths are bounded by communication length. Triangulations with shape constraints for the triangles and the use of Steiner points are considered in mesh generation, see for example the survey by Bern and Eppstein [3].

The problem of placing a minimum number of relays with limited communication range in order to achieve a connected network (a generalization of the classical Steiner tree problem) has been considered by Efrat et al. [8], who gave a number of approximation results for the offline problem (a 3.11-approximation for the one-tier version and a PTAS for the two-tier version, that does allow for the use of relays only, of this problem); see the survey [7] for related problems.

For robot swarms, Hsiang et al. [14] consider the problem of dispersing a swarm of simple robots in a cellular environment, minimizing the time until every cell is occupied by a robot. For the case of a single door, Hsiang et al. present algorithms with optimal makespan. For k doors a $\Theta(\log(k+1))$-competitive algorithm is given. A similar problem from a more practical view was solved by McLurkin and Smith [17].

Instead of considering simple robots for certain tasks, another approach is to consider the minimal necessary capabilities that allow for a certain task, Suri et al. [20] and Brunner et al. [5] classified different robot models along these lines.

Some work has been done on *budget problems*, optimization problems with a hard limit on the total cost. For example, Blum et al. [4] presented the problem of finding a path in a graph with edge costs and vertex rewards, that maximizes the collected reward while keeping the cost below a fixed limit and give a constant factor approximation. See also Averbuch et al. [2].

Our Results are as follows.

- We show that the offline versions of MRTP and MATP are NP-hard.
- For the online MRTP, we give a lower bound of 6/5 for the competitive ratio.
- We give an online strategy for the MRTP with a competitive ratio of 3.
- For an offline version of the MRTP, we give a polynomial-time approximation scheme (PTAS).
- For the online MATP, we show that no strategy can achieve a constant competitive ratio.
- For an offline version of the MATP, we give a polynomial-time approximation scheme (PTAS).

It should be noted that the results for the offline versions provide approximation schemes for *vertex-based* and *area-based* cost functions, whereas classical approximation schemes for geometric optimization problems (such as the ones developed in [1,18]) focus on *length-based* cost functions.

The rest of this paper is organized as follows. Section 2 presents basic definitions and preliminaries. Section 3 sketches the hardness proof for the offline problems. Section 4 considers the online MRTP, while Section 5 gives a polynomial time approximation scheme (PTAS) for the offline version. Section 6 shows that no constant competitive ratio for the online MATP exists. We conclude in Section 7. For lack of space a description of the PTAS for the OMATP will be given in the full version of this paper.

2 Notation and Preliminaries

We are given a polygon (an n-gon) P. Every robot in the swarm has a (circular) communication range r. Within this range, perception of and communication with other robots is possible. For the ease of description we assume that r is equal to 1 (and scale the polygon accordingly).

In the offline *Minimum Relay Triangulation Problem* (MRTP), we are given the n-gon P and a point $z \in P$, and the goal is to compute a set, R (with $z \in R$), of relays within P such that there exists a (Steiner) triangulation of P whose vertex set is exactly the set R and whose edges are each of length at most 1. Necessarily, R contains the set V of n vertices of P. The objective is to minimize the number of relays. We let R_{OPT} denote an optimal (minimum-cardinality) set of relays and let T_{OPT} denote a corresponding optimal triangulation of P using an optimal set of relays; with slight abuse of notation, we also use R_{OPT} to denote the cardinality of the set. For convenience, we refer to a triangulation whose edges are each of length at most 1 as a *unit-triangulation*. The triangulation must not contain edges crossing the boundary of P, reflecting the impossibility of communicating through walls. Thus, the triangulation contains all vertices of P, plus intermediate points. The latter are needed as edges in the triangulation must not have a length exceeding 1.

In the offline *Maximum Area Triangulation Problem* (MATP), we are given the n-gon P and a point $z \in P$, and a budget, k, of relays. The goal is to compute a set, R (with $z \in R$), of $k = |R|$ relays within P such that there exists a connected (Steiner) unit-triangulation within P covering the maximum possible area. Let R_{OPT} denote an optimal set of relays, T_{OPT} the optimal triangulation, and A_{OPT} the total area of T_{OPT}. In some situations, two natural assumptions, rooted in the robots finite size, come into play: the region may be assumed to be free of bottlenecks that are too narrow for robots, and we may already have a discrete set of candidate relay locations.

For the online versions (OMRTP and OMATP), the polygon P is unknown. Each relay may move through the area, and has to decide on a new location for a vertex of the triangulation while still within reach of other relays. Once it has stopped, it becomes part of the static triangulation, allowing other relays to extend the exploration and the triangulation. This is motivated by our application, where it is desirable to partially fix the triangulation as it is constructed, to begin location services in this area even if the polygon is not fully explored yet. This is a crucial property if we assume a huge area that is explored over long times. More precisely, also for the OMRTP we are given the n-gon P and a point $z \in P$, and the goal is to compute a set, R, of relays within P such that there exists a (Steiner) triangulation of P whose vertex set is exactly the set R and whose edges are each of length at most 1. The relays move into the polygon, starting from z. A relay extending the yet established subset $R' \subset R$ must stay within a distance of 1 of at leas one relay $p \in R$. Once it fixed its position it will not move again. No non-triangle edges are allowed in the final construction. For the OMRTP we let R_{OPT} denote the number of relays used by the optimum, for the OMATP A_{OPT} denotes the area covered by the optimum.

3 NP-Hardness

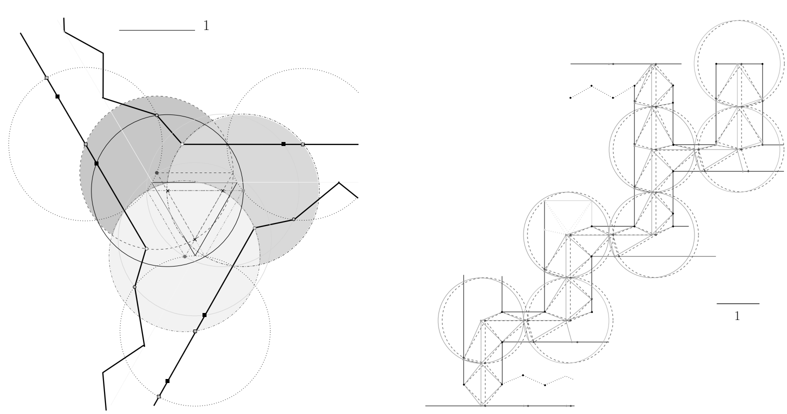
Theorem 1. *The Minimum Relay Triangulation Problem (MRTP) is NP-hard, even without a discrete set of candidate locations.*

Proof Sketch. A complete proof is omitted for lack of space. The proof is based on a reduction of the NP-hard problem `Planar 3SAT`, a special case of 3SAT in which the variable-clause incidence graph H is planar. In particular, there is a rectilinear configuration of the set of variables in a straight line, with the clauses above and below them; see [16]. This layout is represented by a polygon. The different components (for the clauses, the edges and the variables) can be triangulated using two different configurations, corresponding to truth settings for the variables that may or may not satisfy the respective clause. See Figure 1(a) for an example of a clause gadget, and (b) for a variable gadget: the edge corridors of three variables end in the triangular gadget. The boundary is shown in bold black, all other lines are used only to highlight certain distances. A truth setting satisfying the clause is indicated by black squares, a setting not satisfying the clause as black framed gray squares. In case at least one variable satisfies the clause, 3 interior relays are sufficient for the clause component. If all variables do not satisfy the clause, 4 interior relays are required for the clause component.

A considerable number of further technical issues need to be resolved to complete the overall proof. Among them are detailed constructions for corridors connecting the gadgets, which implement the logical structure of the gadgets, while still allowing careful book-keeping for all the involved angles and distances. A full account of these details is given in the full paper.

Using the same construction as in Theorem 1, we can further conclude:

Theorem 2. *The Maximum Area Triangulation Problem (MATP) is NP-hard, even without a discrete set of candidate locations.*



(a) (b)

Fig. 1. Polygonal gadgets for a clause (a) and a variable (b). Circles have radius 1

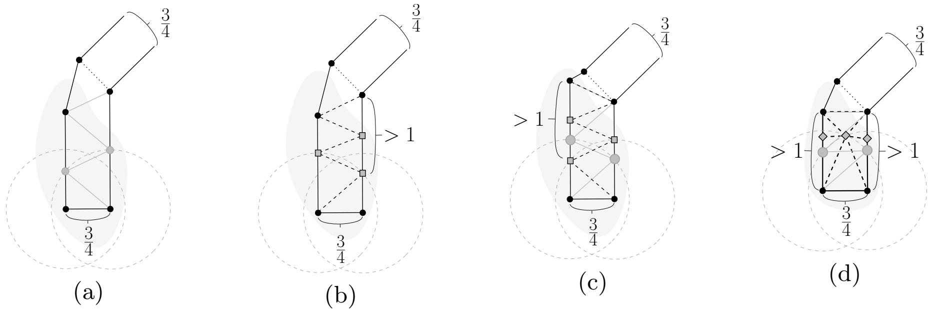


Fig. 2. A lower bound for the OMRTP. Black dots indicate polygon vertices, i.e., mandatory relays; grey disks indicate an optimal solution, while grey squares indicate relays placed by an online strategy.

4 Online Minimum Relay Triangulation

We give a lower bound of 6/5 and present a 3-competitive online strategy for the OMRTP, improving both values we gave in the informal workshop paper [9].

Lower Bound. For the lower bound we use a polygonal corridor of width 3/4. For a complete triangulation, relays must be placed at the vertices, i.e., the position of the first two relays is fixed.

In case the algorithm places the next relay on the right boundary, the polygonal corridor will turn out to look like in Figure 2(a). We need to determine the number of relays up to the two relays connected by the dotted edge (in the area indicated by the light gray shape in Figure 2), those build the two fixed relays of the next polygonal pieces. The optimum needs 5 relays. The distance of the relay placed by the algorithm on the right boundary to the next vertex is larger than 1, thus, the algorithm uses 6 relays; see Figure 2(b). In case the algorithm locates the next relay on the left boundary, the polygonal corridor turns out to look like in Figure 2(c). If, on the other hand, the algorithm places the next relay in the center, the polygonal corridor turns out to look like in Figure 2(d). In both cases with an optimum of 5, and an online solution of 6.

The construction we presented results in the next component connected in 45° to the right. Constructions for a connection within a 45° angle to the left are done analogously (mirrored constructions)—resulting in reflected components. The additional relays ensure that a final corridor of 3/4 is achieved again. Thus, we can iterate the construction. We alternate between the components shown in Figure 2 and the reflected components to avoid a self-overlapping polygon. We conclude

Theorem 3. *No deterministic algorithm for the online minimum relay triangulation problem can be better than* $\frac{6}{5}$*-competitive.*

Online Triangulation. In the following, we describe our algorithm for the online minimum relay triangulation problem. Our construction is based on two components that are glued together into one triangulation: (i) following the boundary of P and (ii) triangulating the interior.

For (i) we place relays within distance 1 along the boundary and on vertices; interior boundaries are dealt with in a similar manner once they are encountered. Let b_{ALG} be the number of relays used in this step, and b_{OPT} the number of relays placed on the boundary by an optimal solution. As any triangulation needs to establish edges along all edges of the polygon P, and the maximum distance of relays is $r = 1$, we conclude:

Lemma 1. $b_{\mathrm{ALG}} \leq b_{\mathrm{OPT}}$.

For (ii), triangulating the interior of P, we build an overlay with an arbitrarily oriented triangular unit grid from our starting point. Whenever we cannot continue this grid but are able to place a relay with distance 1 to all existing interior relays, we do so (and resume the grid construction when possible). Let i_{ALG} be the number of relays used in this step.

Lemma 2. *For an optimal solution for the MRTP with b_{OPT} relays located on the boundary and i_{OPT} located in the interior, the number $\triangle_{\mathrm{OPT}}$ of triangles in an optimal triangulation satisfies $\triangle_{\mathrm{OPT}} = 2 \cdot i_{\mathrm{OPT}} + b_{\mathrm{OPT}} - 2$.*

The proof relies on accounting for interior angles. Comparing grid relays and optimal triangles, we conclude (see proof in the full version of this paper)

Lemma 3. $i_{\mathrm{ALG}} \leq \triangle_{\mathrm{OPT}}$.

Having positioned $b_{\mathrm{ALG}} + i_{\mathrm{ALG}}$ relays, we are left with the task of keeping the explored region triangulated. Whenever we encounter an untriangulated cell bounded by a number of connections between positioned relays, we use additional relays; let their total number be c_{ALG}. We claim that:

Lemma 4. *In total, $c_{\mathrm{ALG}} \leq b_{\mathrm{OPT}}$ additional relays suffice to ensure an overall triangulation.*

Proof. A full proof is omitted for lack of space. As interior relays of degree 0 and 1 can be triangulated without causing further cost, we consider an edge between two relays ($\{r_1, r_2\}$); see Figure 3. We then give a case distinction of possible relay locations: we distinguish several placements of relays, depending on the location of edges and on p_2 and p_3 being included in the triangular gird. Altogether, every relay on the boundary gets charged at most once, concluding the proof.

$\square$

This implies the following theorem.

Theorem 4. *There is a 3-competitive strategy for the online minimum relay triangulation problem in polygons (even with holes).*

The proof is based on the previous lemmas; details are contained in the full version of the paper.

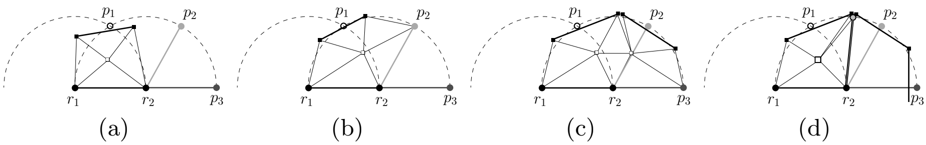


Fig. 3. We consider the black edge $\{r_1, r_2\}$ of a triangulation. The circle position p_1 of the grid is not included. Boundary relays are denoted by black squares, relays placed in phase (ii) as black circles and relays placed to glue the triangulations by black-and-white squares. The resulting triangulations are indicated by fine black lines.

5 Offline Minimum Relay Triangulation

We assume that the relays are restricted to a discrete set, $\mathcal{C}$, of candidate locations. In particular, we assume that $V \subset \mathcal{C}$ and that, for a fixed (small) parameter $\beta > 0$, $\mathcal{C}$ includes the set of grid points within P, with spacing β, whose points lie at positions $(i\beta, j\beta)$ in the plane, and that $\mathcal{C}$ includes the points where lines of the form $x = i\beta$ and $y = j\beta$ intersect edges of P.

Further, in order to address the most realistic form of the problem, we assume that P is *δ-accessible*, for some fixed $0 < \delta < 1$: The δ-medial axis is topologically equivalent to the medial access, and each point of P is within geodesic distance $O(\delta)$ of some disk of radius δ within P. Here, the *δ-medial axis* is the locus of all centers of medial disks (i.e., disks within P that are in contact with the boundary of P at two or more points) that are of radius δ or greater. Domains P that are δ-accessible can be fully accessed by robots of some fixed size δ: any path within P for a point has a homotopically equivalent path for a disk of radius δ, for all of its length except possible length $O(\delta)$ at each of its ends.

Let $BB(P)$ denote the axis-aligned bounding box of P. Without loss of generality, we can assume that $BB(P)$ has bottom left corner at (0,0); let (x_{max}, y_{max}) denote the upper right corner of $BB(P)$. We let X denote the set of x-coordinates of V, together with the multiples of β, $i\beta$, for $i = 1, 2, \ldots, \lfloor x_{max}/\beta \rfloor$; we define the set Y of y-coordinates similarly. An axis-parallel line ℓ is a *cut* if it is defined by the candidate coordinates X (for vertical lines) or Y (for horizontal lines). An axis-aligned rectangle, $\rho \subseteq BB(P)$, is *(X, Y)-respecting* if its left/right sides are defined by x-coordinates of X and its top/bottom sides are defined by y-coordinates of Y.

Let T denote an arbitrary triangulation of P. The *m-span, $\sigma_m(\ell, \rho, T)$, of ℓ with respect to rectangle ρ and triangulation T* is defined as follows. Assume that ℓ is vertical; the definition for horizontal cuts is similar. If $\ell \cap \rho$ intersects at most $2m$ edges of T, then the m-span is empty ($\sigma_m(\ell, \rho, T) = \emptyset$). Otherwise, let a be the topmost point in the mth intersection between ℓ and edges of T, from the top of ρ, along $\ell \cap \rho$. (Each intersection between ℓ and edges of T is either a single point, where ℓ crosses an edge, or is an edge e of T, in the case that ℓ contains the (vertical) edge e of T.) Similarly, let b be the bottommost point in the mth intersection between ℓ and edges of T, from the bottom of ρ, along $\ell \cap \rho$. Then, the m-span is defined to be the segment ab: $\sigma_m(\ell, \rho, T) = ab$;

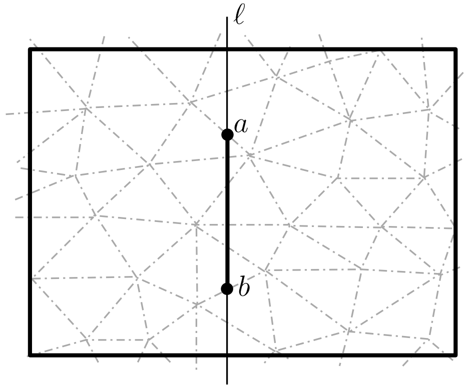


Fig. 4. The m-span (in bold), $\sigma_m(\ell, \rho, T)$, $(m = 2)$ of ℓ with respect to rectangle ρ and triangulation T

see Figure 4. Note that $\sigma_m(\ell, \rho, T) \cap P$ is a union of potentially many $(O(n))$ vertical segments along ℓ, all but two of which are vertical chords of P.

We describe a PTAS for the MRTP problem by developing the m-guillotine method [18], with several key new ideas needed to address the triangulation problem. We fix an $\epsilon > 0$ and let $m = \lceil 1/\epsilon \rceil$. Our method employs a structure theorem, which shows that we can transform an arbitrary (Steiner) unit-triangulation T of P into a (Steiner) unit-triangulation T_G that is "m-guillotine", with the number of vertices of T_G at most $(1 + \epsilon)$ times the number of vertices of T. The m-guillotine structure is a special recursive structure that allows a dynamic programming algorithm to optimize over all m-guillotine unit-triangulations with vertices in $\mathcal{C}$. Since our algorithm will find an m-guillotine unit-triangulation of P having a minimum number of vertices, and our structure theorem shows that any unit-triangulation (in particular, an optimal unit-triangulation) can be transformed into an m-guillotine unit-triangulation having approximately the same number of vertices (within factor $(1 + \epsilon)$), it follows that the unit-triangulation found by our algorithm yields a PTAS for determining R_{OPT}. We say that a (Steiner) triangulation T of P is *m-guillotine* if the bounding box $BB(P)$ can be recursively partitioned into (X, Y)-respecting rectangles by "m-perfect cuts". (At the base of the recursion are rectangles of dimensions $O(\delta)$, for which a brute-force enumeration of a constant number of cases in the dynamic programming algorithm will suffice.) A cut ℓ is *m-perfect with respect to a rectangle ρ* if its intersection with the triangulation has the following special structure: (i) ℓ intersects ρ, and (ii) the m-span of ℓ with respect to ρ is either empty or, if nonempty, is *canonically partitioned by the triangulation T*, in the following sense. Assume that the cut ℓ is vertical; the horizontal case is handled similarly. Let pq be one segment of $\sigma_m(\ell, \rho, T) \cap P$, with p (on an edge of T) vertically above q (also on an edge of T). Then, we say that the m-span is *canonically partitioned* by T if each component segment pq of the set $\sigma_m(\ell, \rho, T) \cap P$ that has length $|pq| \geq 2\delta$ is a union of $k = \lceil |pq|/\delta \rceil$ vertical edges of T, each of length exactly $|pq|/k$ (which is at most δ). We refer to the sequence of edges along the

m-span (each component pq) as a *Steiner bridge.* Those segments pq of length $|pq| < 2\delta$ define *small pockets* of P – such a segment bounds a simple polygon (since p and q must be on the same connected component of the boundary ∂P, and there can be no hole in the pocket, in order that the δ-medial axis have the same topology as the medial axis, as we assume in δ-accessibility), and this simple polygon has geodesic diameter $O(\delta)$ (again by δ-accessibility, since each point of P is within distance $O(\delta)$ of a disk of radius δ within P). Our main structure theorem for the PTAS is:

Theorem 5. *Let P be a δ-accessible polygonal domain with n vertices. For any fixed $\epsilon > 0$, let $m = \lceil 1/\epsilon \rceil$. Let T be a Steiner unit-triangulation of P whose $t = |T|$ vertices lie in the set $\mathcal{C}$ of candidates. Then, there exists an m-guillotine (Steiner) unit-triangulation, T_G, of P with $t_G \leq (1+\epsilon)t$ vertices in the set $\mathcal{C}$.*

The following lemma (whose proof is in the full paper) is utilized in the proof of the structure theorem.

Lemma 5. *Let ab be the m-span, $\sigma_m(\ell, \rho, T)$, of a cut ℓ through rectangle ρ and unit-triangulation T of P. Then, we can transform T to a new unit-triangulation T' that is canonically partitioned along ab by adding $O(|ab|/\delta)$ new Steiner points at candidate locations in $\mathcal{C}$.*

The Algorithm. The main algorithm is based on dynamic programming to compute a minimum-vertex m-guillotine (Steiner) unit-triangulation. A subproblem is specified by an (X, Y)-respecting rectangle ρ, and various boundary information specifying how the unit-triangulation of P within ρ must interface with the unit-triangulation outside of ρ. This boundary information includes up to $2m$ edges (each of length at most 1) per side of ρ; since these edges have endpoints that lie on the grid of candidate Steiner points, $\mathcal{C}$, we know that there are only a polynomial number of possibilities for these edges. Importantly, the m-span on each side of ρ is partitioned into a *canonical* set of edges, which is determined solely by the location of the cuts bounding ρ, and their interactions with the (fixed) geometry of P. This means that the interface specification, between subproblems, is *succinct* (specifiable with a constant, $O(m)$, of data), as it must be for a polynomial-time dynamic program.

Theorem 6. *Let P be a multiply connected polygonal domain with n vertices. Assume that P is δ-accessible, for some fixed $0 < \delta < 1$. Then, for any fixed $\epsilon > 0$, there is an algorithm, with running time polynomial in n, that computes a unit-triangulation, T_G, of P having at most $(1+\epsilon)R_{\mathrm{OPT}}$ vertices.*

6 Online Maximum Area Triangulation

Theorem 7. *There is no competitive algorithm for the Online Maximum Area Triangulation Problem.*

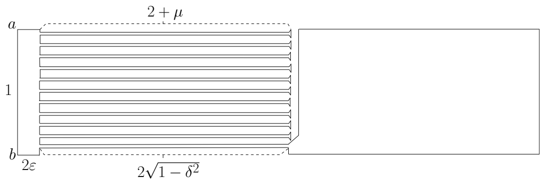


Fig. 5. An example for the polygon construction

Proof Sketch. A full proof is omitted for lack of space. For ℓ given relays we construct a polygon with $x = \lceil \frac{\ell-2}{4} \rceil$ narrow corridors, $x-1$ of which end in a small structure and one in a large polygonal piece that allows for the placement for ℓ unit triangles. Every online algorithm for the OMATP will use all relays in the corridors, while the offline optimum needs a few relays on the way to the large polygonal piece and then places unit triangles only. Hence, every online algorithm for the OMATP covers less than $\frac{8}{\sqrt{3}}\varepsilon$ of the area that the optimal offline algorithm OPT covers for any given ε (using k relays); see Figure 5 for the general idea of the construction.

7 Conclusions

In this paper we have presented a number of online and offline results for natural problems motivated by exploration and triangulation of a region by a swarm of robots. A variety of open problems and issues remain.

Can we further improve the upper and lower bounds on the competitive factor? We believe that the final answer should be a factor of 2. On the other hand, the lower bound of 6/5 applies to *any* algorithm; it may be possible to bring this closer to 2 by using corridor pieces of varying width. For an online strategy that separately considers boundary and interior, such as our algorithm, we believe that 2 is best possible.

As discussed above, the OMATP does not allow any strategy with a constant competitive factor, as some robots need to commit to a location before further exploration is possible. It may be interesting to consider variants in which robots may be allowed to continue exploration in a connected fashion before being required to settle down. However, this changes the basic nature of the problem, and will be treated elsewhere.

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