A MILP Model for Rescheduling Freight Trains under an Unexpected Marshalling-Yard Closure

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Abstract
This study is about rescheduling freight trains to reduce the effects of major interruptions. In this paper, we consider that the interruption is an unexpected marshalling-yard closure. We develop a macroscopic Mixed-Integer Linear Programming (MILP) model to reschedule railway timetables. One important principle is that we simultaneously reschedule several trains, instead of one-by-one. Furthermore, we consider a rescheduling strategy of letting trains wait on the way when the destination yard have a closure. The model considers stopping restrictions and the capacity of each segment and station. The order of the trains affected by the interruption is not fixed. We present experimental results of three different cases, which are all based on artificial data.

Keywords
Railway timetable rescheduling, Major interruption, Mixed-integer linear programming

1 Introduction

Over decades, the mobility of both persons and goods has increased. The demand for railway transportation has also grown. However, not all railway systems have expanded their capacity accordingly. This has lead to situations where the railway network is congested, and disturbances easily spread from train to train through the network. Thus, it becomes more and more important to have a stable railway schedule. This can be achieved by improving the robustness of timetables and the ability to recover to normal state after an interruption. Our main interests in this paper are traffic-management related, including the optimization models and their potential to handle major interruptions. In our application, we consider an interruption only affecting a subset of the trains, for example, a temporarily closure of the shunting yard.

During a closure of the marshalling yard, the majority of the trains are not able to be driven in or out. Since these closures lead to a major interruption that in turn can lead to cancellation of trains, congestion in the surrounding railway lines, and an indirect impact on other trains.The process of rescheduling the trains may be similar for other unexpected interruptions, which are often unpredictable and wide-ranging, such as natural disasters and other incidents. Examples of such incidents previously mentioned in the literature include flooding (Liu et al., 2018; Hamarat et al., 2021; Menan Hasnayn et al., 2017) and derailment (Huang et al., 2021; Nelldal, 2014). In this paper, we consider the unexpected interruptions in marshalling yards that lead to closures. Louwerse and Huisman (2014) mentioned that interruptions lasting longer than one hour happen three times a day on average in the Netherlands. Nelldal (2014) indicates that major interruptions lasting up to several days occurred once a year on average in Sweden from 2000 to 2013, and some of them happened within marshalling yards. Marshalling yards consist of tracks and switches where different wagons are sorted and combined with new trains driving to different destinations.

Our scenario is that a severe interruption occurs in a marshalling yard so that it has to be closed and there is many freight trains along the rail line that need to be rescheduled. We consider that there are a limited number of alternative formation marshalling yards available in the surrounding area and that the nearest one is several hours away. Due to the interruption, the capacity for the incoming trains in the interrupted yard is limited.

The goal is to reschedule the affected traffic on a railway line connected to the interrupted yard while disturbing other trains as little as possible and study the effects on the railway line that connects with the affected yard. Thus, the rescheduling strategy includes extending the stopping time in stations along the corridor for temporary parking and delaying the departure time at the departing yard.

To summarize, we propose a mathematical model for rescheduling a railway timetable when a group of freight trains must be postponed due to an unexpected marshalling-yard closure. The model does not reschedule other trains, e.g., passenger trains, that are assumed to be unaffected by the interruption. Capacity restrictions on tracks and in stations are respected, and so are technical limitations in, e.g., speed and minimum headway.

This paper is organized as follows. In Section 2, we present related research. We describe the research problem in Section 3. In Section 4, we introduce the mathematical model. We analyse the computational experiments in Section 5 and present the conclusions in Section 6.

2 Related Work

In this section, we discuss different studies. These studies either have the topic of focusing on rescheduling train schedules under major interruptions or the models that can be applied on rescheduling the timetables.

Cacchiani et al. (2010) discussed the different definitions between minor disturbances and major interruptions. Minor disturbances usually refer to a series of delays caused by a minor delay of one train, which can easily be solved by rescheduling the timetable. A major interruption refers to a series of severe delays and a large number of cancellations caused by external incidents.

To our knowledge, the research on rescheduling at major interruptions is limited. Current studies are mainly based on cyclic timetables since the research topic is mainly passenger-oriented. Louwerse and Huisman (2014) mentioned that the rescheduling approach under a major interruption normally starts with a timetable adjustment, which is followed by rolling stock rescheduling and then crew rescheduling. They also consider the schedule of rolling stock, since the focus is on rescheduling passenger trains with cyclic schedule. Delaying a passenger train may affect the subsequent schedules of this train vehicle.

Other operation strategies for rescheduling passenger trains under major interruptions include reordering trains, cancelling trains that have not departed, and shunting in stations

(Louwerse and Huisman, 2014; Zhan, 2015; Veelenturf et al., 2016; Cavone et al., 2019). Furthermore, there are two main types of blockage in these studies. The two blockage types are full blockages and partial blockages on the rail lines between two consecutive stations. We consider the railway lines between stations as segments and each two parallel rail lines as separate tracks. Veelenturf et al. (2016) mention that partial blockages refer to only some tracks of one segment being blocked and other tracks are still functional, while a full blockage is that all the tracks of one segment are blocked. Zhan (2015), Veelenturf et al. (2016) and Cavone et al. (2019) only considered full blockages on the railway lines, while Louwerse and Huisman (2014) studied both partial and full blockage scenarios.

There are three phases in the interruption management process. Phase one is transiting from the original timetable to the rescheduled one. Phase two is running the rescheduled timetable on the rail lines, and phase three is transiting from the rescheduled timetable to the original one. Louwerse and Huisman (2014) only consider phase two, which is applying the rescheduled timetable on the rail lines, while Zhan (2015), Veelenturf et al. (2016) and Cavone et al. (2019) involved all three phases of the interruption management process.

Louwerse and Huisman (2014) developed an Integer Programming (IP) model to maximize the service level for passengers by minimizing the number of cancelled trains and the delay of uncancelled trains. It includes rolling stock constraints in the model to make the result easier to modify into a feasible rolling stock schedule. The model was tested on real-world data. Veelenturf et al. (2016) extended Louwerse and Huisman's model to an Integer Linear Programming (ILP) model under different scenarios of partial or full blockages at different locations at the same time and the option for rerouting trains and balancing the timetable. The capacity of each station is considered in the model by tracking the availability of each track in each station.

Cavone et al. (2019) considered full blockage of the rail line between two marshalling yards. They introduced a MILP model and an innovative bi-level rescheduling algorithm to fill the gap between macroscopic and microscopic modelling. They also included the rolling stock in the yard through shunting constraints.

The adopted operation strategy for major interruption by Zhan (2015) is slightly different. They rescheduled the timetable only by retiming, reordering, and cancelling trains without considering the options for rerouting, short-turning or rolling stock constraints. They formulated a Mixed Integer Programming (MIP) model with the objective function of minimizing the weighted sum of the number of cancelled trains and the delay for other trains. They also tracked the stations' capacities by calculating the difference in the number of ingoing and outgoing trains to check the availability of tracks whenever there is a new train planning to drive into this station.

Some of the models primarily developed for minor disturbances are also applicable to major interruptions. For rescheduling the timetable under small disturbances, several types of models have been suggested, including both non-cyclic (Törnquist and Persson, 2007; Gestrelius et al., 2015) and cyclic models (Jiang et al., 2014; Tan et al., 2020a; Tan et al., 2020b). Törnquist and Persson (2007) introduced a MILP-based model to reschedule timetables on a multiple tracked railway network during disturbances. The start time and end time of every event are included as continuous variables, and the train order is considered as a binary variable. The purpose with the model is to reduce the delay as much as possible, thus, the two objective functions are minimizing the total delay for all trains at their destination station, and the weighted sum of both total final delays and the additional delay penalty. This model is extended by Törnquist Krasemann (2015) to include the options of passing through and stopping at the stations.

Furthermore, there are some studies about inserting additional train service into an existing railway timetable, other than rescheduling the complete railway timetable (Burdett and Kozan, 2009; Cacchiani et al., 2010; Jiang et al., 2017; Ljunggren et al., 2020). However, it is very time-consuming to inserting a large number of trains.

Although different authors have studied the topic of handling major interruptions in the railway system, to our knowledge, there is no model available that is applicable in our case. Most of the previously proposed models aim at optimising the timetable from the passenger's view. Their scenarios mainly consider partial or complete closures at rail lines between two stations. Furthermore, the schedule of passenger trains is different from the schedule of freight trains. Freight trains have longer travel times, are typically composed of waggons for several destinations, that need to be shunted or coupled/decoupled along the route. Our rescheduling strategy focuses on rescheduling and extending the duration time at stations instead of cancelling and short turning. Furthermore, the major interruptions within the marshalling yard have, to the best of our knowledge, not yet been considered.

To summarize, we face several problems when trying to apply previously presented models to our problem:

(i) The rail corridor involves both freight and passenger trains, while only some freight trains (interrupted trains) require a completely new timetable whereas we allow limited adjustments of the existing schedules of other trains.

(ii) The interrupted trains need to depart from the original yard within a certain time range and are only allowed to arrive at the destination yard after the interruption is over.

(iii) The freight trains can have longer stopping time at some stations along the path to extend their total travel time.

(iv) The computational time for the model should remain in an acceptable range.

3 Problem Description

We consider a rail corridor connecting Yard A with Yard B, There is a major interruption at Yard B with anticipated duration and all trains coming to yard B during this time are completely blocked. We allow the freight trains to stop at some selected sidetracks in each station for both delaying the arrival time at yard B of interrupted trains and increasing the rescheduling flexibility of the non-interrupted freight trains, so that we can avoid the cancellation of freight trains and congestion in yard A.

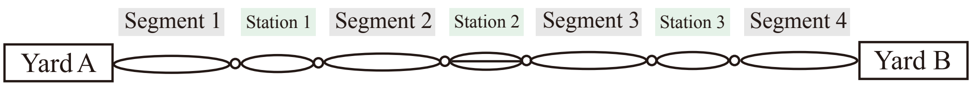

Figure 1: A rail-line example with two segments and three stations

In this paper, a segment refers to a set of tracks between two points, which can be either a station, a yard, or track segment. A rail corridor refers to the long passage between two yards. We denote the freight trains with a destination at the interrupted yard as interrupted freight trains, all other freight trains are denoted as non-interrupted freight trains and all passenger trains as non-interrupted passenger trains. We show an example of a rail line with four rail segments and three stations in Figure 1. Both segments are double-tracked. Station 1 and 3 have two tracks and Station 2 has three tracks.

To simplify the problem, we assume that:

(i) All tracks are bi-directional.

(ii) It is possible to access all outgoing tracks from any track at the end of each segment and station.

(iii) No train can be cancelled.

Figure 4 shows the capacity change for the incoming trains into Yard B over time. The curve represents the reality whereas the green blocks depict how it is modelled. We consider a maximum capacity for incoming trains into Yard B. Before the interruption, we assume the capacity for incoming trains into Yard B, which is set to zero during the interruption. After interruption occurs, the capacity immediately drops down to 0.

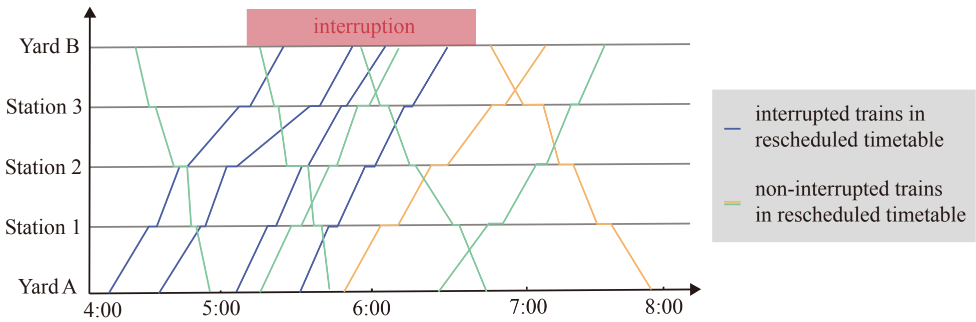

Figure 2: Original timetable

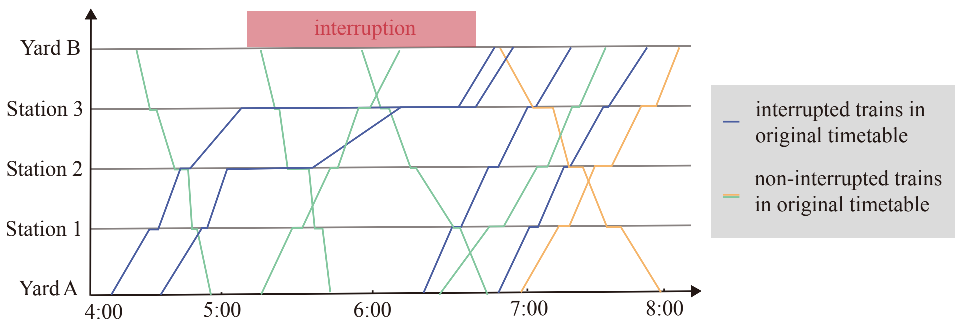

Figure 3: Rescheduled timetable

Figure 2 and Figure 3 give an example of rescheduling a timetable under a major interruption in Yard B. In Figure 2, we show the original timetable on the line between Yard A and Yard B. In Figure 3, we show the rescheduled timetable. Non-interrupted trains are shown in green and yellow, and the interrupted trains are shown in blue. Orange refers to the non-interrupted trains with different timetable after rescheduling, and green refers to the non-interrupted trains with an unchanged timetable after rescheduling. There is a major interruption in Yard B and the interrupted freight trains are either already on the way towards Yard B or remain in Yard A when this interruption occurs. Thus, these interrupted trains need to be rescheduled such that they arrive at Yard B after the interruption ends.

4 Mathematical Model

There are many optimization models for rescheduling railway timetables and the most common modelling approach is to build a MILP model. Törnquist and Persson (2007) presented a MILP model that creates timetables on a multiple-tracked railway network and Törnquist Krasemann (2015) extended it with options for passing through and stopping at stations, where the train order is discrete, and time is continuous. Törnquist and Persson's model has constraints on basic train restrictions, technical restrictions at each track, and operator preferences. We propose a mathematical model for rescheduling the railway timetable based on Törnquist and Persson's model. Their rescheduling model is an event-based MILP model for a network with multiple tracks. The events include both the departure and arrival at any station or line segment.

In our model, we make two main modifications:

(i) Stations' and segments' capacity: We track the occupation on each track in all segments and stations by introducing a new binary variable $\gamma_{i,j,k}$. Variable $\gamma_{i,j,k}$ indicates whether train i travels through track k on segment j. The conflict constraints in the model ensure that each track can only be occupied by one train at a time. These conflict constraints are only activated when two trains are assigned to the same track. By solving the conflicts on each track in each segment and station separately, we can avoid the situation that either a segment or station contains more trains than possible because of its capacity limit.

(ii) Travel duration: Freight trains may need to stop at sidetracks along the path during the interruption to extend the total travel time and postpone the arrival time at the destination yard till the interruption ends. However, there are some technical restrictions on stopping freight trains at some sidetracks, for example, the travelling direction, the train length etc. In our model, we check whether a freight train is allowed to stop at a track in a station by introducing a new binary parameter $tt_{i,j,k}$. This parameter can affect the stopping duration of each train at each station. $tt_{i,j,k}$ is set to 1 if train i may stop at track k of segment j. If a train needs to pass a station, $tt_{i,j,k}$ determines whether this train has to pass without stopping or can stop at this station.

We assume the original timetable is conflict-free. We also suppose that our model is applied after the interruption has started. Thus, our rescheduling model only takes care of the second and third phases of the interruption management process with fixed paths for all trains. In Figure 4, the second phase refers to the time duration between b^{inter} and e^{inter}, which is the interruption. The third phase refers to the time duration between e^{inter} and t^{r}, which is called recovery time.

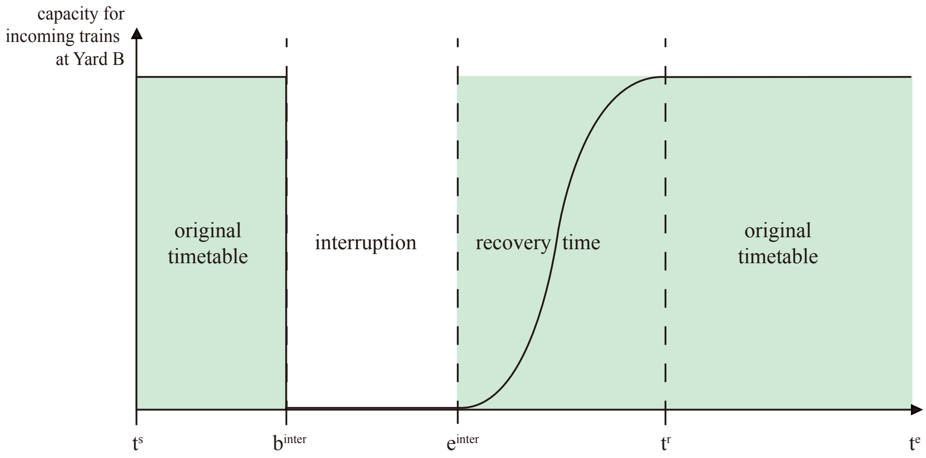

Figure 4: The capacity change over time

Table 1: Sets

Index and Set	**Description**
i	Train i
j	Segment j
k	Track k
$\ell_{i,j}$	The event number for train i at segment j
T	Set of all trains i, where $T = T^{\text{fre}} \cup T^{\text{pas}}$
T^{fre}	Set of all freight trains
T^{pas}	Set of all passenger trains and the freight trains that have departed
J	Set of railway segments and stations, where $J = J^{\text{t}} \cup J^{\text{s}}$
J^{t}	Set of railway segments
J^{s}	Set of railway stations
K_j	Set of tracks on segment j
K^{pas}	Set of track segments that passenger trains are allowed to pass at all stations
K^{fre}	Set of track segments that non-interrupted freight trains are allowed to pass at all stations
L	Set of events $\ell_{i,j}$ for all trains

Table 2: Parameters

Parameters	**Description**
nL_i	Event number for each train i
Δ_j	Smallest headway on segment j, i.e., a train i cannot enter segment j before the previous train has left and at least Δ_j time units have passed
$d_{i,j}$	Minimum duration for train i on segment or station j

$b_{i,\ell_{i,j}}$	Planned start time for train i on segment or station j
$e_{i,\ell_{i,j}}$	Planned end time for train i on segment or station j
b_i^{win}	Maximum allowed deviation from the desired departure time for train i from the departure station
$tt_{i,j,k}$	Equals 1, if train i is allowed to stop at the track k on segment j, and 0 otherwise
t^{s}	The initial time of the origin timetable without any crew and rolling stock schedule
t^{e}	The end time of the origin timetable without any crew and rolling stock schedule
b^{inter}	The time point when the major interruption starts
e^{inter}	The time point when the major interruption ends
t^{r}	The time point when the recovery process should end
M^{dur}	Big M. M^{dur} is a large constant for the duration constraints and it should be larger than the maximum allowed stopping time length at one station.
M^{con}	Big M. M^{con} is a large constant for the conflict constraints and it should be larger than the total running time for two trains on one segment.

Table 3: Variables

Variables	**Description**
$x_{i,\ell_{i,j}}^{\text{start}}$	Start time for event $\ell_{i,j}$ of train i
$x_{i,\ell_{i,j}}^{\text{end}}$	End time for event $\ell_{i,j}$ of train i
$\lambda_{i,v,j}$	Binary, event order on segment j: equals 1 if the event of train i occurs before the event of train v, equals 0 otherwise
$\gamma_{i,j,k}$	Binary, equals 1 if train i passes the track k on segment j

4.1 Constraints

Constraint (1) ensures that the events of each train are tightly connected, which means that the end time for each event of each train equals the start time for this train's next event.

$$x_{i,\ell_{i,j}+1}^{\text{start}} = x_{i,\ell_{i,j}}^{\text{end}} \qquad \forall i \in T; j \in J; \ell_{i,j} \in L \tag{1}$$

Constraints (2) and (3) fix the timetable for all passenger trains. Constraints (4) and (5) fix the schedule before the interruption happens and after the recovery time.

$$x_{i,\ell_{i,j}}^{\text{start}} = b_{i,\ell_{i,j}} \qquad \forall i \in T^{\text{pas}}; j \in J; \ell_{i,j} \in L \tag{2}$$

$$x^{\text{end}}_{i,\ell_{i,j}} = e_{i,\ell_{i,j}} \qquad \forall i \in T^{\text{pas}}; j \in J; \ell_{i,j} \in L \tag{3}$$

$$x^{\text{start}}_{i,\ell_{i,j}} = b_{i,\ell_{i,j}} \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : b_{i,\ell_{i,j}} \geq t^{\text{r}} \vee b_{i,\ell_{i,j}} \leq b^{\text{inter}} \tag{4}$$

$$x^{\text{end}}_{i,\ell_{i,j}} = e_{i,\ell_{i,j}} \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : b_{i,\ell_{i,j}} \geq t^{\text{r}} \vee b_{i,\ell_{i,j}} \leq b^{\text{inter}} \tag{5}$$

Duration Constraints

The parameter $d_{i,j}$ refers to the minimum duration for each train per track at a segment or station. Constraint (6) makes sure that if the train passes one track of a rail segment, then its duration time should be at least as long as the minimum duration on that segment. We have $x^{\text{end}}_{i,\ell_{i,j}} - x^{\text{start}}_{i,\ell_{i,j}} \geq d_{i,j}$ when $\gamma_{i,j,k} = 1$. If $\gamma_{i,j,k} = 0$, then we have $x^{\text{end}}_{i,\ell_{i,j}} - x^{\text{start}}_{i,\ell_{i,j}} \geq 0$. Constraints (7) and (8) ensure that if the train is allowed to stop on a track at a station, then it can either run through or stop at a track. We have $x^{\text{end}}_{i,\ell_{i,j}} - x^{\text{start}}_{i,\ell_{i,j}} \geq d_{i,j}$ when $\gamma_{i,j,k} = 1$ and $tt_{i,j,k} = 1$. If the train is not allowed to stop on a track of the station, then it can only run on this track. Thus, we have $x^{\text{end}}_{i,\ell_{i,j}}$-$x^{\text{start}}_{i,\ell_{i,j}} = d_{i,j}$ when $\gamma_{i,j,k} = 1$ and $tt_{i,j,k} = 0$.

$$x^{\text{end}}_{i,\ell_{i,j}} \geq x^{\text{start}}_{i,\ell_{i,j}} + \gamma_{i,j,k} d_{i,j} \qquad \forall i \in T; j \in J^{\text{t}}; k \in K_j; \ell_{i,j} \in L \tag{6}$$

$$x^{\text{end}}_{i,\ell_{i,j}} \geq x^{\text{start}}_{i,\ell_{i,j}} + \gamma_{i,j,k} d_{i,j} + M^{\text{dur}}(\gamma_{i,j,k} - 1) \qquad \forall i \in T; j \in J^{\text{s}}; k \in K_j; \ell_{i,j} \in L \tag{7}$$

$$x^{\text{end}}_{i,\ell_{i,j}} \leq x^{\text{start}}_{i,\ell_{i,j}} + \gamma_{i,j,k} d_{i,j} + M^{\text{dur}}(1 - \gamma_{i,j,k} + tt_{i,j,k}) \qquad \forall i \in T; j \in J^{\text{s}}; k \in K_j; \ell_{i,j} \in L \tag{8}$$

Conflicts and Headway Constraints

Constraints (9) and (10) ensure that each track can only be occupied by one train at a time. Constraint (11) and (12) limit the value selection of variable $\lambda_{i,v,j}$. If the start time of a train i is earlier than another train v at a segment or station j, then $\lambda_{i,v,j} = 1$, and $\lambda_{i,v,j} = 0$ if the start time of a train i is later than the start time of train v. Constraint (13) enforces that only one track can be assigned to a train in a segment or station when this train passes this segment or station. Constraint (14) and (15) limit the track number that passenger and freight trains can choose at each station. The passenger and freight trains can only choose the tracks that are included in the set K^{pas} and K^{fre} accordingly.

$$x^{\text{start}}_{v,\ell_{v,j}} - x^{\text{end}}_{i,\ell_{i,j}} \geq \Delta_j + M^{\text{con}}(\lambda_{i,v,j} + \gamma_{i,j,k}\gamma_{v,j,k} - 2) \qquad \forall i \in T; v \in T; j \in J; k \in K_j; \ell_{i,j} \in L \tag{9}$$

$$x^{\text{start}}_{i,\ell_{i,j}} - x^{\text{end}}_{v,\ell_{v,j}} \geq \Delta_j + M^{\text{con}}(\gamma_{i,j,k}\gamma_{v,j,k} - \lambda_{i,v,j} - 1) \qquad \forall i \in T; v \in T; j \in J; k \in K_j; \ell_{i,j} \in L \tag{10}$$

$$(x^{\text{start}}_{i,\ell_{i,j}} - x^{\text{start}}_{v,\ell_{v,j}})\lambda_{i,v,j} \leq 0 \qquad \forall i \in T; v \in T; j \in J; \ell_{i,j} \in L \tag{11}$$

$$(x^{\text{start}}_{i,\ell_{i,j}} - x^{\text{start}}_{v,\ell_{v,j}})(1 - \lambda_{i,v,j}) \geq 0 \qquad \forall i \in T; v \in T; j \in J; \ell_{i,j} \in L \tag{12}$$

$$\sum_{k \in K_j} \gamma_{i,j,k} = 1 \qquad \forall i \in T; j \in J; \ell_{i,j} \in L \tag{13}$$

$$\gamma_{i,j,k} = 0 \qquad \forall i \in T^{\text{pas}}; j \in J^{\text{s}}; k \in K \backslash K^{\text{pas}}; \ell_{i,j} \in L \tag{14}$$

$$\gamma_{v,j,k} = 0 \qquad \forall i \in T^{\text{fre}}; j \in J^{\text{s}}; k \in K \backslash K^{\text{fre}}; \ell_{i,j} \in L \tag{15}$$

Constraint (16) gives the range for variables $x^{\text{start}}_{i,\ell_{i,j}}$ and $x^{\text{end}}_{i,\ell_{i,j}}$. These two variables are continuous variables, which are larger or equal to t^{s}. Constraints (17) and (18) indicate $\lambda_{i,v,j}$ and $\gamma_{i,j,k}$ are binary variables.

$$x^{\text{start}}_{i,\ell_{i,j}}, x^{\text{end}}_{i,\ell_{i,j}} \geq t^{\text{s}} \qquad \forall i \in T; j \in J; \ell_{i,j} \in L \tag{16}$$

$$\lambda_{i,v,j} \in \{0, 1\} \qquad \forall i \in 1, ..., |T| - 1; v \in T; j \in J \tag{17}$$

$$\gamma_{i,j,k} \in \{0, 1\} \qquad \forall i \in 1, ..., |T| - 1; j \in J; k \in K_j \tag{18}$$

Deviation Constraints

Constraint (19) and (20) limit the end time for all interrupted trains at the destination yard within a certain range: to within the interval $[e^{\text{inter}}, e^{\text{inter}} + b^{\text{win}}]$. Constraint (21) limits the start time at the origin yard for the interrupted trains. The interrupted trains can only be delayed at the origin yard. The non-interrupted trains are only allowed to depart later than the originally scheduled time, which is enforced in Constraint (22).

$$x^{\text{end}}_{i,nL_i} \geq e^{\text{inter}} \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : b^{\text{inter}} \leq e_{i,\ell_{i,nL_i}} \leq e^{\text{inter}} \wedge \ell_{i,nL_i} = nL_i \tag{19}$$

$$x^{\text{end}}_{i,nL_i} \leq e^{\text{inter}} + b^{\text{win}} \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : b^{\text{inter}} \leq e_{i,ev(i,nL_i)} \leq e^{\text{inter}} \wedge \ell_{i,nL_i} = nL_i \tag{20}$$

$$x^{\text{start}}_{i,1} \geq b_{i,1} \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : b^{\text{inter}} \leq e_{i,\ell_{i,nL_i}} \leq e^{\text{inter}} \wedge \ell_{i,nL_i} = nL_i \tag{21}$$

$$x^{\text{start}}_{i,\ell_{i,j}} \geq b_{i,\ell_{i,j}} \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : e^{\text{inter}} \leq b_{i,\ell_{i,nL_i}} \vee b_{i,\ell_{i,nL_i}} \leq b^{\text{inter}} \vee \ell_{i,nL_i} \neq nL_i \tag{22}$$

4.2 Objectives

We consider three objective functions in this model. The first one is to minimize the deviation from the original schedule for all non-interrupted freight trains. We will use 'Deviation' to represent this objective in the remainder of this paper. The variable z_i is a continuous variable, which equals the absolute value of the difference between $x^{\text{start}}_{i,\ell_{i,j}}$ and $b_{i,\ell_{i,j}}$ for all non-interrupted trains at each event. We sum up for all the freight trains that are either originally scheduled to arrive the interrupted yard after interruption ($e^{\text{inter}} \leq b_{i,\ell_{i,nL_i}}$) or before interruption ($b_{i,\ell_{i,nL_i}} \leq b^{\text{inter}}$) or run towards the opposite direction ($\ell_{i,nL_i} \neq nL_i$).

$$f_1 = \sum_{i \in T^{\text{fre}}: e^{\text{inter}} \leq b_{i,\ell_{i,nL_i}} \vee b_{i,\ell_{i,nL_i}} \leq b^{\text{inter}} \vee \ell_{i,nL_i} \neq nL_i} z_i$$

$$x^{\text{start}}_{i,\ell_{i,j}} - b_{i,\ell_{i,j}} \leq z_i \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : e^{\text{inter}} \leq b_{i,\ell_{i,nL_i}} \vee b_{i,\ell_{i,nL_i}} \leq b^{\text{inter}} \vee \ell_{i,nL_i} \neq nL_i \tag{23}$$

$$x^{\text{start}}_{i,\ell_{i,j}} - b_{i,\ell_{i,j}} \geq -z_i \qquad \forall i \in T^{\text{fre}}; j \in J; \ell_{i,j} \in L : e^{\text{inter}} \leq b_{i,\ell_{i,nL_i}} \vee b_{i,\ell_{i,nL_i}} \leq b^{\text{inter}} \vee \ell_{i,nL_i} \neq nL_i \tag{24}$$

$$z_i \geq 0 \qquad \forall i \in T^{\text{fre}} : e^{\text{inter}} \leq b_{i,\ell_{i,nL_i}} \vee b_{i,\ell_{i,nL_i}} \leq b^{\text{inter}} \vee \ell_{i,nL_i} \neq nL_i \tag{25}$$

In the second objective function, the goal is to minimize the sum of the total running time for all interrupted trains, which we call 'Transport Time'. We sum up for all the freight trains that are both originally scheduled to arrive the interrupted yard during interruption

($b^{\text{inter}} \leq e_{i,\ell_{i,nL_i}} \leq e^{\text{inter}}$) and run in the direction towards the interrupted yard ($\ell_{i,nL_i} = nL_i$).

$$f_2 = \sum_{i \in T^{\text{fre}} : b^{\text{inter}} \leq e_{i,\ell_{i,nL_i}} \leq e^{\text{inter}} \wedge \ell_{i,nL_i} = nL_i} (x^{\text{end}}_{i,nL_i} - x^{\text{start}}_{i,1})$$

In the third objective function, the goal is to find a feasible solution. We use ‘Feasible’ to represent this objective function.

$$f_3 = 1$$

The problem formulation can be summarized as:

$$\begin{array}{ll} \min & f \\ \text{subject to:} & \text{constraint (1) - (22)} \\ & \text{constraint (23) - (25) , if } f = f_1 \\ \text{where} & f \in \{f_1, f_2, f_3\} \end{array}$$

5 Computational Experiments

In this section, we run our computational experiments based on artificial data and discuss the results with different parameter settings. This section aims to show the feasibility of our proposed model and to illustrate its performance. The data sets used in the case studies are artificial, moreover, the data set for Case Study III is of a reasonable size for a real-world application. The model was solved using AMPL and the server Gurobi 7.5.2 on a computational server. In the case studies, we consider 'Yard B' is the interrupted marshalling yard.

5.1 Case Study I

In this case, the instance is a timetable that includes 20 passenger trains and 5 freight trains. The schedule of passenger trains is homogeneous and symmetrical. Four of the freight trains run towards Yard B and one freight trains runs to Yard A. We assume that all freight trains are allowed to stop at any two side tracks in each station. Furthermore, the arrival time deviates within a given range for all interrupted trains at the Yard B. All the stations have 4 tracks and segments have two tracks. The interruption duration is set to 30 minutes, and the recovery time is set to 1 hour. The parameter b^{win} is 50 minutes, and we assume that the freight trains can only take half of the tracks in all stations. Figure 6 shows the artificial timetable as the input of the model.

In Table 4, we display the average and maximum computational time with different objective functions. As we can see from Table 4, the computational time with objective function ‘Feasible’ (f_3) is the shortest with on average 0.33 seconds. The computational time for ‘Deviation’ is slightly larger than that for ‘Feasible’; for 'Transport Time' it is around 200 seconds.

Table 4: The computational times with different objective functions

Objective Function	Computation Time (s)	
	Average	Max
'Feasible' (f_3)	0.33	0.34
'Deviation' (f_1)	2.11	2.40
'Transport Time' (f_2)	200.13	200.21

Both Table 5 and 6 show results for the objective function of 'Transport Time'. Table 5 includes the average and maximum deviation of start time at the origin yard for all interrupted trains, and the average and maximum delay at the destination yard for all non-interrupted trains. When the value of b^{win} increases, the start-time deviation of interrupted trains and the delay for non-interrupted trains also increase.

Table 5: Results with different value of b^{win}

b^{win}	Objective function (min)	Interrupted trains at Yard A		Non-interrupted trains at Yard B	
		Average departure time deviation (min)	Maximum departure time deviation (min)	Average delay (min)	Maximum delay (min)
10	73	6	13	1	2
20	72	11	23	3	9
30	72	11	23	3	9
40	72	11	23	4	10
50	72	11	23	4	10
60	72	21	43	4	11

Table 6: results with different values of $tt_{i,j,k}$

$\sum_{k \in K_j} tt_{i,j,k}$	Interrupted trains	Non-interrupted trains		
	Average arrival time deviation at yard B (min)	Average delay from Yard B (min)	Average increase of stopping time (min)	Average increase of the running time (min)
Full	27.5	8	6	1
Half	23.5	4	3	0
Single	22.5	4	0	0

Table 6 shows the average arrival time deviation at the destination yard for interrupted trains, the average delay from destination yard, average increase of stopping time and running time for all non-interrupted trains. 'Full' indicates that the freight trains are allowed stop at all tracks at each station, which can be represented with $\sum_{k \in K_j} 1$. 'Half' indicates that the freight trains are allowed to stop at half of the tracks at all stations, which can be represented with $0.5 \cdot \sum_{k \in K_j} 1$. As for 'Single', it indicates that the freight trains are only allowed to stop on one track at each station. In Table 6, the average arrival time deviation at the destination yard and the average delay to the destination yard decrease with a decrease of $\sum_{k \in K_j} tt_{i,j,k}$—which is an unexpected behavior. A possible explanation is that we have different schedules with the same optimal objective value and we do not steer towards a specific value for these metrics.

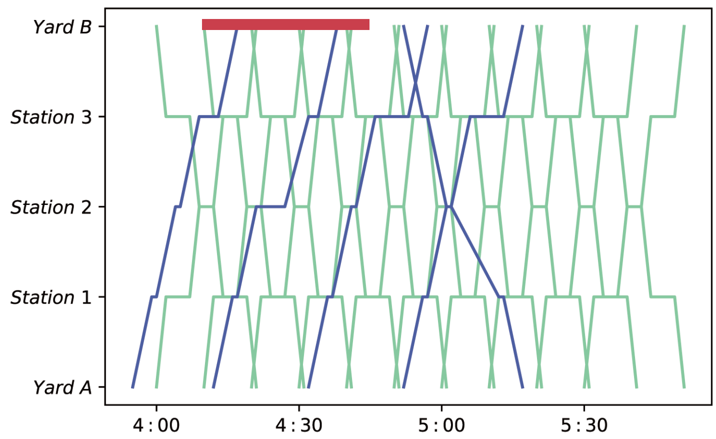

Figure 5: The original timetable (Case I)

Figure 5 shows the original timetable in Case I. The interruption duration is marked as a red block. Figure 6 and Figure 7 dispict the rescheduled timetable with the objective function of 'Deviation' f_1 and 'Transport Time' f_2, respectively. Freight trains are shown in blue, passenger trains are shown in yellow. In Figure 6, the interrupted trains stop at Station 3 for a while and then run towards Yard B. Some non-interrupted freight trains are also delayed in Station 2, Station 3 and Yard B. In Figure 7, the non-interrupted trains have less stopping time in total, but the total deviation of the arrival time for all non-interrupted trains is larger than for the rescheduled result of 'Deviation'.

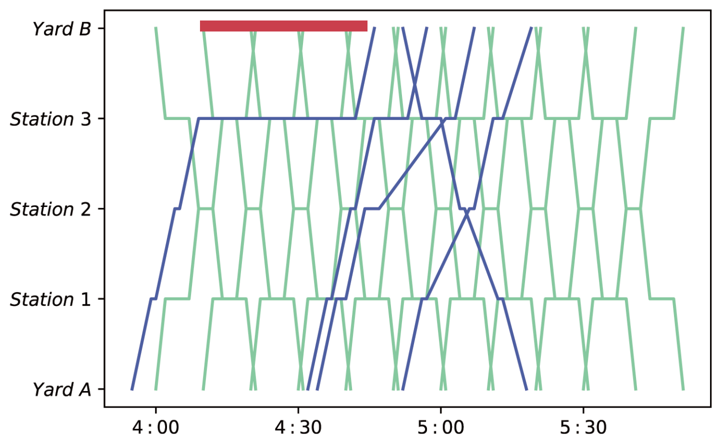

Figure 6: The rescheduled timetable with objective function of f_1 'Deviation'

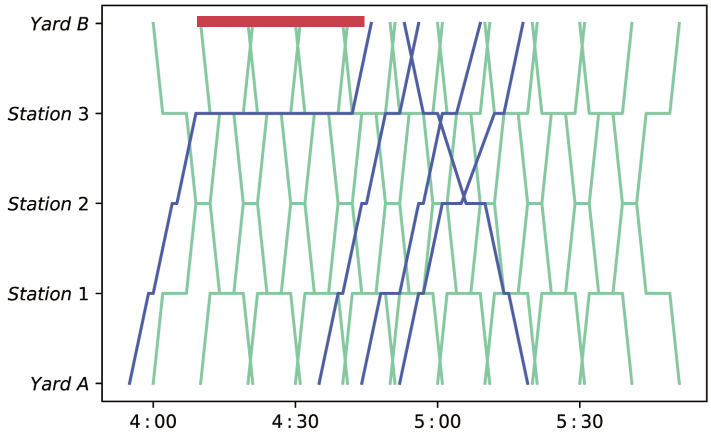

Figure 7: The rescheduled timetable with objective function of f_2 'Transport Time'

5.2 Case Study II

In Case II, we consider the number of segments and stations remains the same as in Case I, so is the number of tracks in each segment and station. The initial timetable has 20 passenger trains and 5 freight trains. In Figure 8, freight trains are shown in blue and passenger trains are shown in yellow and green with different speed.

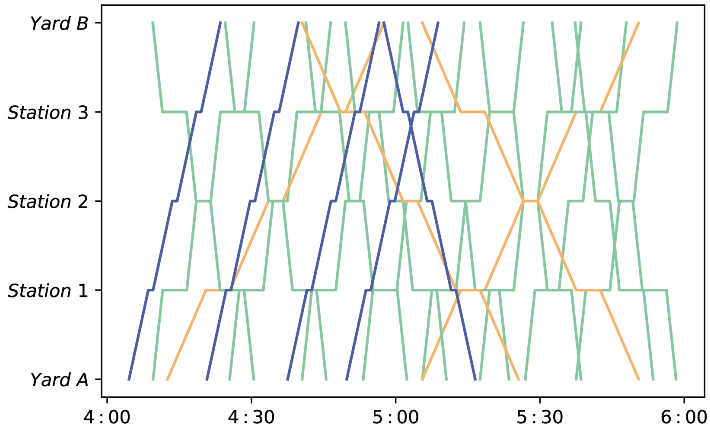

Figure 8: The original timetable (Case II)

In Table 7, we show the computational result with different interruption, recovery times, and different objective functions. The result includes the number of interrupted trains, the average delay of interrupted trains for the objective function f_1 'Deviation', and the average delay of non-interrupted trains for f_2 'Transport Time'. The number of interrupted trains increases while the interruption time increases. The average of interrupted trains and the average delay of non-interrupted trains also increase while the interruption time increases.

Table 7: Results with different interruption and recovery time

Interruption time (min)	Recovery time (min)	Average number of interrupted trains	'Deviation' Average delay of interrupted trains	'Transport Time' Average delay of non-interrupted trains
10	20	1.0	13.0	10.7
10	30	1.0	24.7	9.3
20	20	1.3	34.7	10.7
20	40	1.3	34.0	12.7
20	60	1.0	35.7	24.3
30	30	1.7	35.0	27.0
30	60	1.7	43.7	35.7

5.3 Case Study III

The instance is a timetable that includes 120 passenger trains and 30 freight trains with 76 segments and 75 stations along the rail line. We assume that each rail segment is double-tracked, and each station has four tracks. Figure 9 shows the artificial timetable. Freight trains are shown in blue and passenger trains are shown in green. There are 60 passenger trains and 20 freight trains in one direction, and 60 passenger trains and 10 freight trains in the other direction. The departure-time difference from the same yard between two consecutive passenger trains is 40 minutes. The departure-time difference from Yard B between two consecutive freight trains is 4 hours, and 2 hours from Yard A. The stopping time for passenger trains at station 37 is set to 10 minutes, and freight trains are set to 20 minutes.

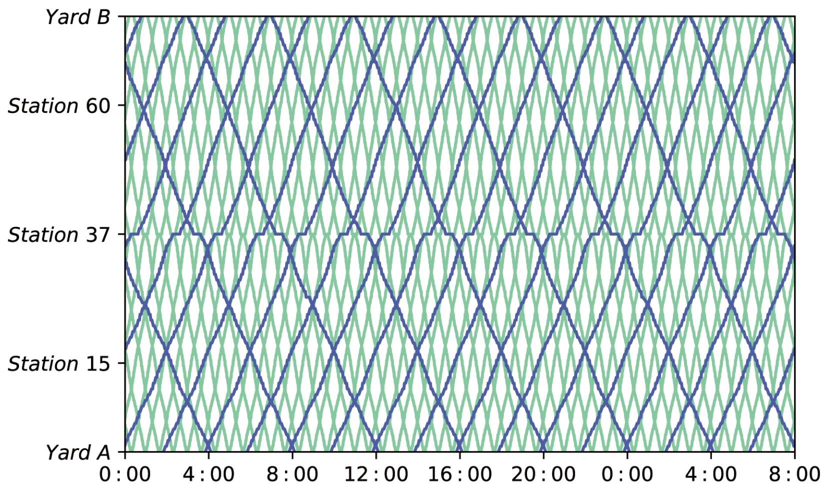

Figure 9: The original timetable (Case III)

In Table 8, we show the minimum, average and maximum computational time and the number of variables and constraints. Delayed events refer to the number of events for all freight trains with delayed schedule. Total average delay refers to the average delay at the destination yard for all freight trains. The objective function remains as the deviation of all

Table 8: Results with different values of interruption and recovery time

Interruption (h)	Recovery Time (h)	Computational Time (s) Min	Average	Max	Variable number	Constraint number	Delayed Events	Total Average Delay
1	1	16.8	24.0	28.4	78670	5810920	15	14
1	2	46.4	47.4	49.2	94681	5872420	39	14
1	3	57.6	62.4	67.5	112870	5940510	12	14
2	2	60.6	61.7	62.3	112870	5940550	88	74
2	4	78.3	79.1	80.6	146104	6068580	12	292
3	3	107.8	112.0	120.1	146103	6067670	101	127.5
3	6	1679.5	1838.1	1996.7	195416	6258560	76	87.5

non-interrupted freight trains, and the value of b^{win} equals to the recovery time duration. All freight trains are allowed to stop at only two side tracks at each station. In this case, the computational time also grows as the duration of recovery time grows. Furthermore, the number of variables and constraints increase significantly if the interruption and recovery time lengths increase. The number of variables doubled when the total duration of interruption and recovery time doubled from 2 hours to 4 hours. Furthermore, when the sum of total duration of interruption and recovery time is the same, the numbers of variables and constraints are also roughly the same.

6 Conclusion and Further Research

In this paper, we introduced a MILP model for rescheduling a railway timetable during a major interruption, which postpones a group of (freight) trains. Other (passenger) trains may not be moved to accommodate the rescheduled train paths. In the MILP model, we consider the capacity limit for each segment and station by forbidding the conflicts at segments and stations. We also limit the train types that are allowed to stop on each track in each station.

In our numerical experiments with a fictive case, the computational time is strongly affected by the objective function. The calculation time for 'Deviation' is approximately 6 times longer than for 'Feasible' and for 'Transport Time' 100 times longer than for 'Deviation'. The computational test also showed that the increasing delay of the non-interrupted freight trains is affected to a great extent by the rise of both interruption and recovery duration. The objective function 'Deviation' causes more severe delay for the interruption freight trains than non-interrupted trains. Furthermore, the value changing of parameter b^{win} and $tt_{i,j,k}$ has a severe impact on the delay and the stopping time at all stations in the rescheduled timetable. We noticed that when the interruption and recovery time increase, the space and computational time increase a lot accordingly.

This model still needs further improvements in terms of model size and computation time, if it is to be applied in a practical case. As the amount of data increases in the experiments, both space and time complexity are increasing rapidly.

For future research, there are three main aspects. Firstly, we consider to reduce the size of the model, especially the number of binary variables. Secondly, we consider to extend the current model aiming to apply it on a network instead of a rail corridor. Thirdly, we consider the possibility to include alternative geographical routes for some of the trains.

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