Guarding Polyominoes Under k-Hop Visibility

Christiane Schmidt

Malmö University, December 5, 2024

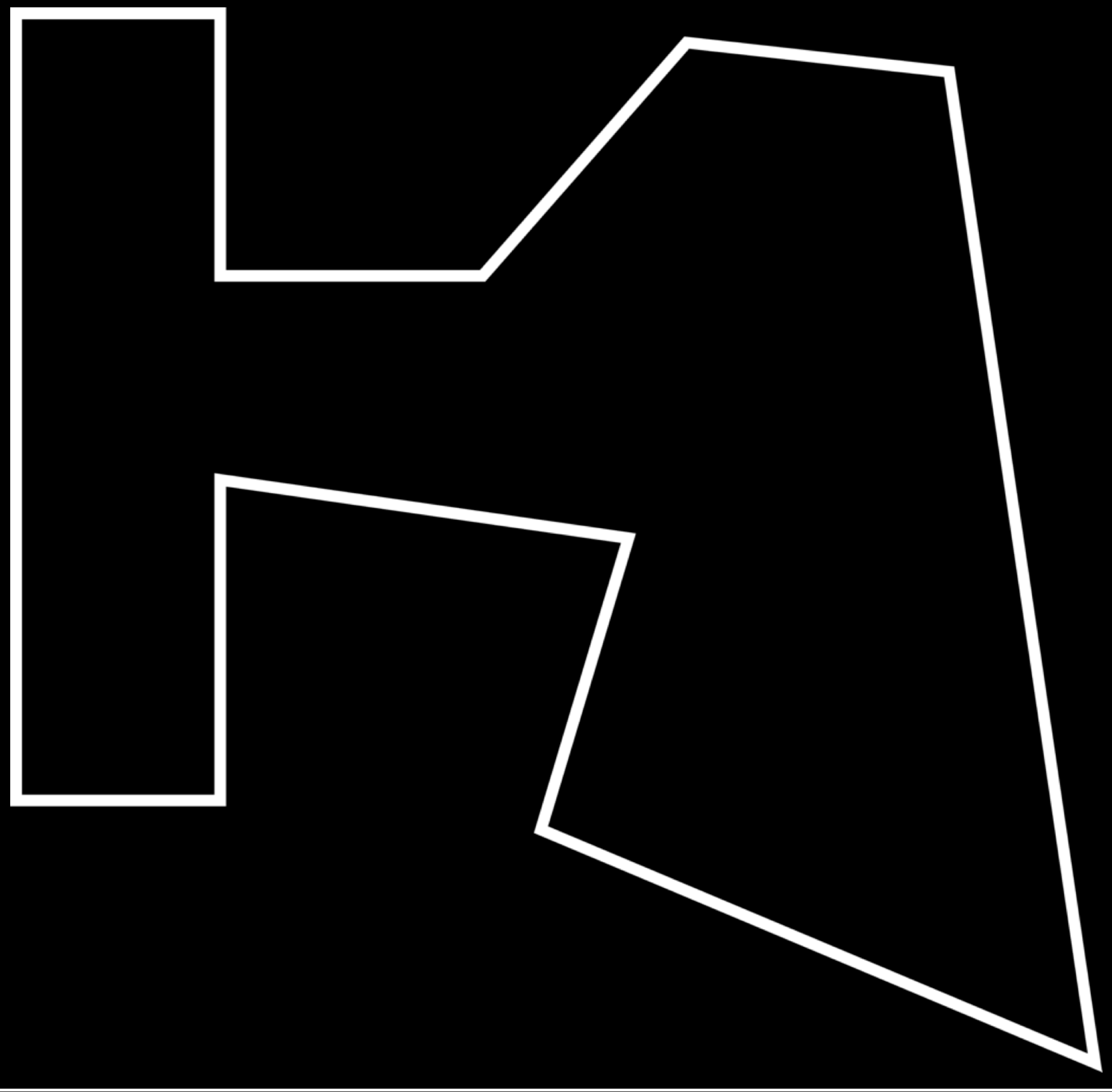
Based on joint work with

Omrit Filtser, Erik Krohn, Bengt J. Nilsson, and Christian Rieck

Agenda

- The Art Gallery Problem and Its Variants
- *k*-Hop Visibility
- What Do We Know About Guarding Polyominoes + Thin Polygons and About the Minimum k-Hop Dominating Set Problem? Aka: Related Work
- Some More Definitions
- VC Dimension
- Computational Complexity: NP-Completeness for 1-Thin Polyominoes with Holes
- A Linear-Time 4-Approximation for Simple 2-Thin Polyominoes
- Outlook

The Art Gallery Problem (AGP)



Given: Polygon P

How many guards do we need to monitor P?

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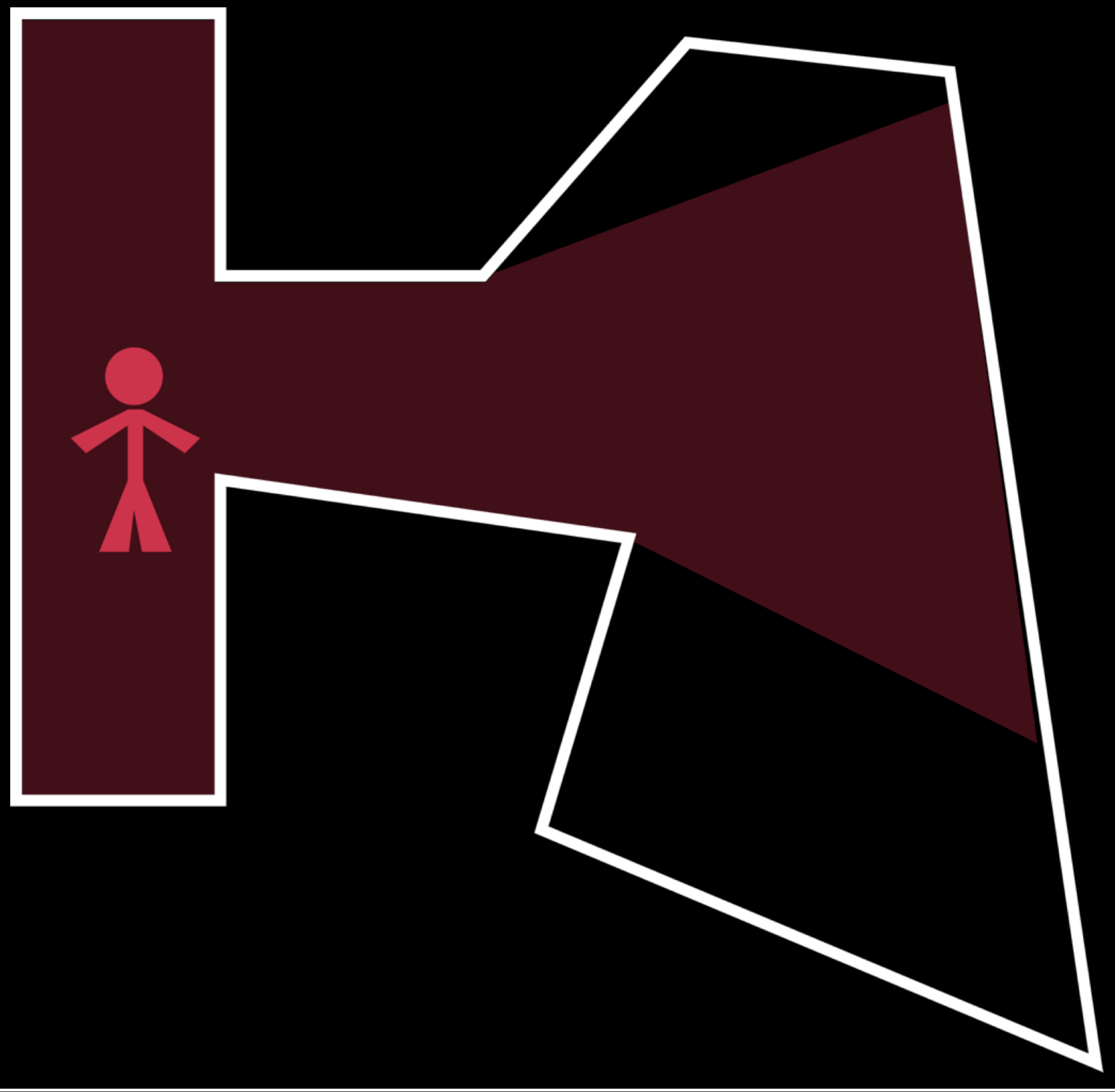

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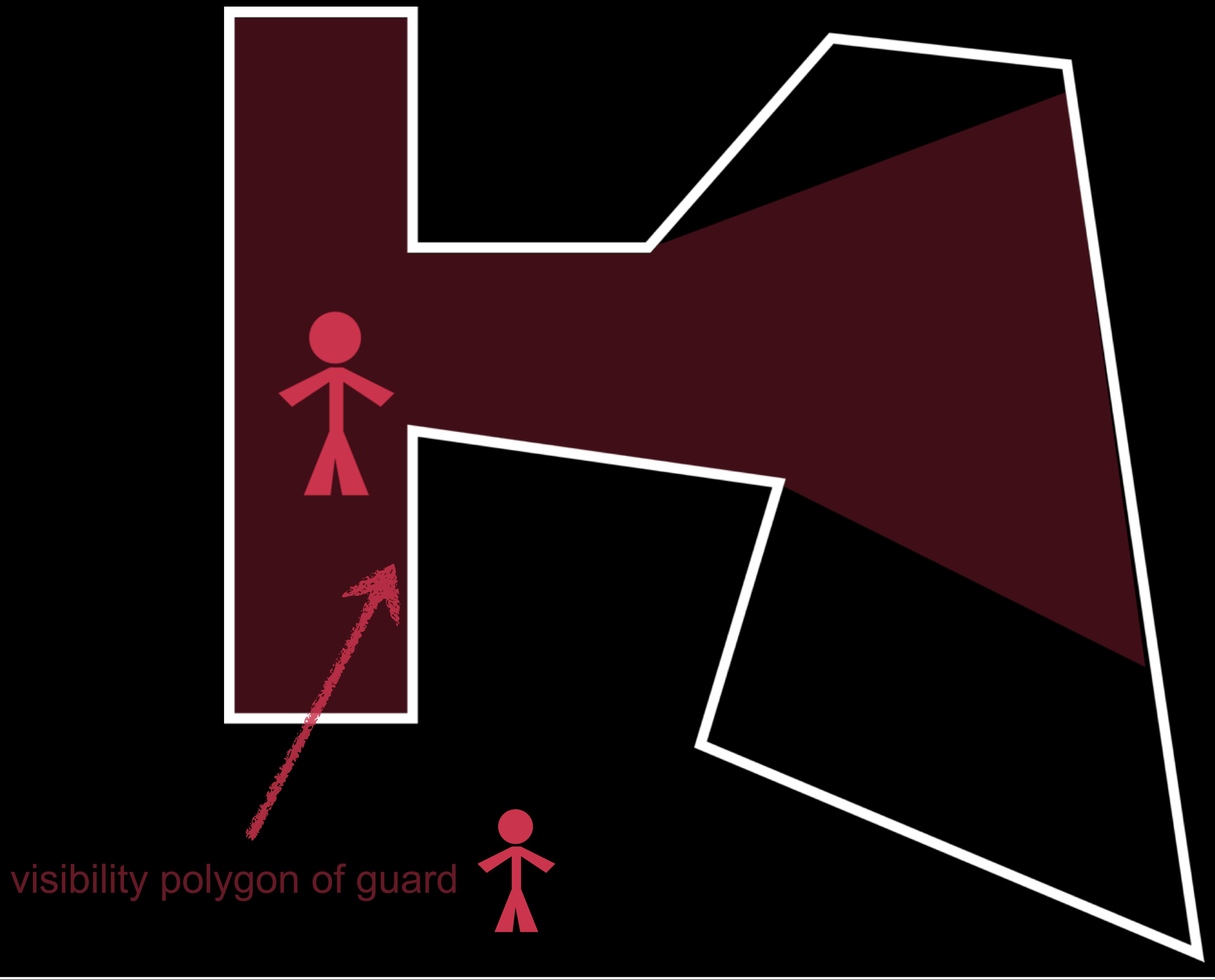

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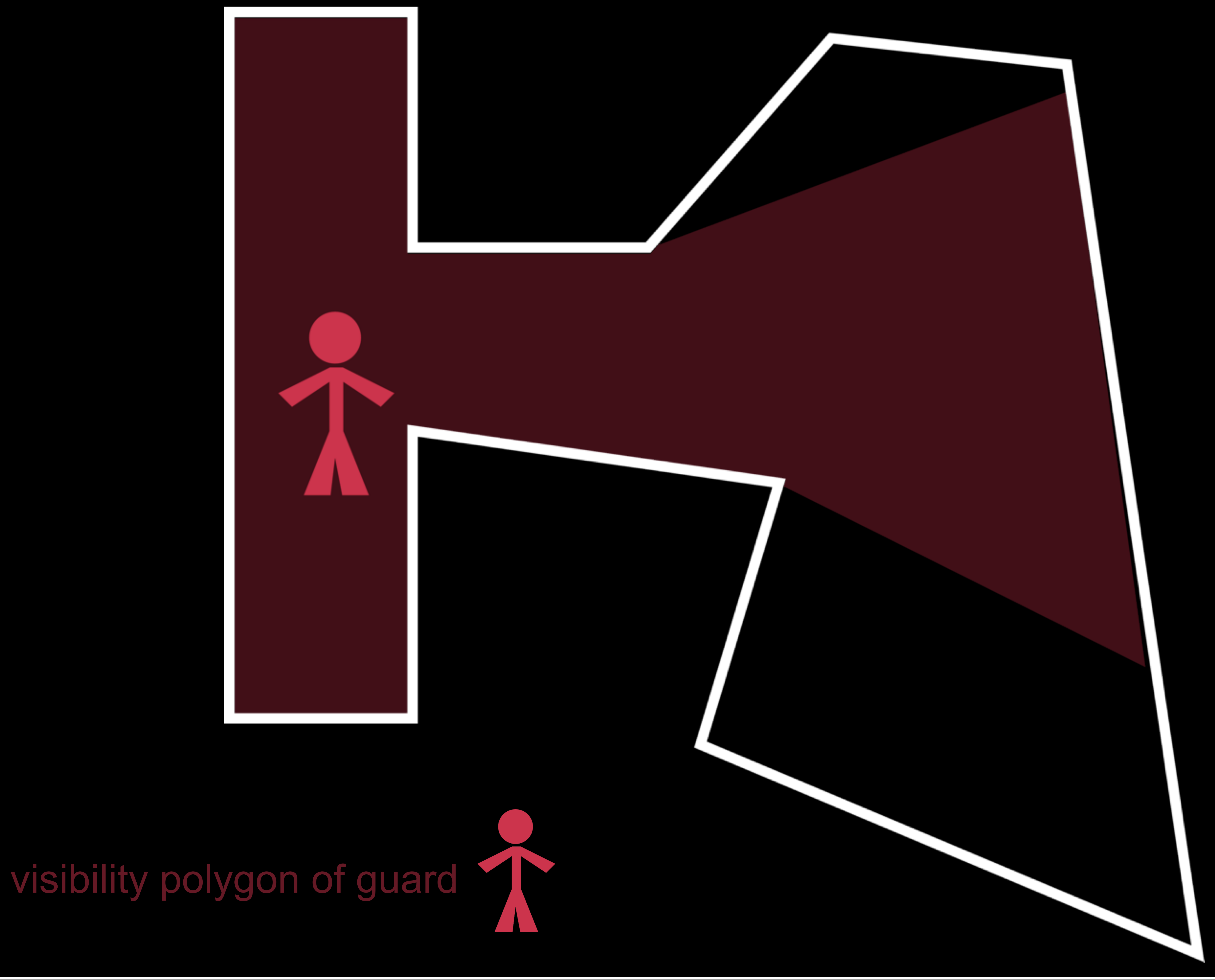

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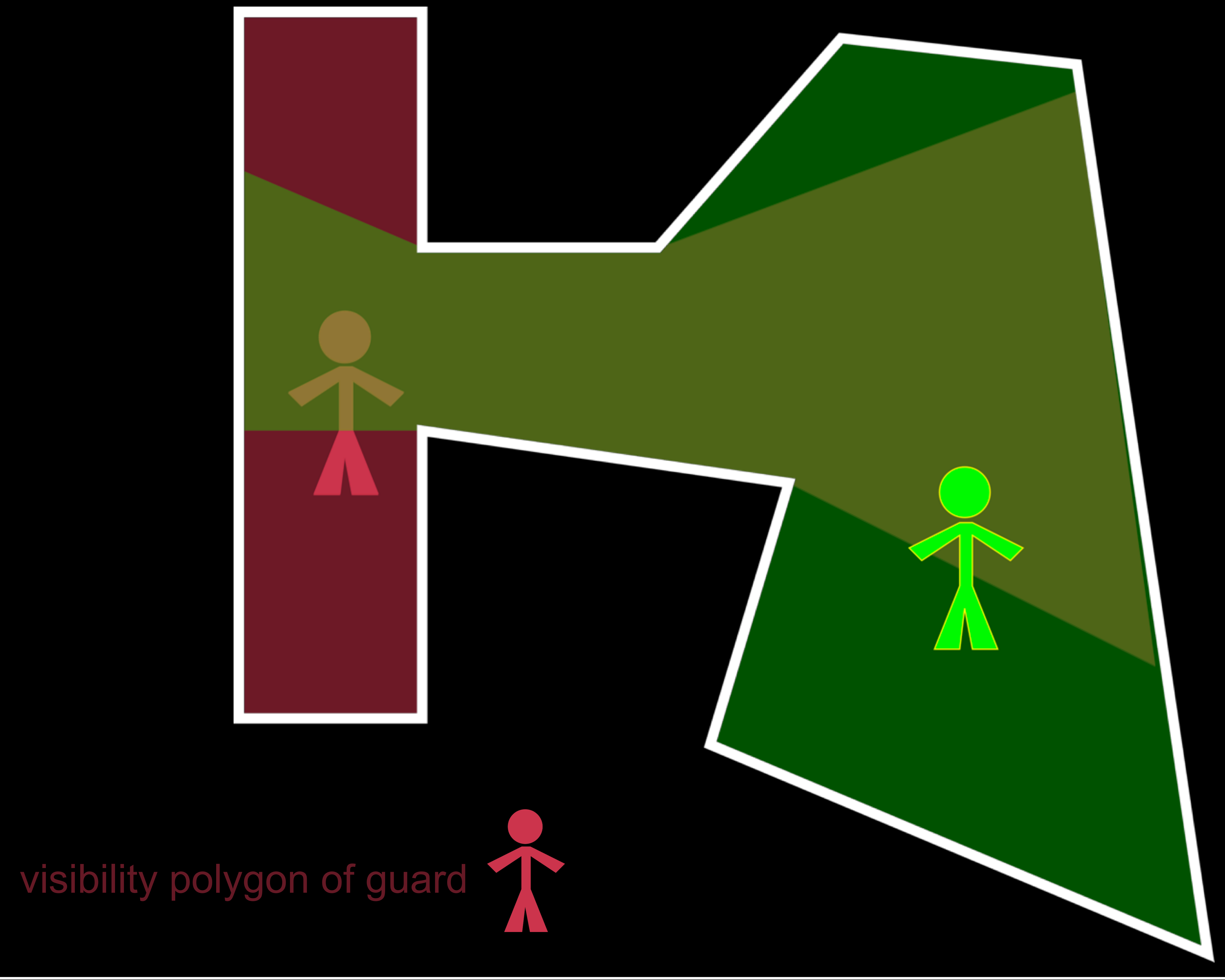

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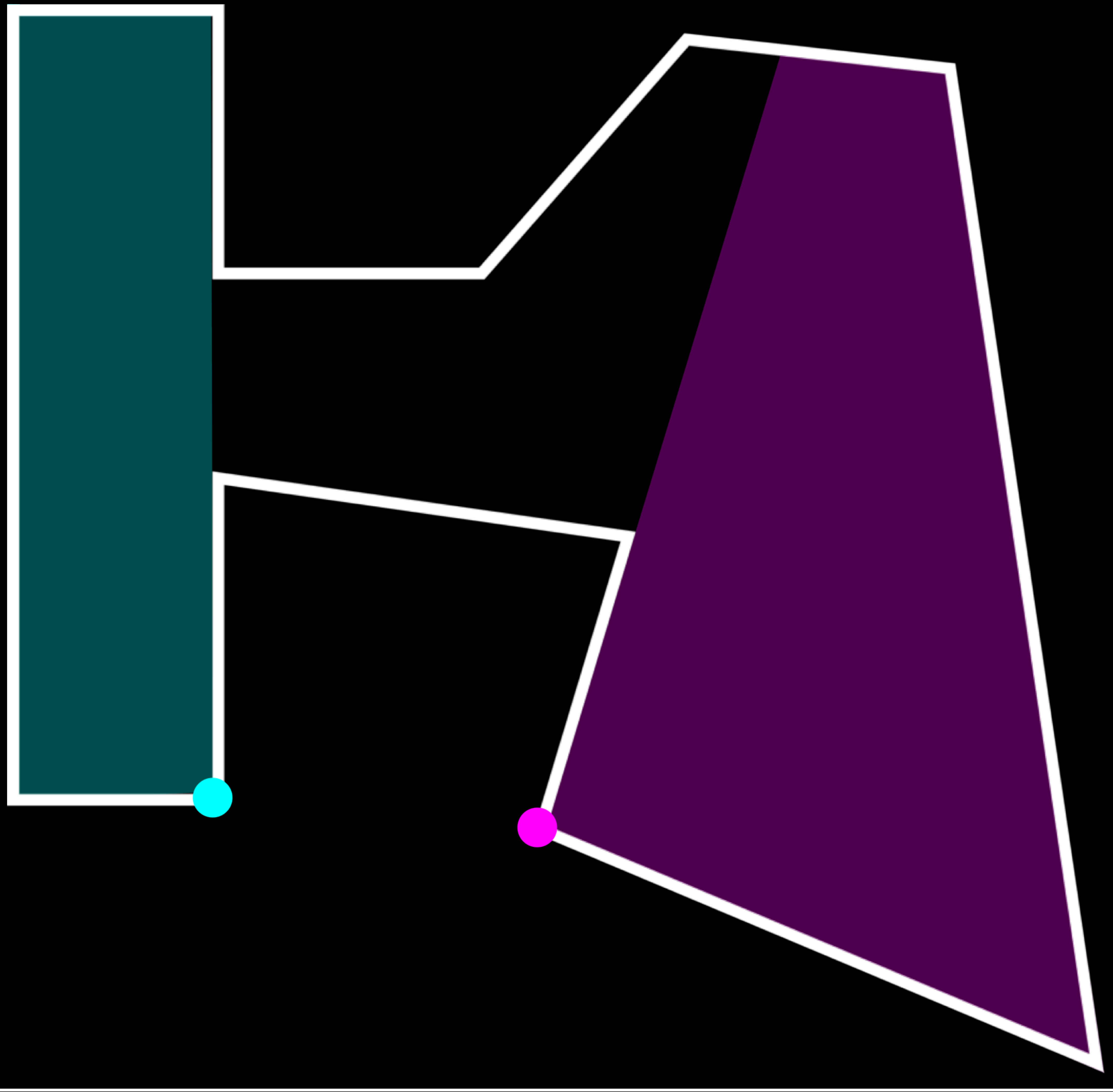

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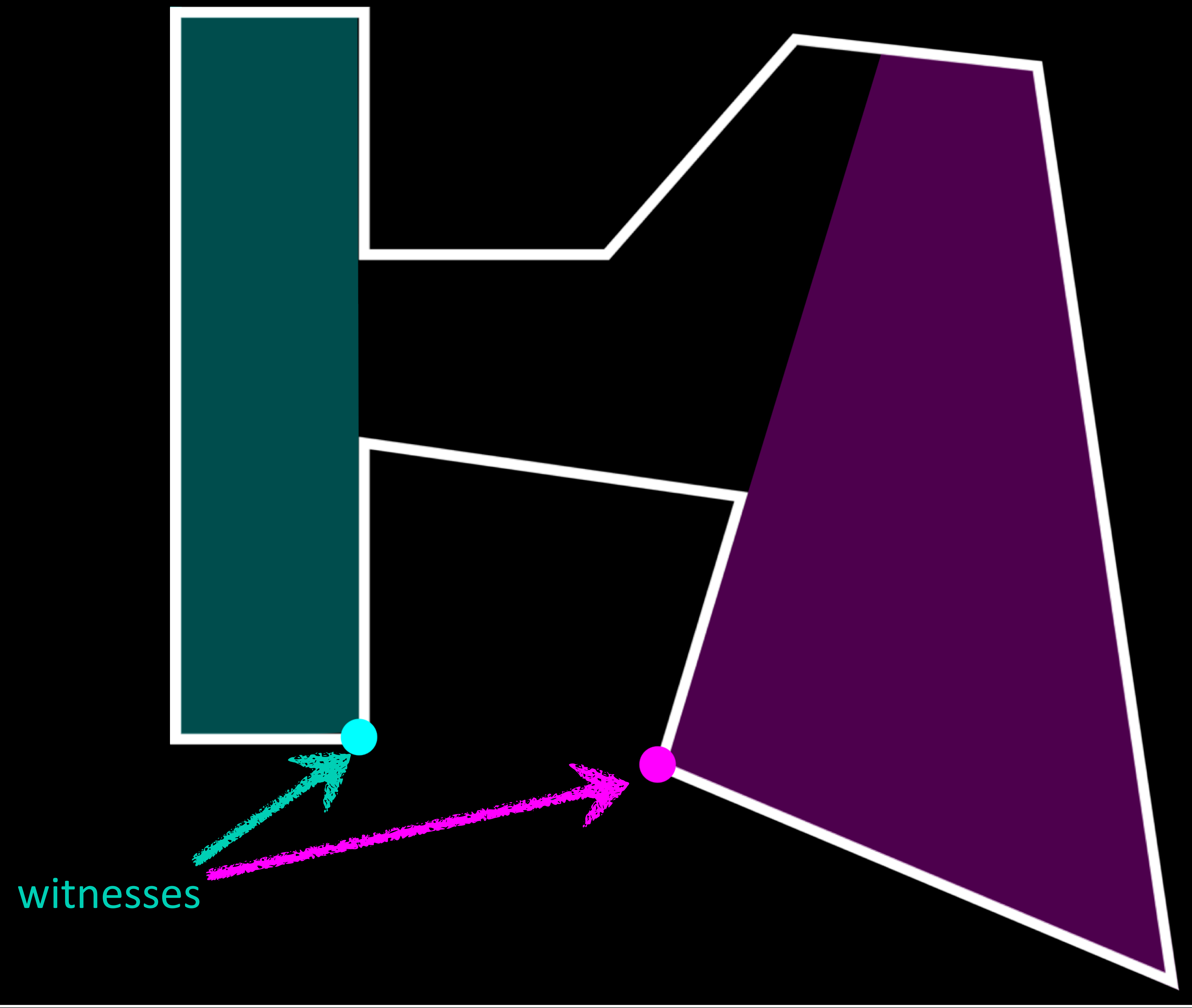

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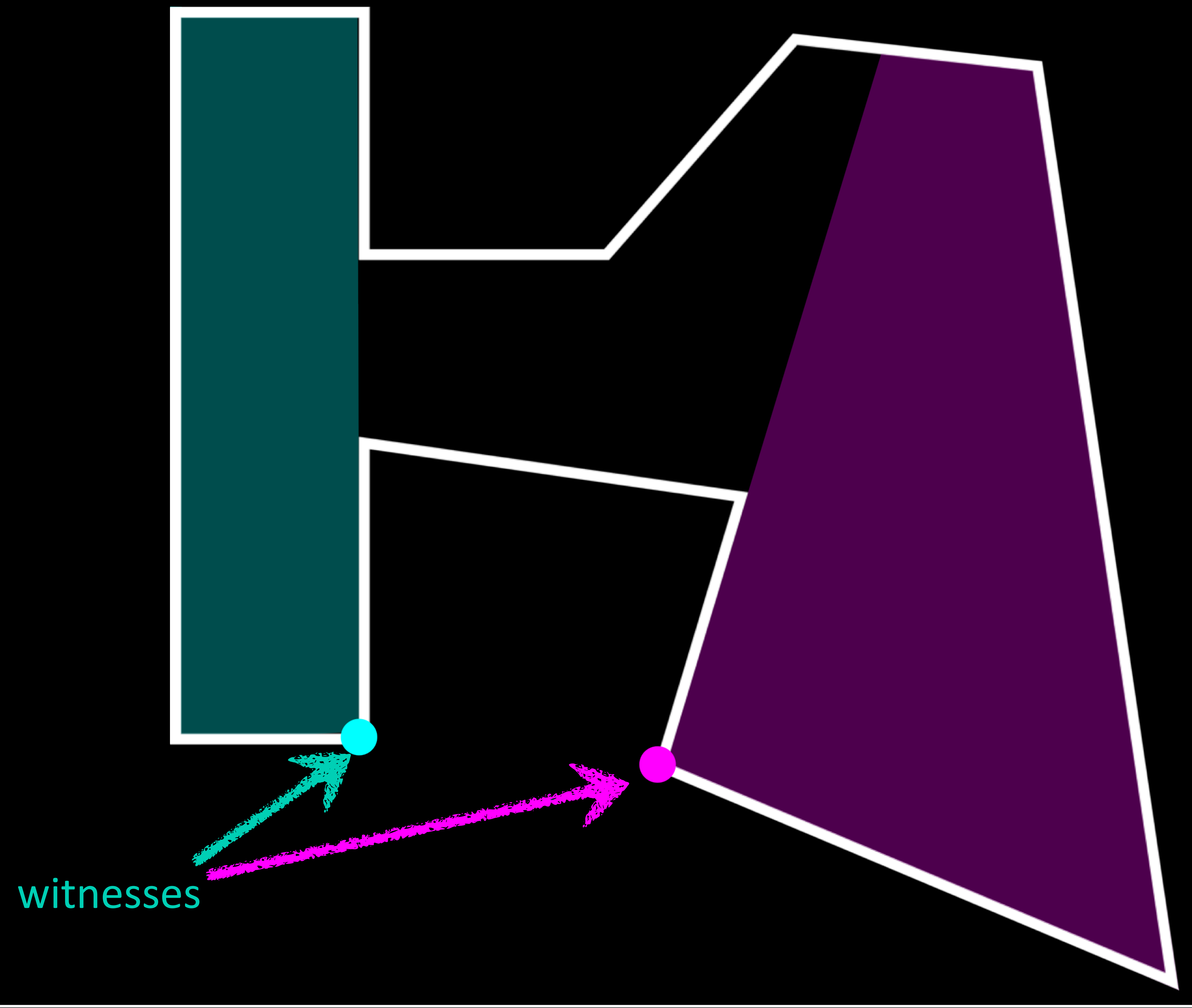

➜ Lower bound of 2
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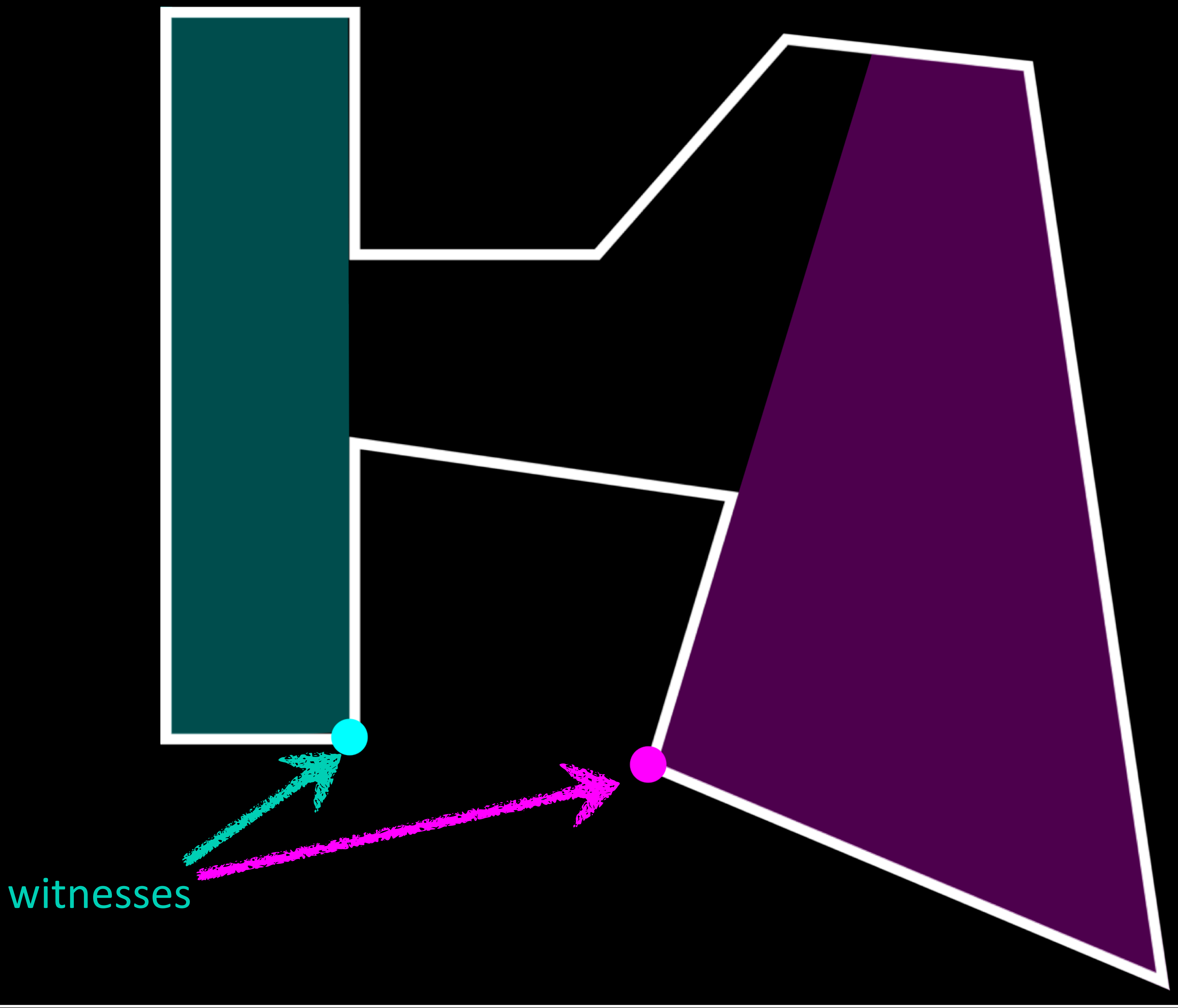

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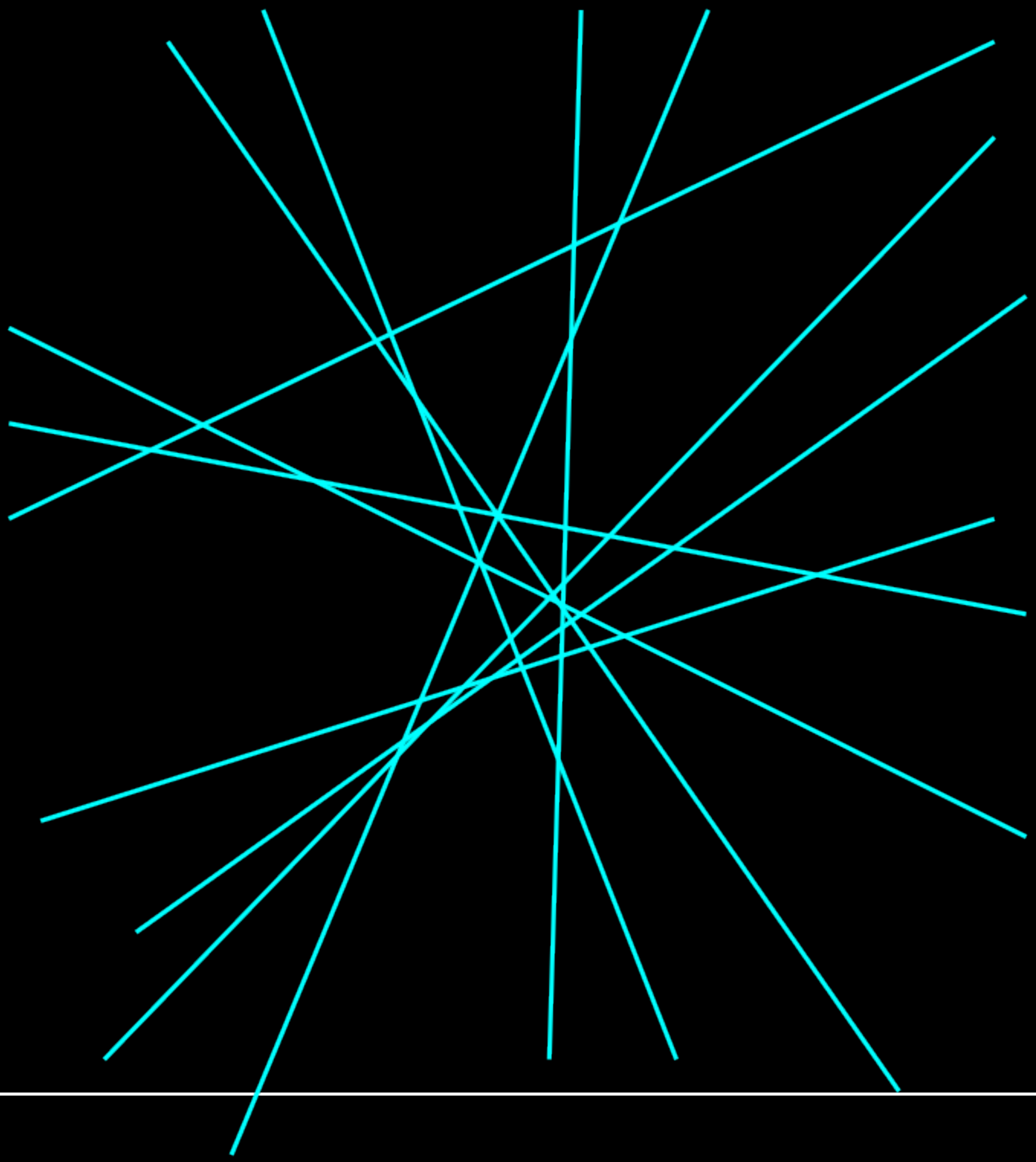

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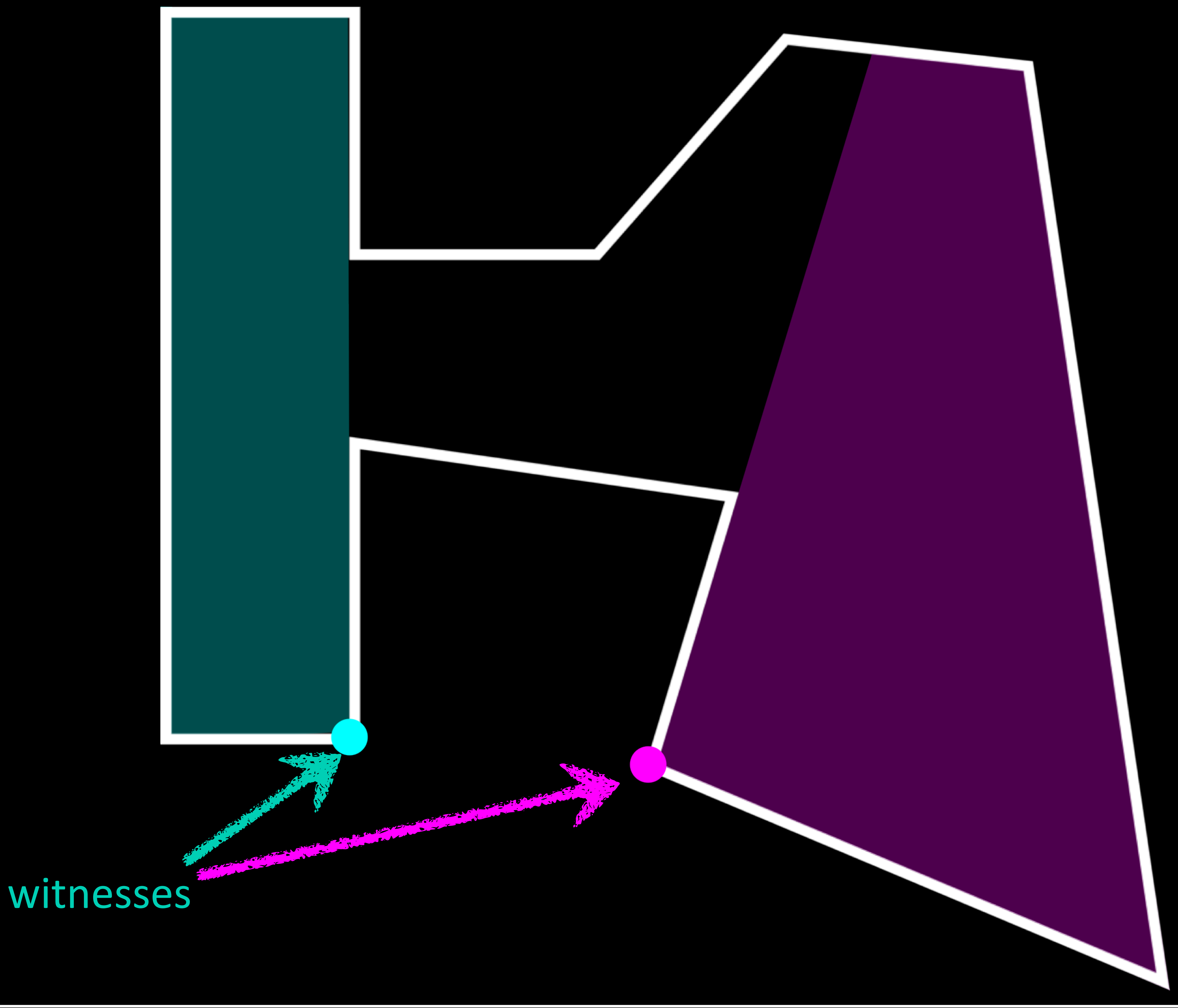

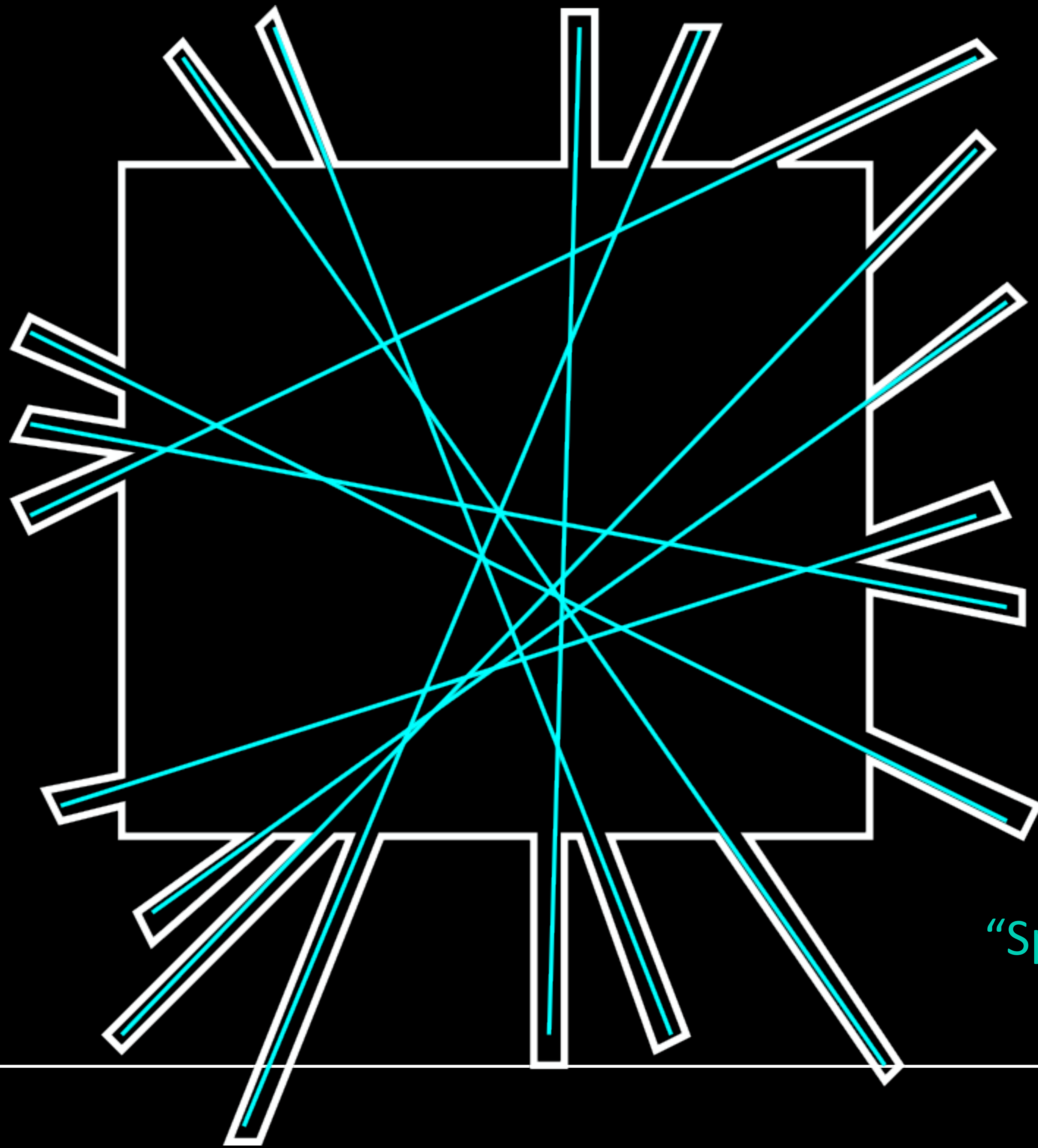

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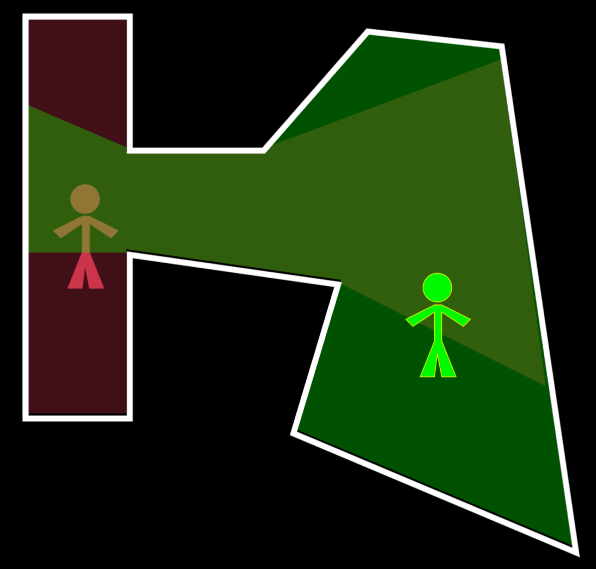

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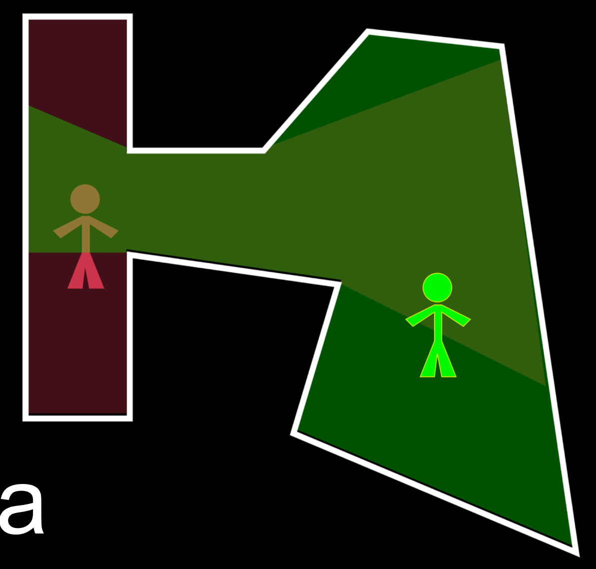

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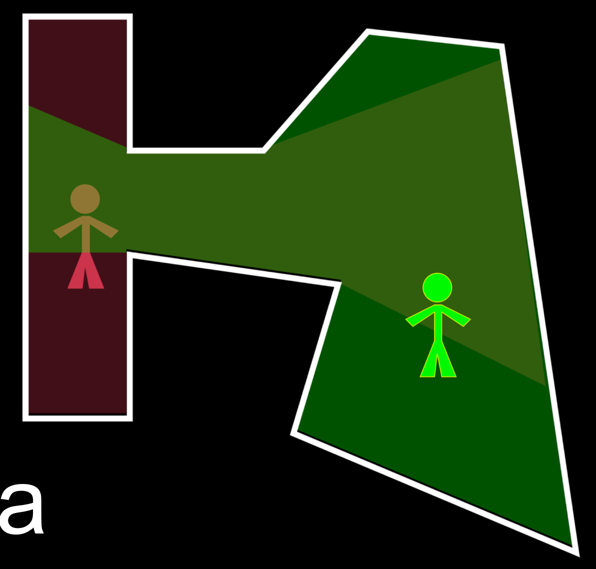

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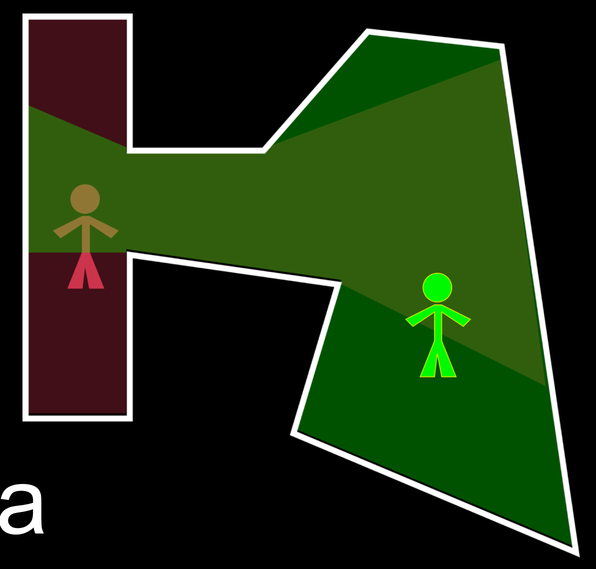

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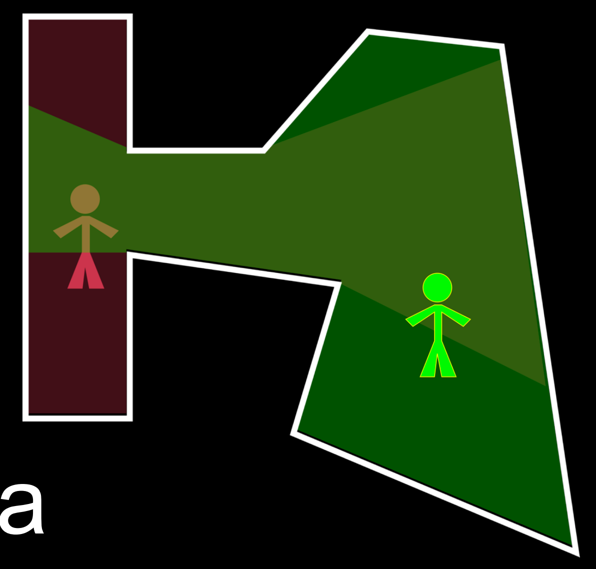

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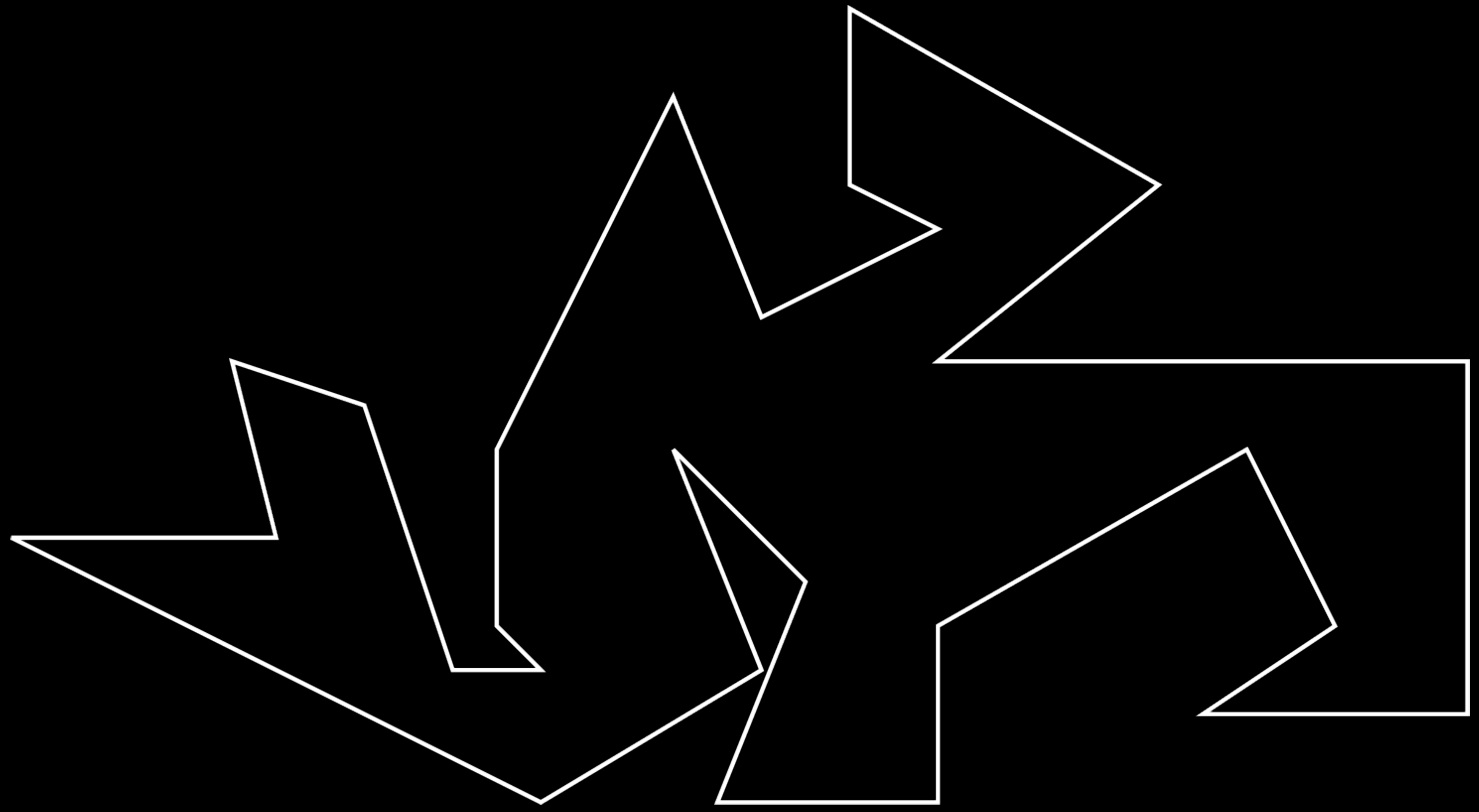

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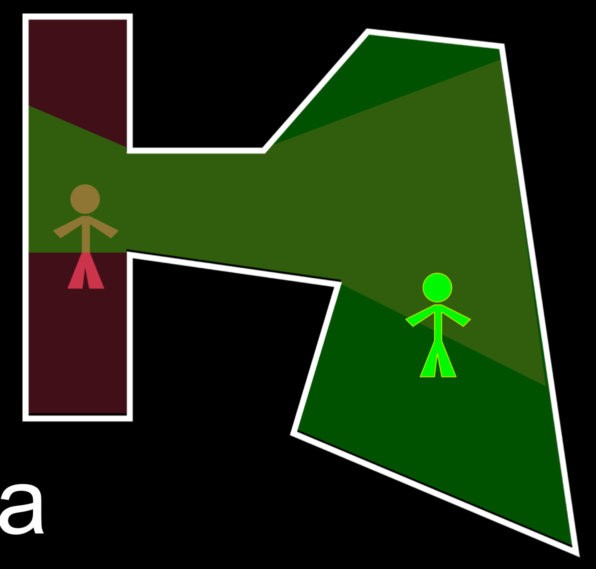

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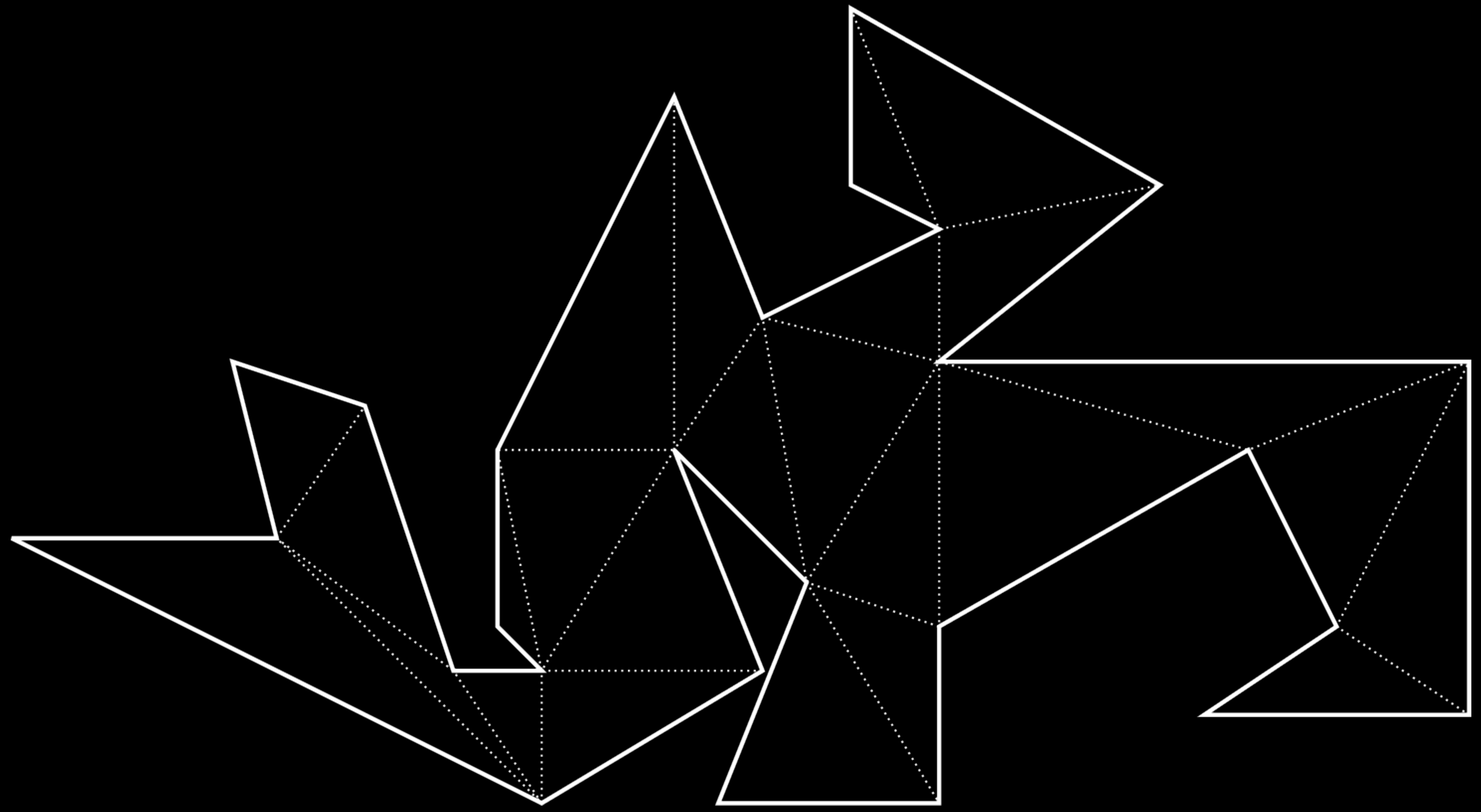

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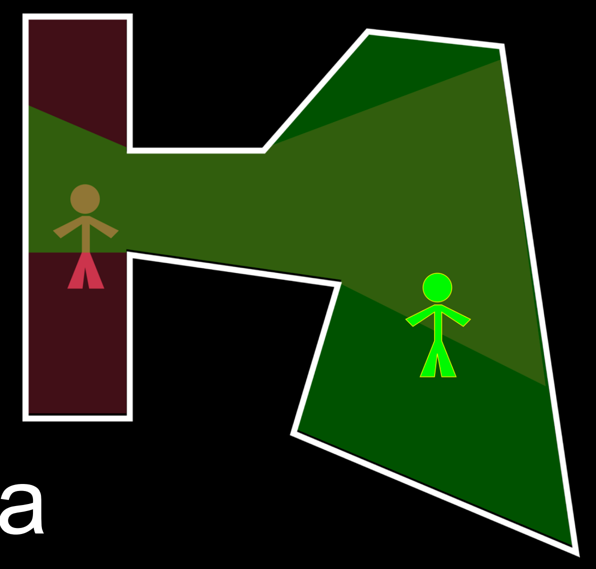

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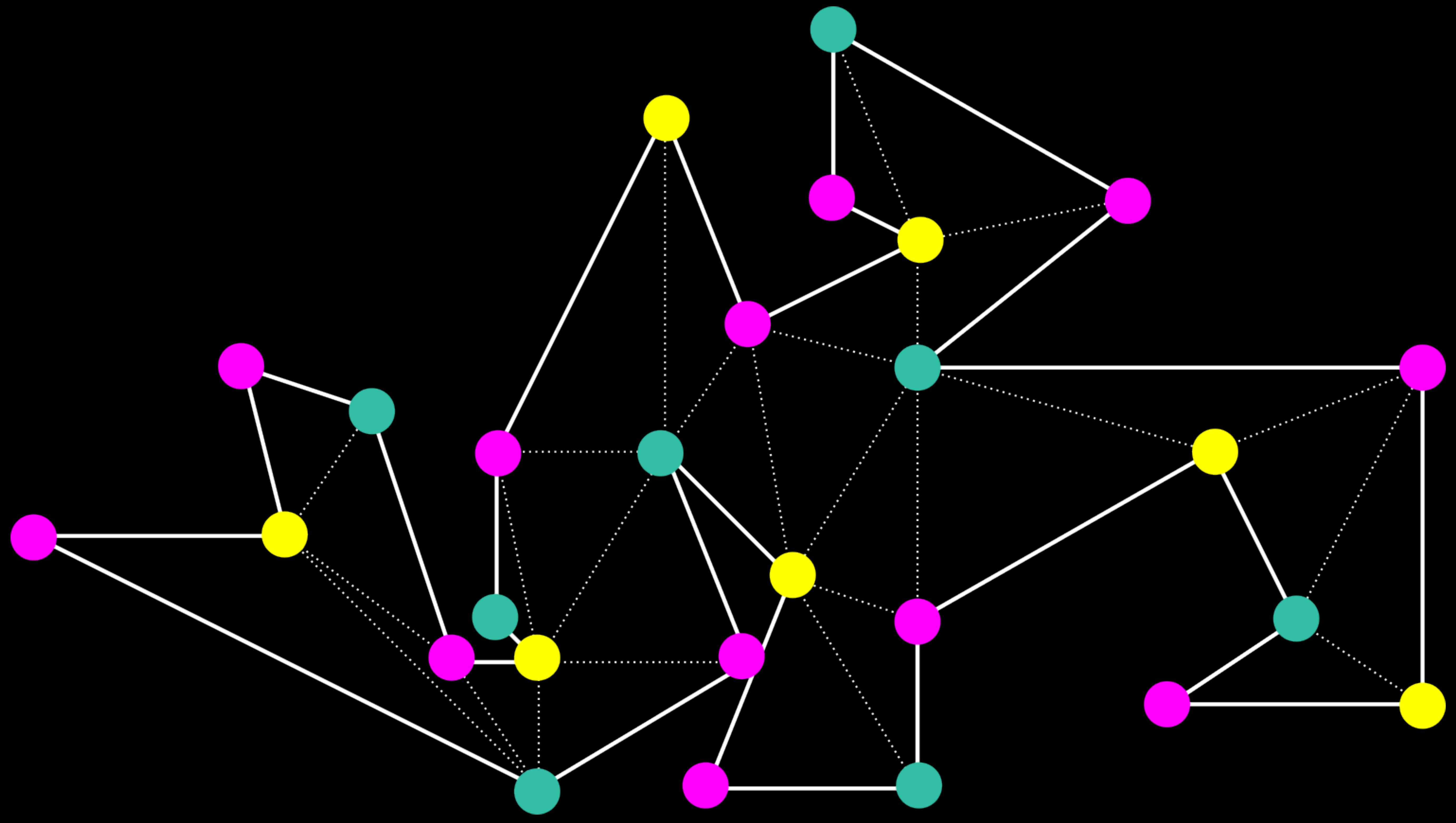

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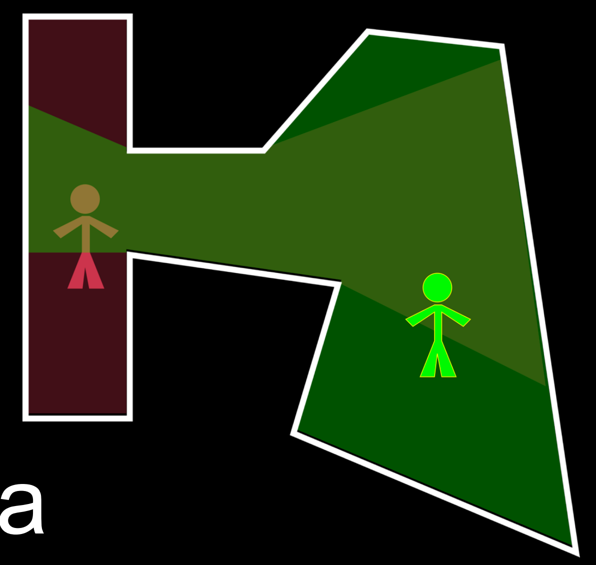

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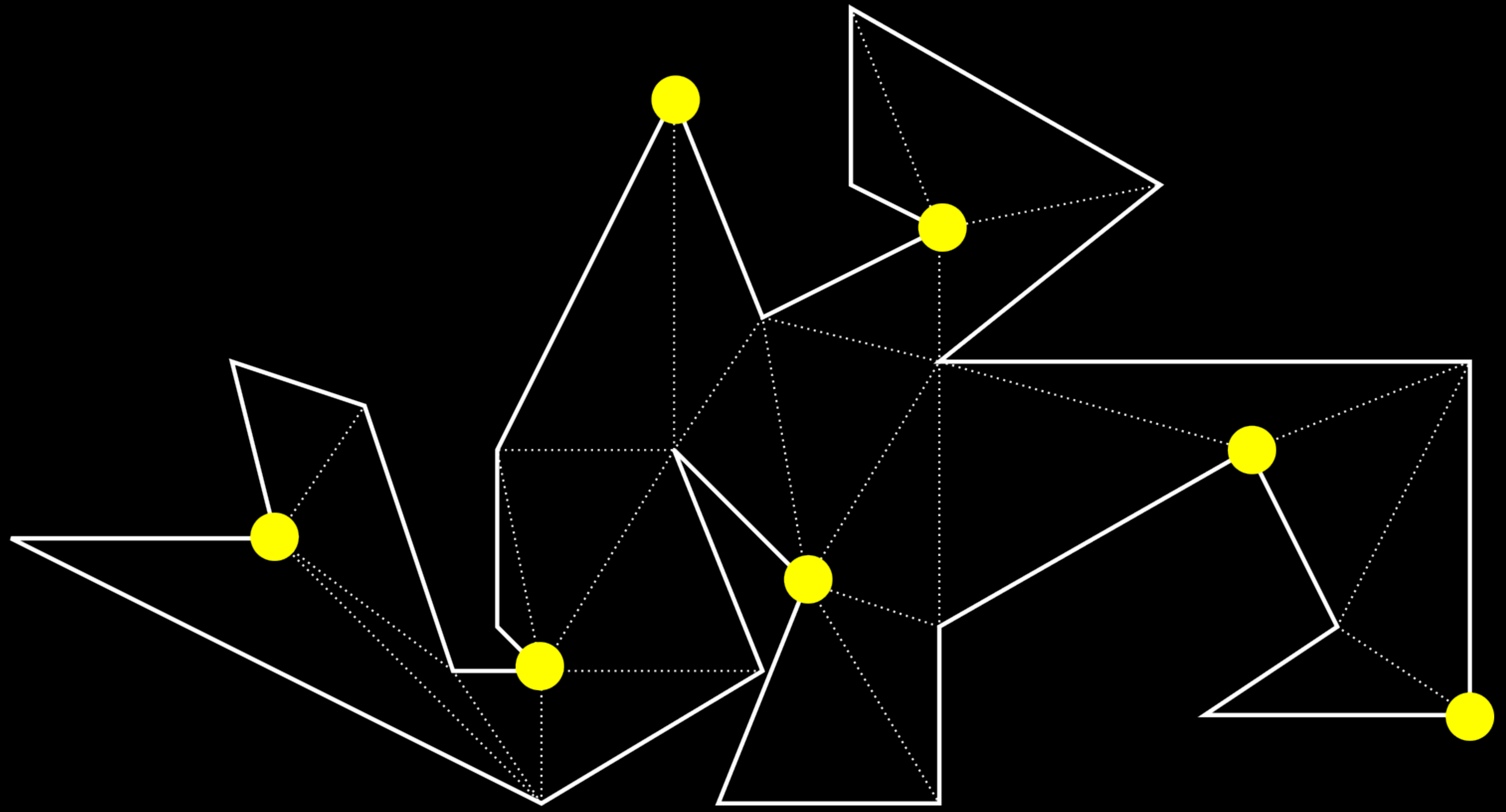

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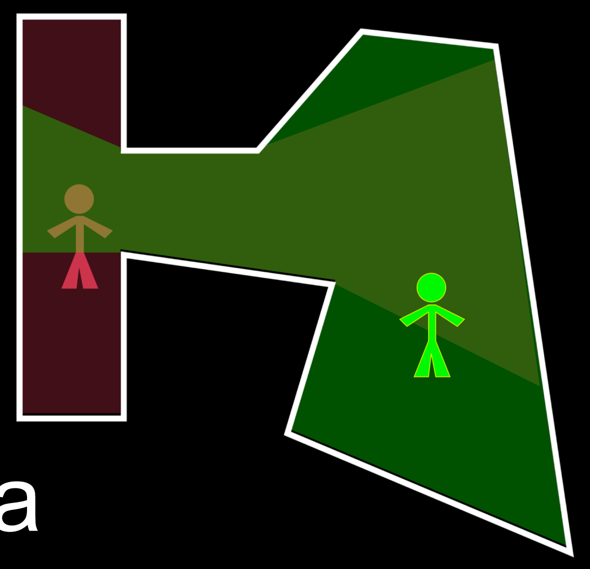

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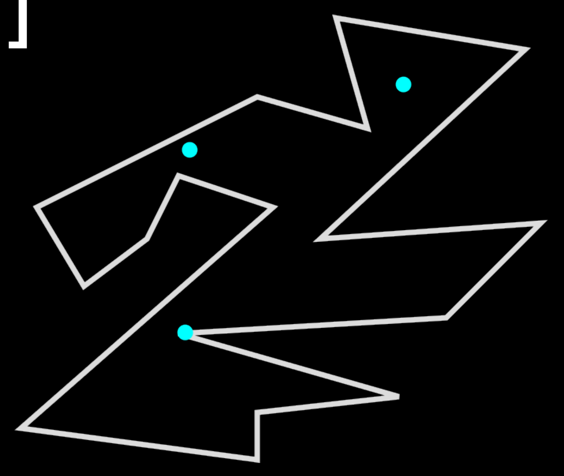
- The AGP is NP-hard for point guards with holes [O’Rourke & Supowit 1983] , vertex guards without holes [Lee & Lin 1986], point guards without holes [Aggarwal 1986]; point guards without holes is ∃R-hard [Abrahamsen et al. 2021]

point guards



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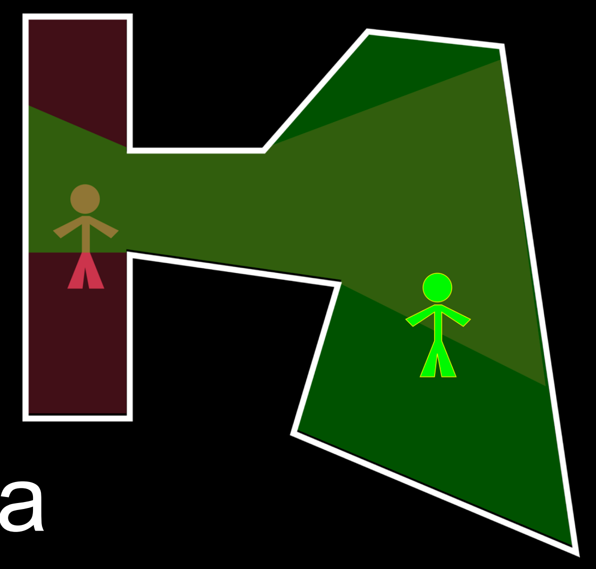

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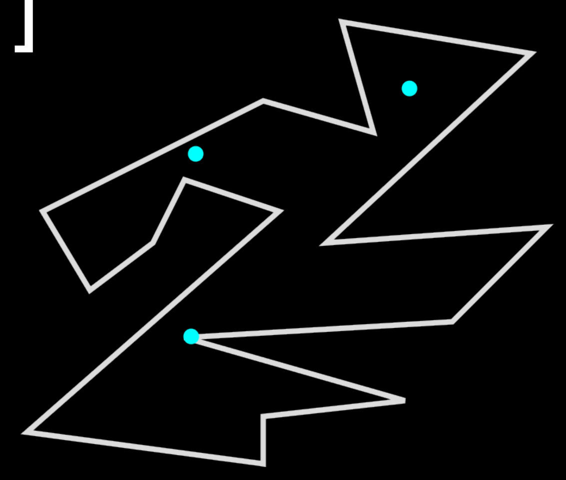
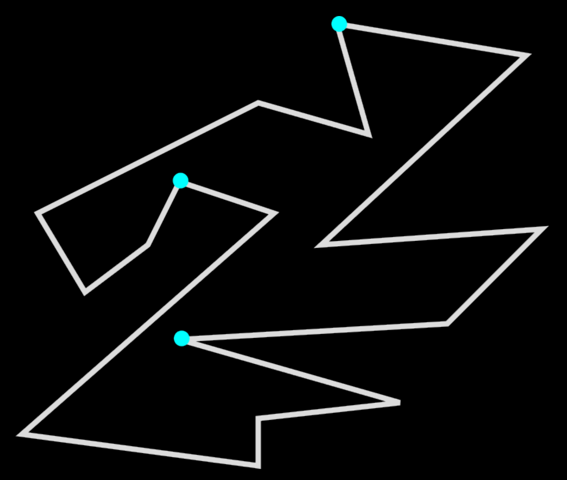
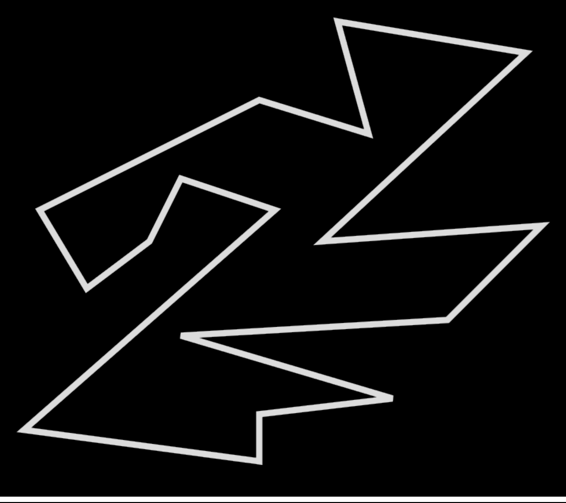
point guards   vertex guards

Simple polygon:

- Does not intersect itself
- No holes



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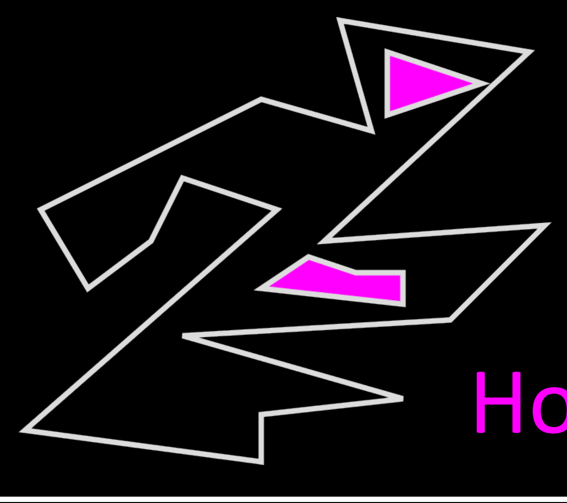

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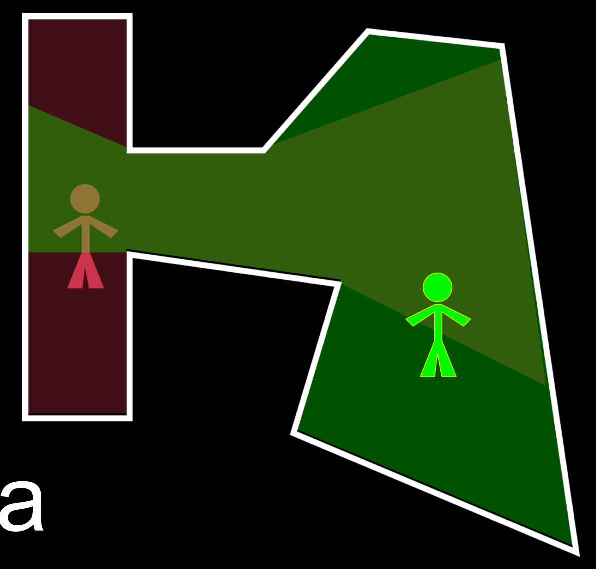

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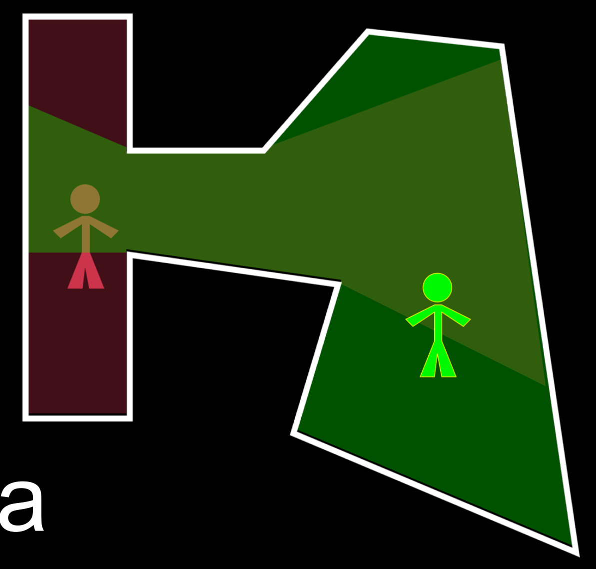

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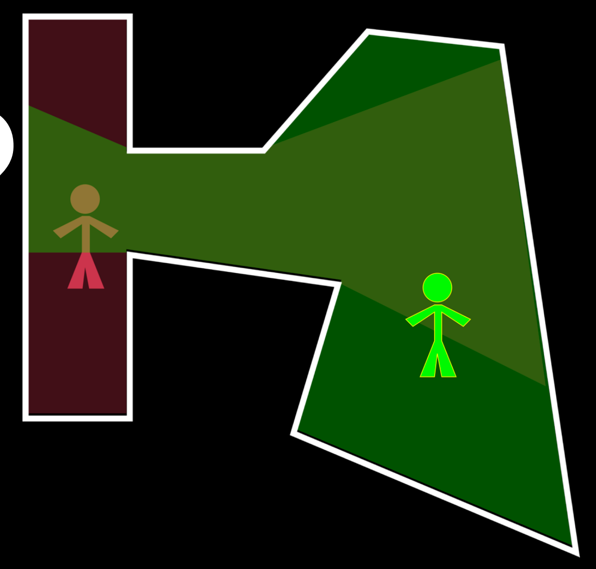
Other structural results

The Art Gallery Problem and Its Variants

The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

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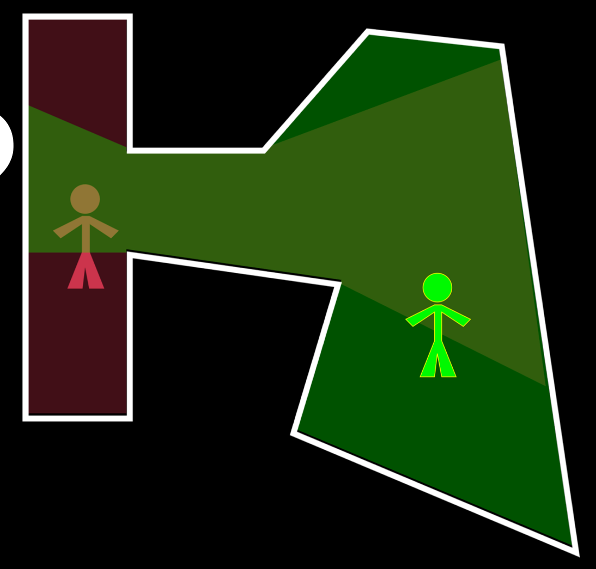

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- Capabilities of the guards
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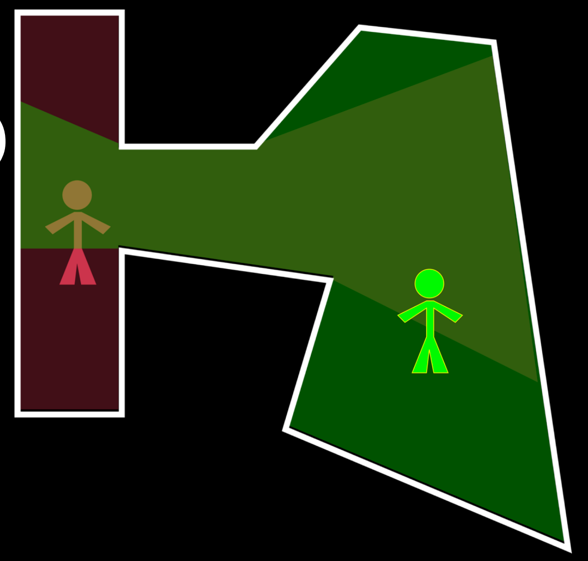

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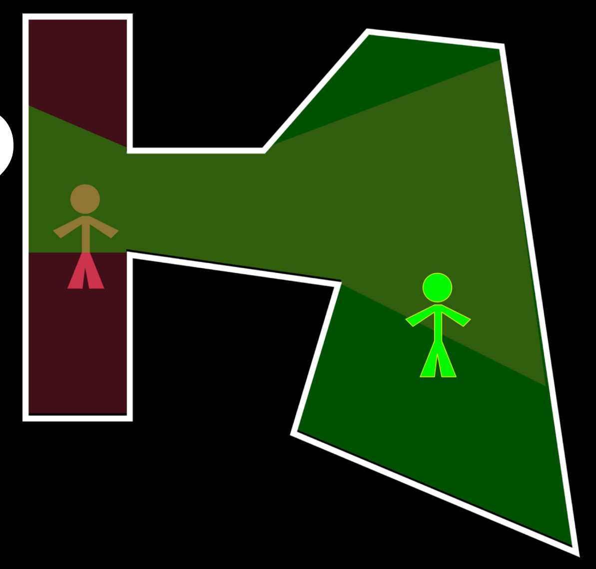

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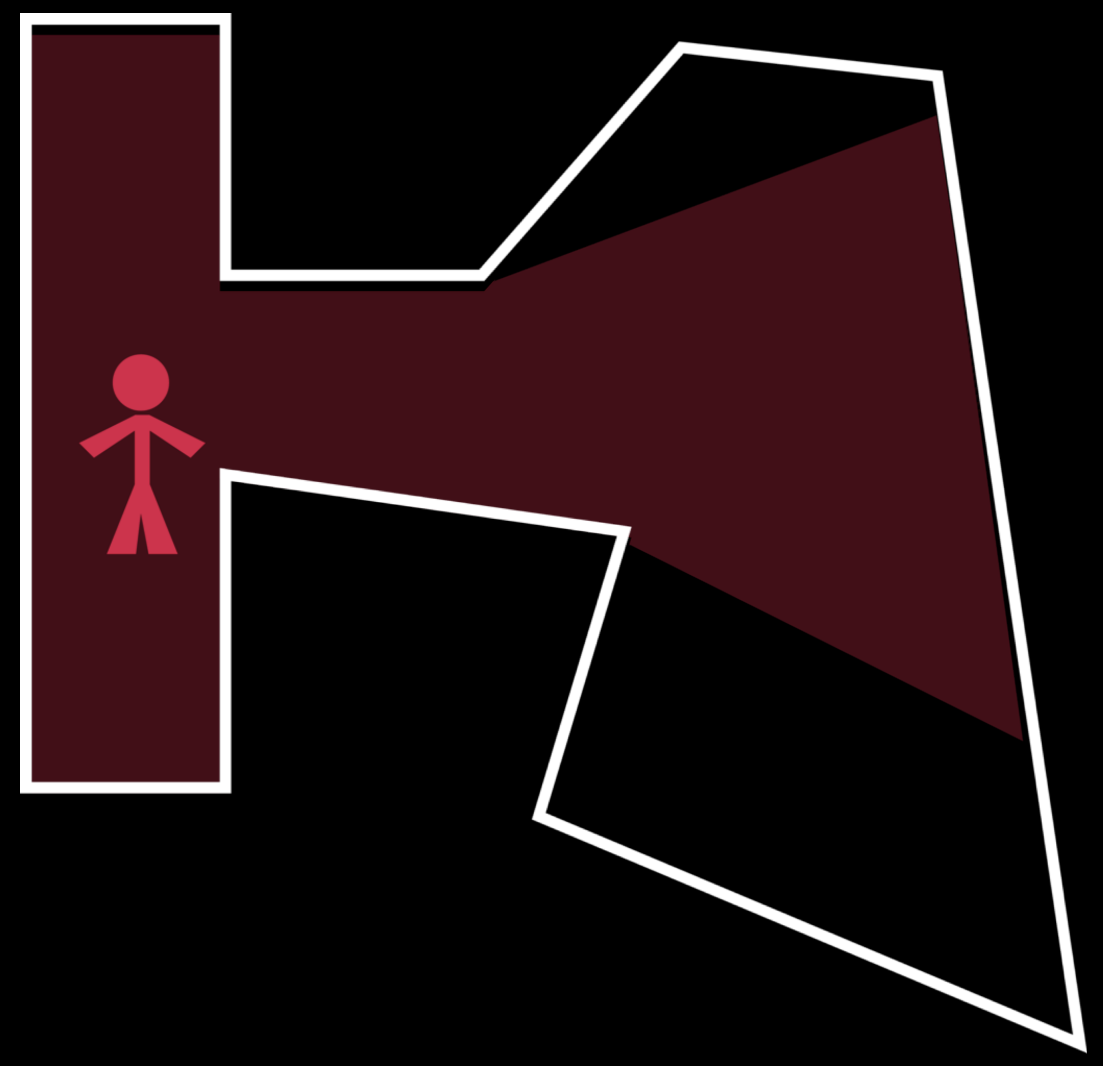

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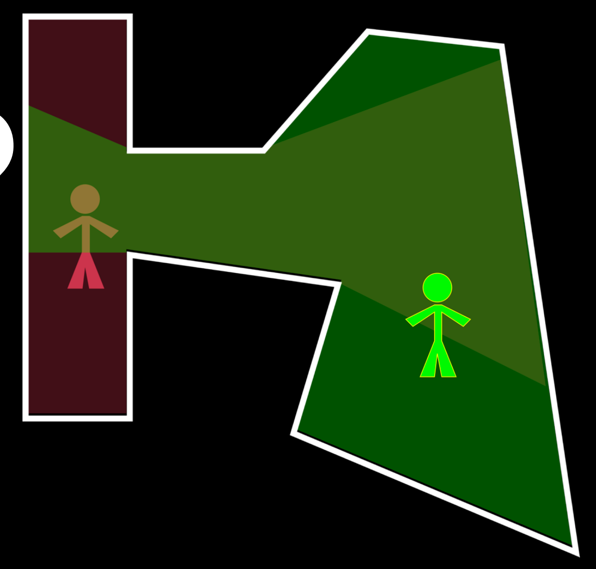

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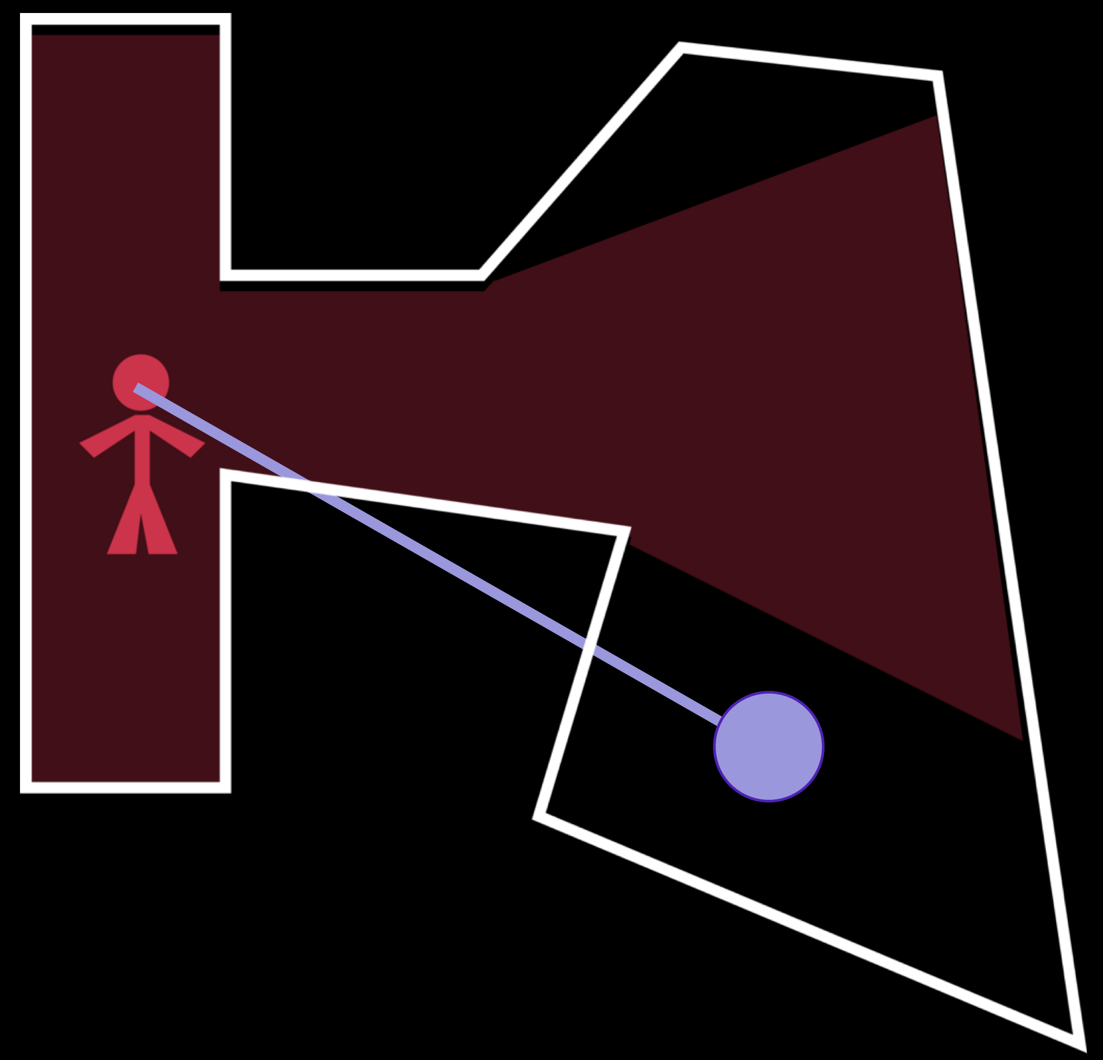

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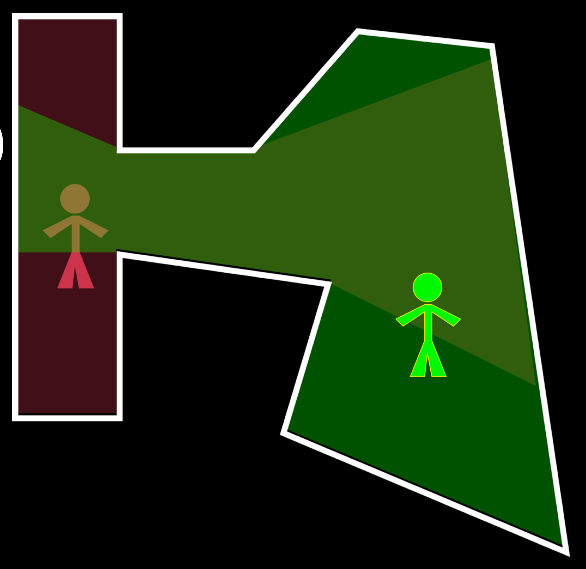

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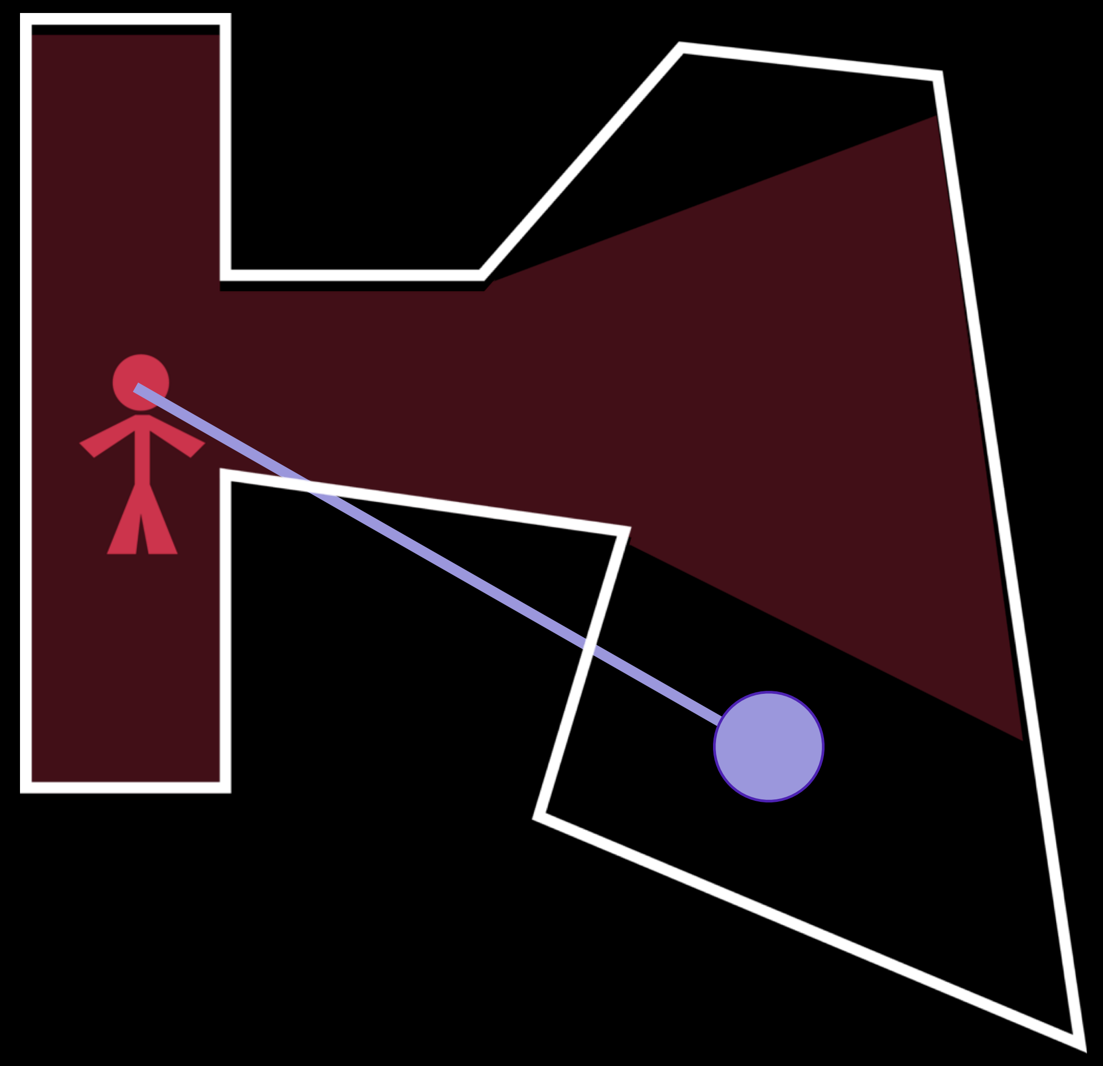

Line crosses at most 2 walls
⇒visible from the 2-transmitter

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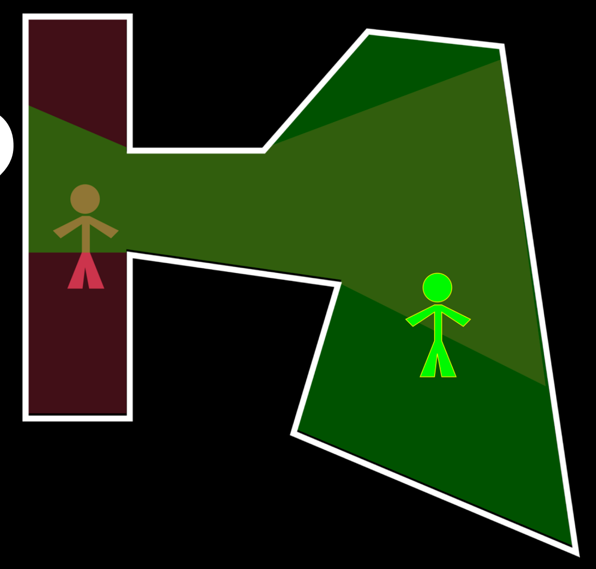

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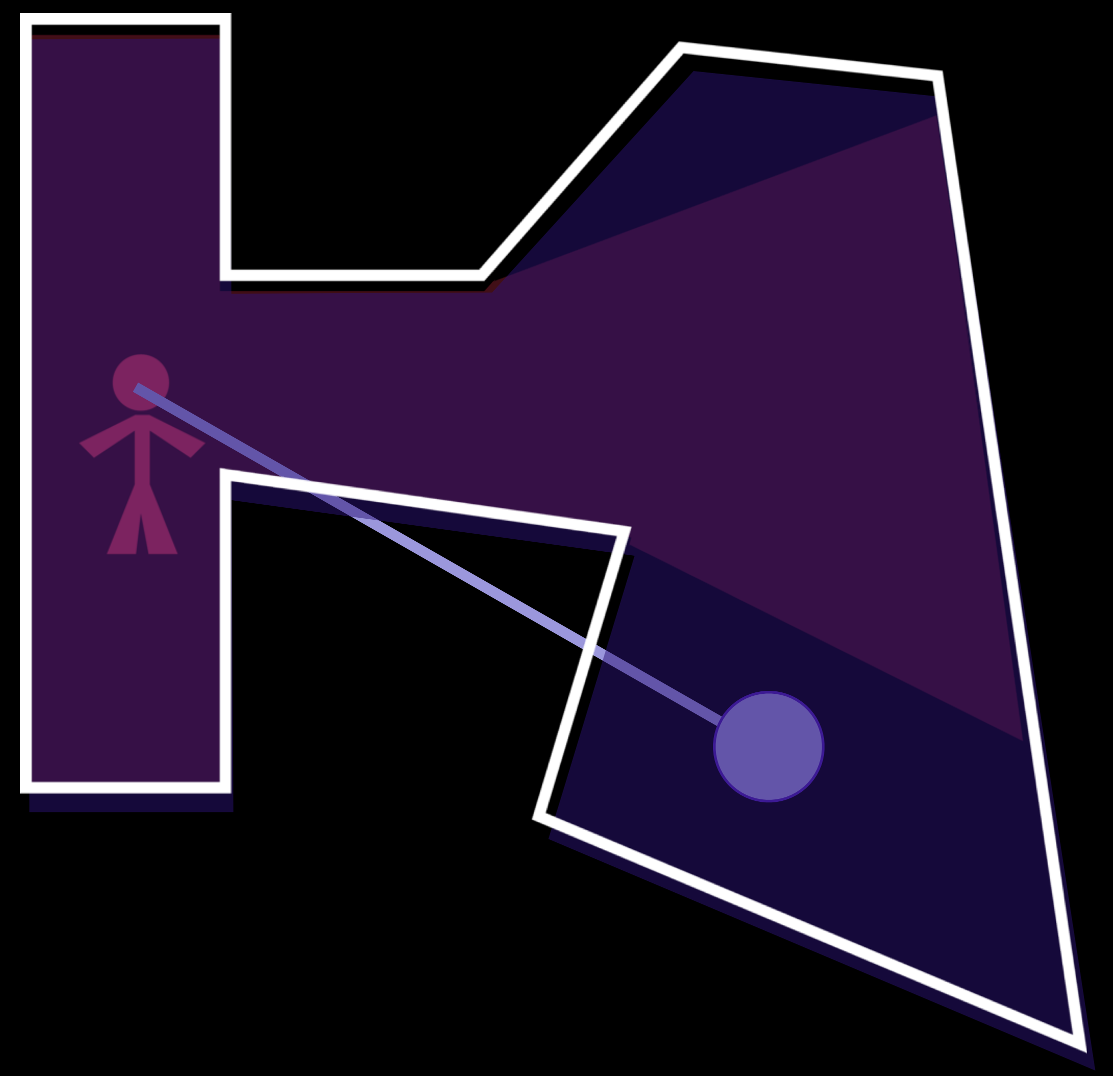

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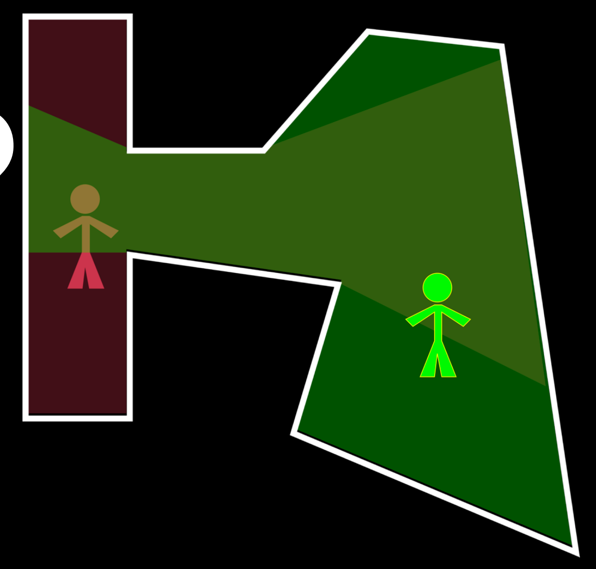

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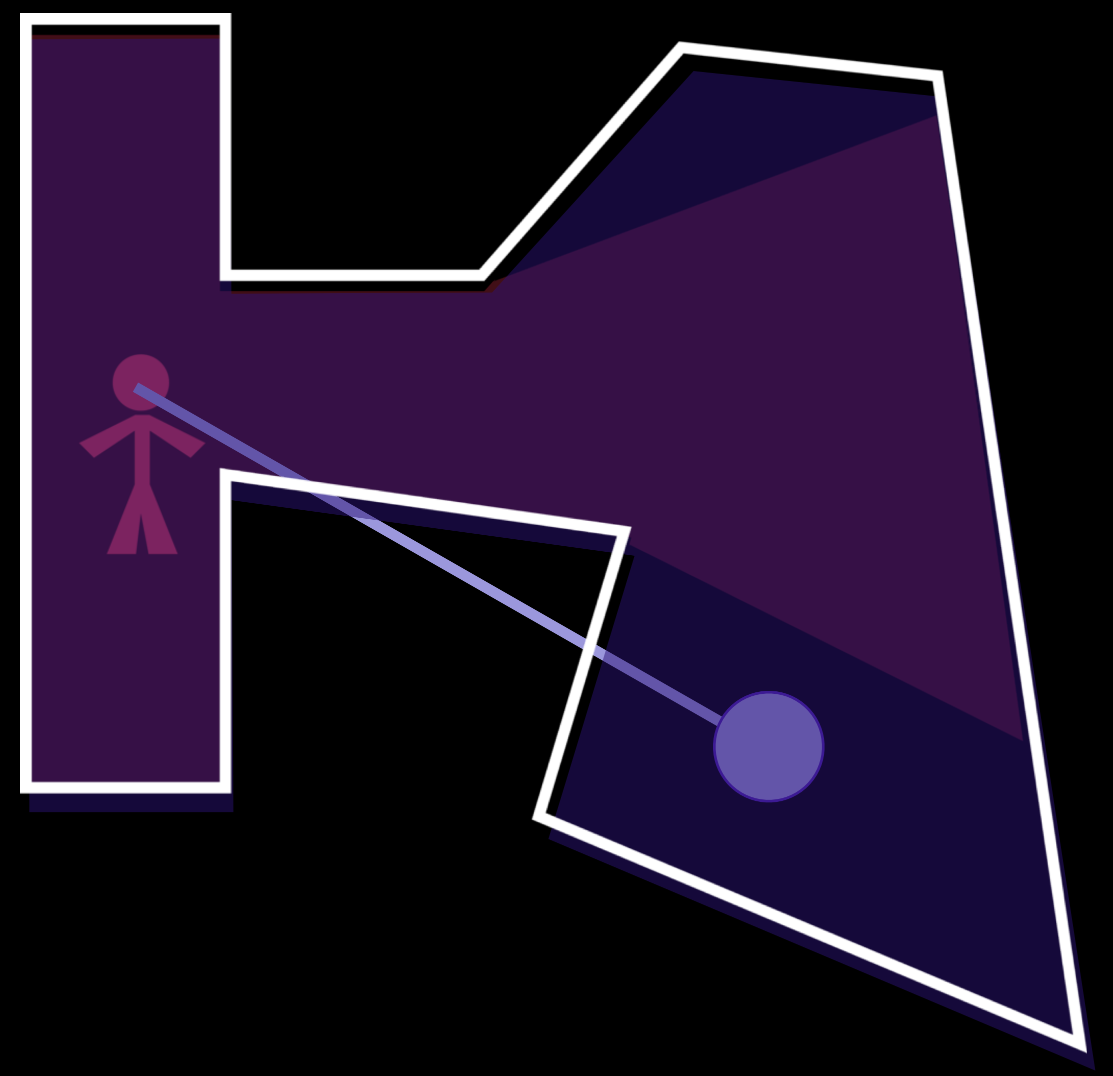

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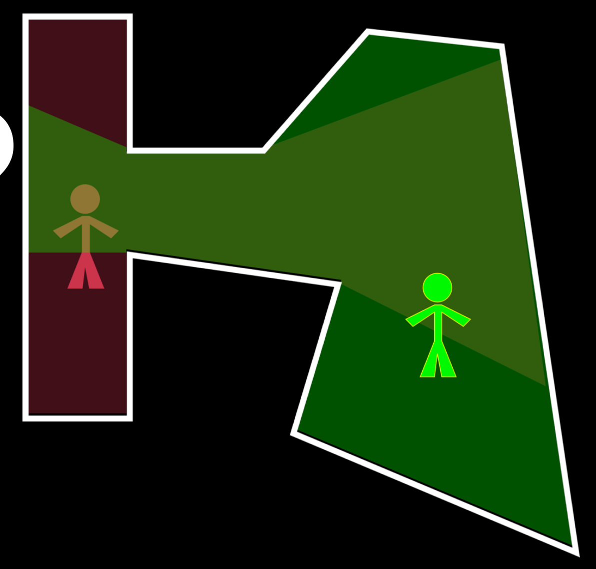

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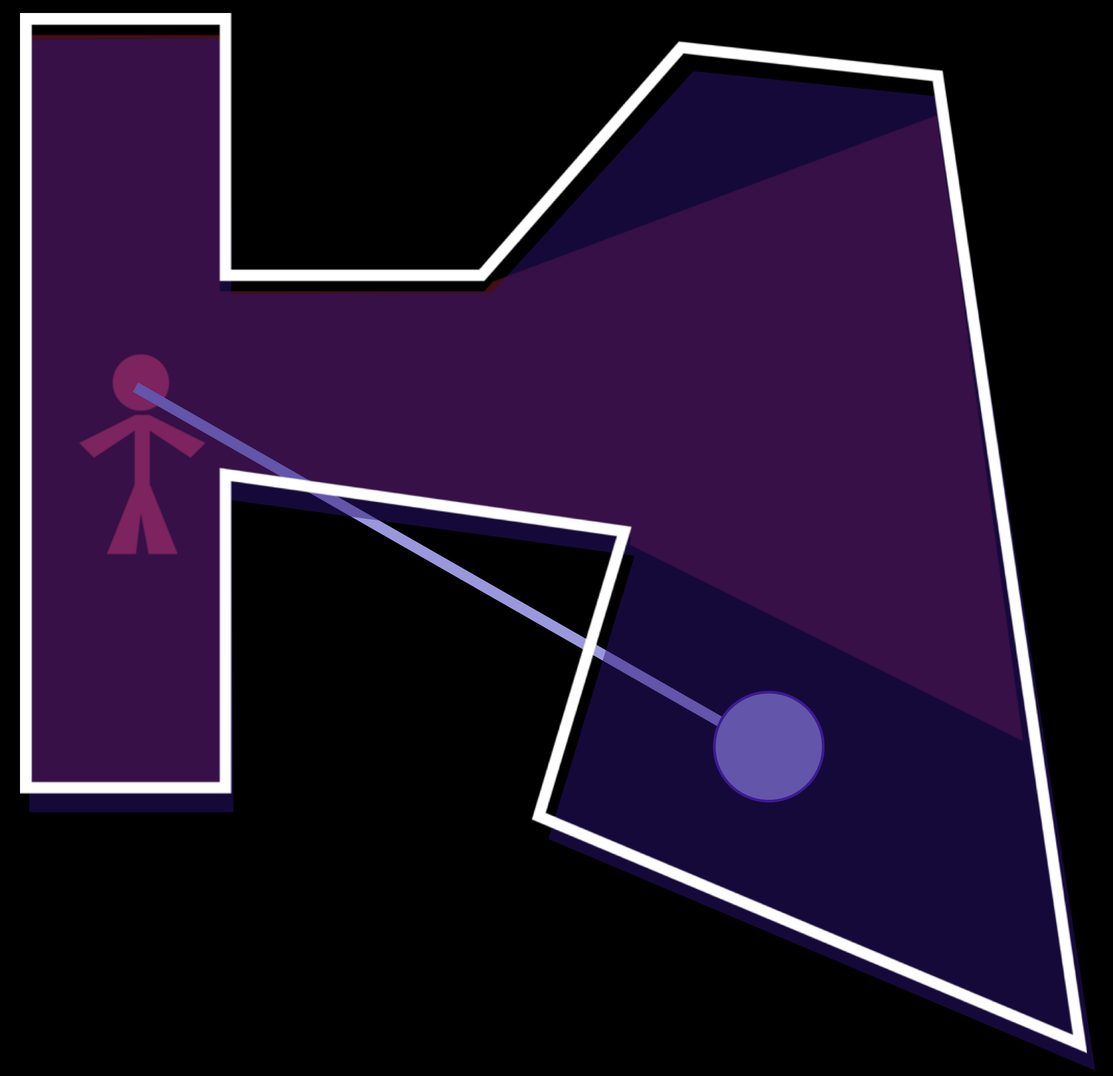

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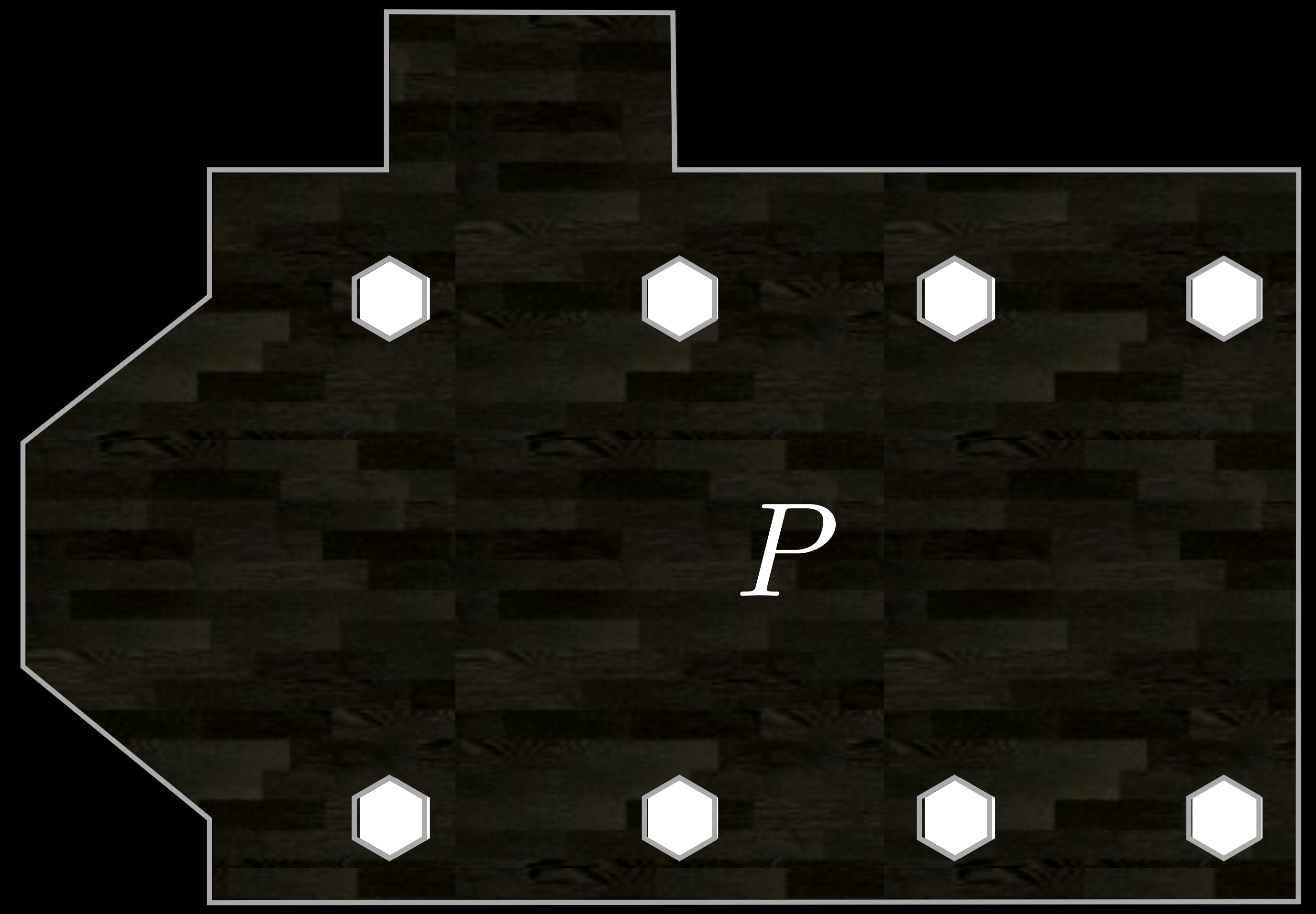

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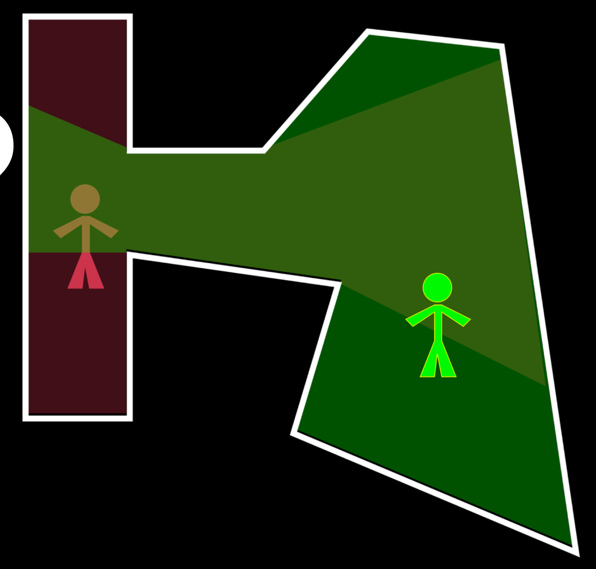

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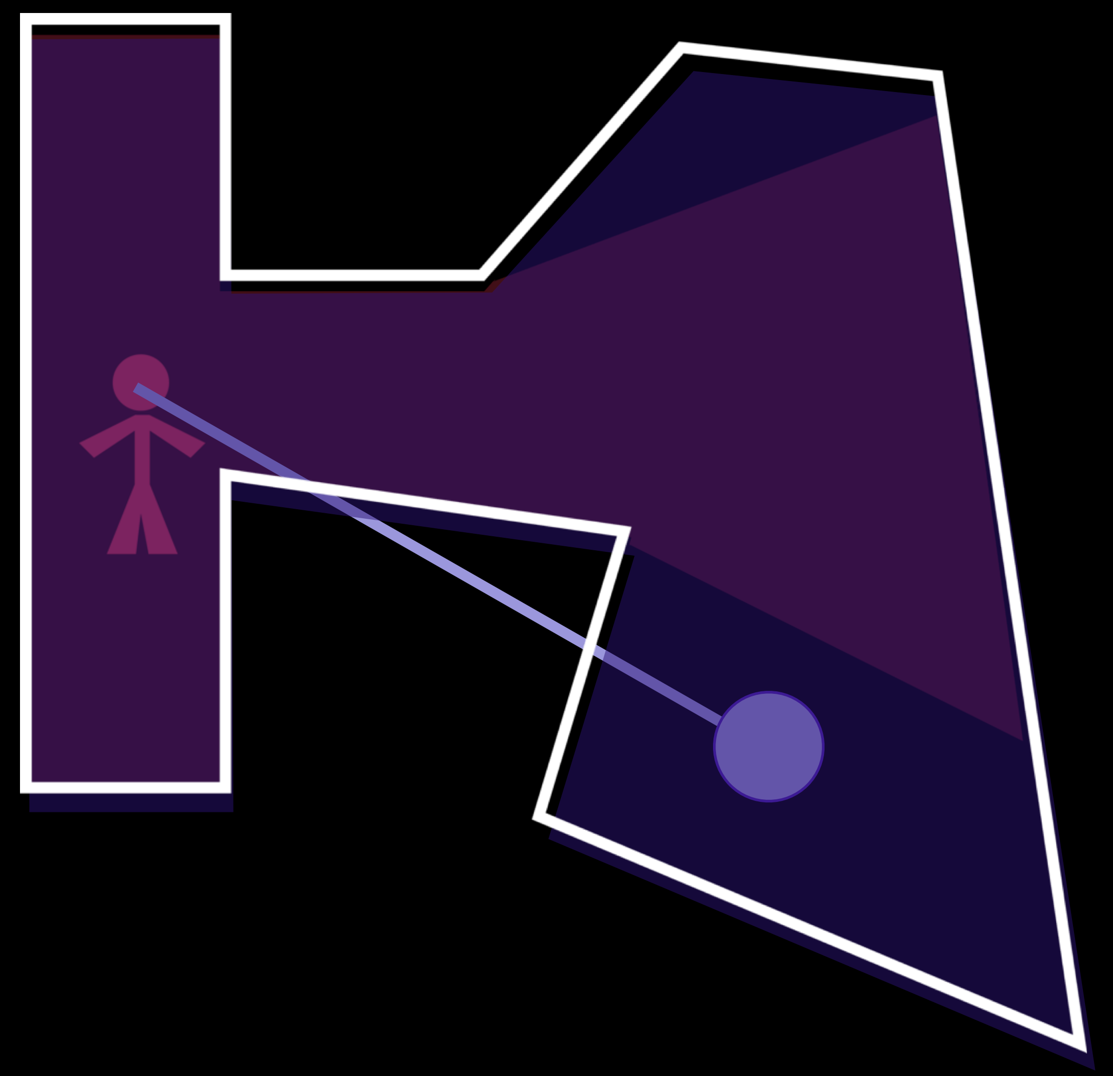

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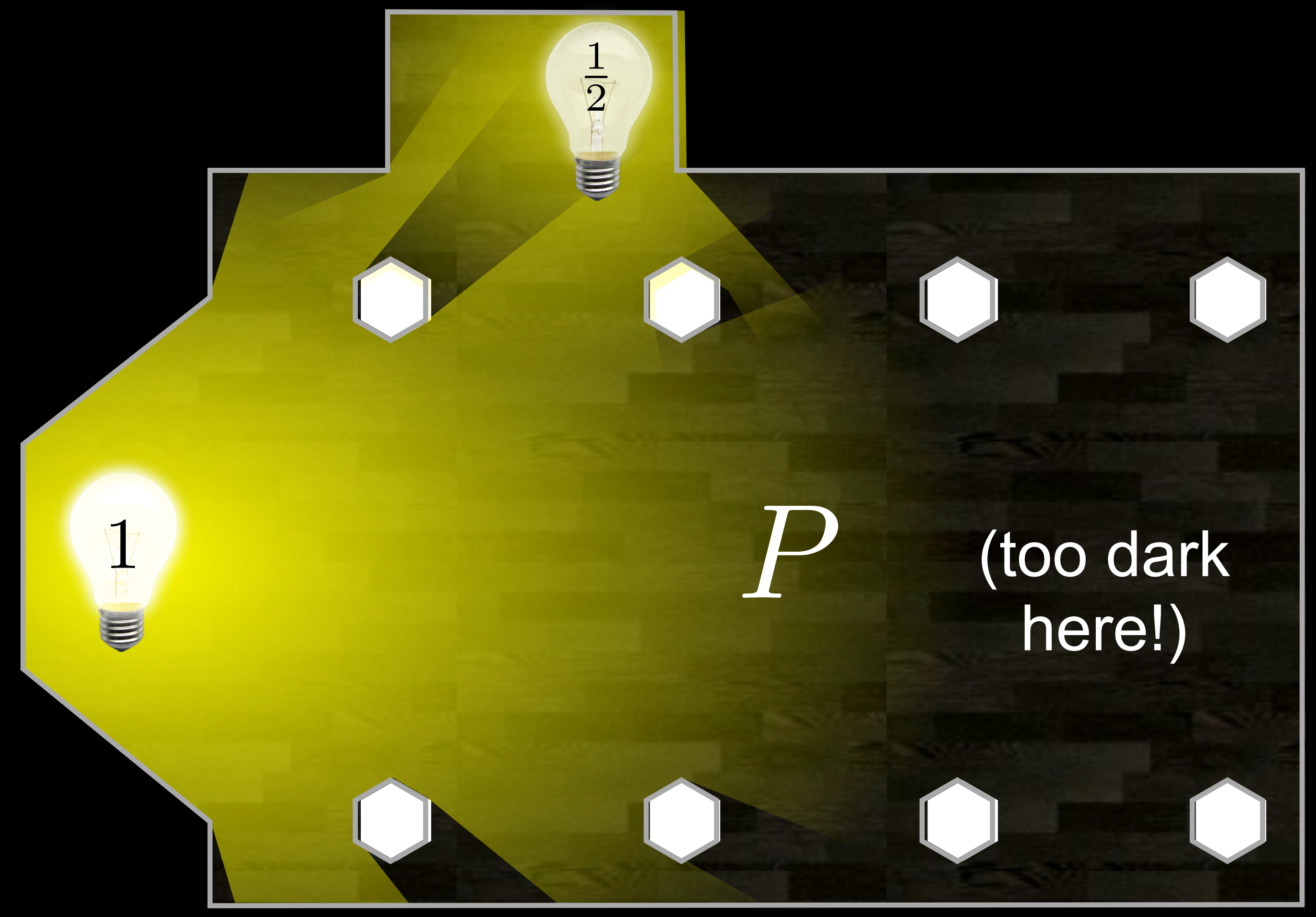

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assign energy (= brightness).

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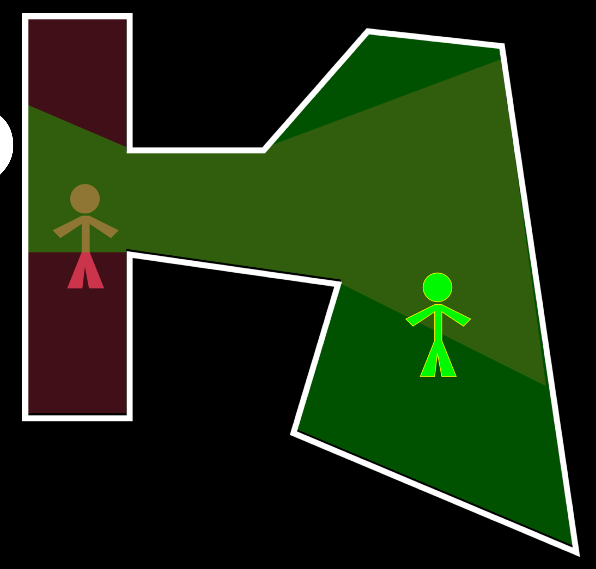

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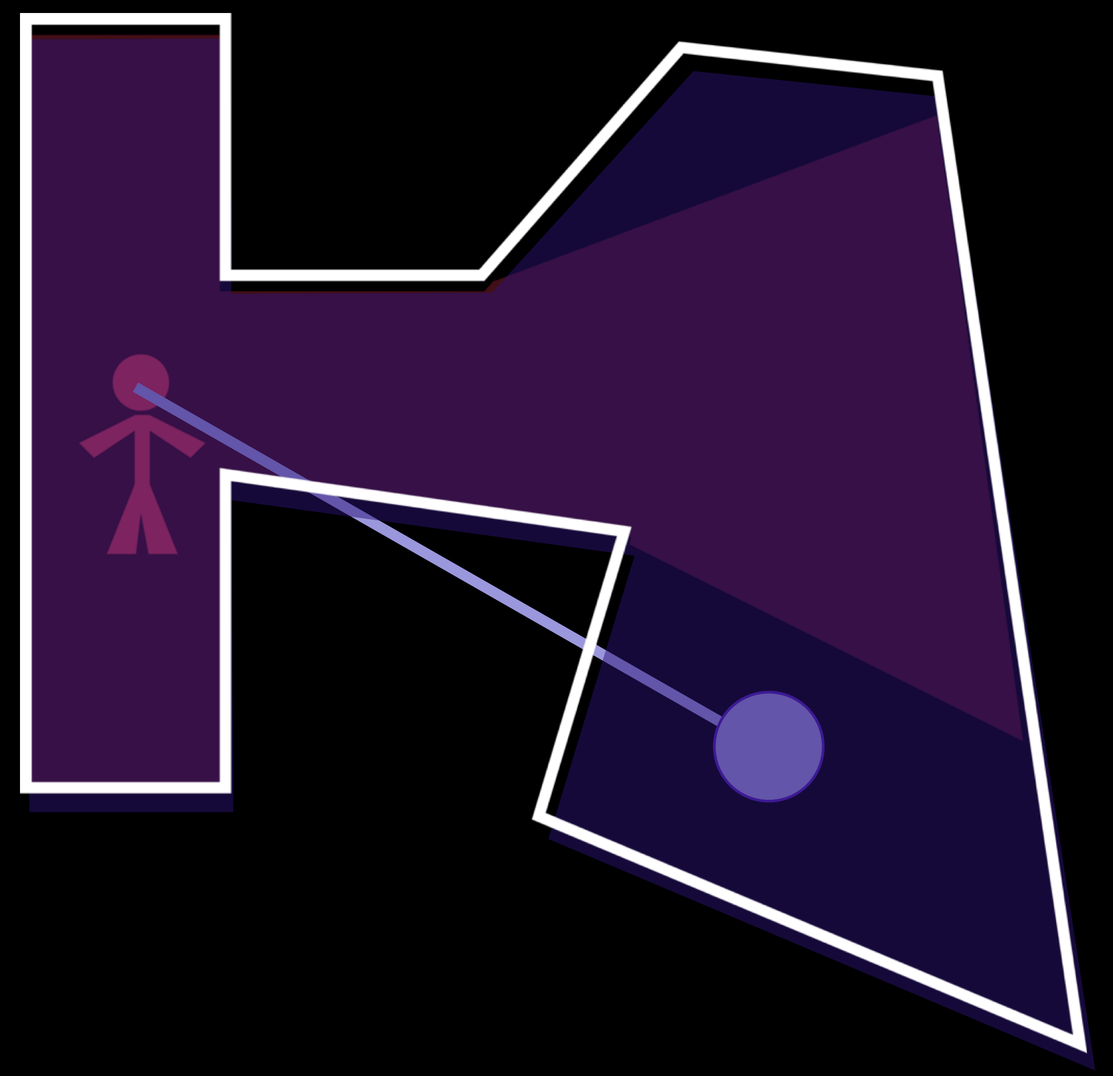

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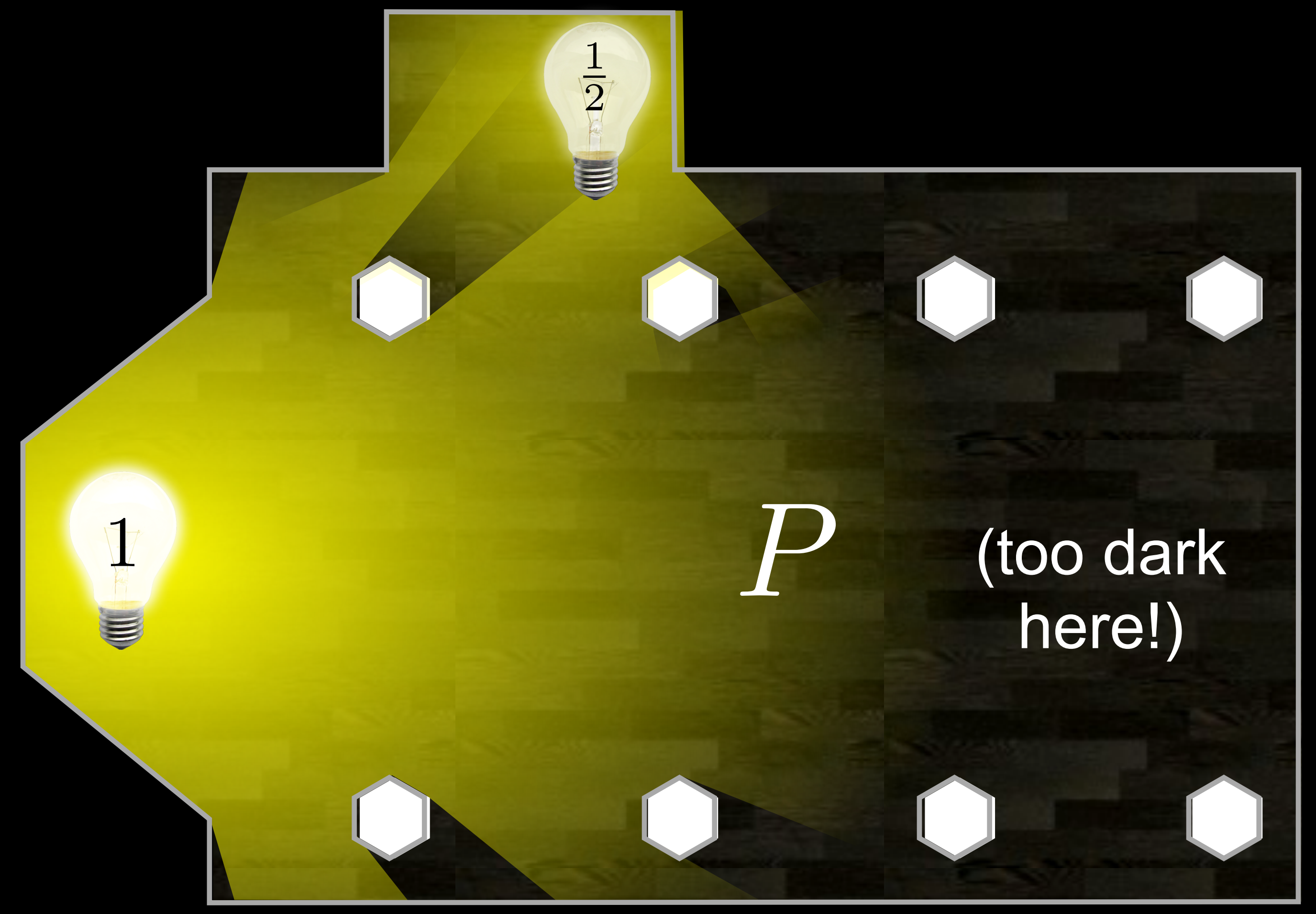

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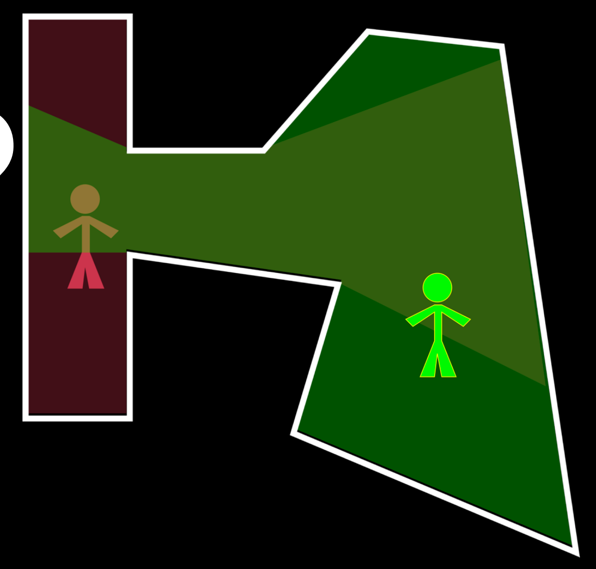
"Sufficiently" (normalize to 1) light everything — with fading!

The Art Gallery Problem (AGP)—and Its Variants?



We can alter:

- Capabilities of the guards
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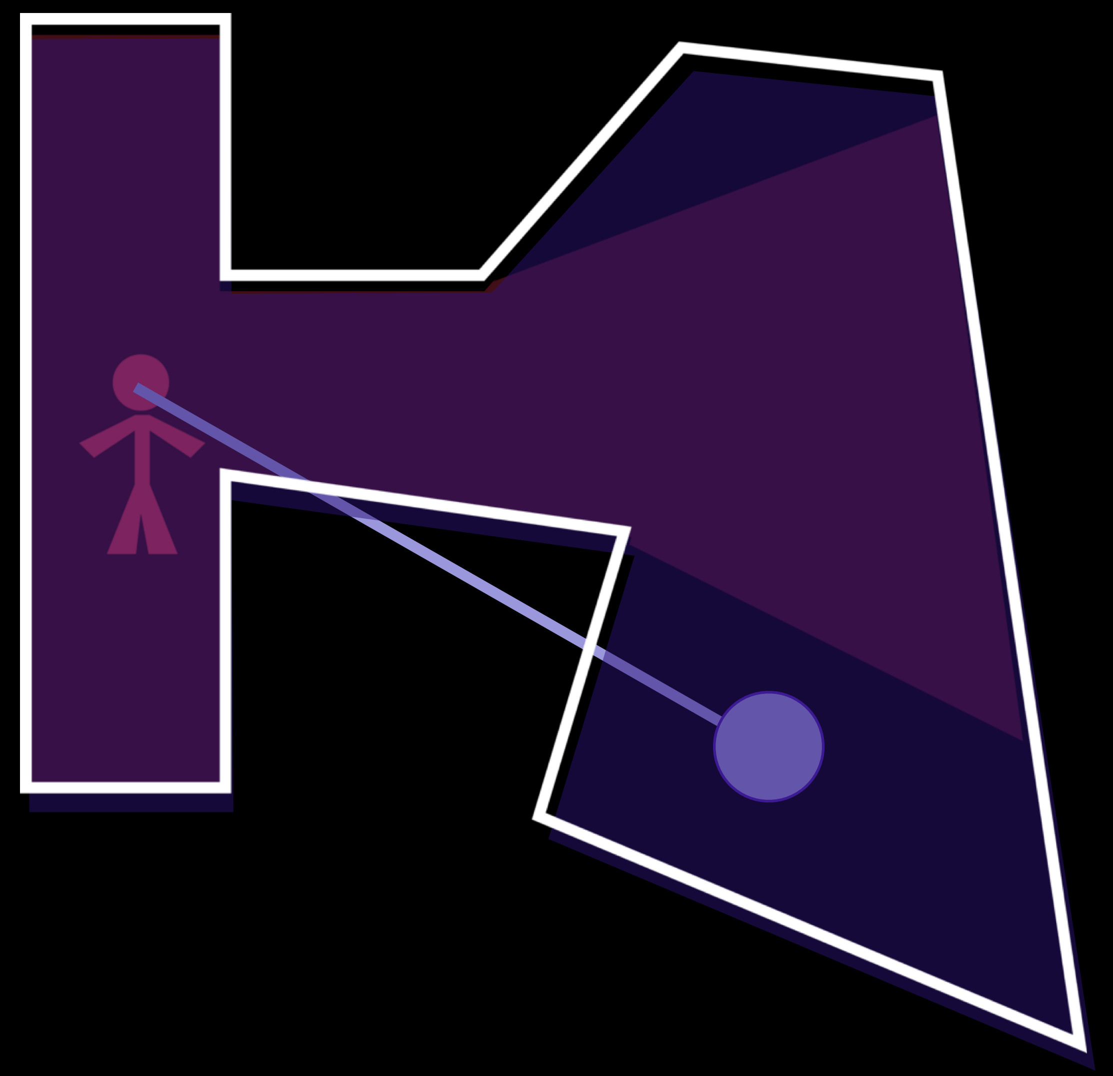
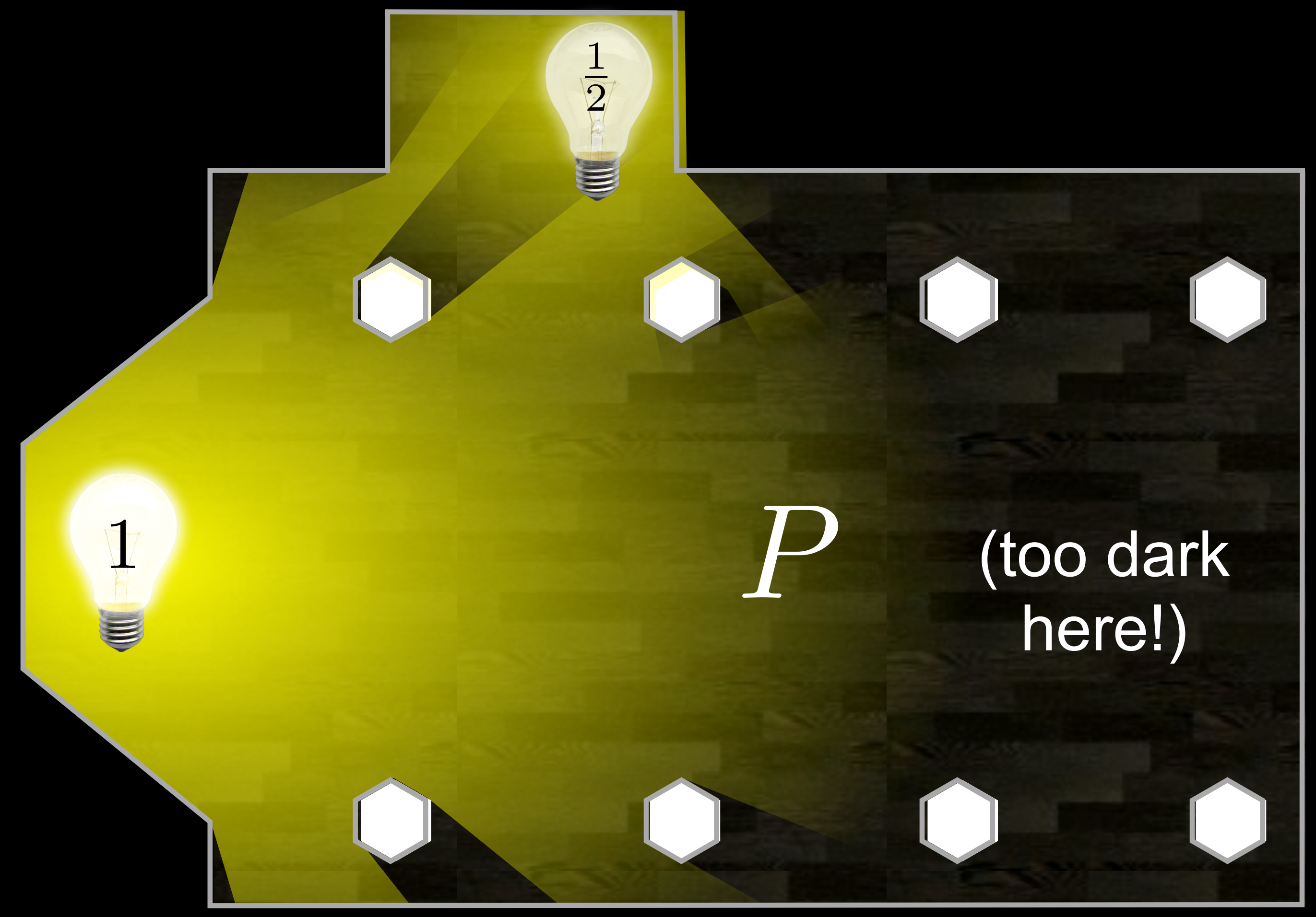
k-transmitter:



Line crosses at most 2 walls
⇒visible from the 2-transmitter

Fading:



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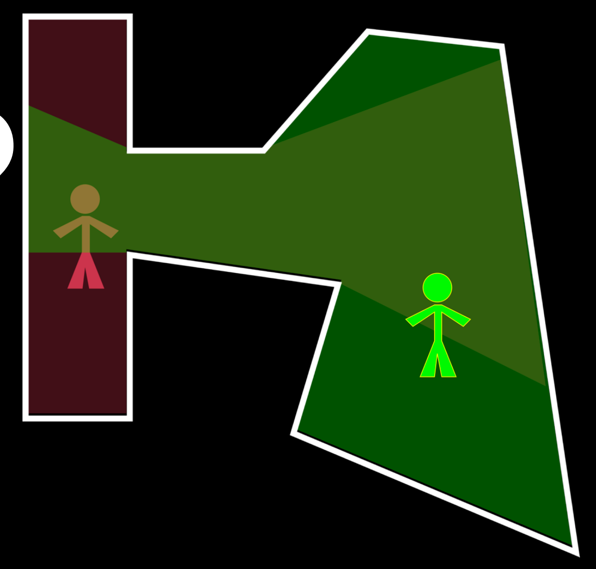

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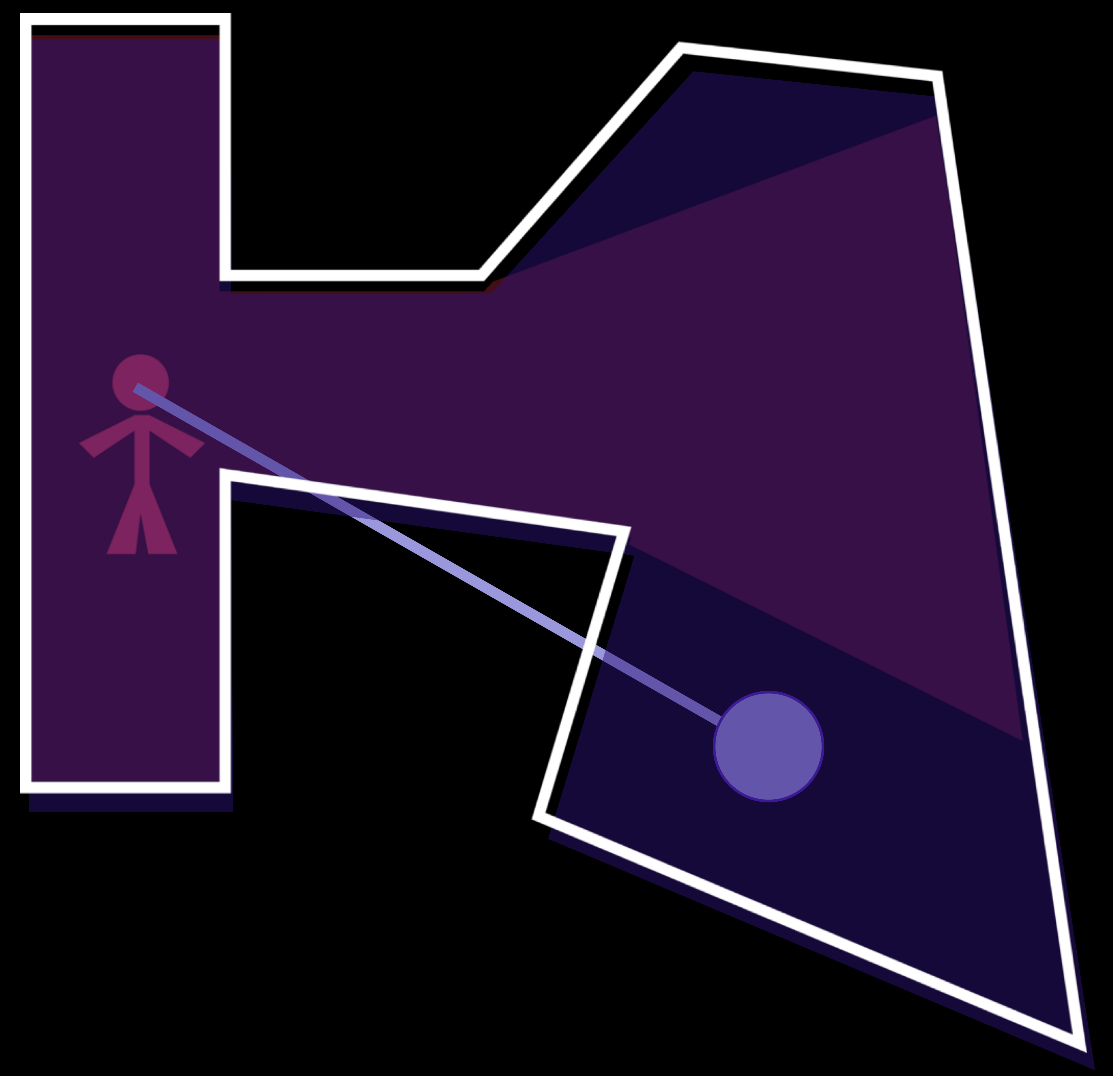

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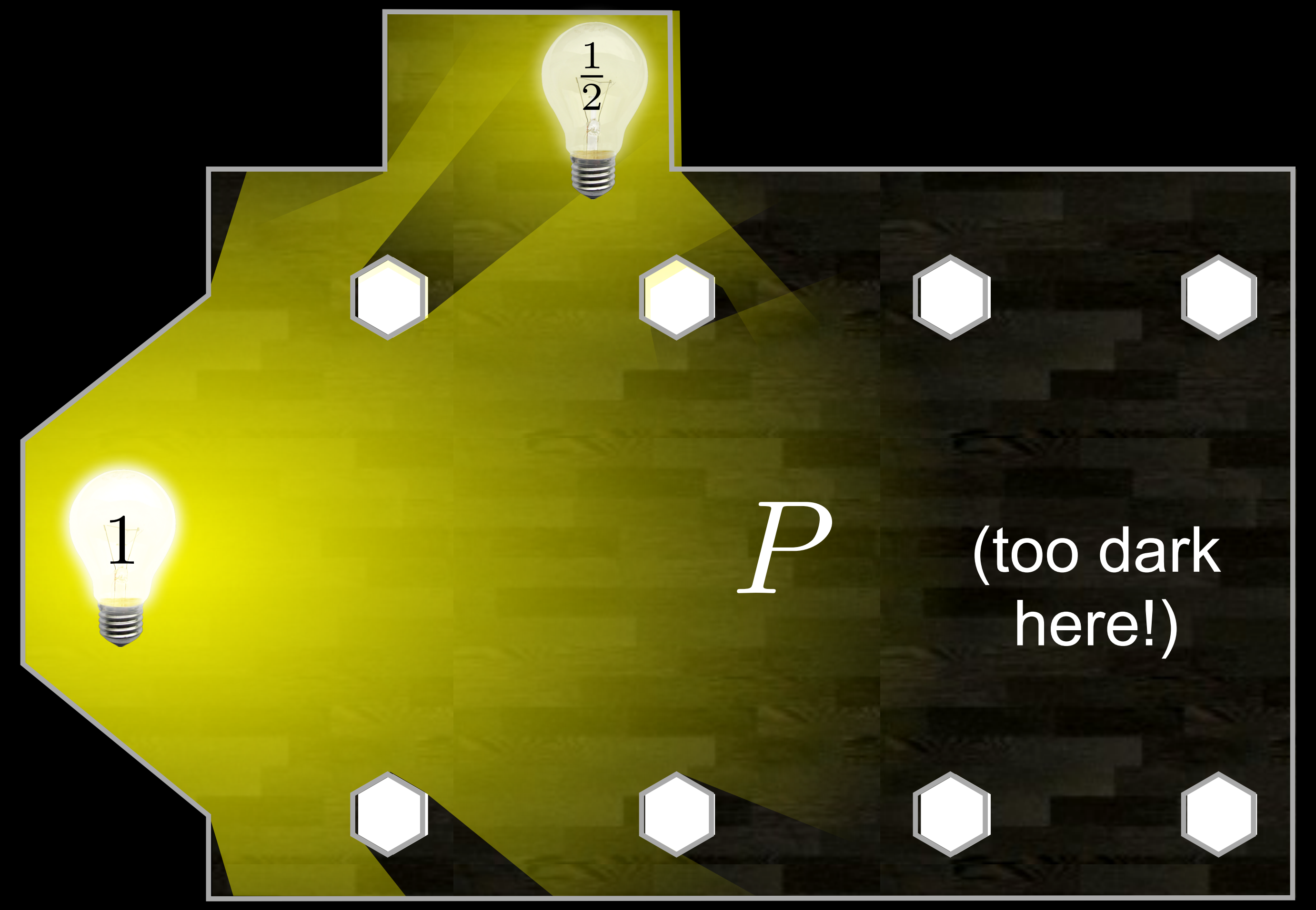

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Given: a polygon P

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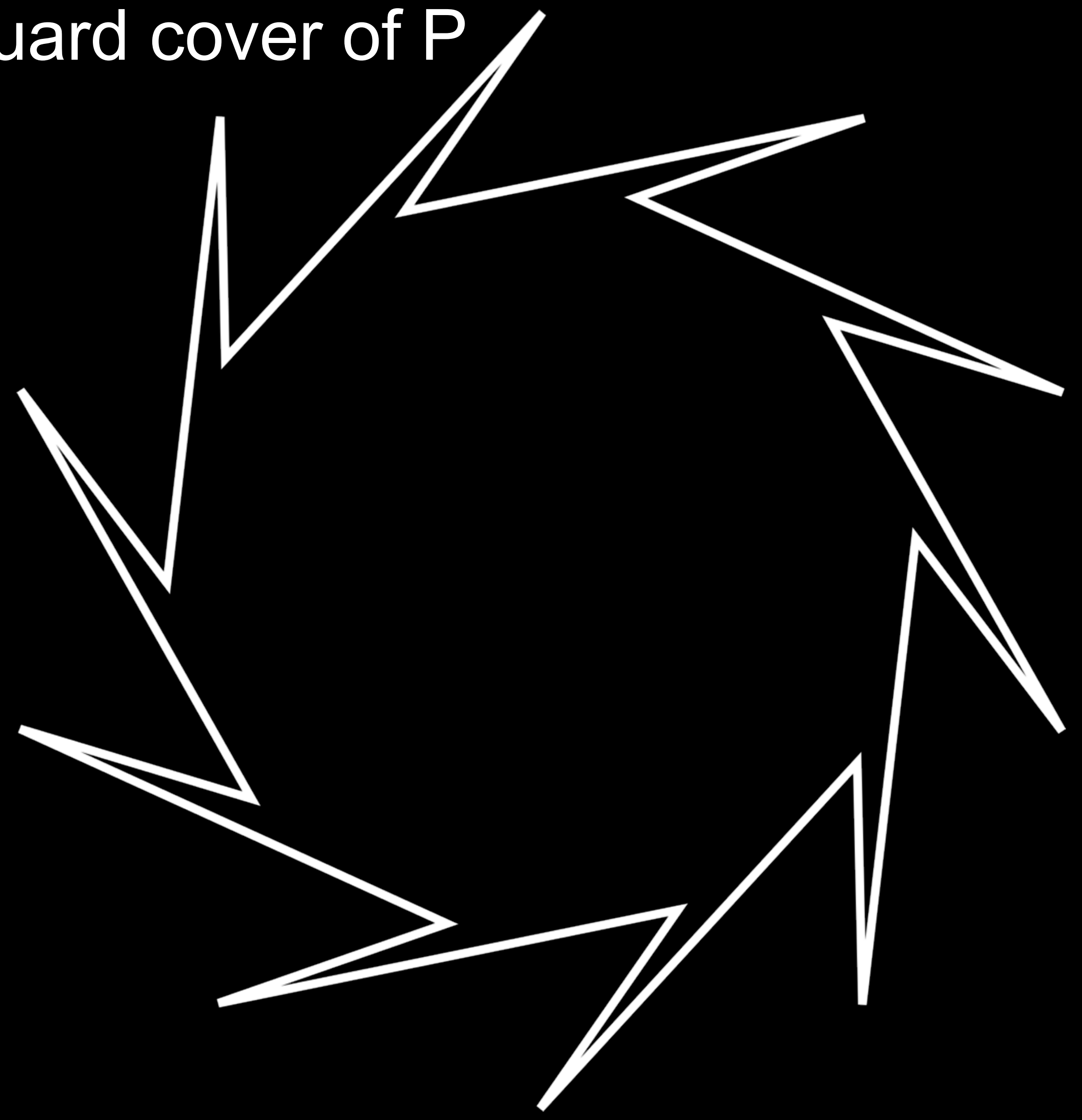

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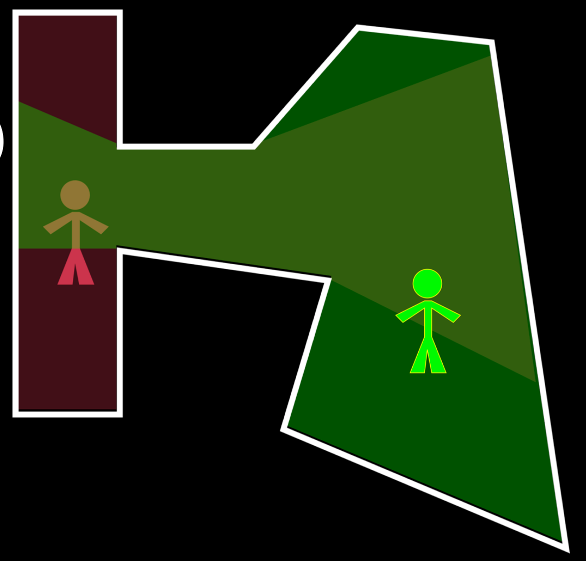

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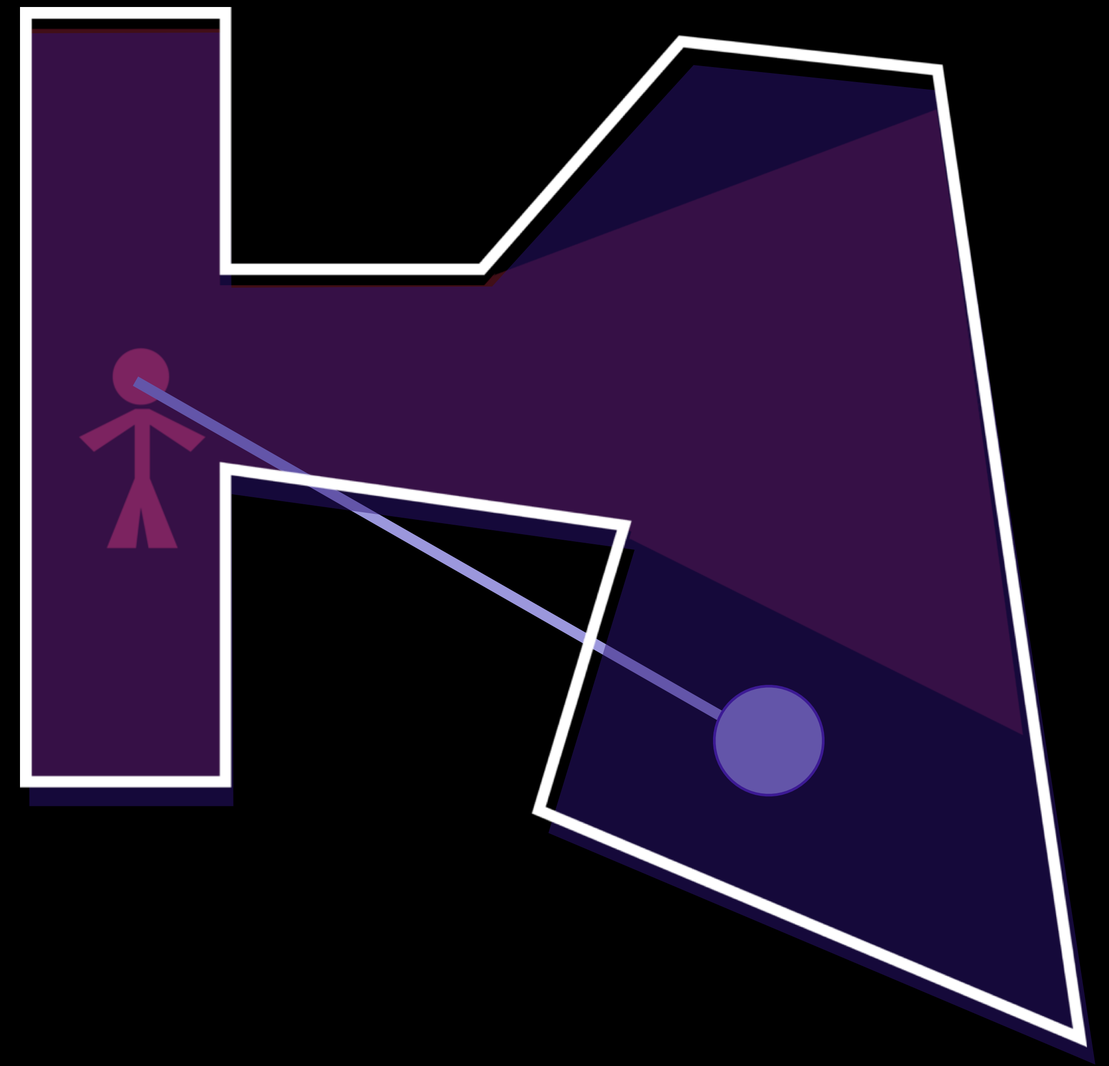

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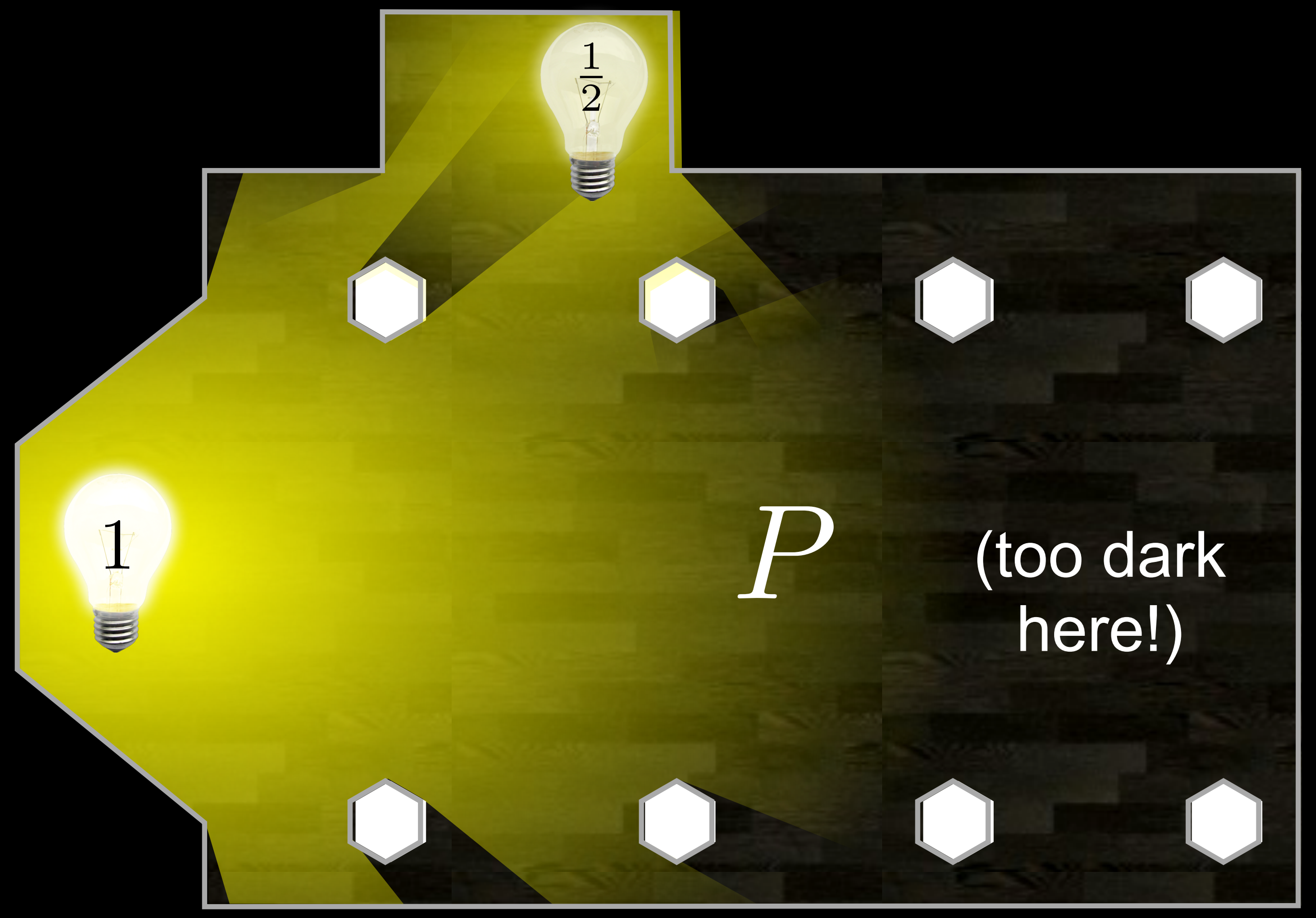

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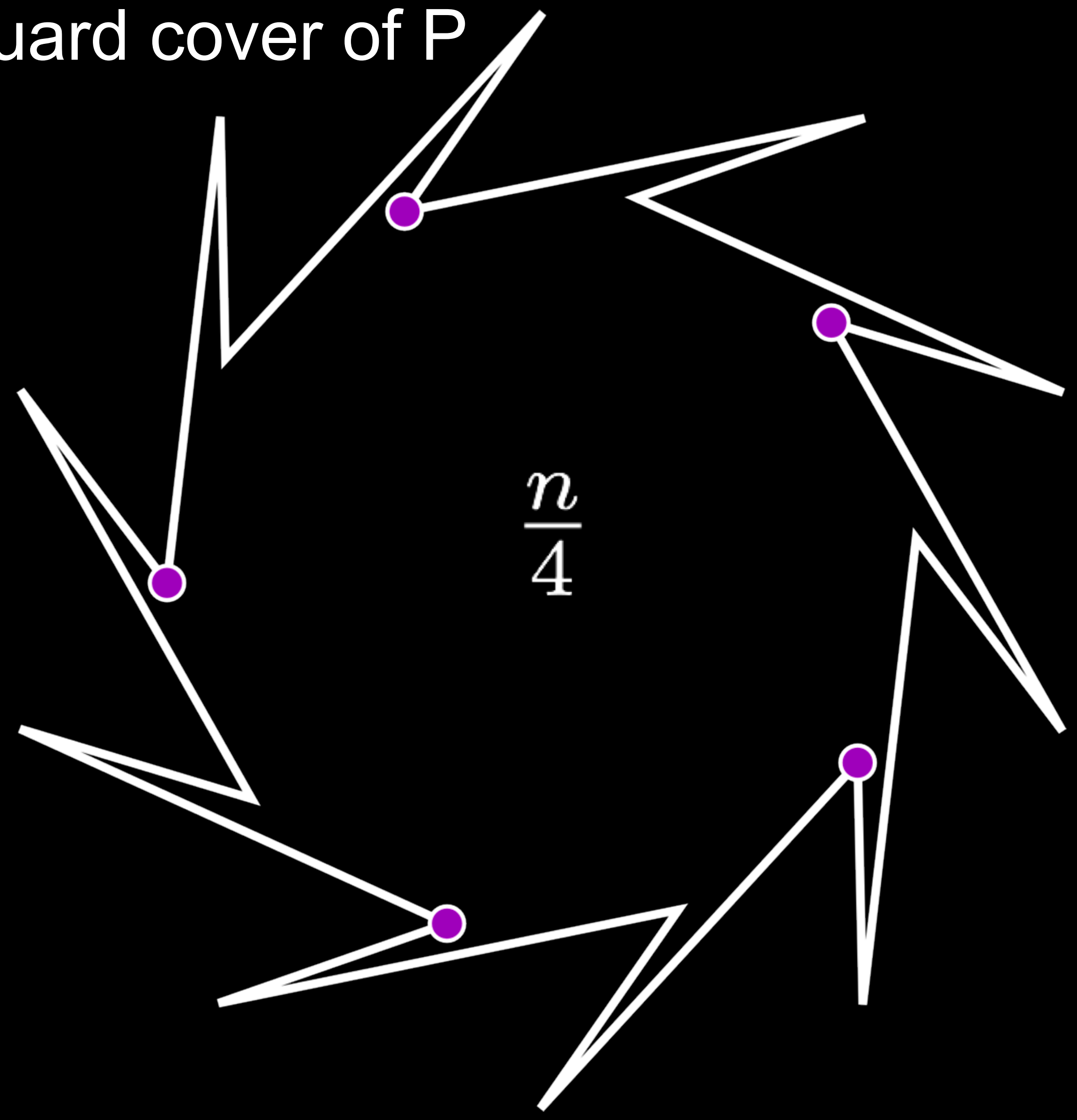

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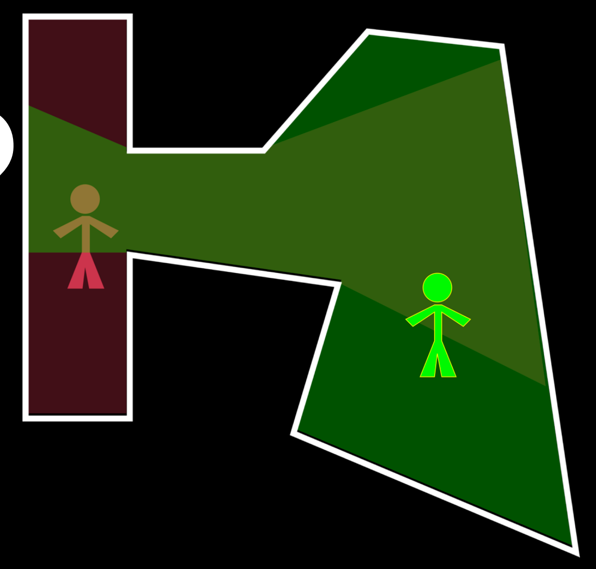

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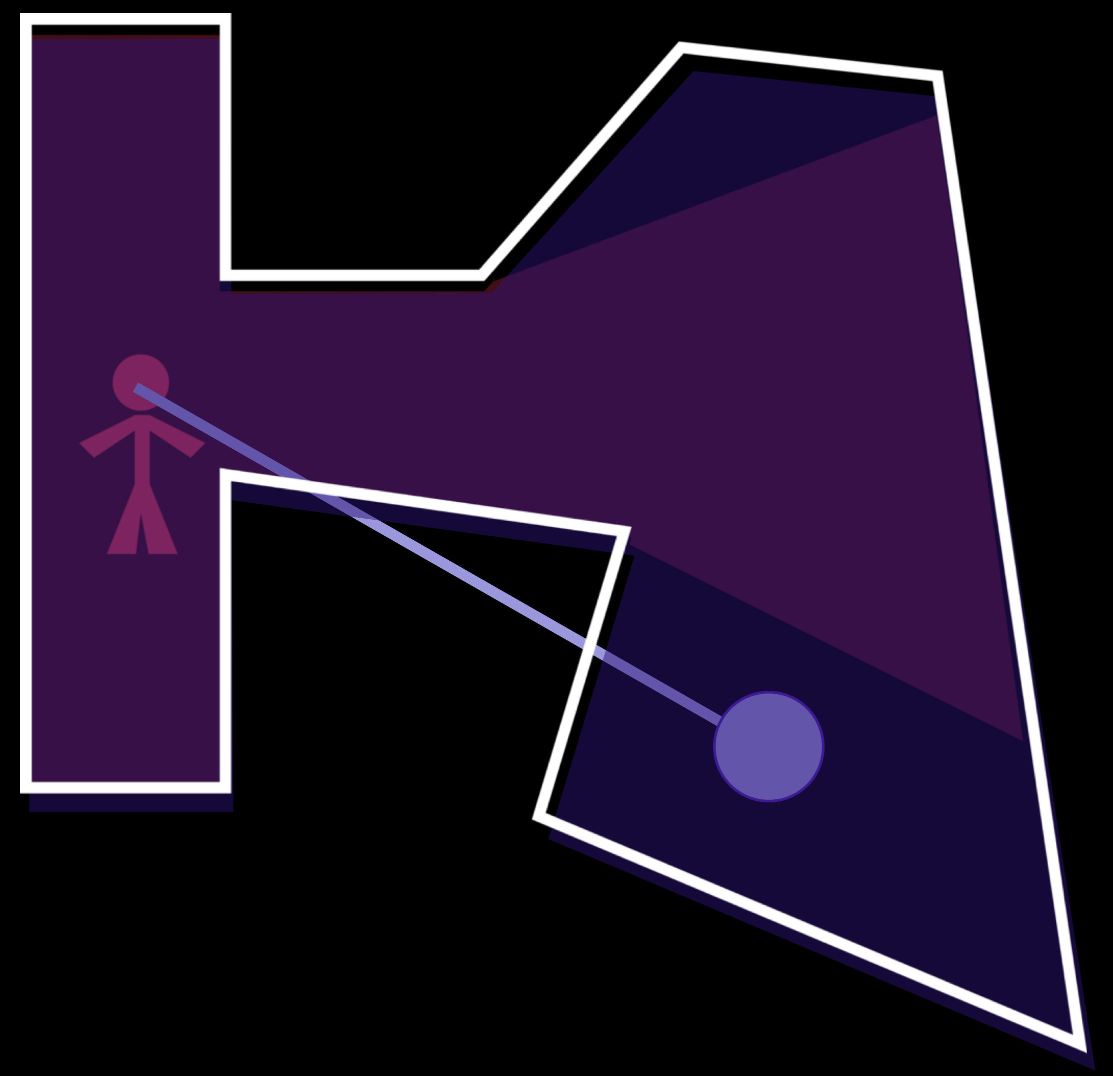

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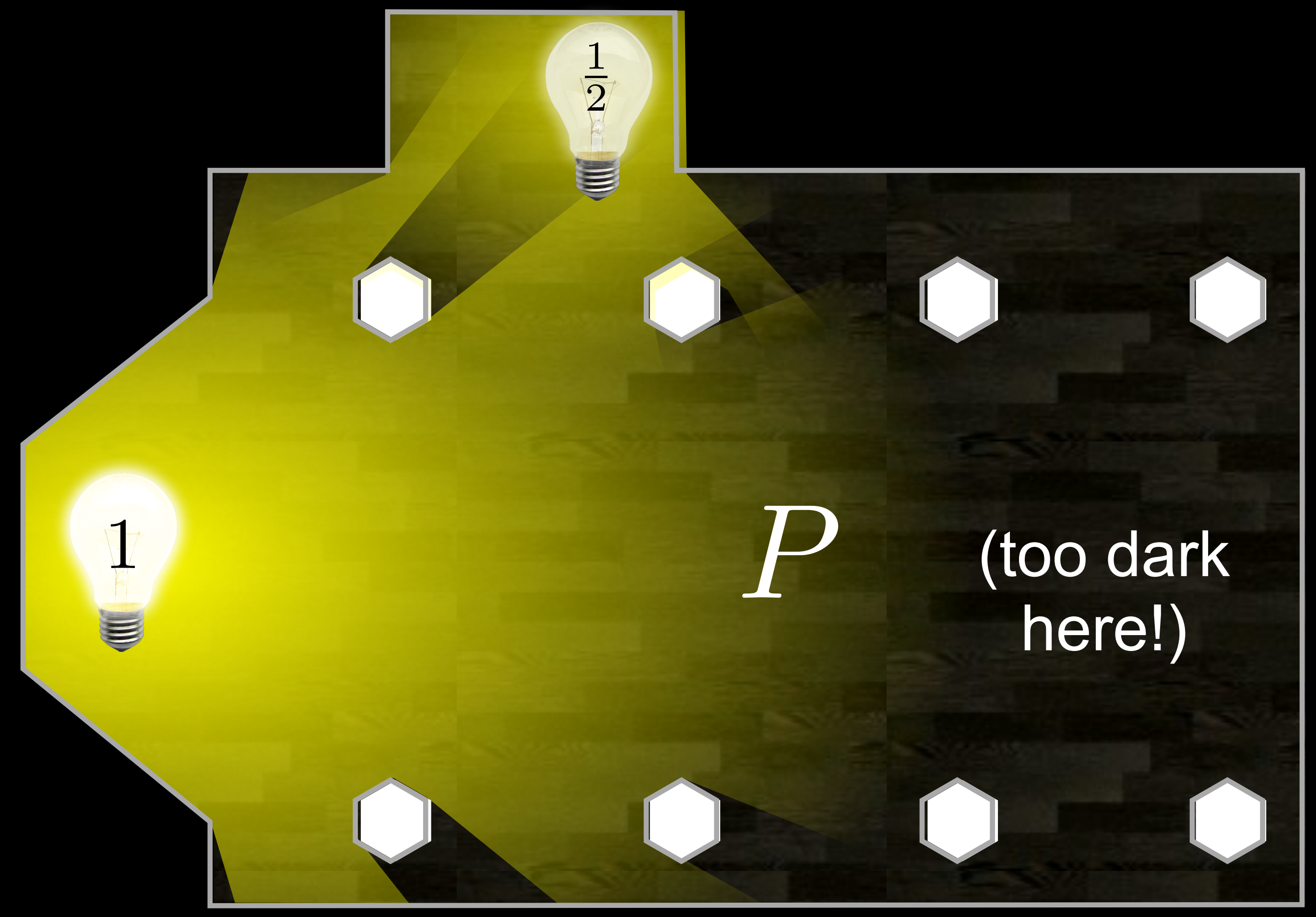

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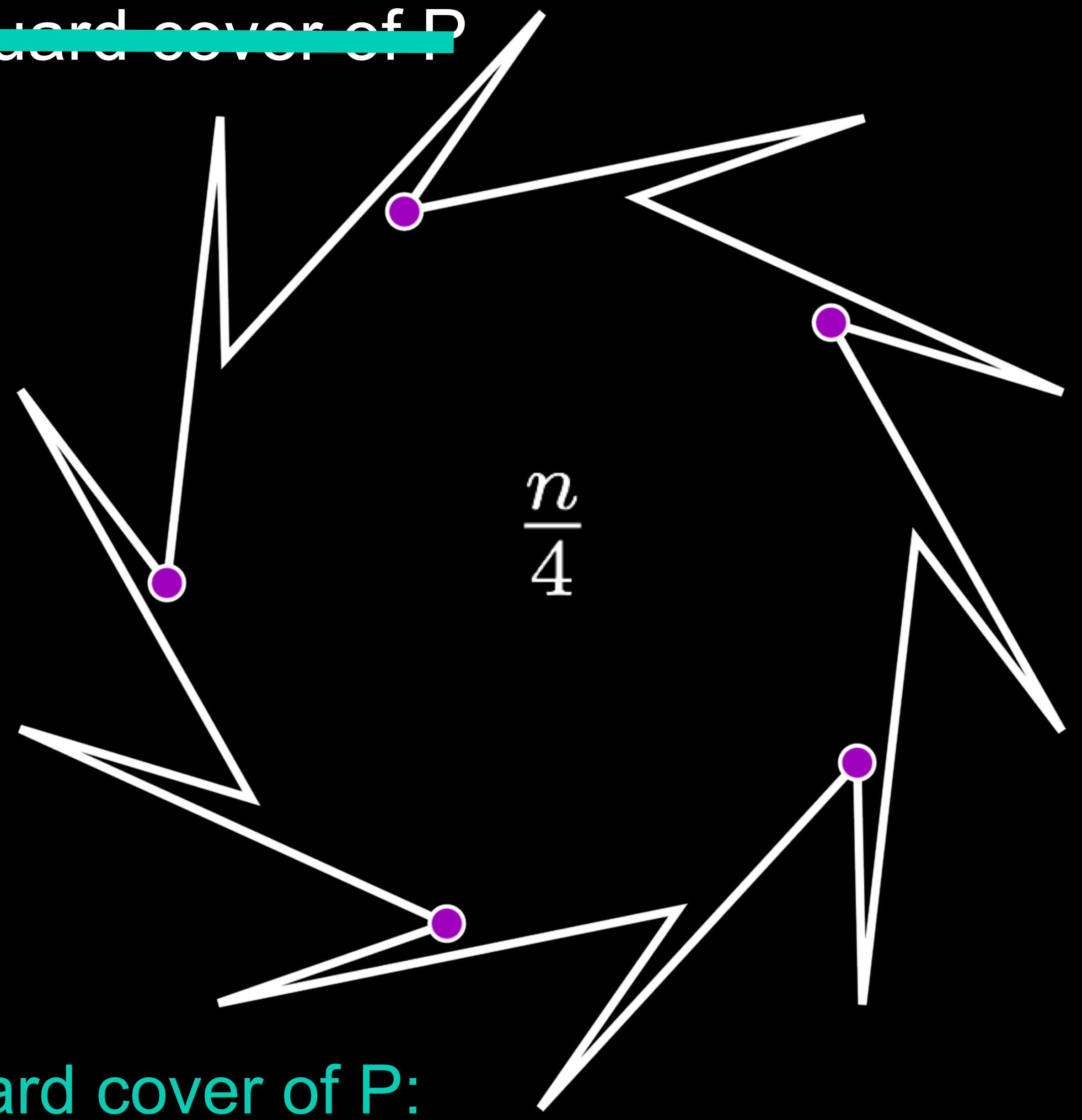

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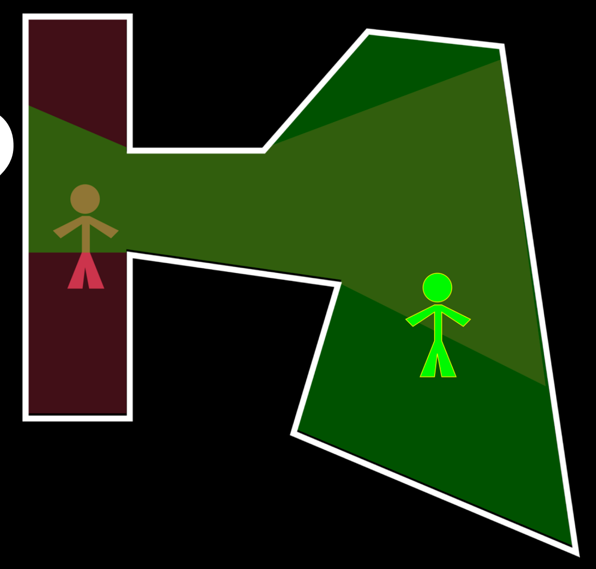

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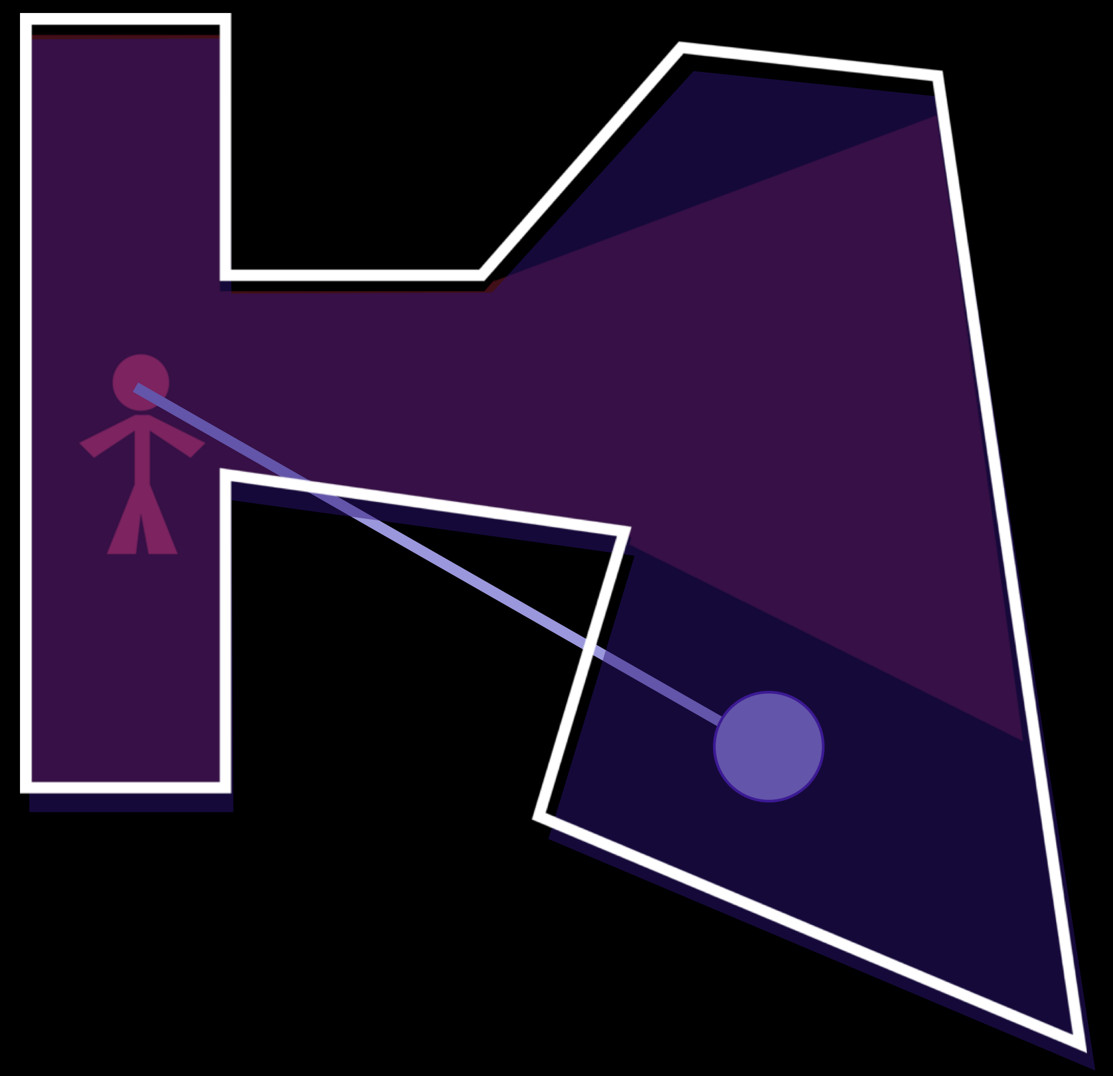

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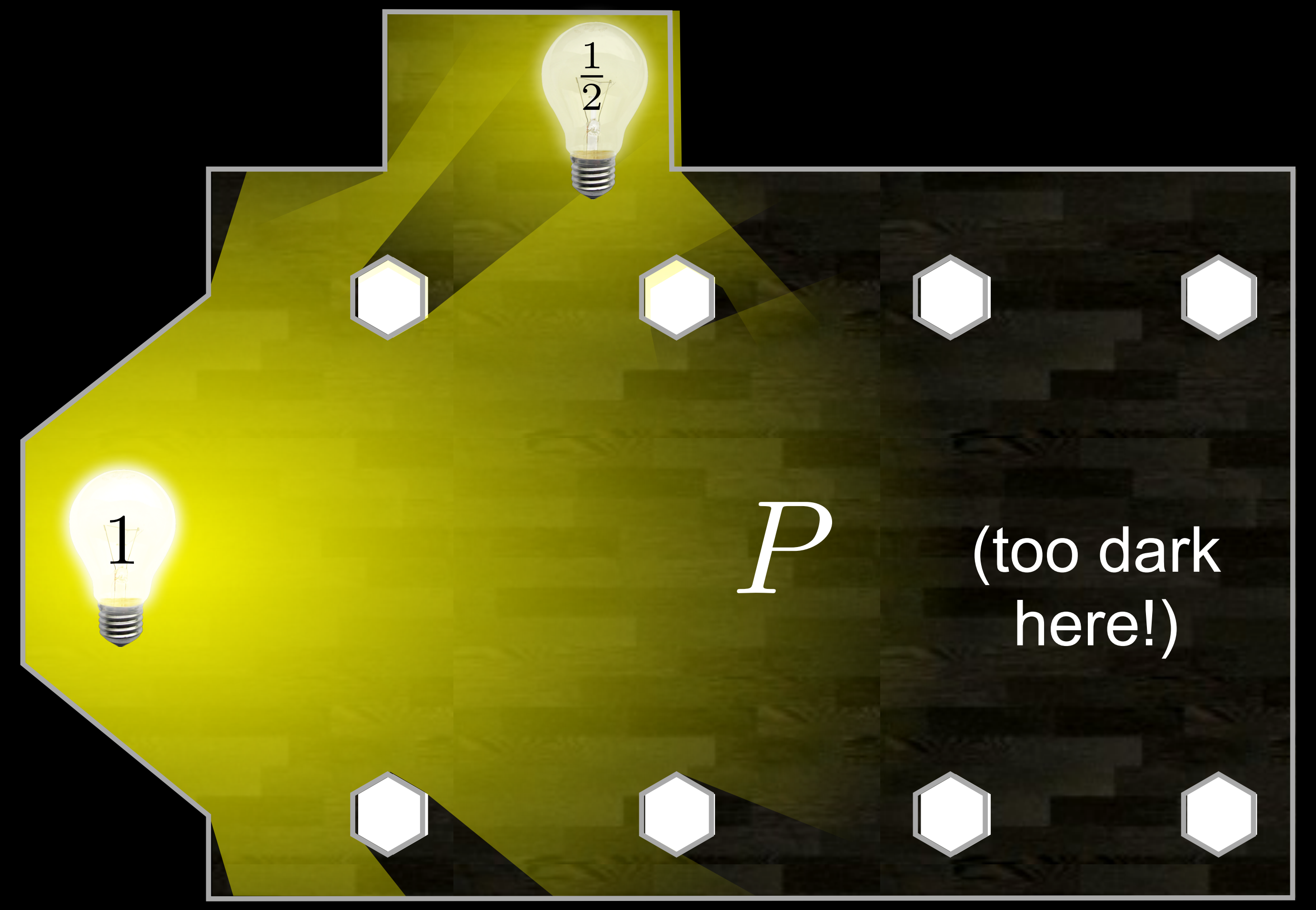

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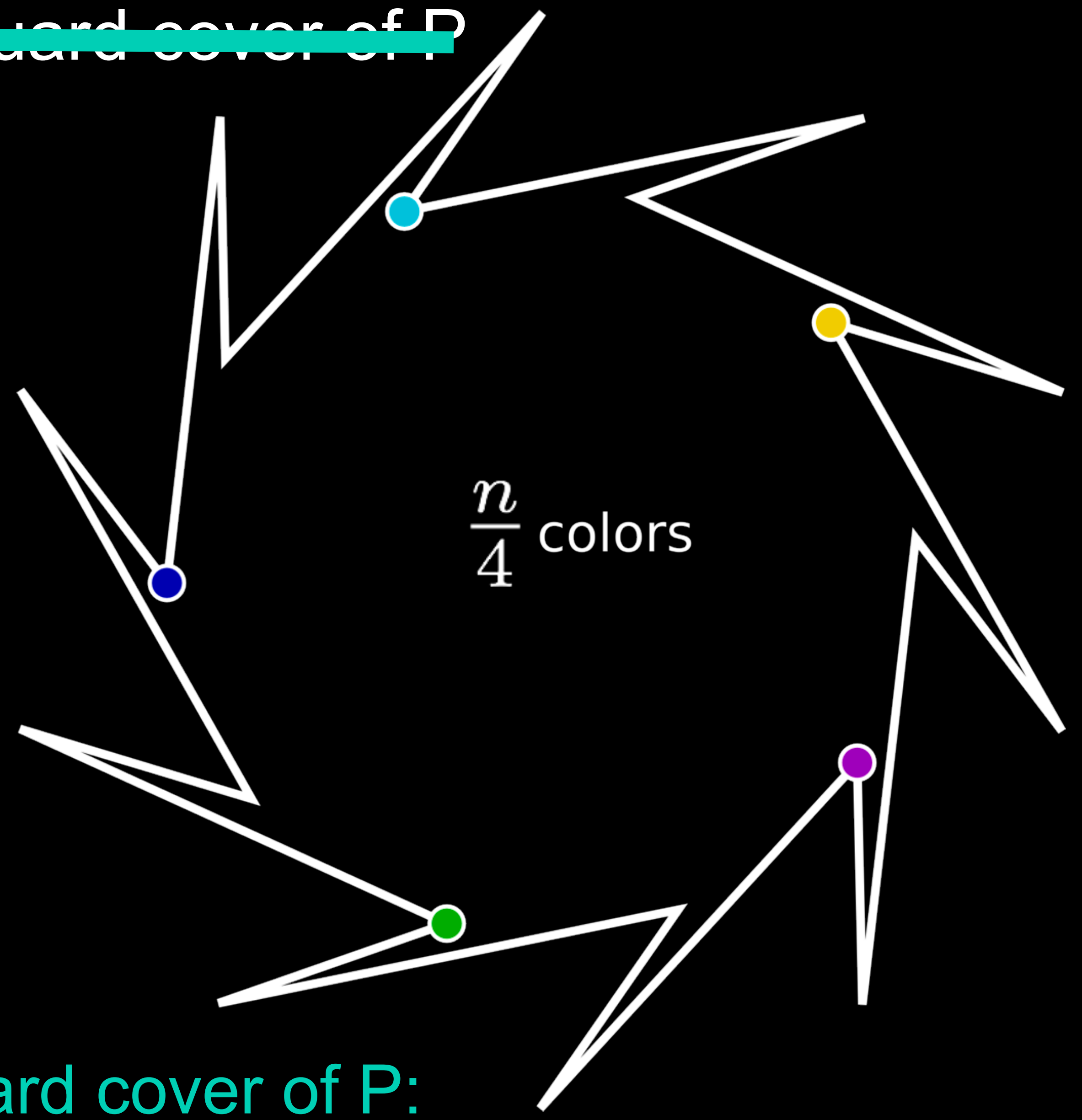

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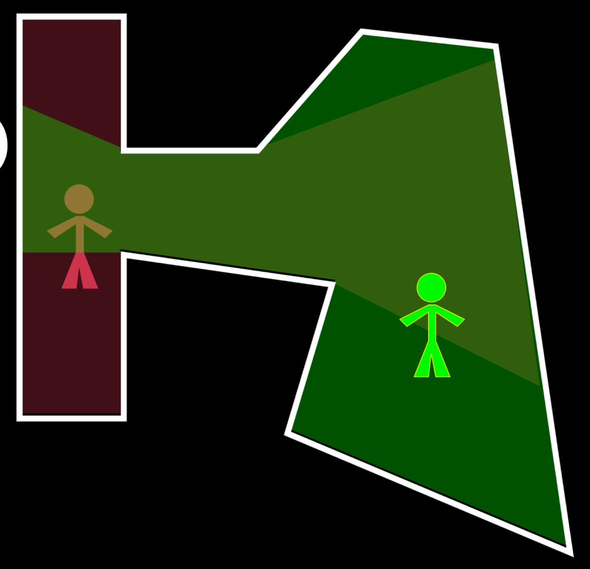

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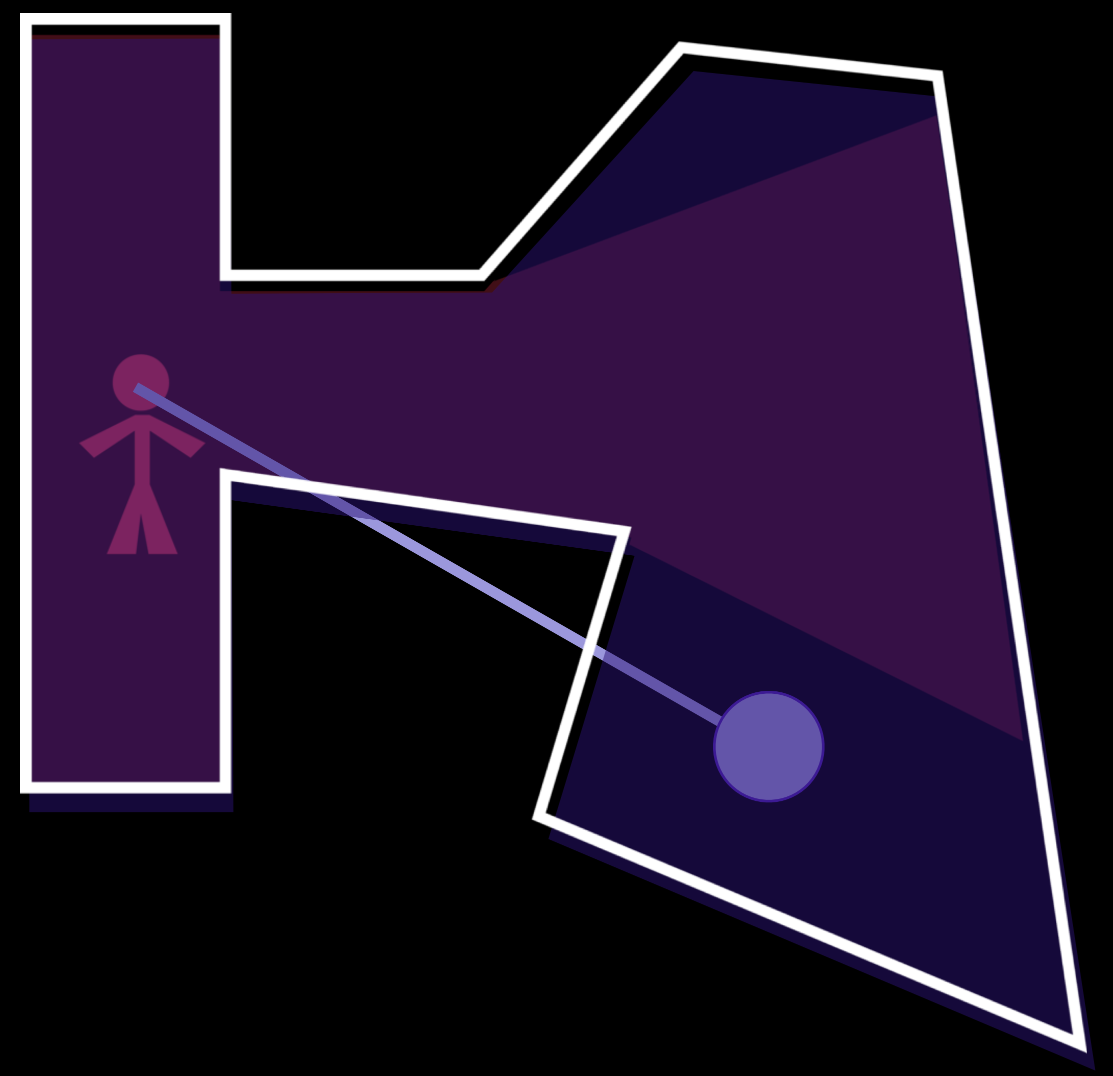

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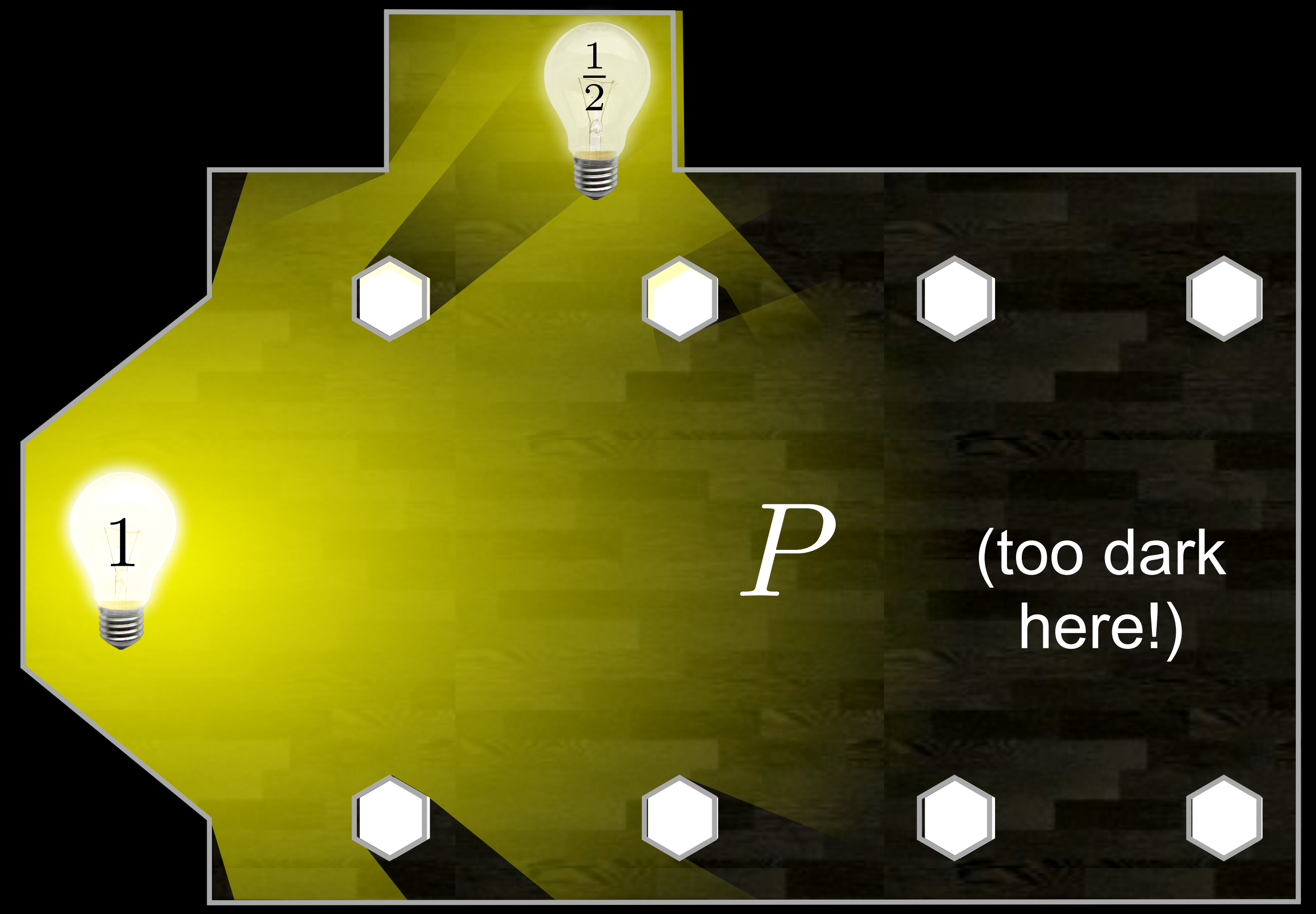

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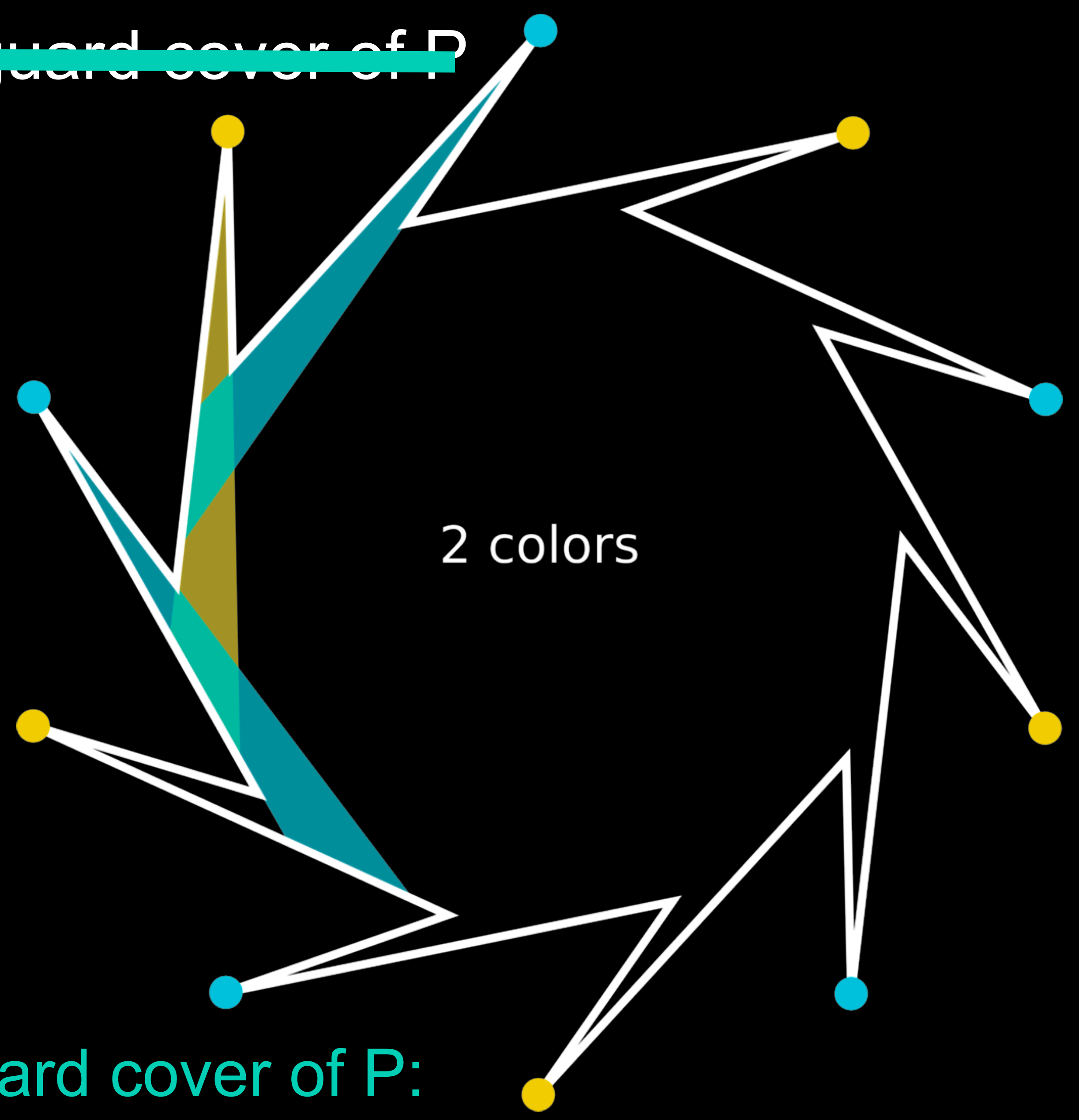

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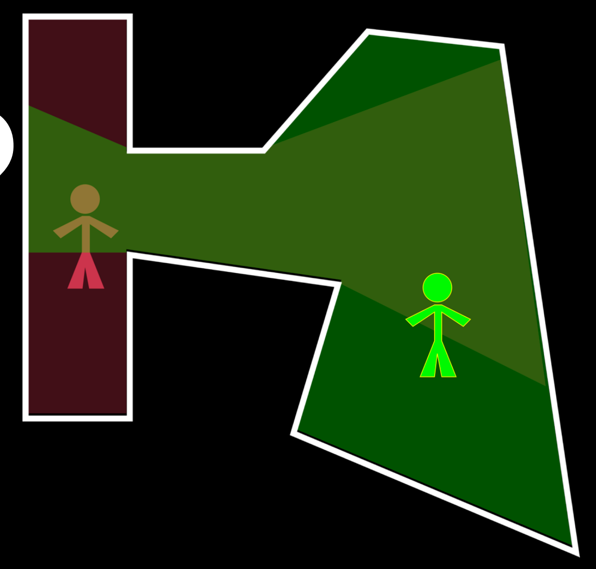

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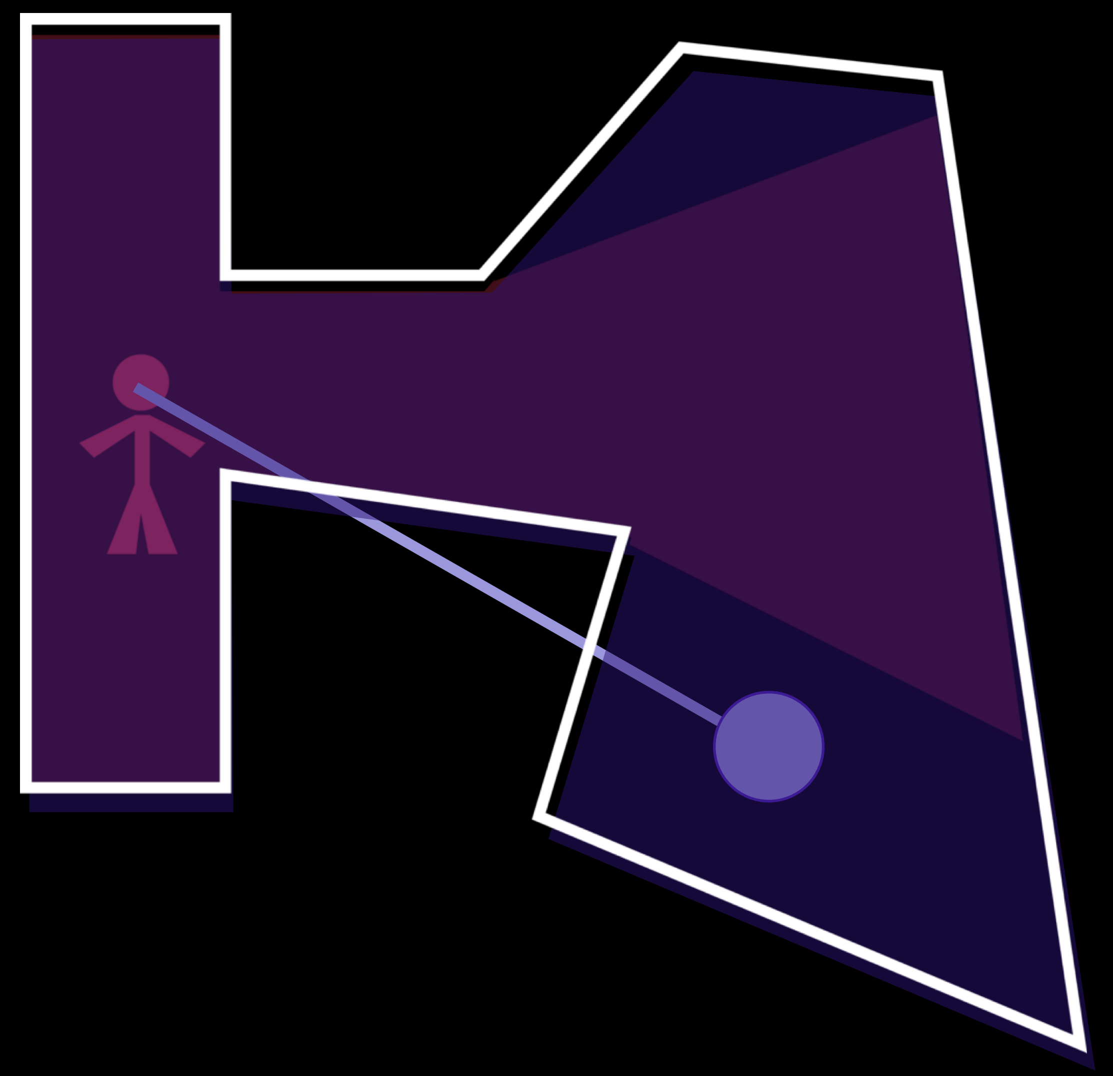

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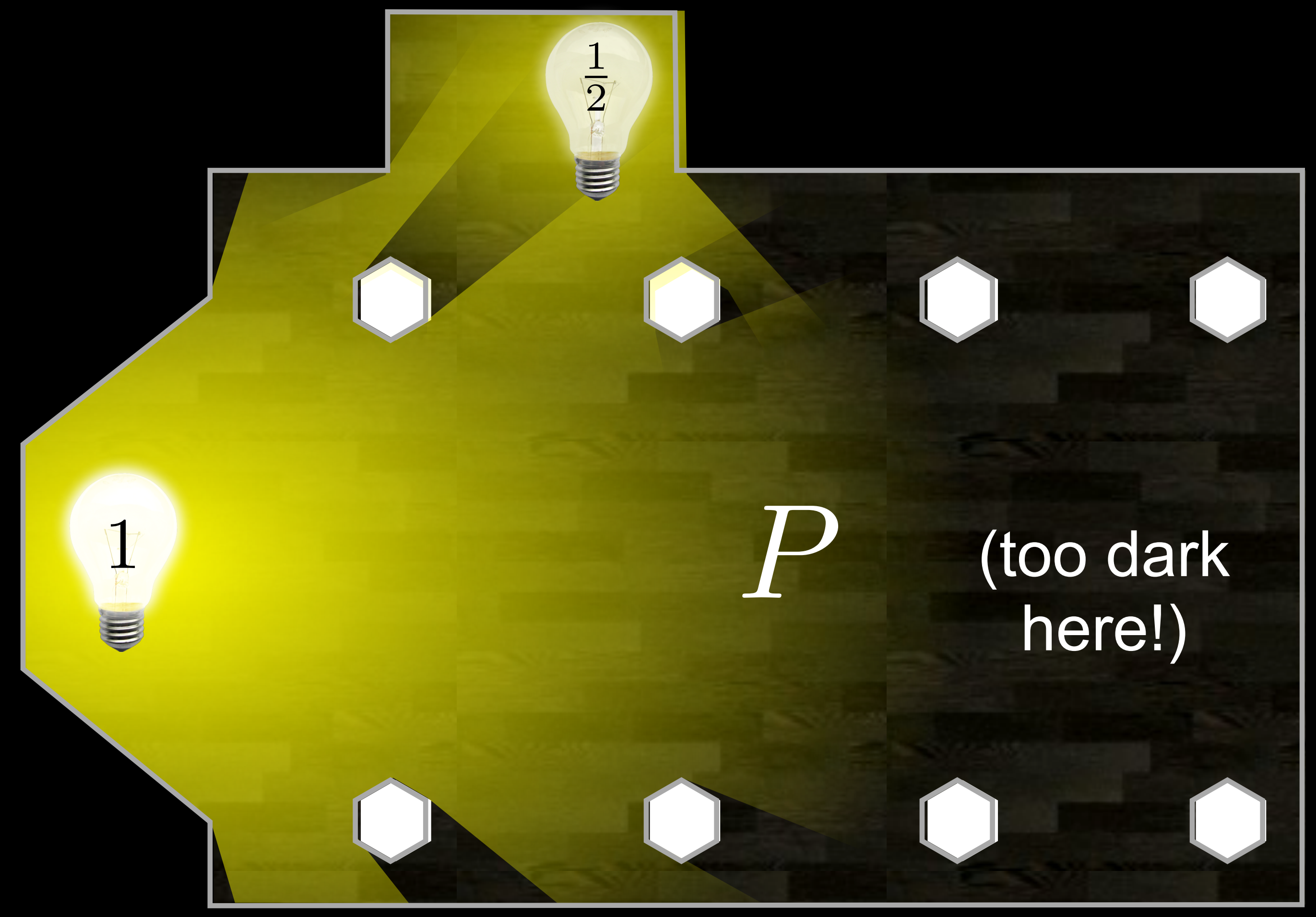

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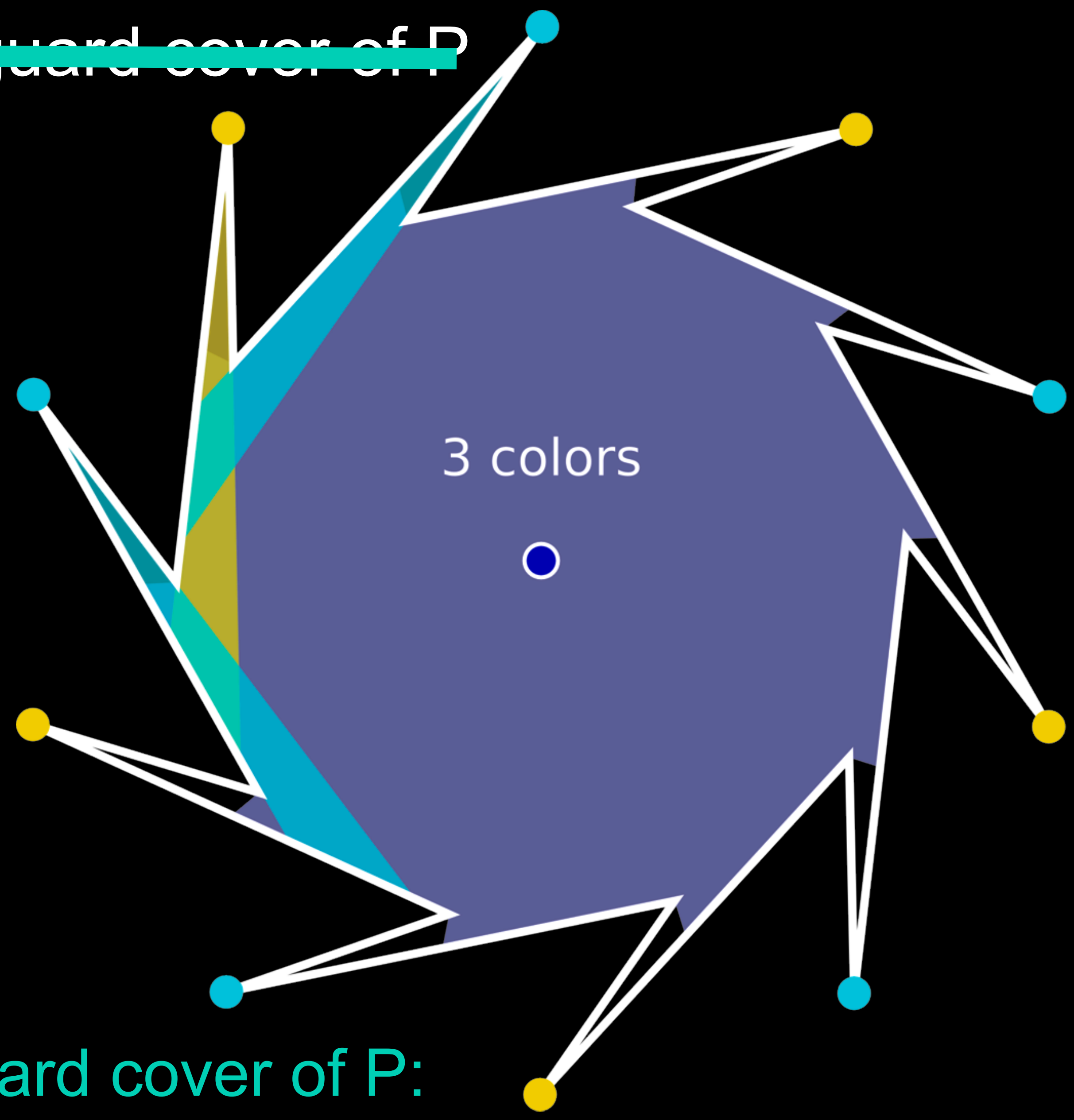

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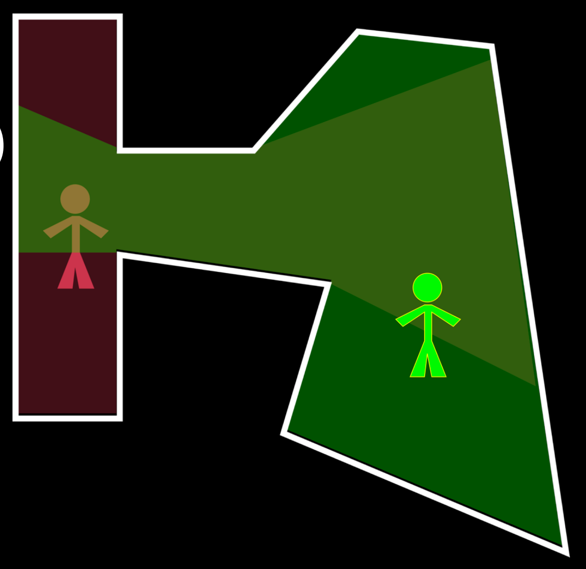

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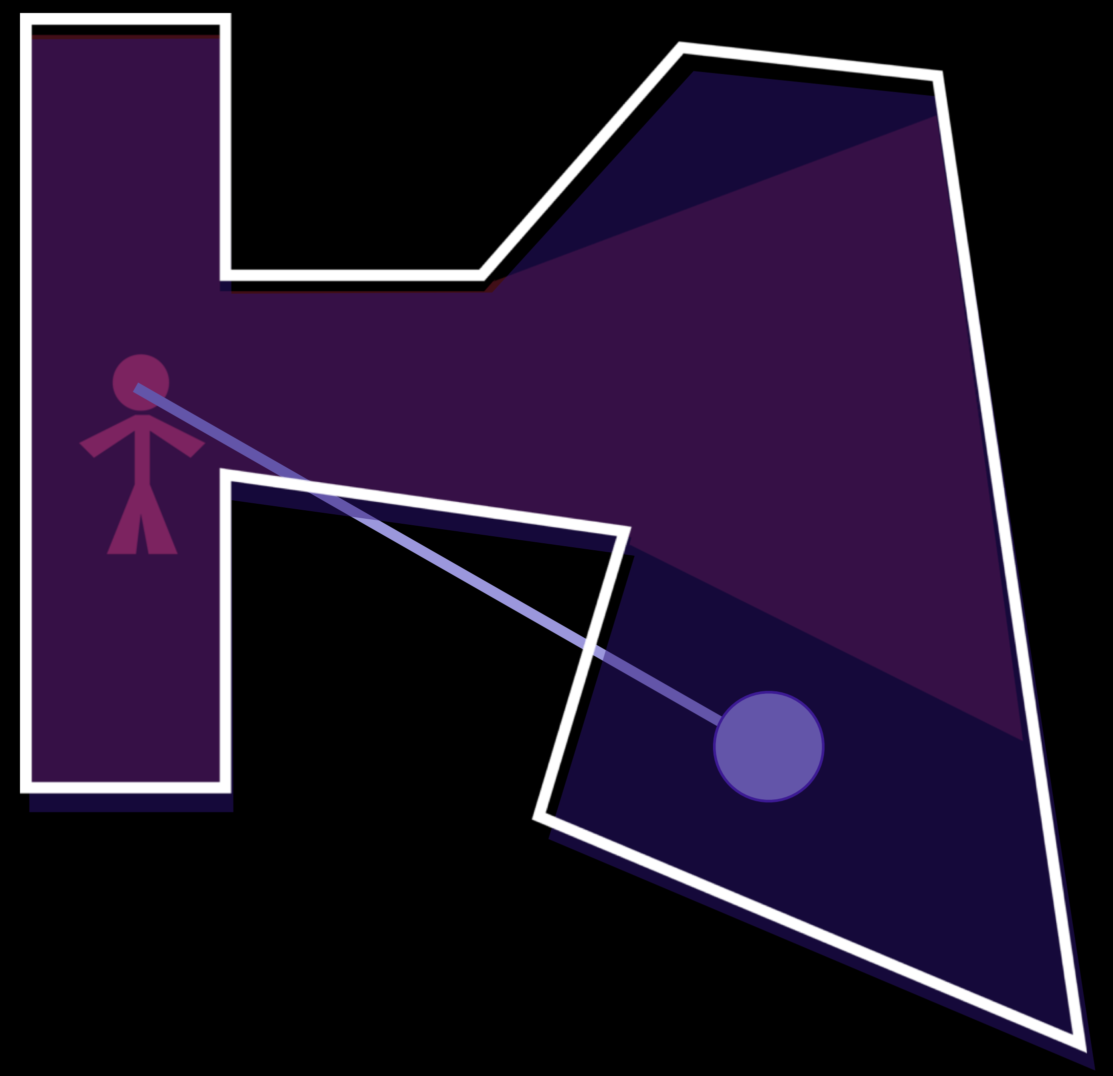

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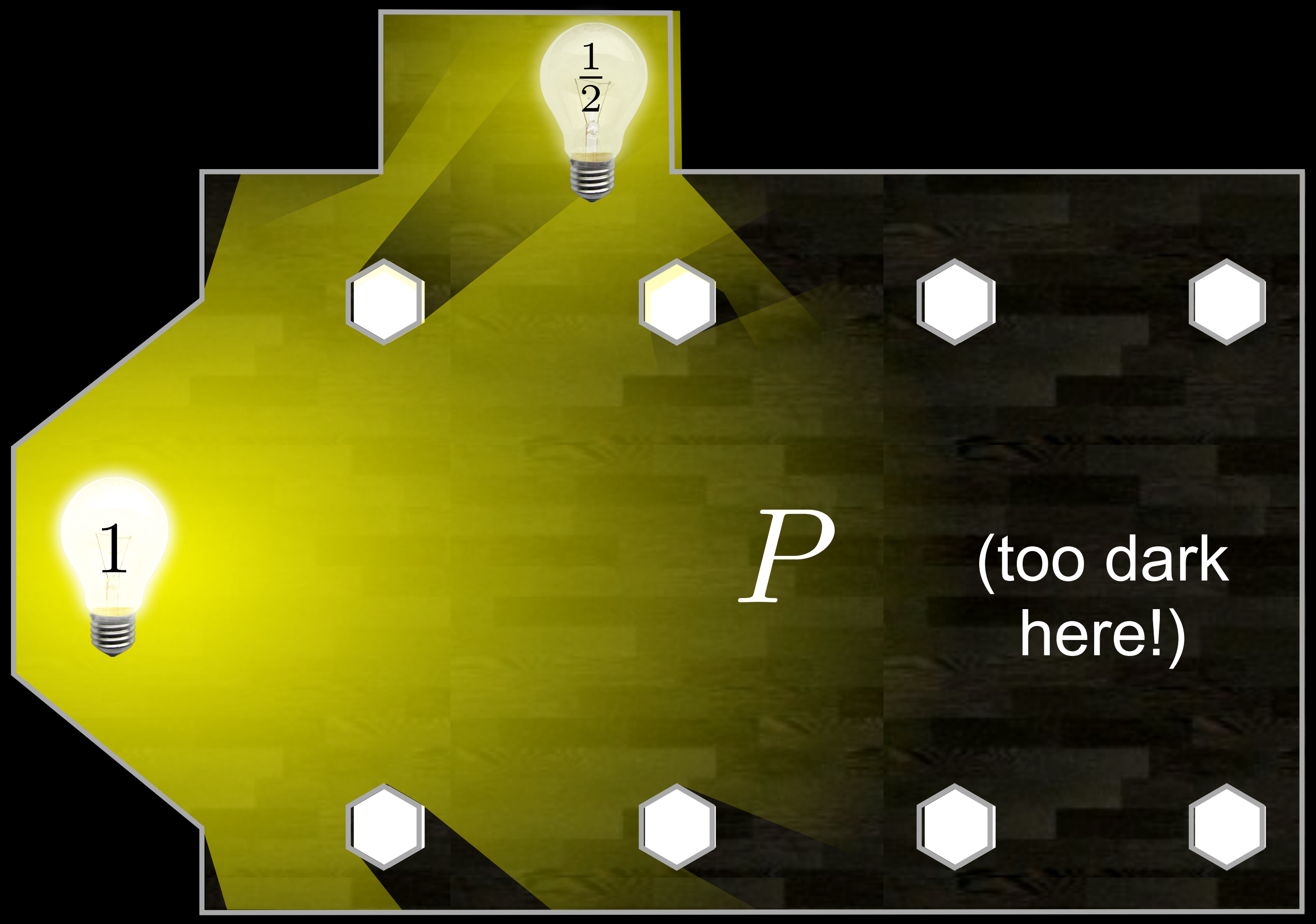

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Chromatic AGP:

We do not care about the number of guards, but about the number of colors!

Given: a polygon P

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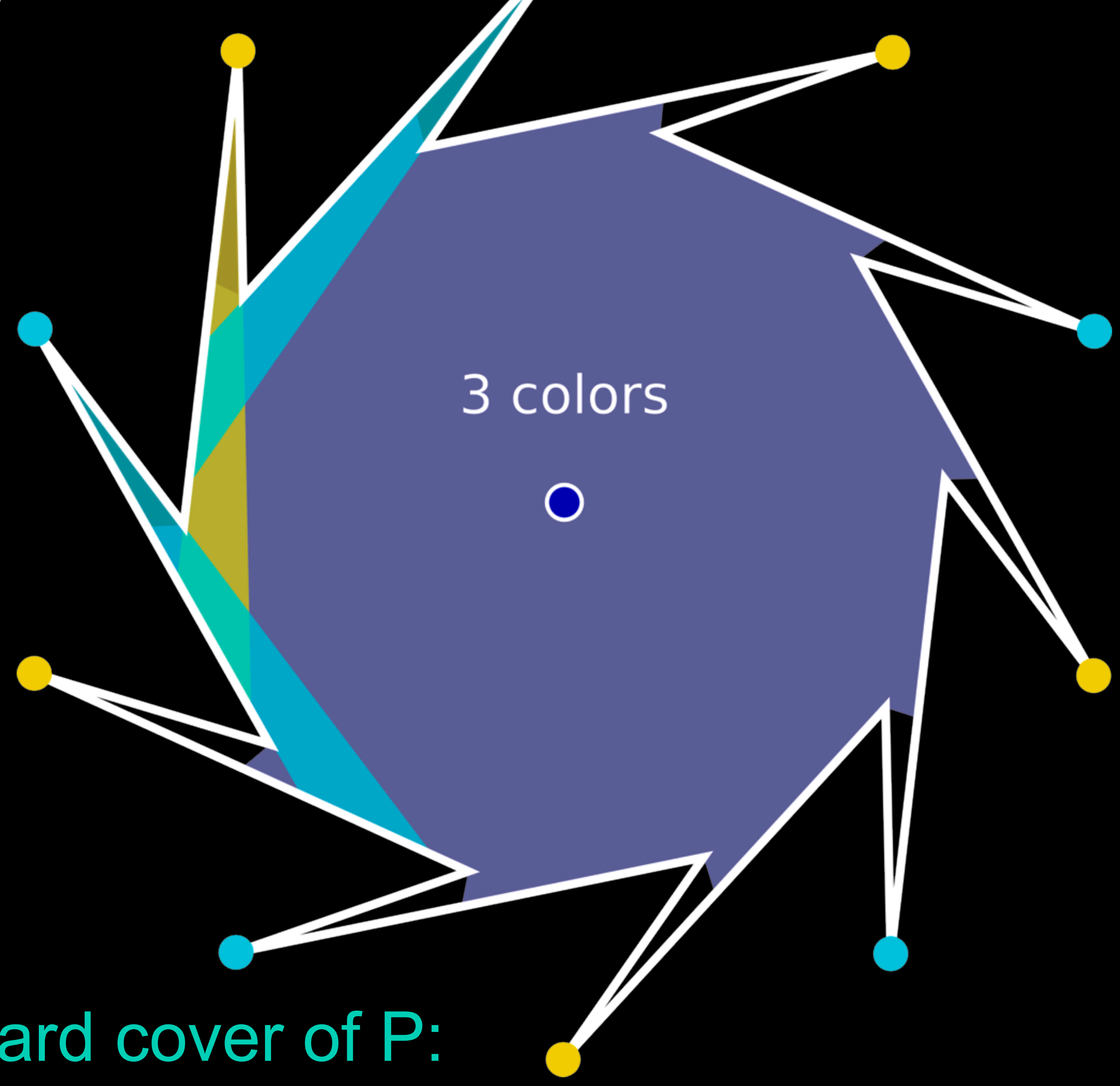

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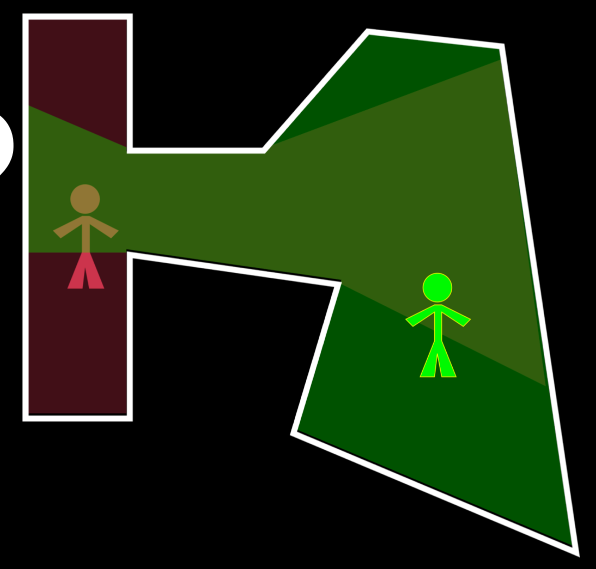

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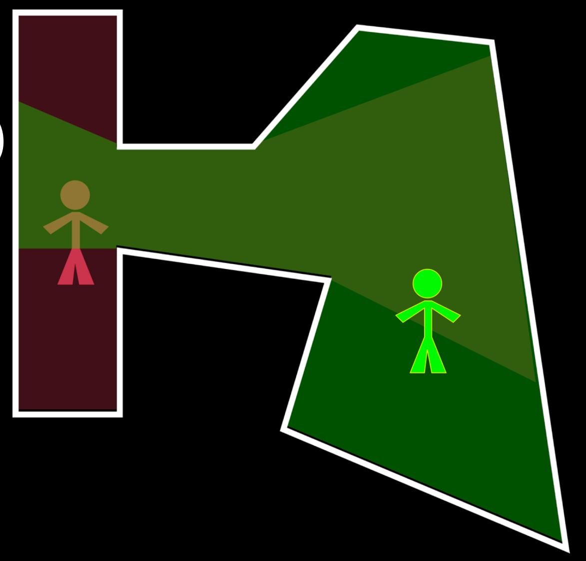

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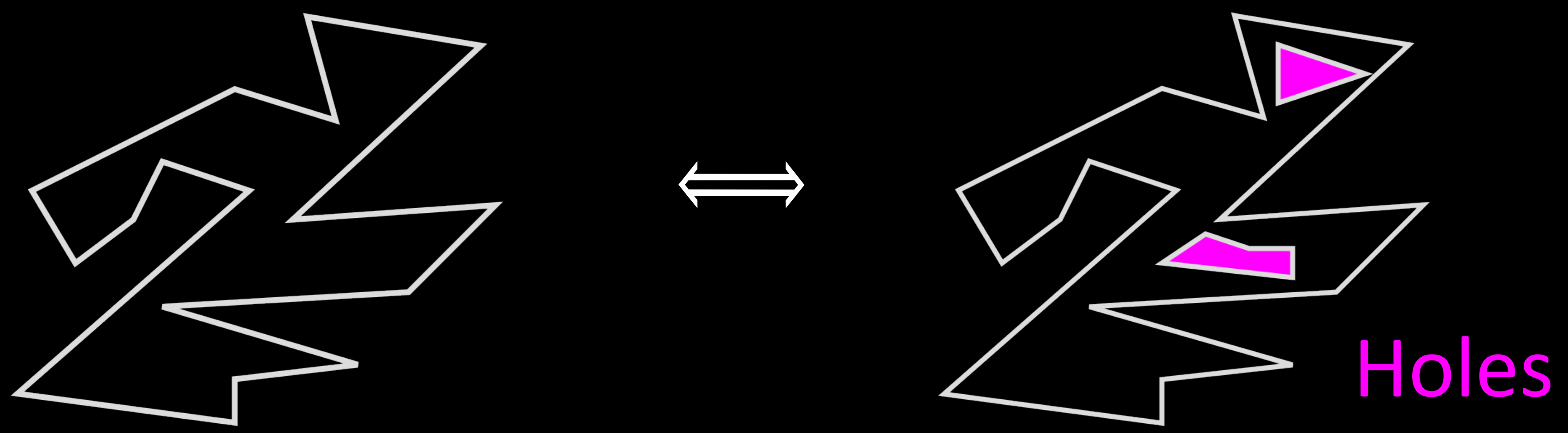
Alter the polygon class:

Traditionally:
Simple polygons or polygons with holes



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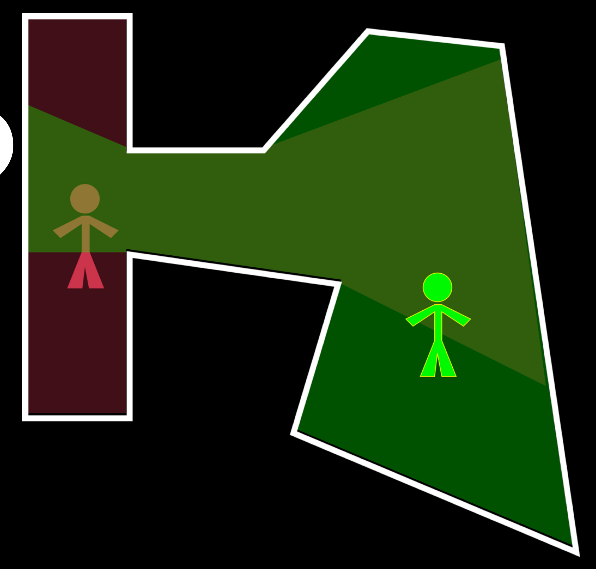

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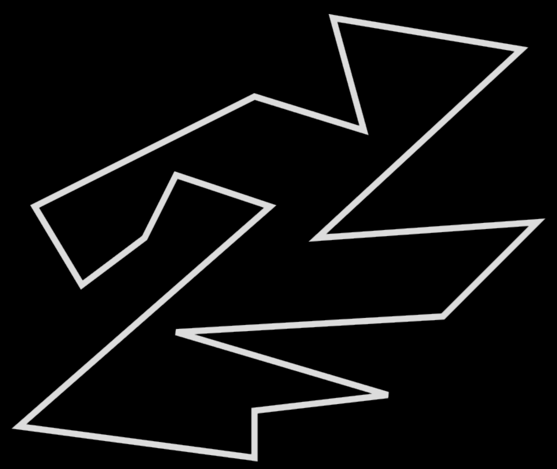
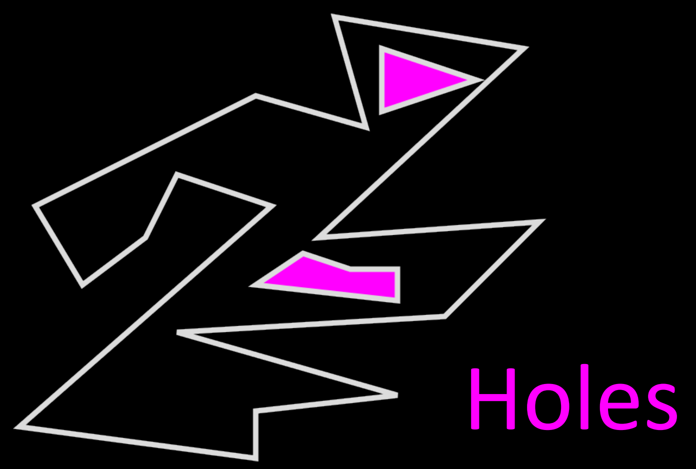


⟺



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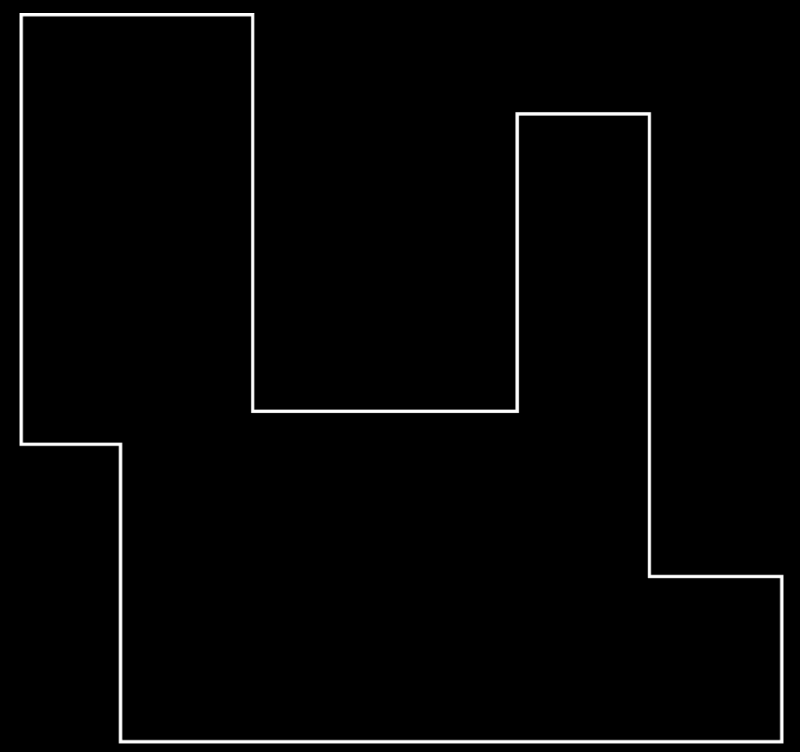
Rectilinear polygons



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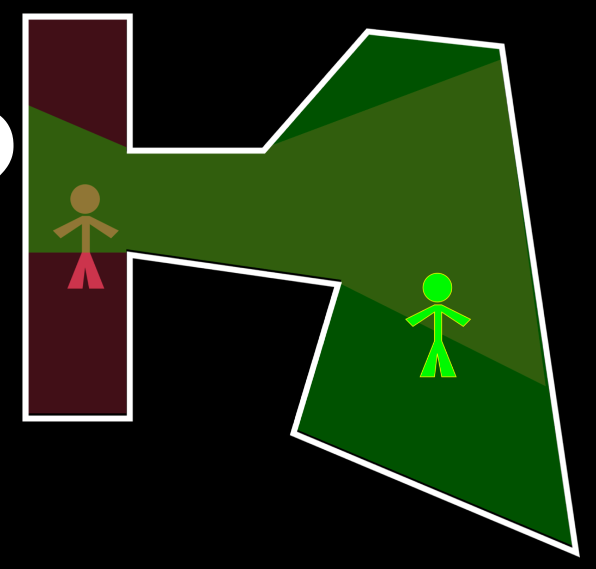

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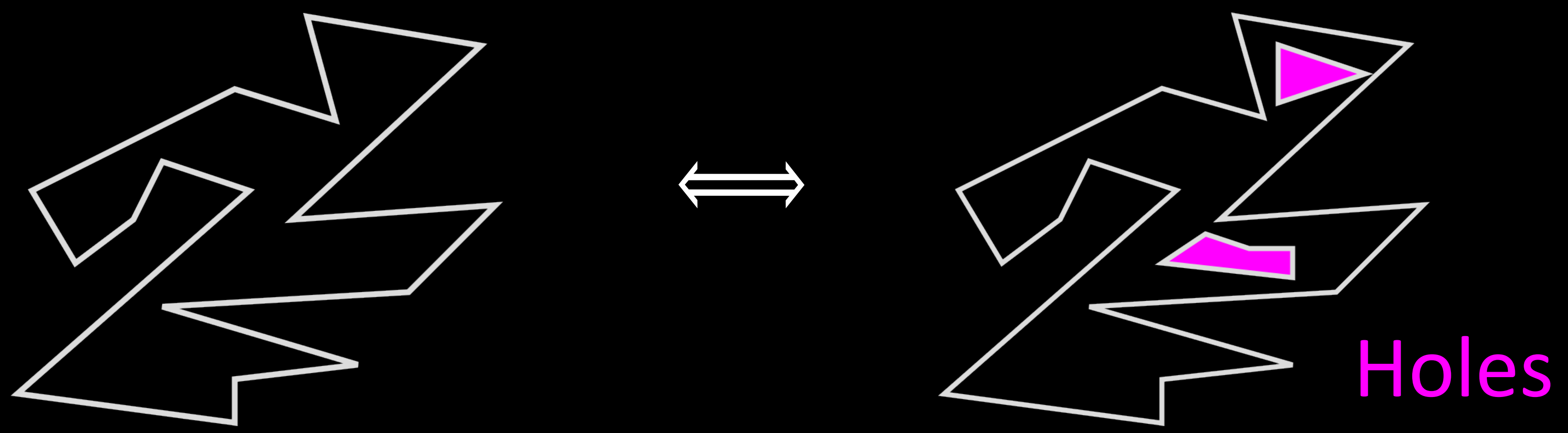

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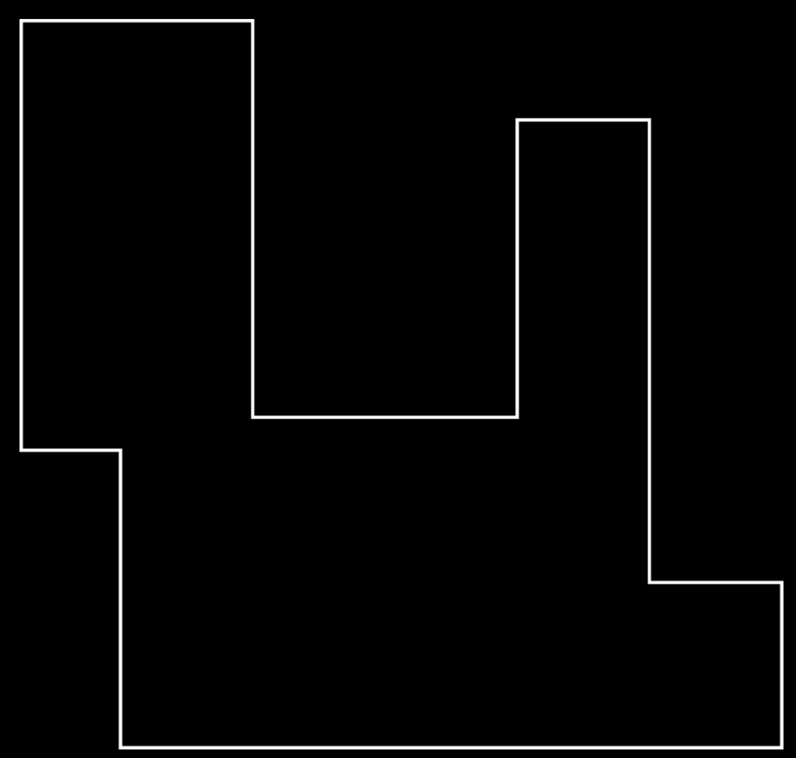
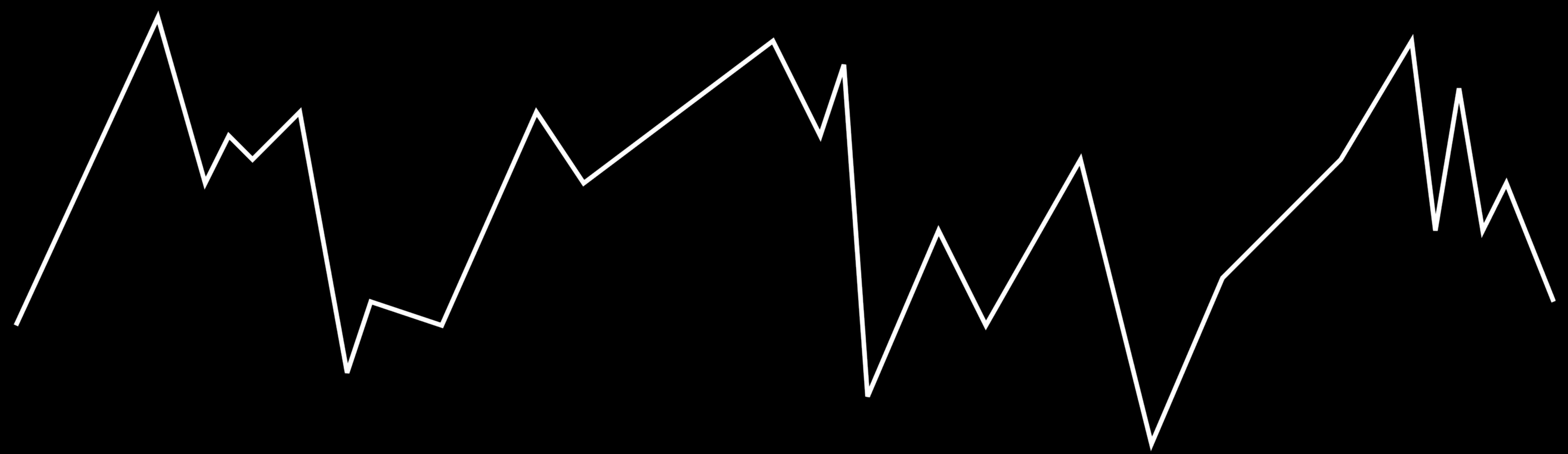


Guard a 1.5D-Terrain



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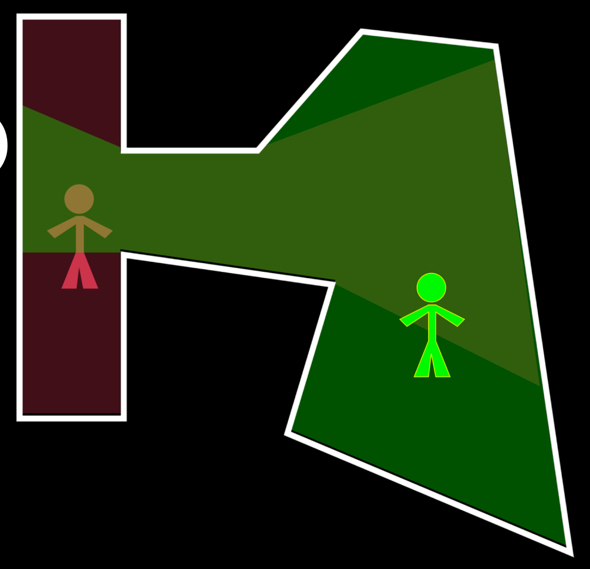

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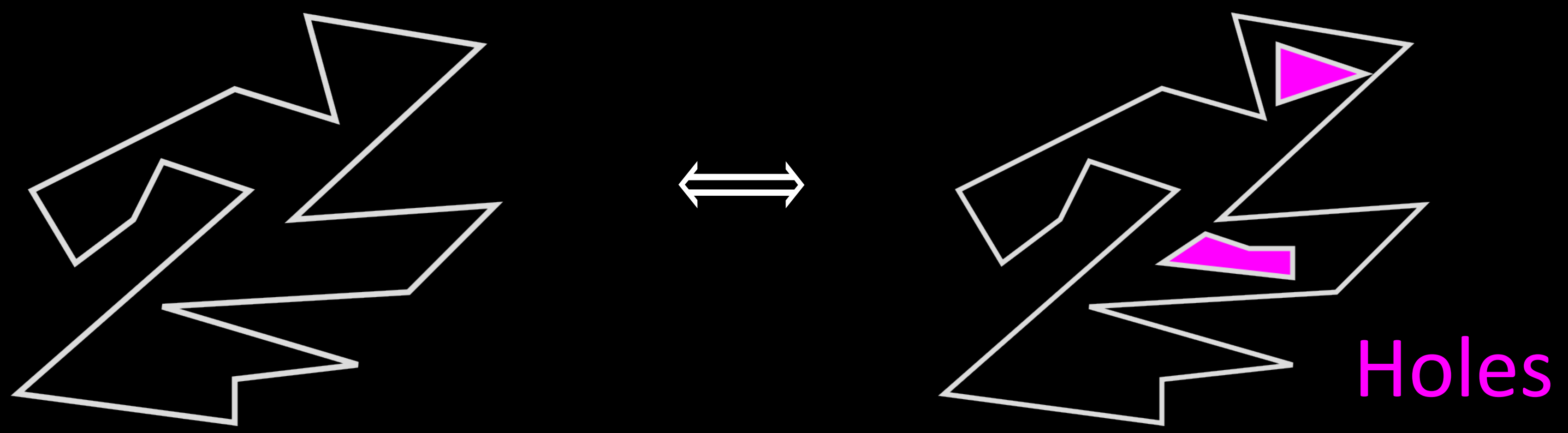

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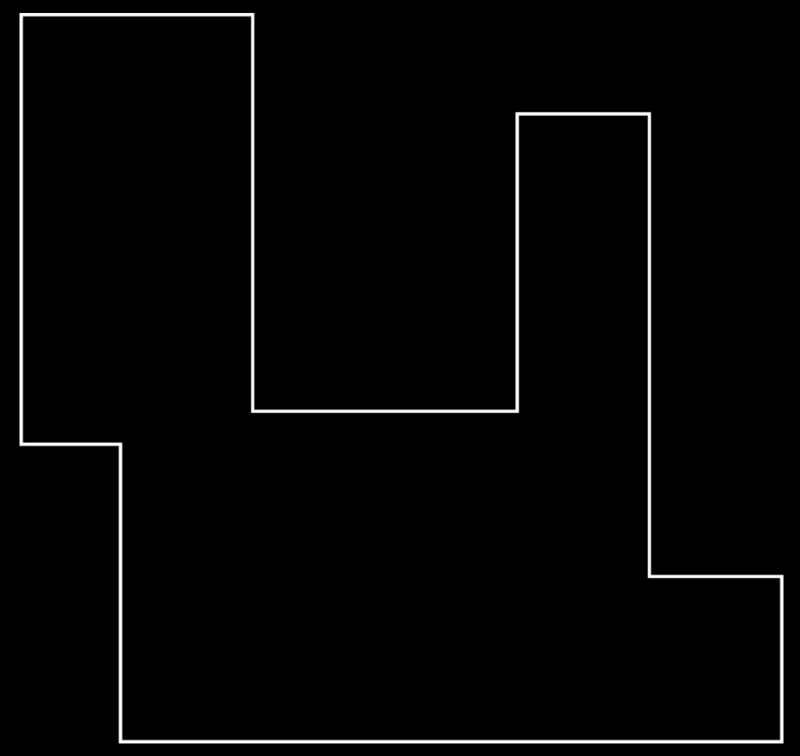

Guard a 1.5D-Terrain

- With guards on the terrain

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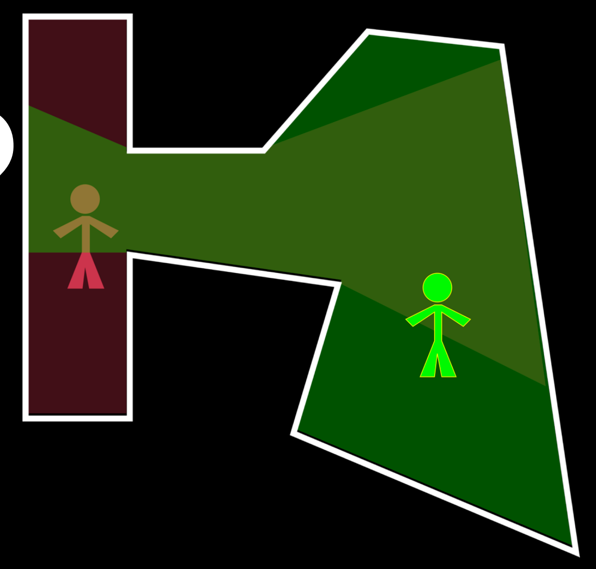

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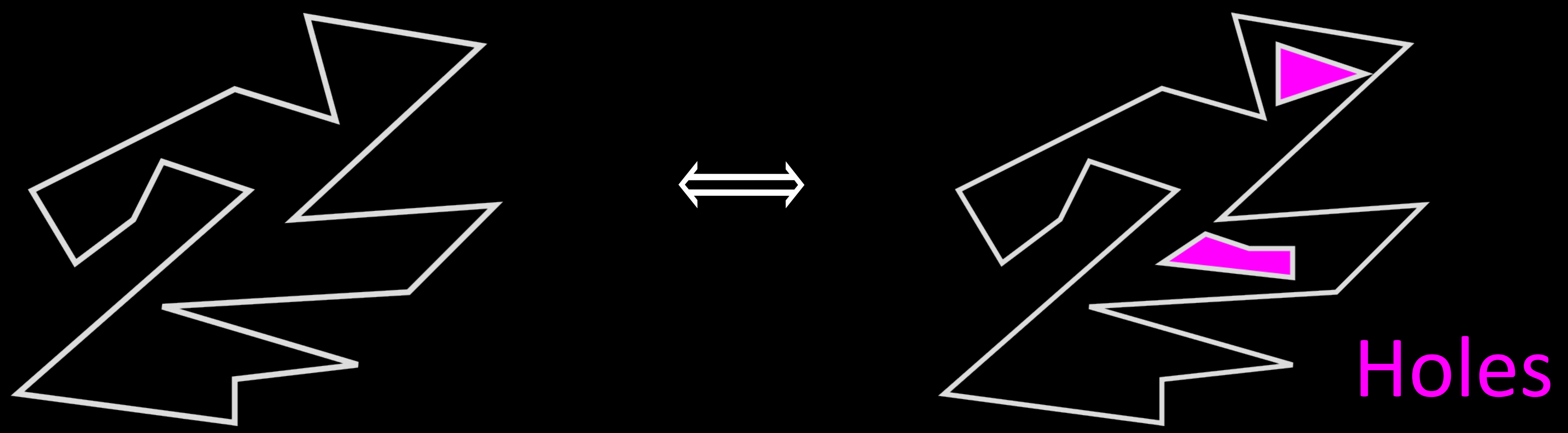

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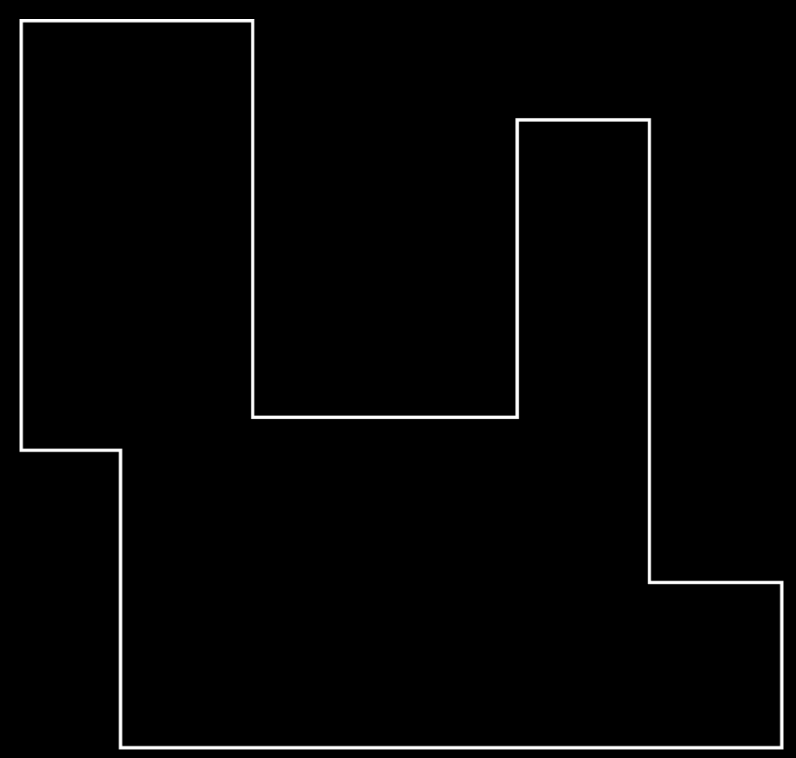
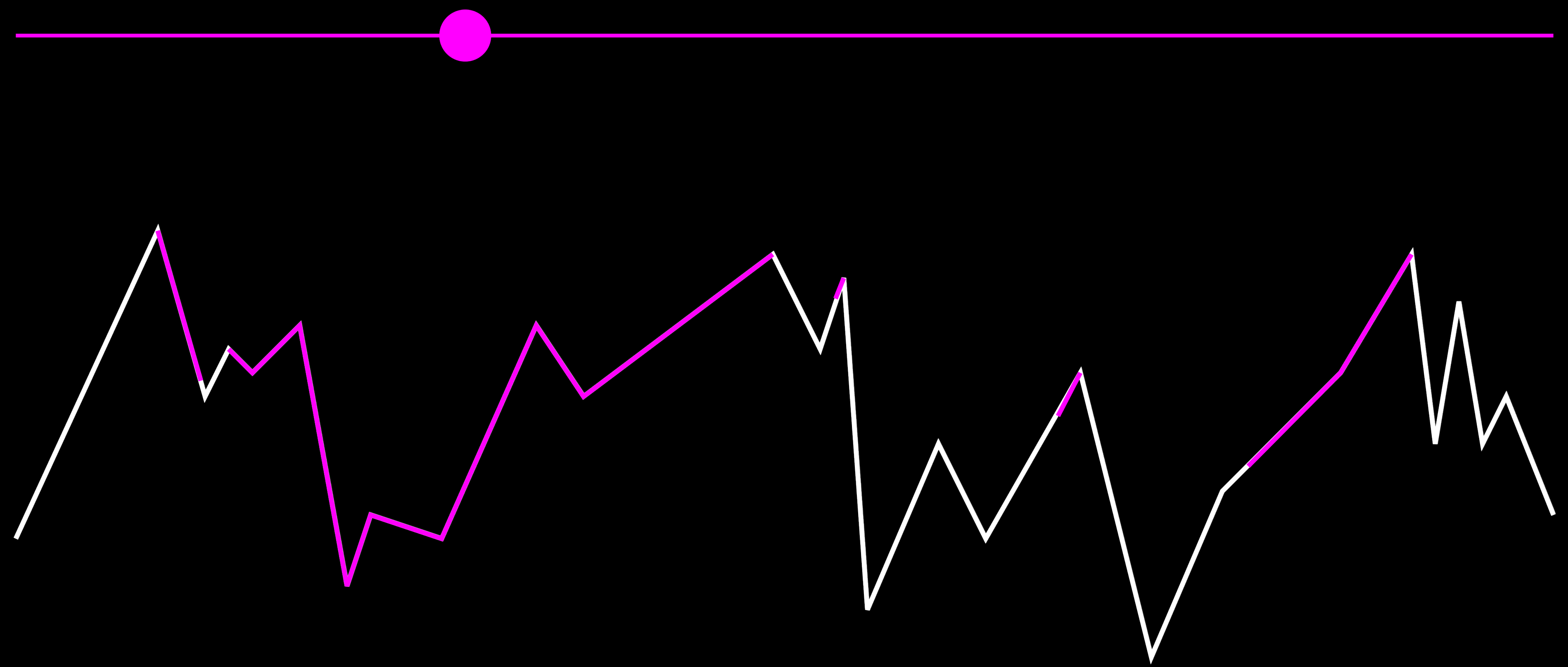


Guard a 1.5D-Terrain

- With guards on the terrain
- With guards on an altitude line above the terrain



k-Hop Visibility

Motivation

- Serve a city with carsharing (CS) stations:

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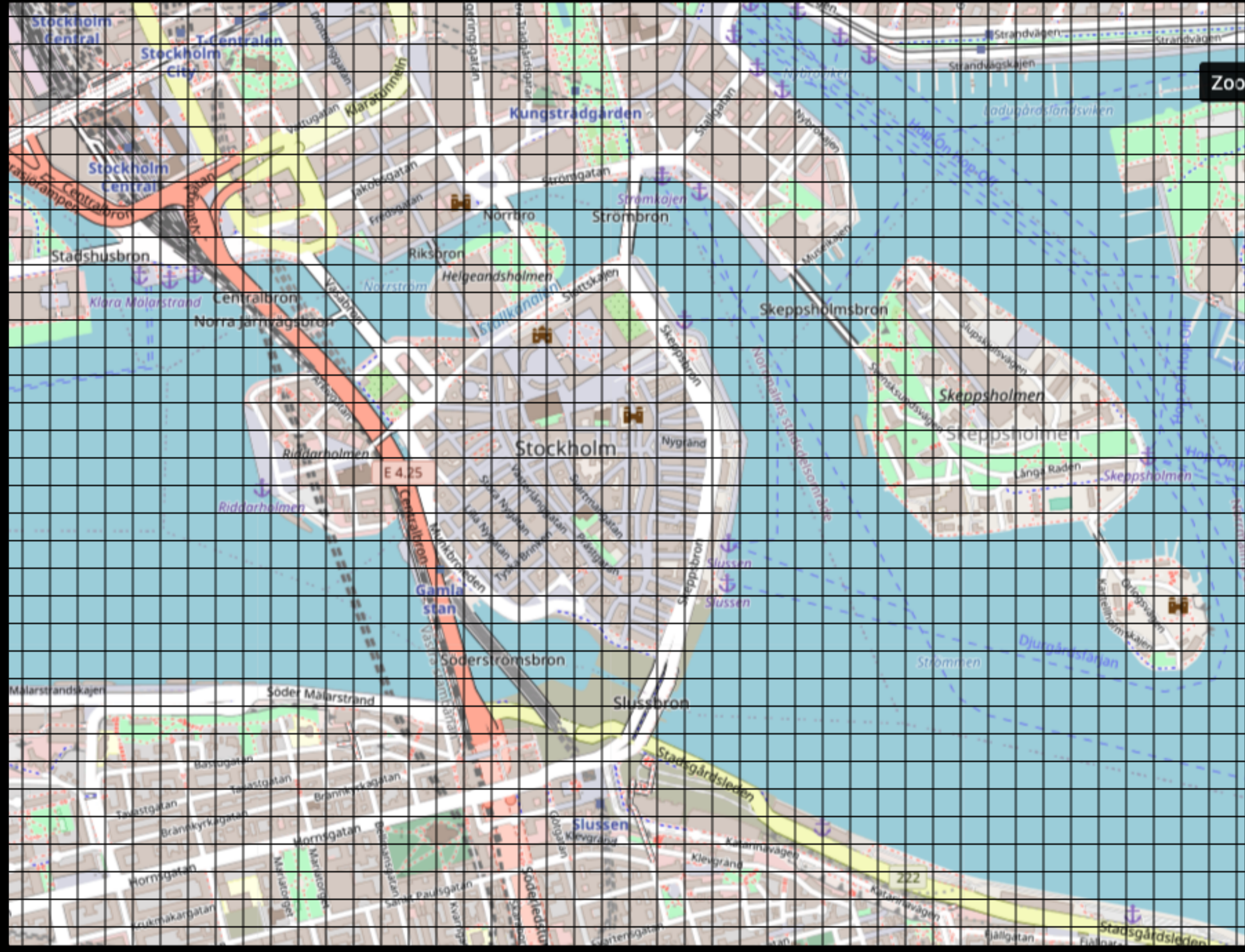

Motivation

- Serve a city with carsharing (CS) stations:
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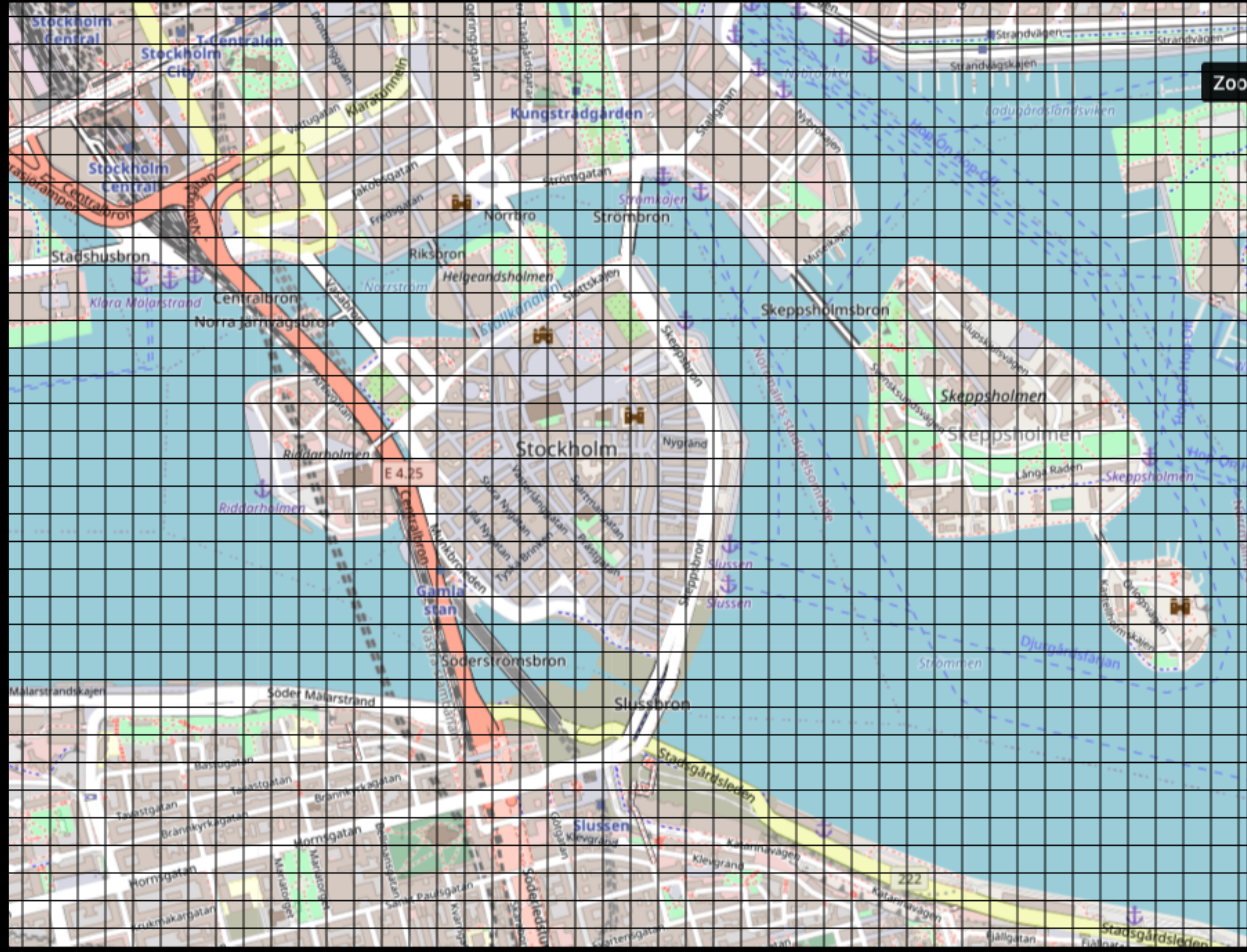
  - Customers willing to walk a certain distance



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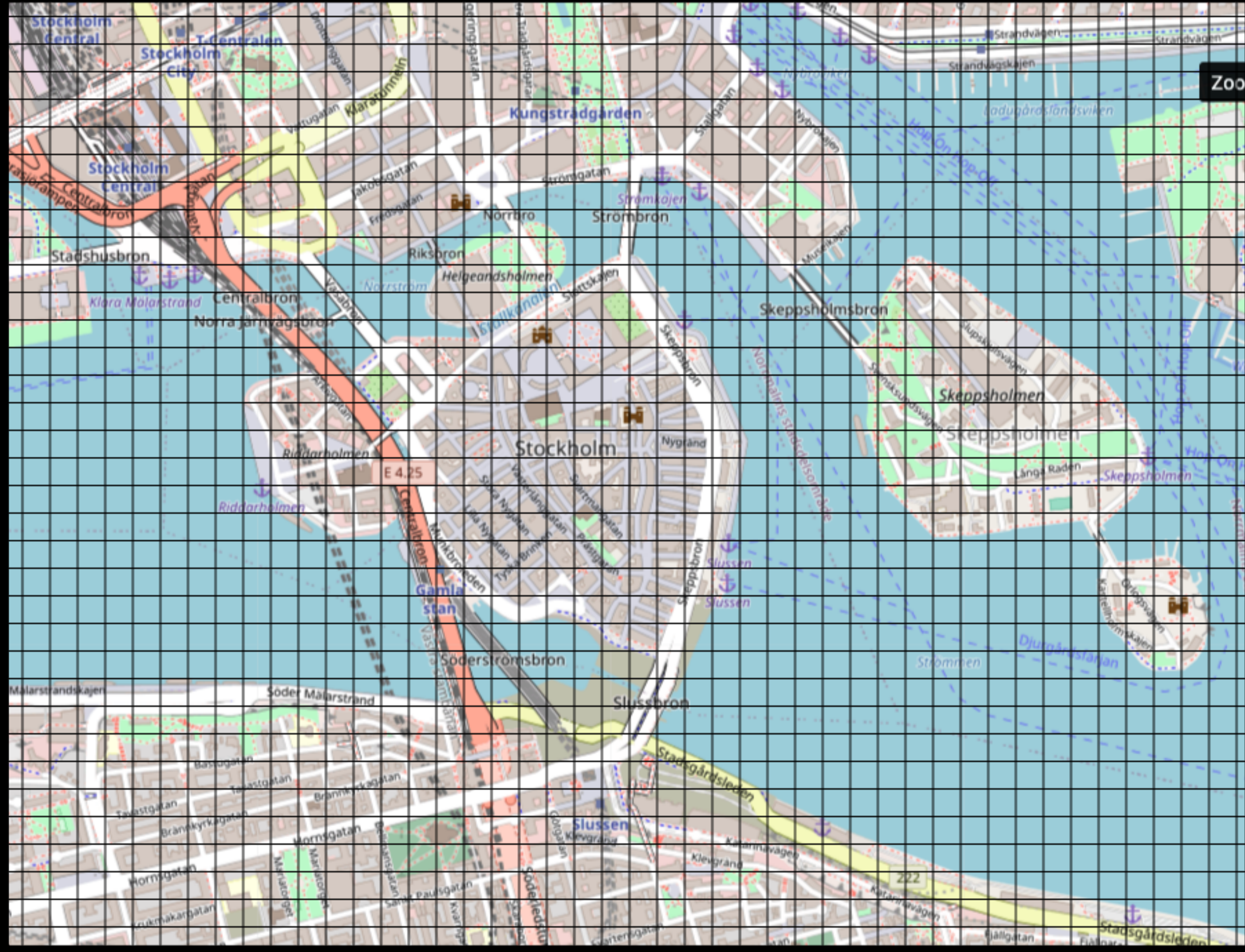

- Serve a city with carsharing (CS) stations:
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  - Same distance bound for the complete city

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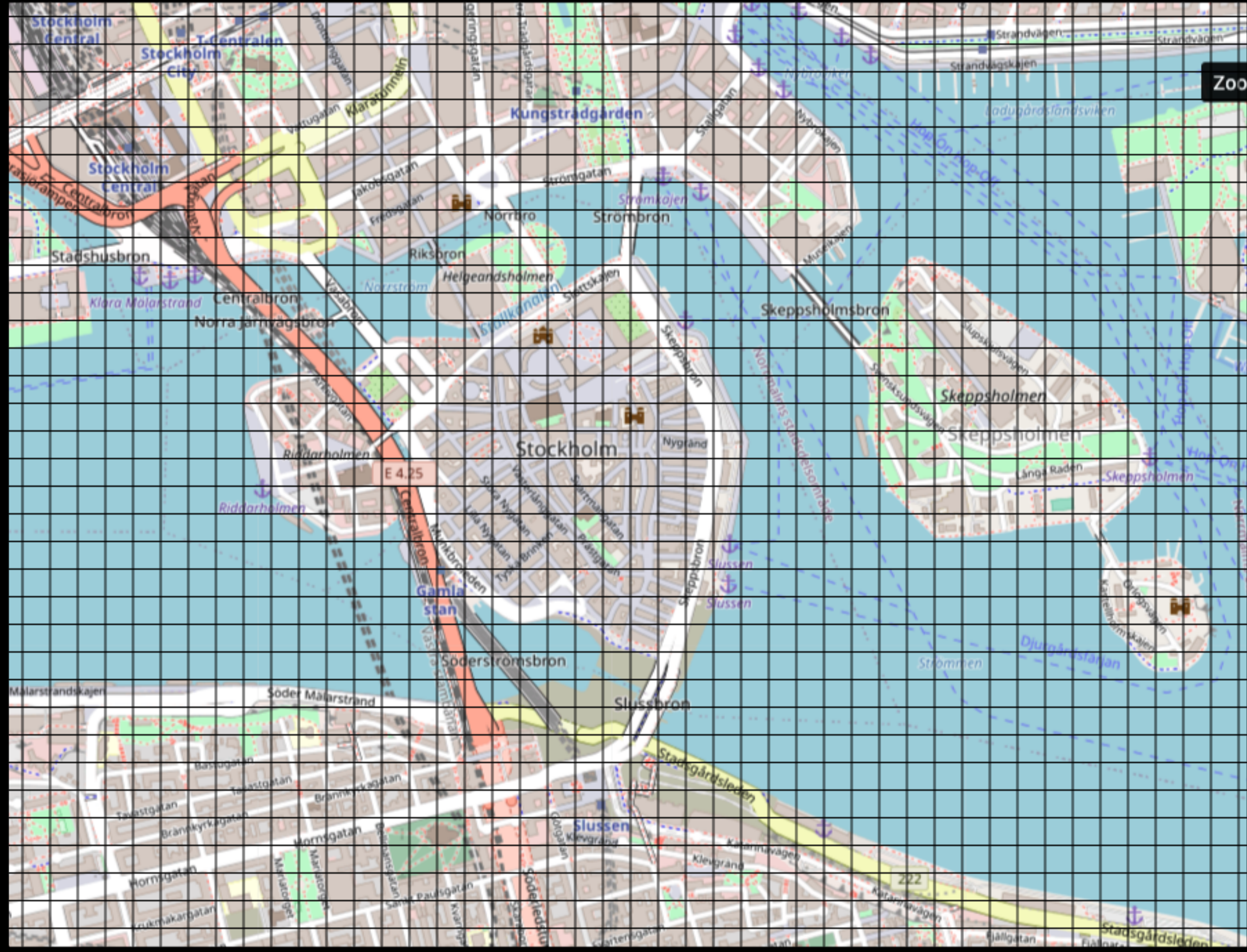

Image source: Gunnar Flötteröd, Waterborne Urban Mobility, Final Project Report

LINKÖPING UNIVERSITY

Motivation

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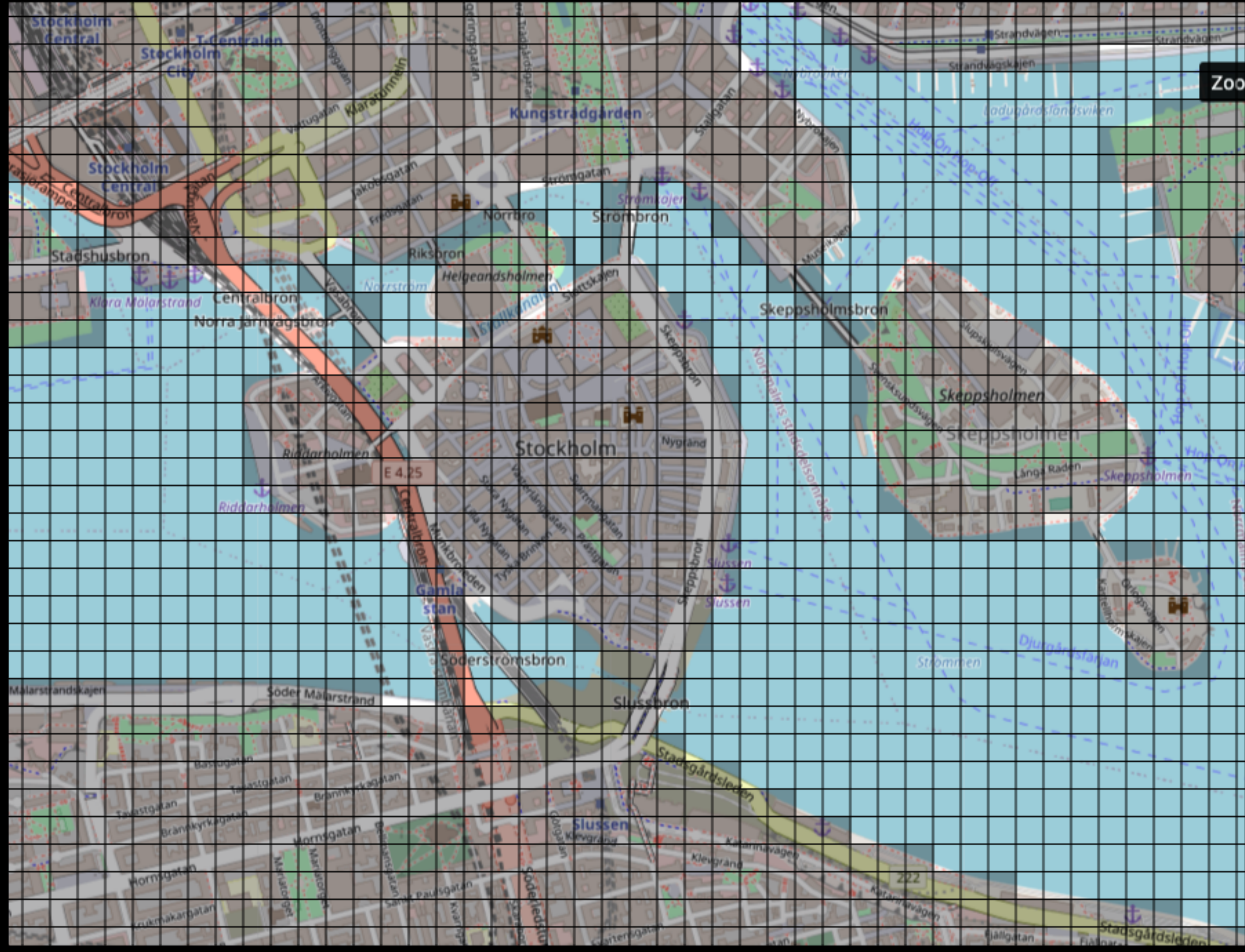
  - City ➜ Polyomino (➜ alter the environment)



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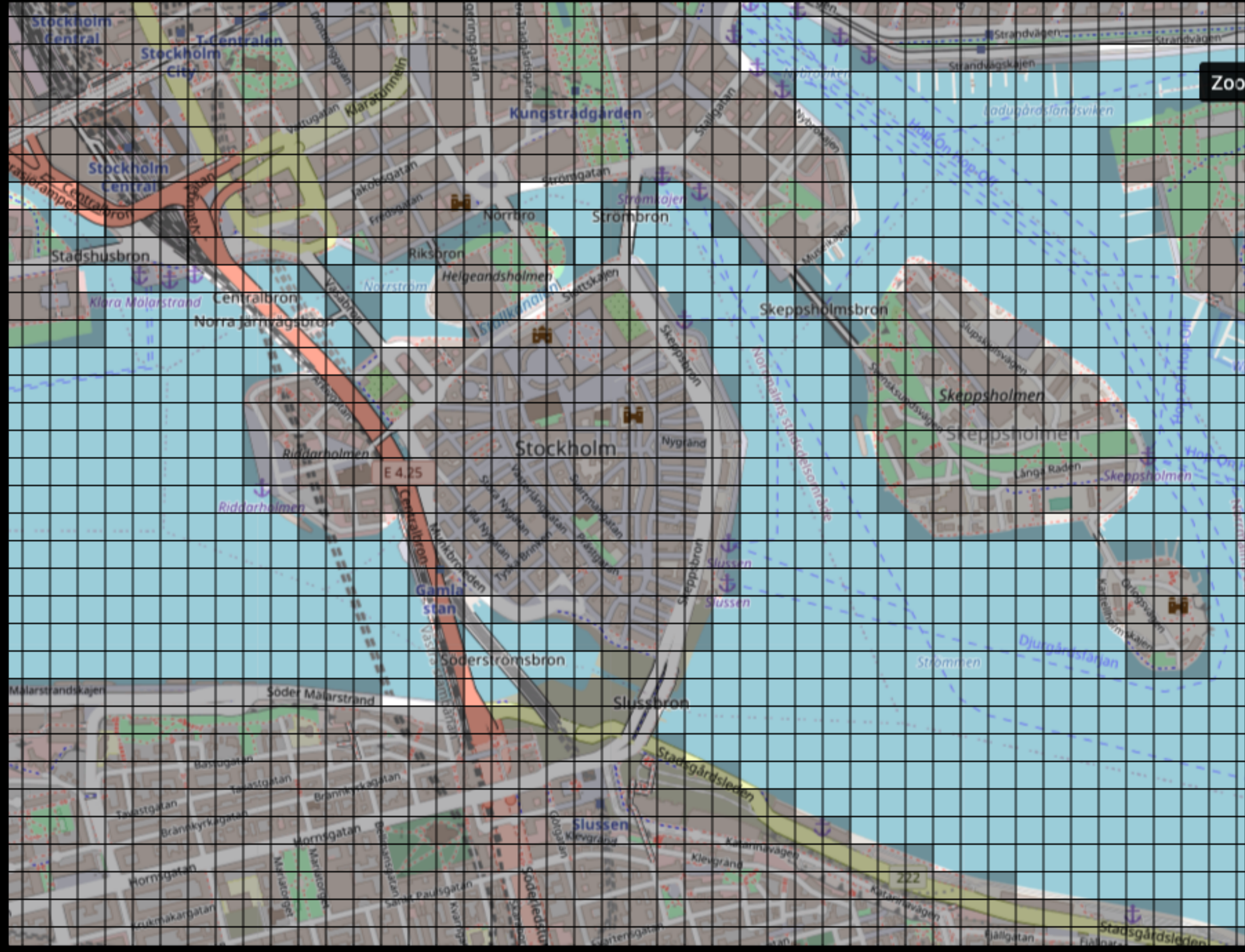

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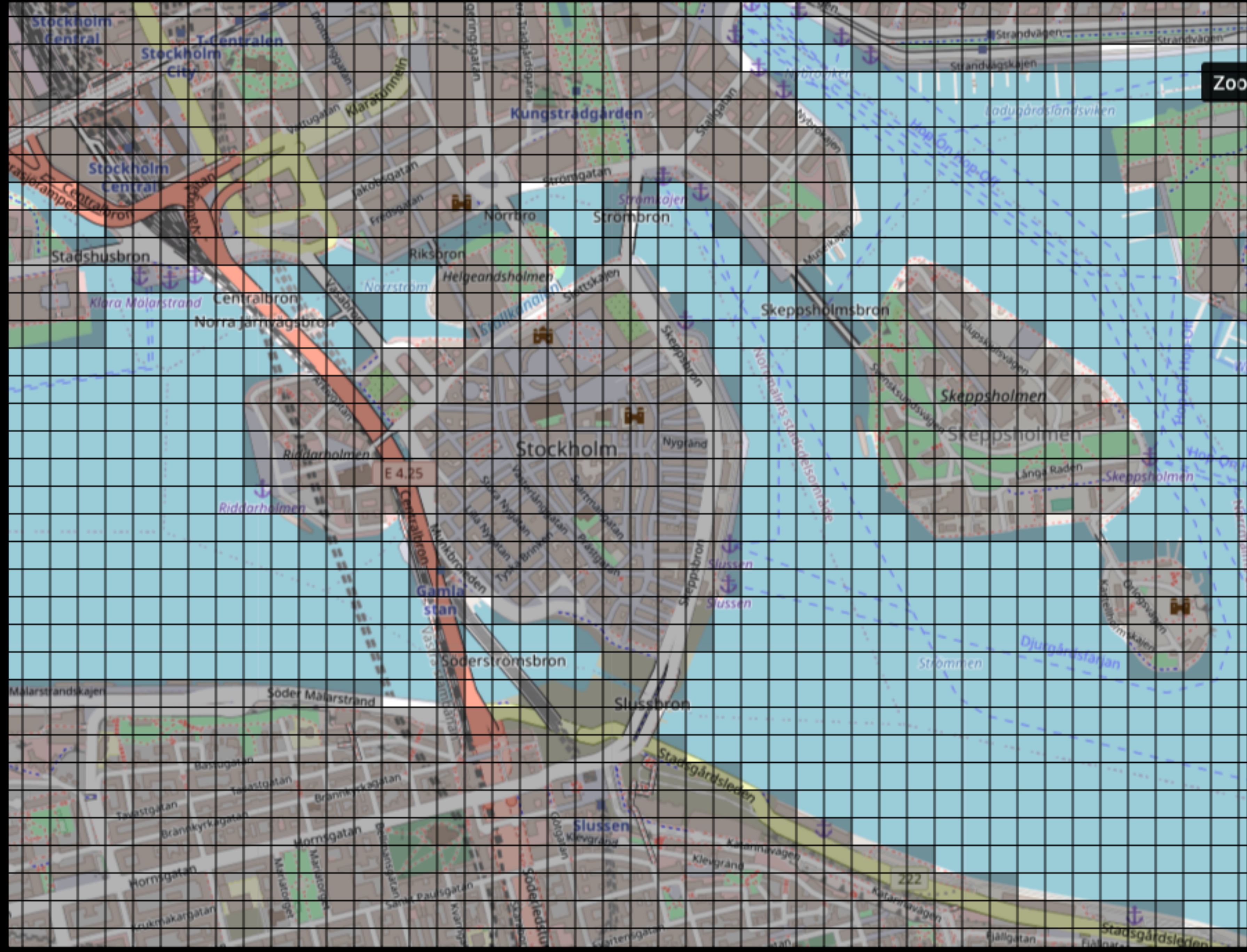

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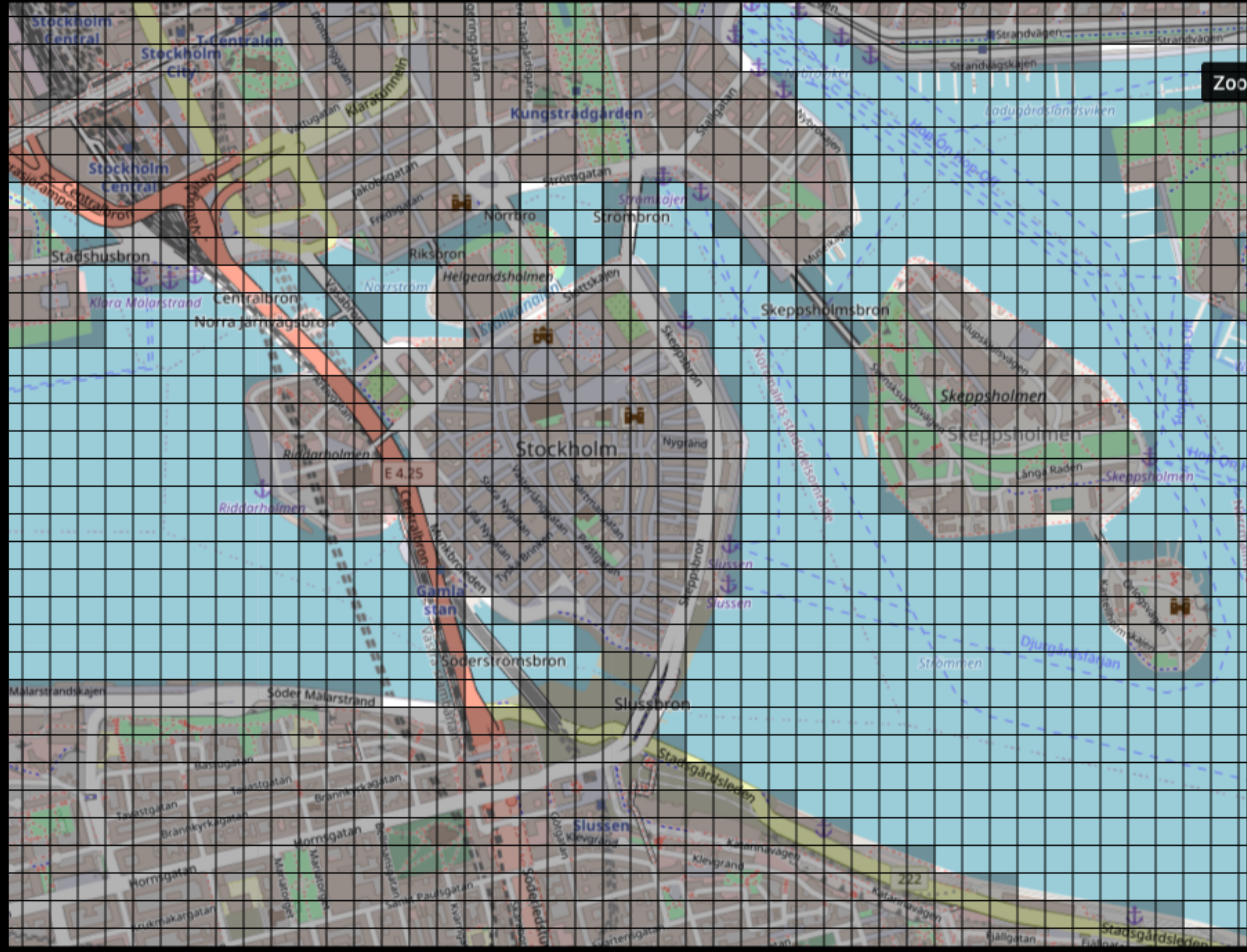
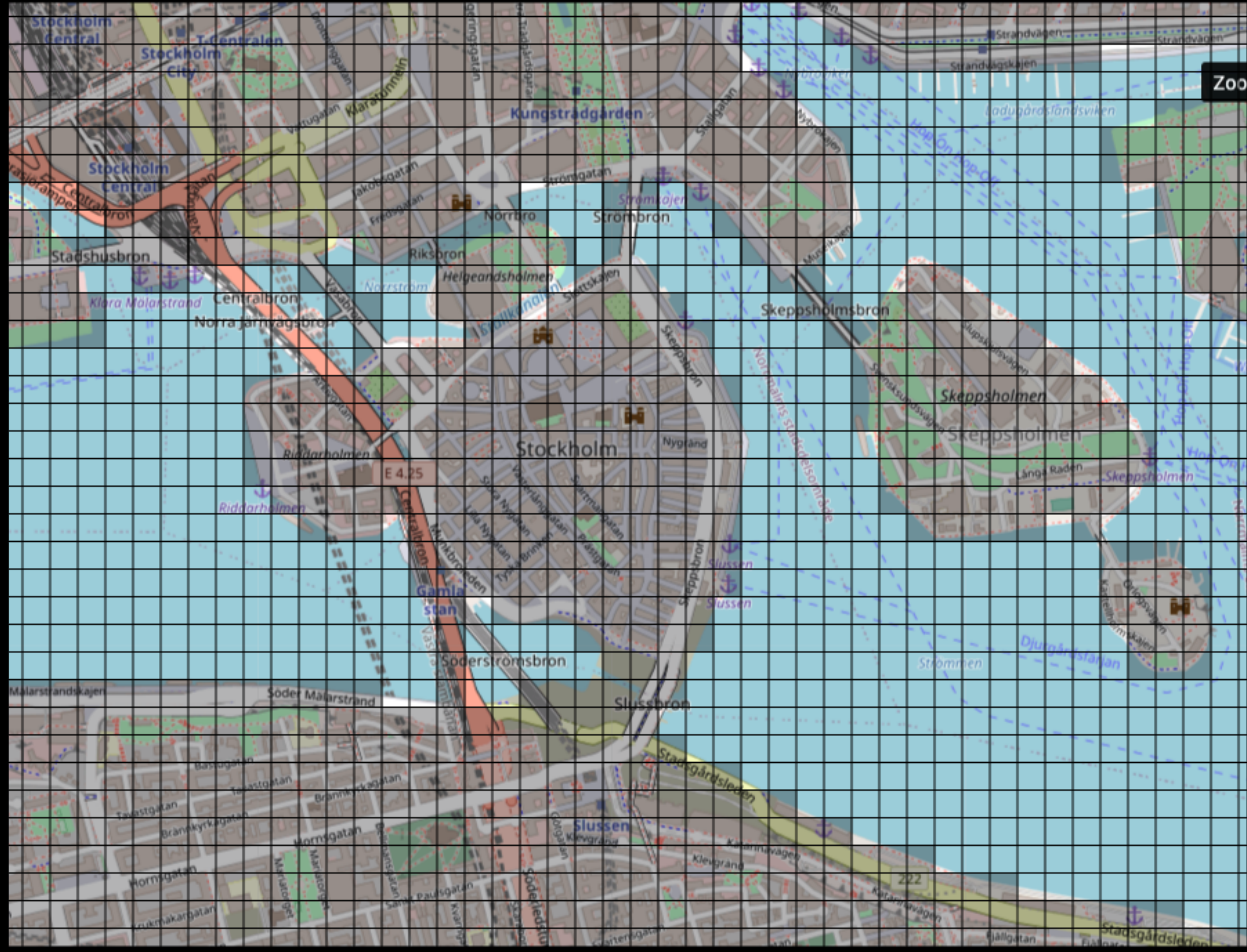
- Goal: Place as few CS stations as possible to serve the complete city

Motivation


Kungsträdgården
Stadshusbron
Centralbron
Riksbron
Norrbro
Strömbron
Helgeandsholmen
Skeppsholmsbron
Stockholm
Skeppsholmen
Riddarholmen
Söderströmsbron
Slussbron
Slussen

Motivation

- So, what can a station serve?



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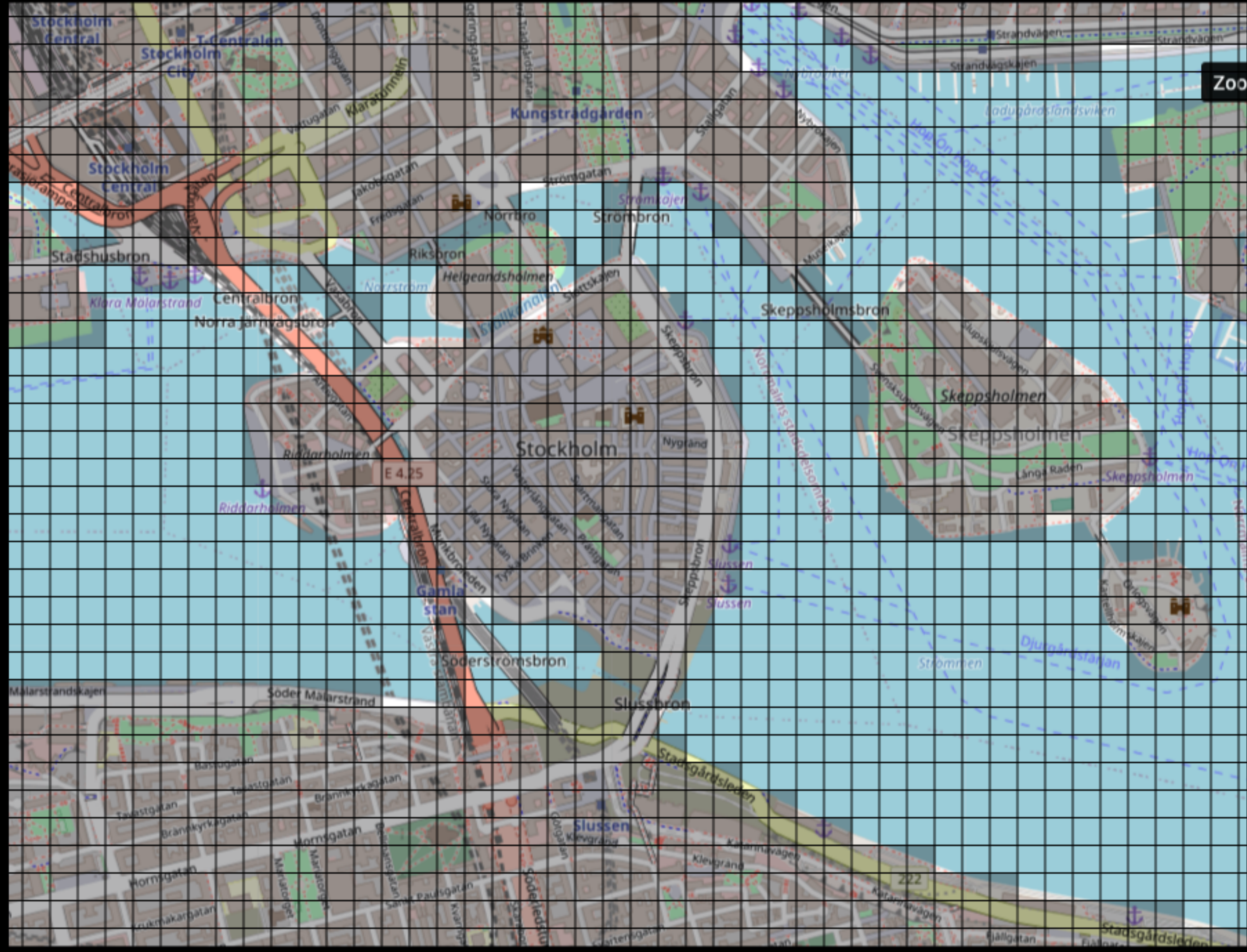

- So, what can a station serve?
  - All unit squares of the polyomino reachable when walking inside the polyomino for at most the given walking range

Motivation

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k=5

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  - Walking range *k*
  - “Visibility”: We can look around corners for $k \geq 2$

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Motivation

k=5

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  - All unit squares of the polyomino reachable when walking inside the polyomino for at most the given walking range
  - Walking range k
  - "Visibility": We can look around corners for $k \geq 2$
  - ➡ Alter the capabilities of the guard

k-Hop Visibility: Formal Definition

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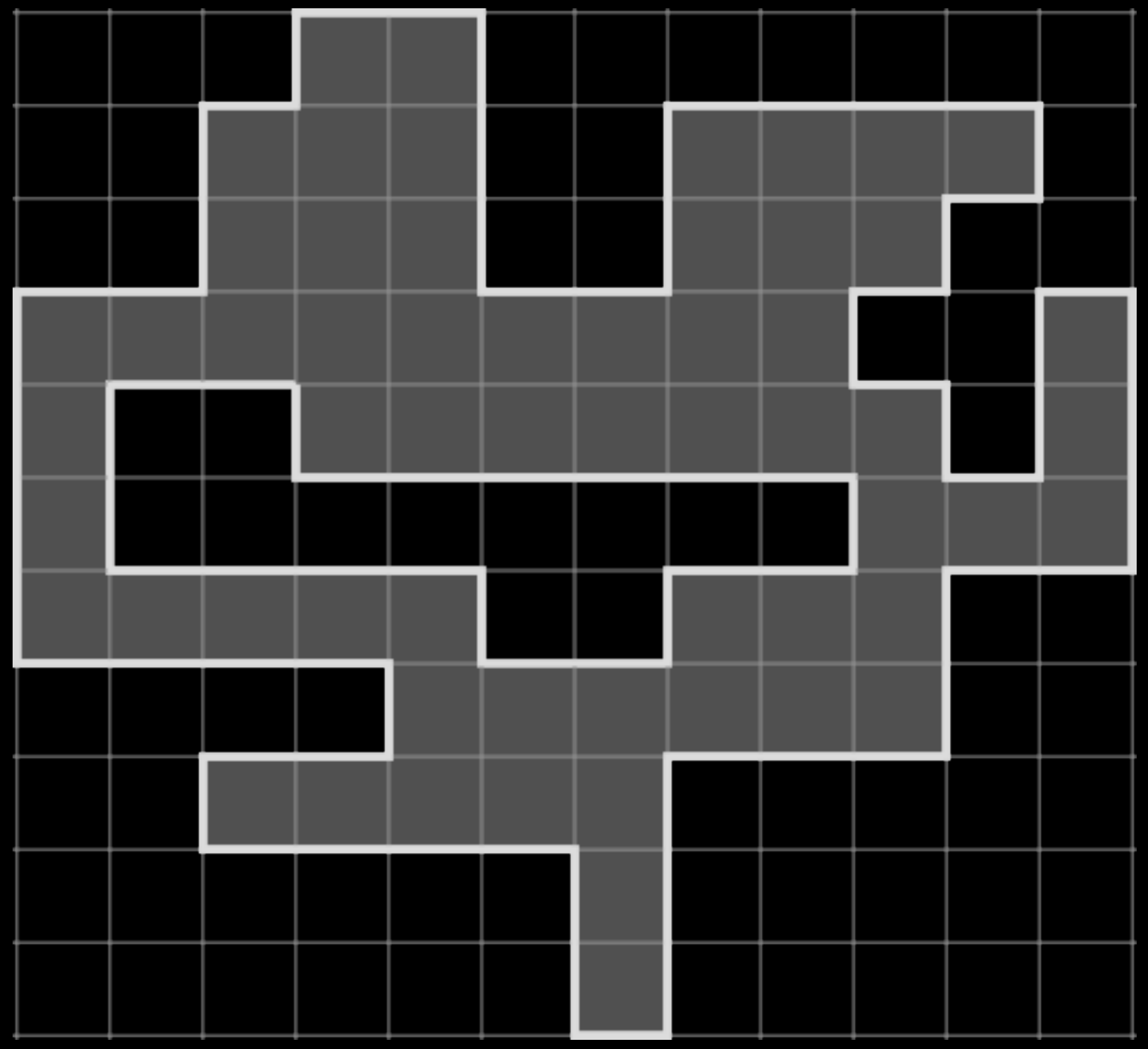

- Polyomino: connected polygon P in plane, formed by joining m unit squares on the square lattice

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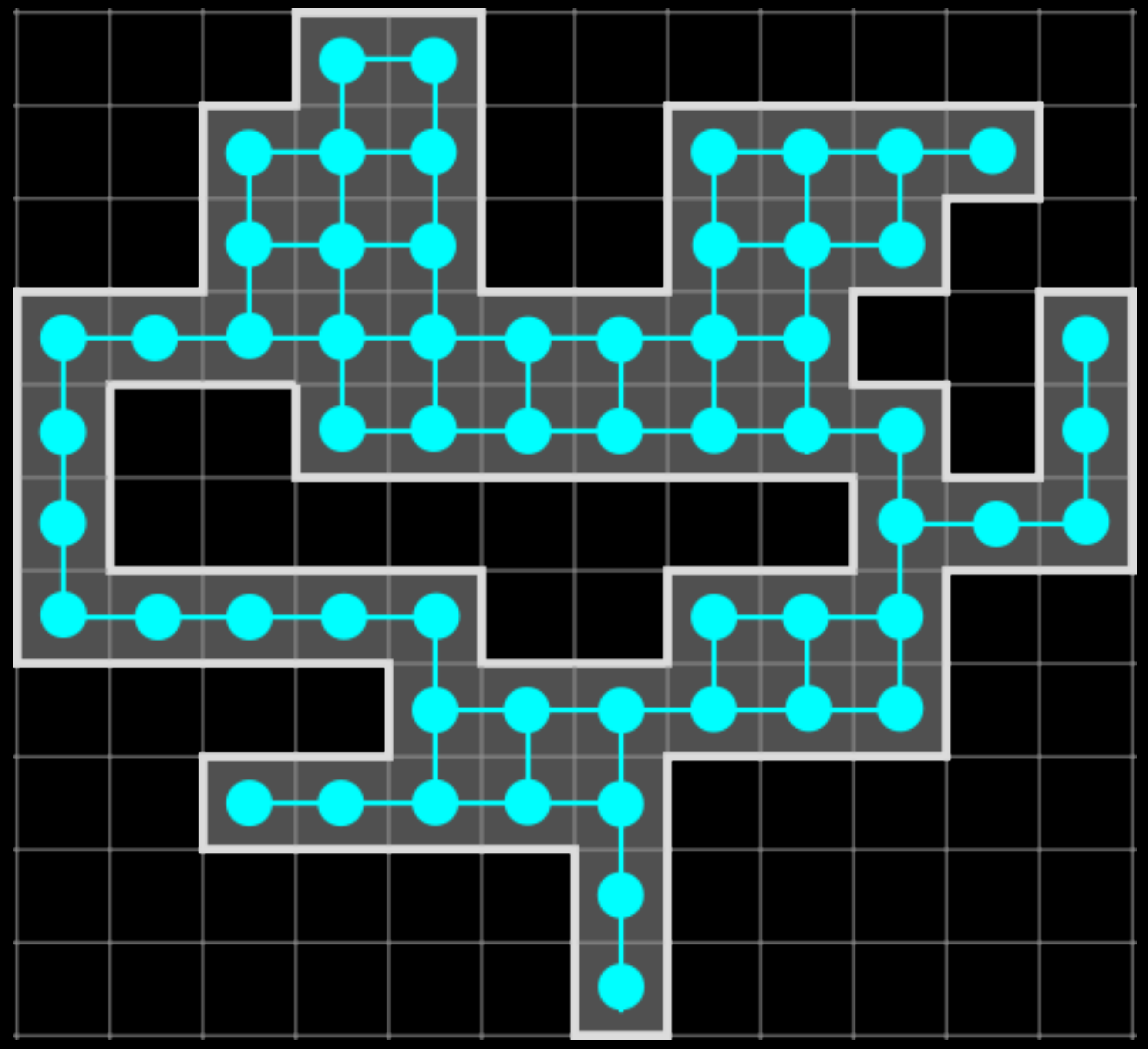
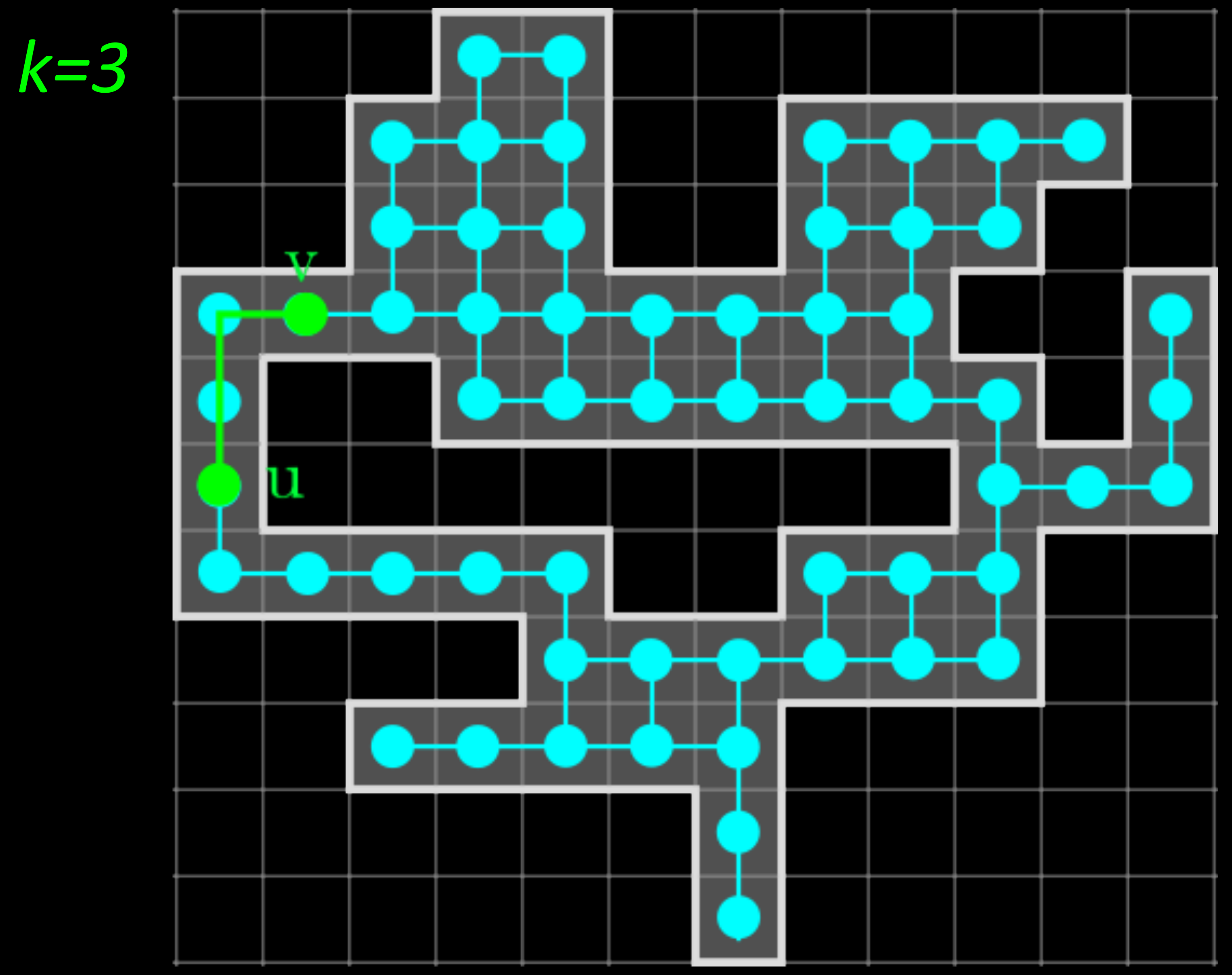
k-Hop Visibility: Formal Definition



- Polyomino: connected polygon P in plane, formed by joining m unit squares on the square lattice
- Dual graph G_P is a grid graph



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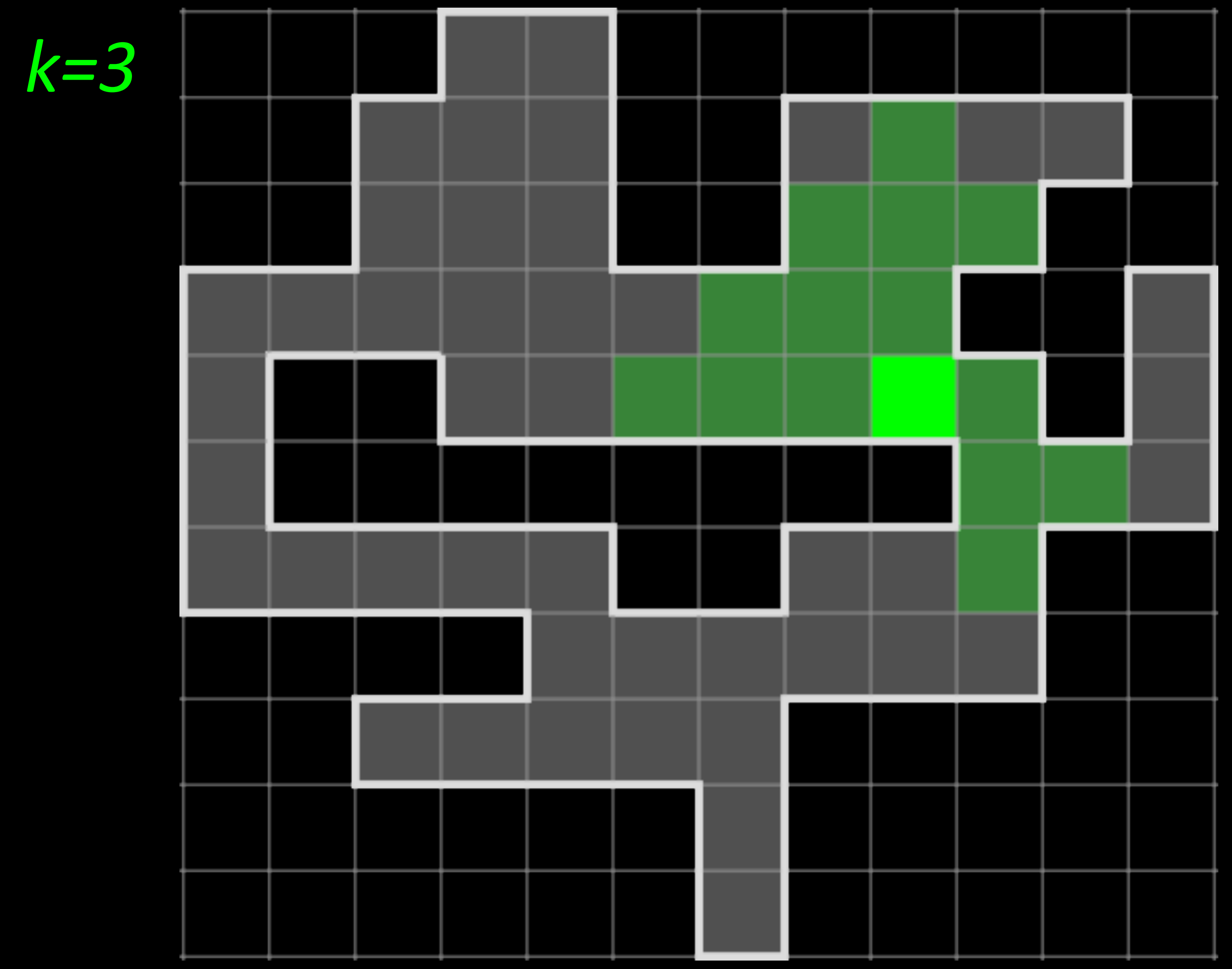

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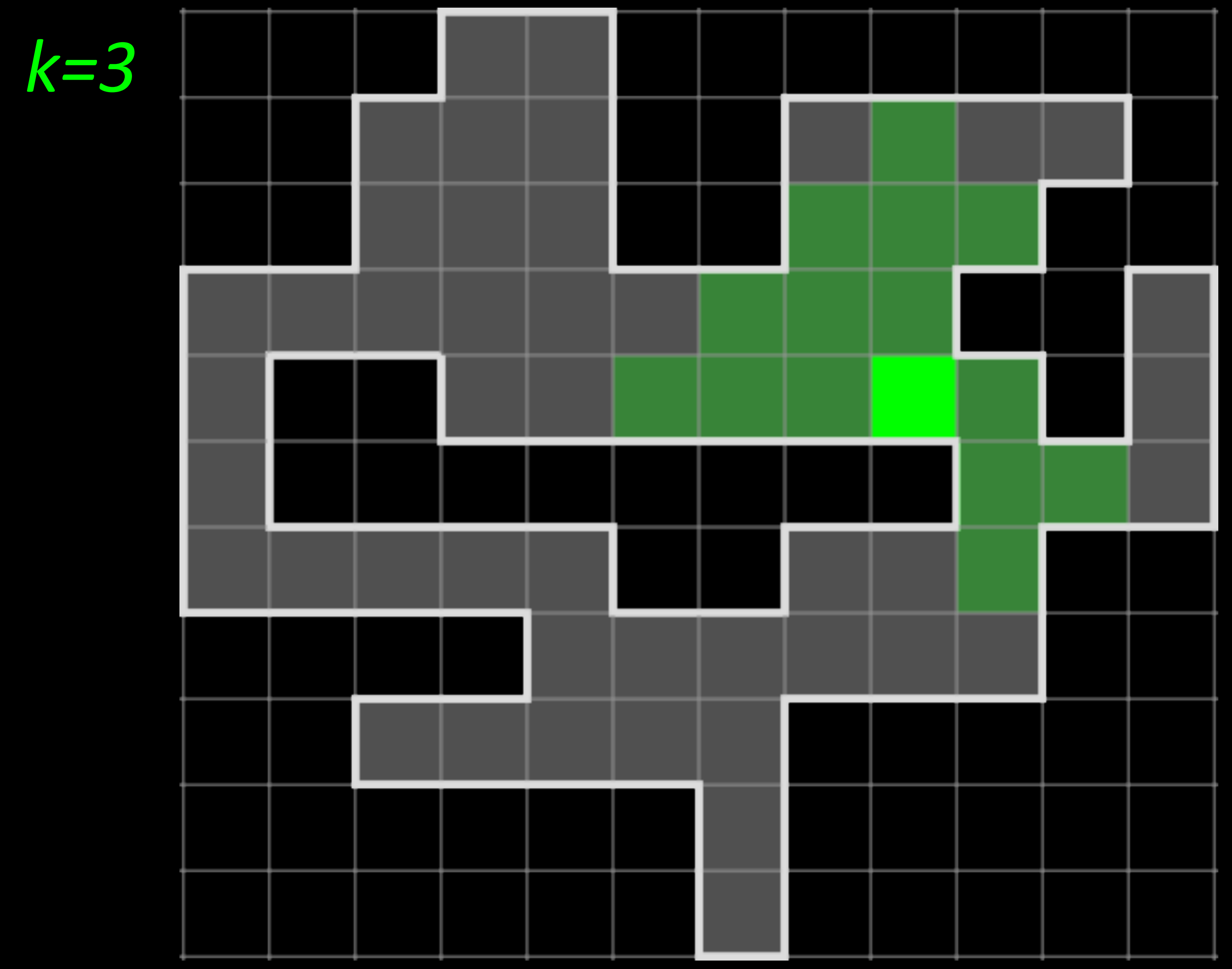

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Minimum *k*-hop Guarding Problem (M*k*GP)

Given: Polyomino *P*, range *k*

Find: Minimum cardinality unit-square guard cover in *P* under *k*-hop visibility.

Alternative Formulation

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Minimum *k*-Hop Dominating Set Problem (M*k*DSP)

Given: Graph G

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➡ We want to solve M*k*DSP in grid graphs

What Do We Know About Guarding Polyominoes + Thin Polygons and About the Minimum k-Hop Dominating Set Problem?

If the dual graph of the partition obtained by extending all polygon edges through incident reflex vertices is a tree

Guarding Polyominoes + Thin Orthogonal Polygons

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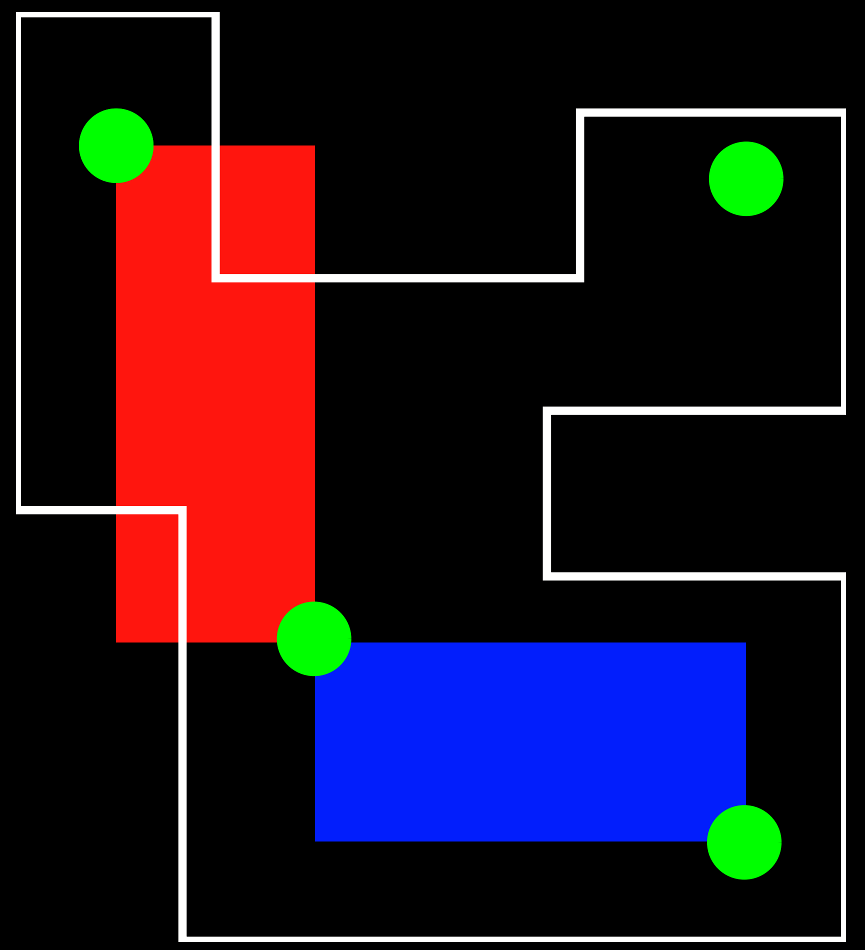
Rectilinear visibility/ r-visibility:



Two points are r-visible to each other if there exists a rectangle in P that contains both points.

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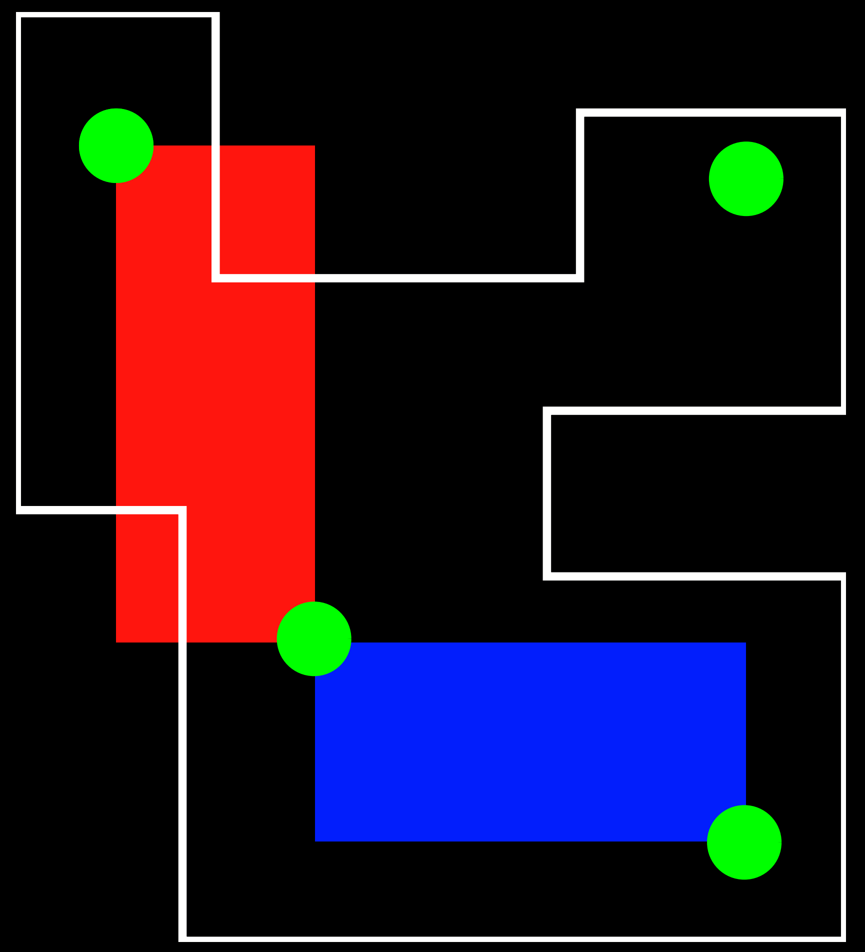

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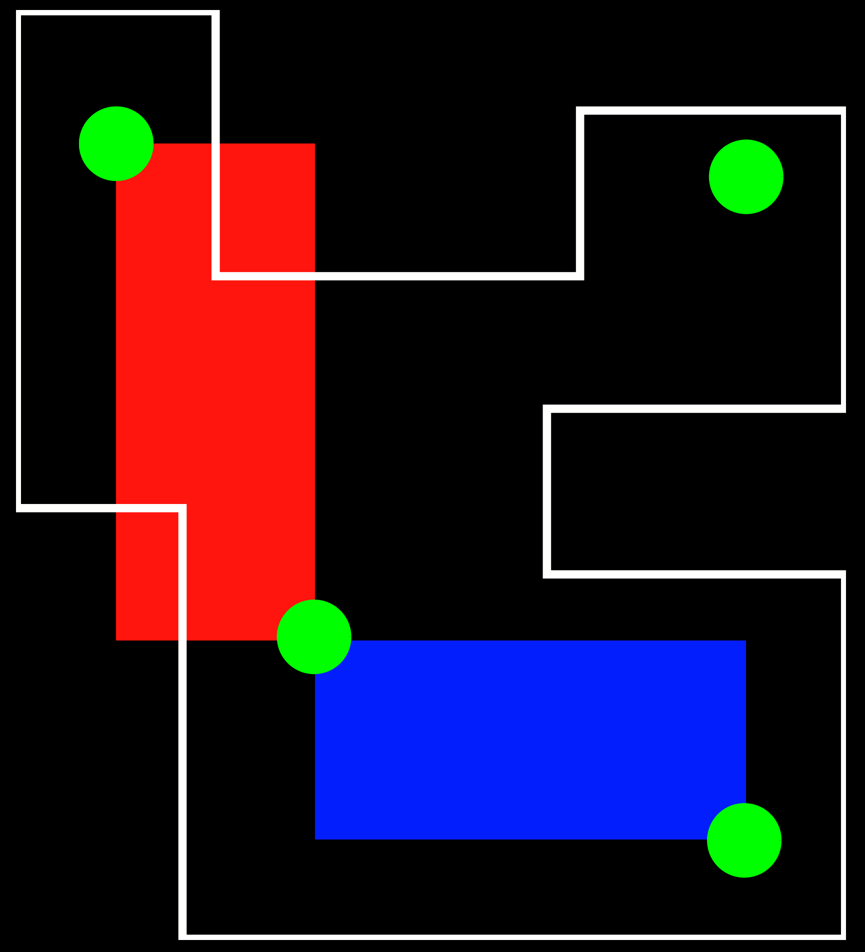

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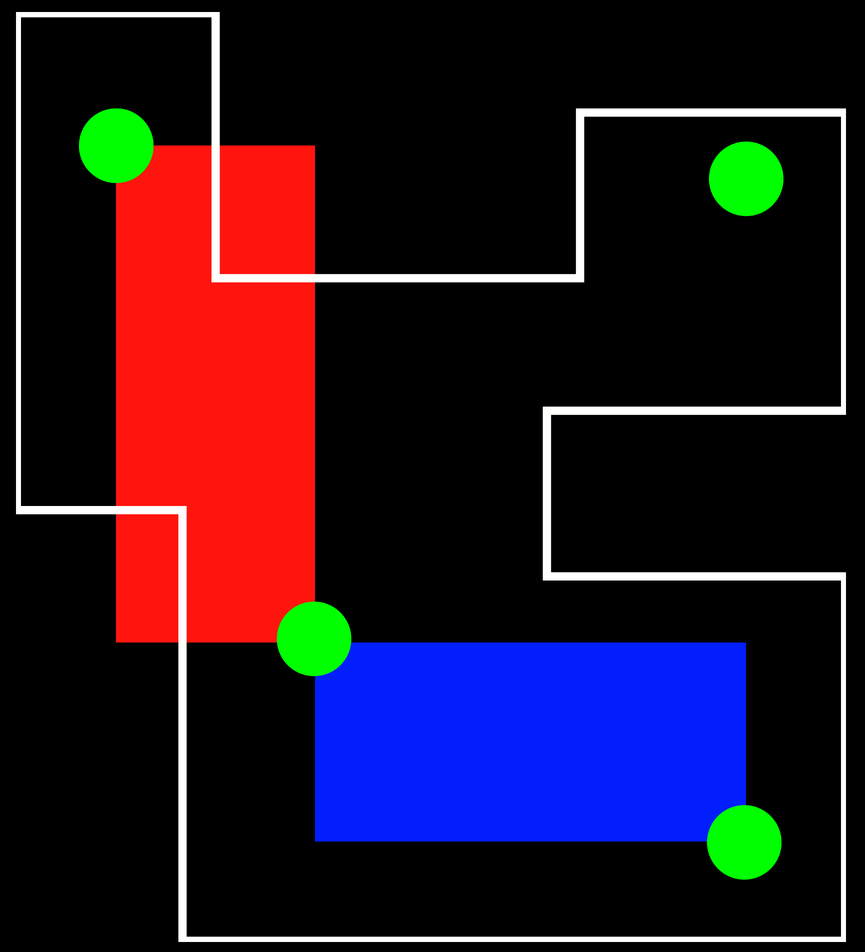
  - NP-hard in polygons with holes
  - Linear-time algorithm for tree polygons
  - Generalizes to polygons with h holes or thickness t (dual graph does not contain an induced $(t+1)$x$(t+1)$ grid)➜ fixed-parameter tractable in $t+h$



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- Orthogonal polygons with bounded treewidth [*]:
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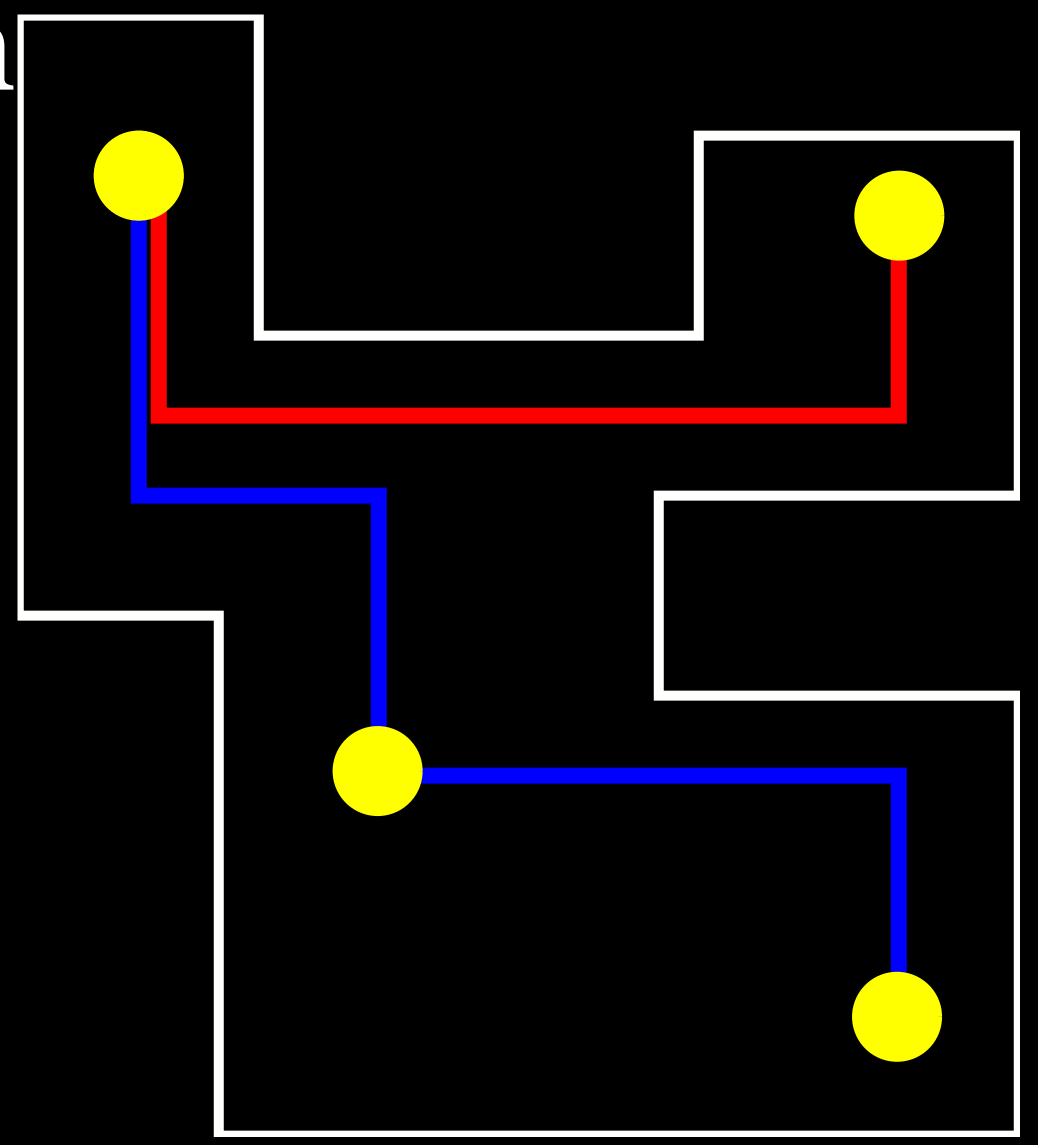
Staircase visibility/ s-visiblity:



Two points are s-visible to each other if there exists a staircase path in P that connects them.

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Guarding Polyominoes + Thin Orthogonal Polygons

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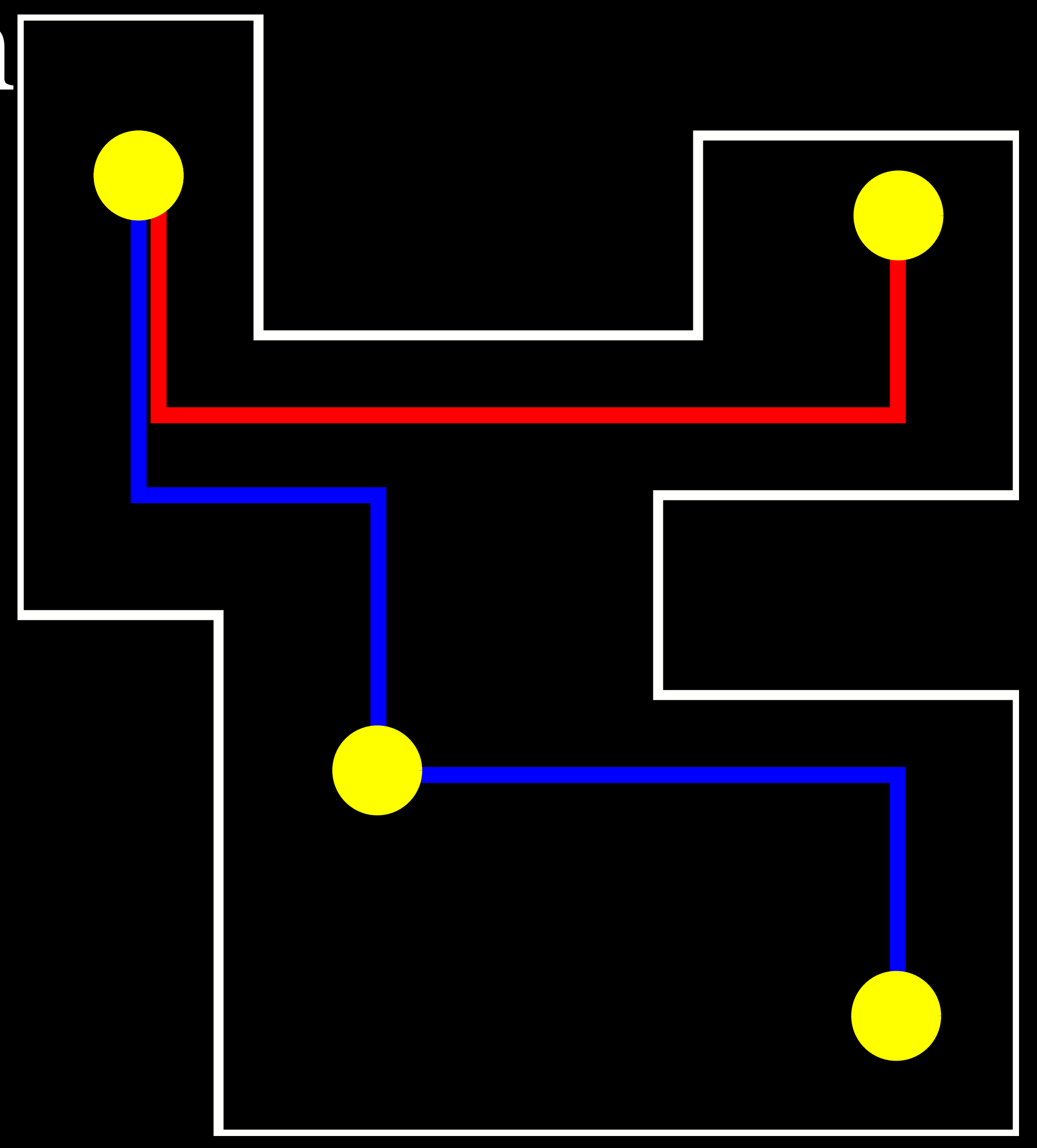

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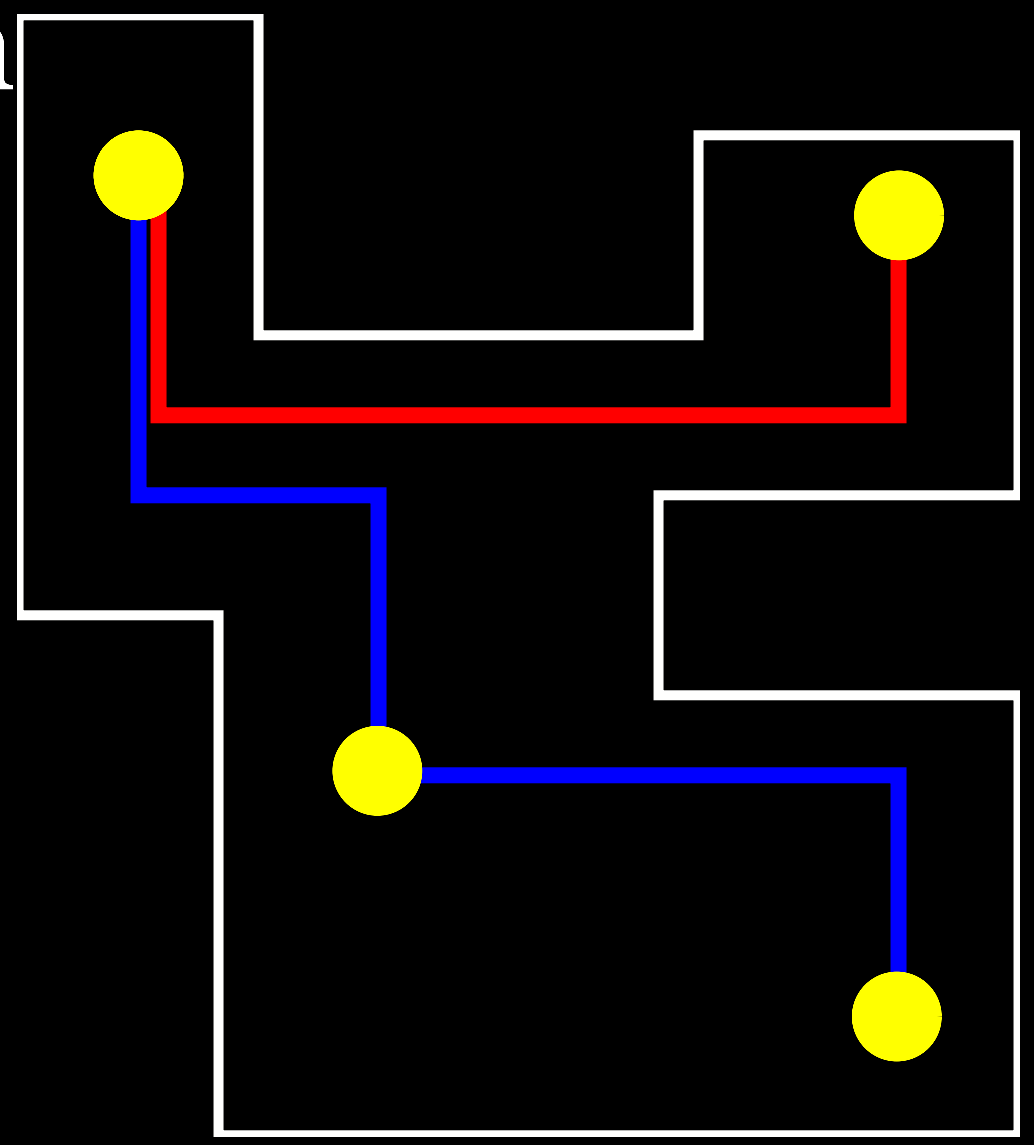
- Rectilinear visibility [#]:



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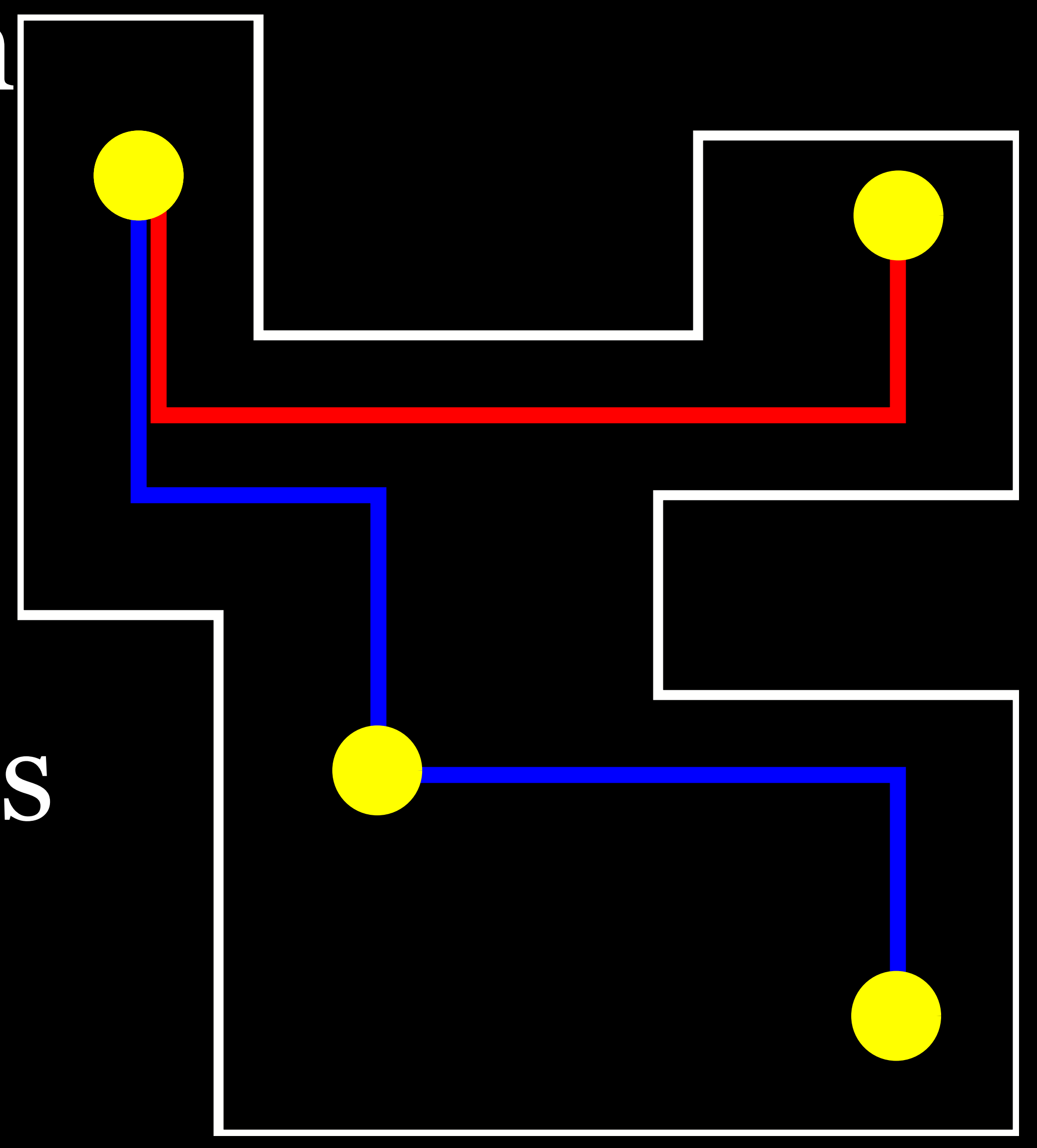

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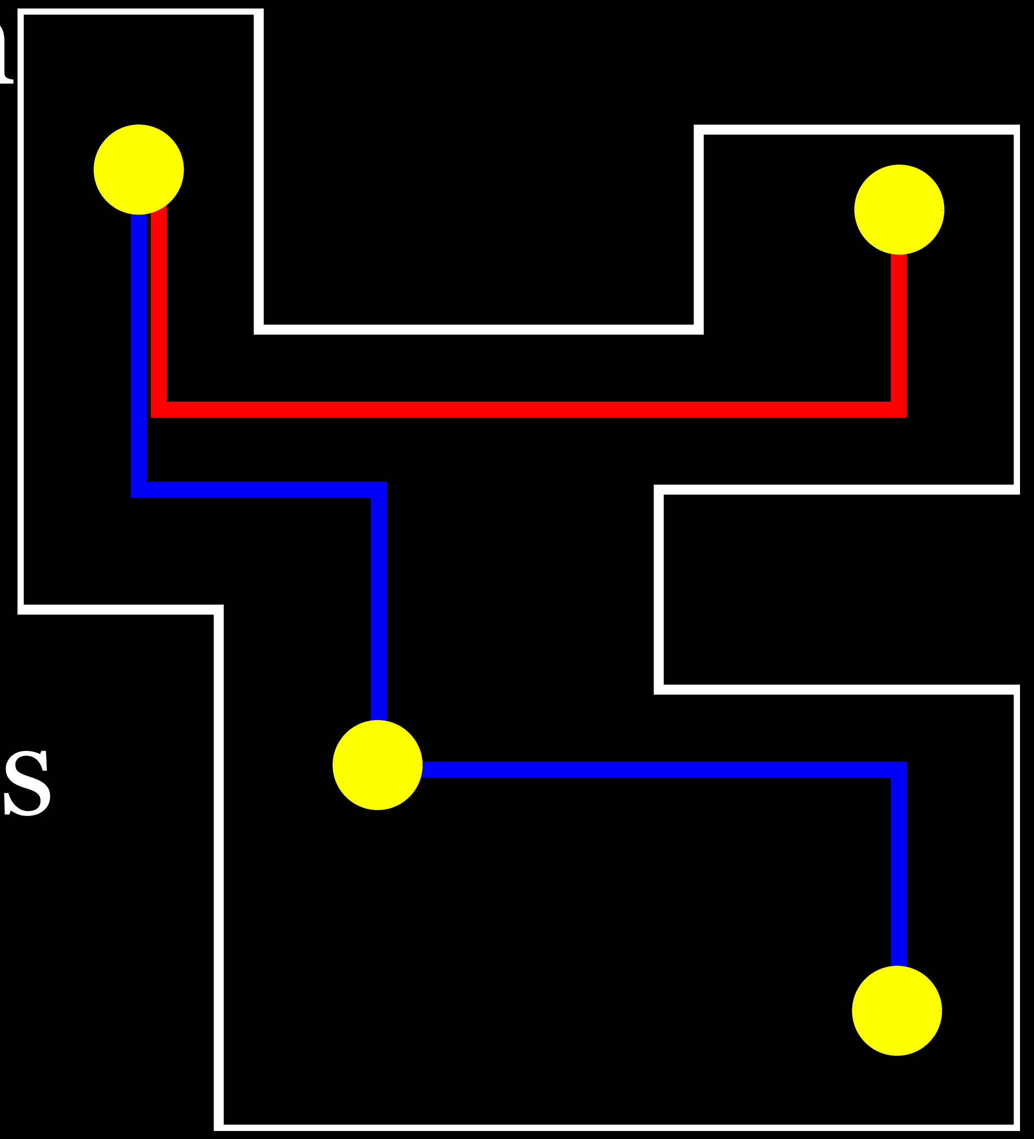
  ➡ Linear-time algorithm



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Unit square visible from a guard if and only if the guard sees all points of the unit square under ordinary visibility

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- Simple polyominoes, all-or-nothing visibility and ordinary visibility[*]: NP-hard

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- Polyominoes with holes, rectilinear visibility [#]: NP-hard

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- Generalized to polycubes [+]

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Minimum k-Hop Dominating Set Problem

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- NP-complete in general graphs [*,#]

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Minimum k-Hop Dominating Set Problem

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Some More Definitions

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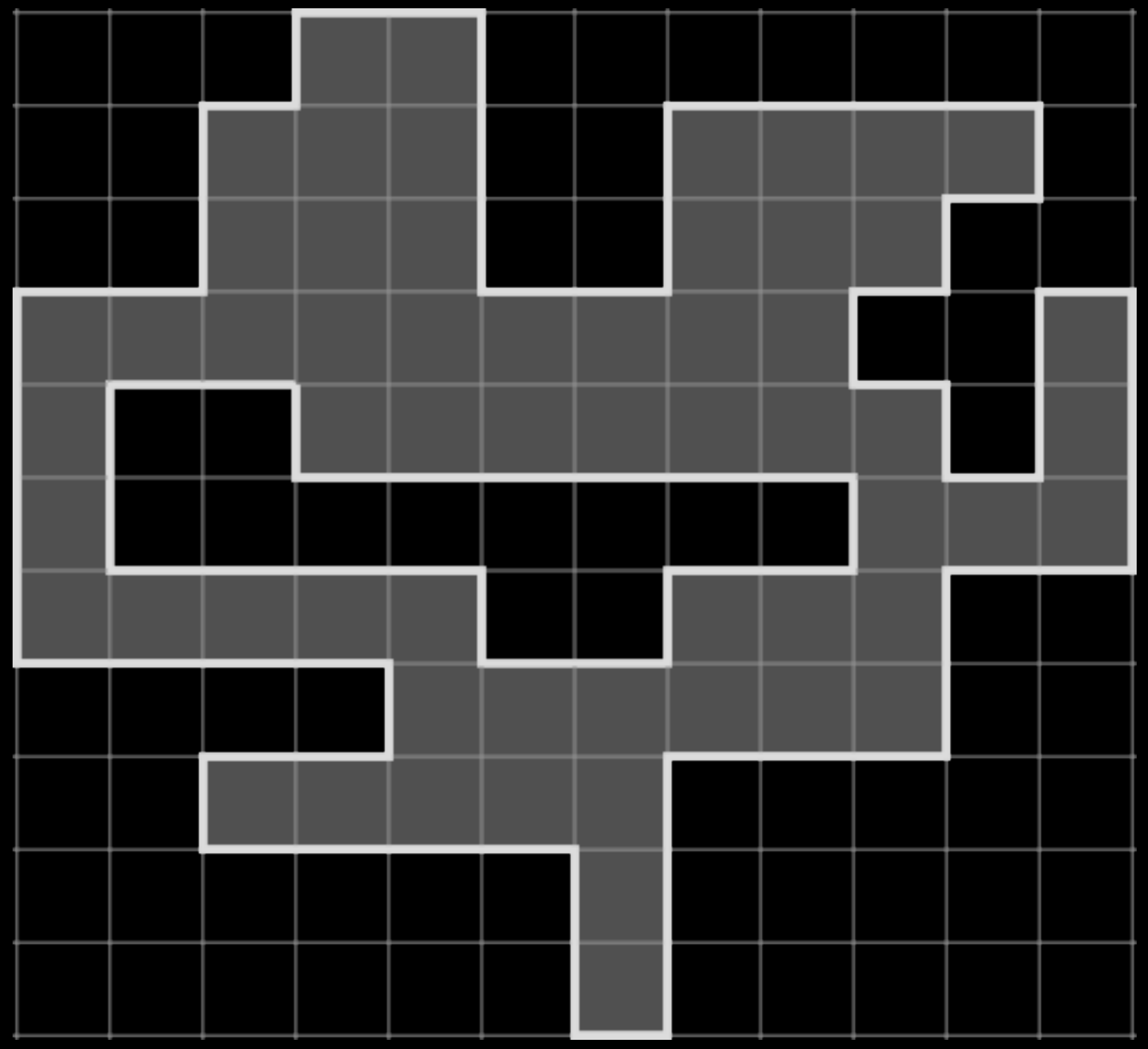

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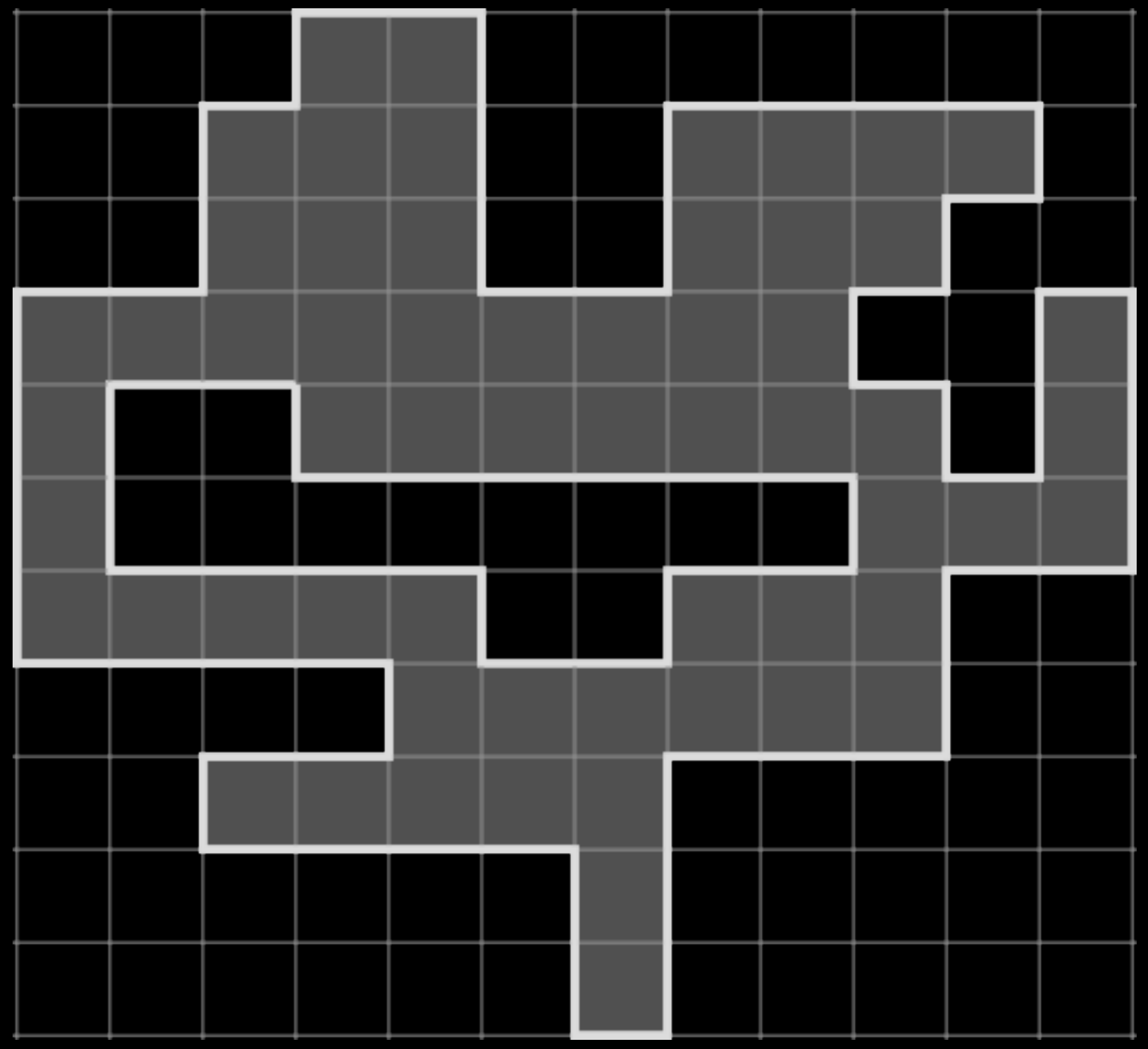

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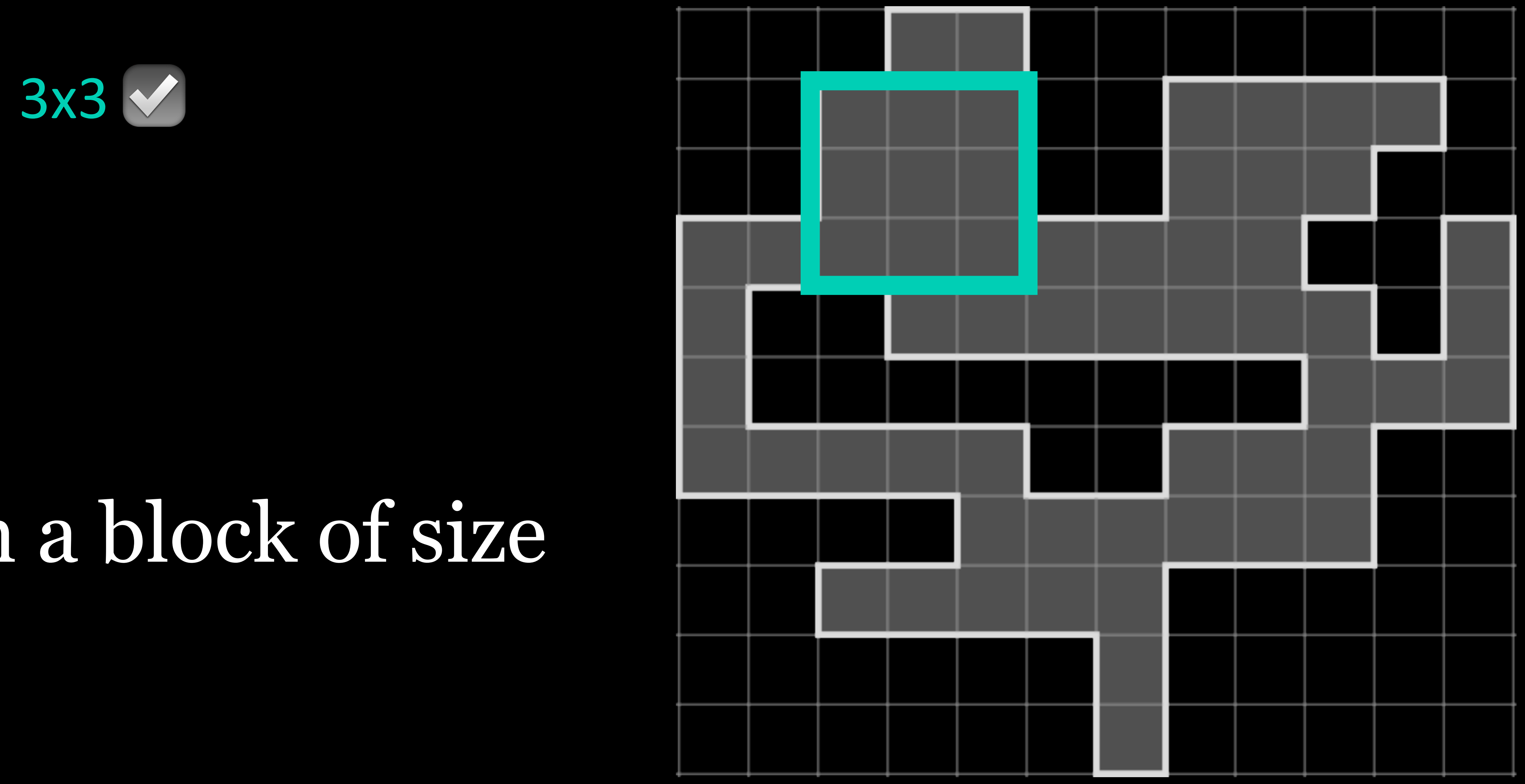

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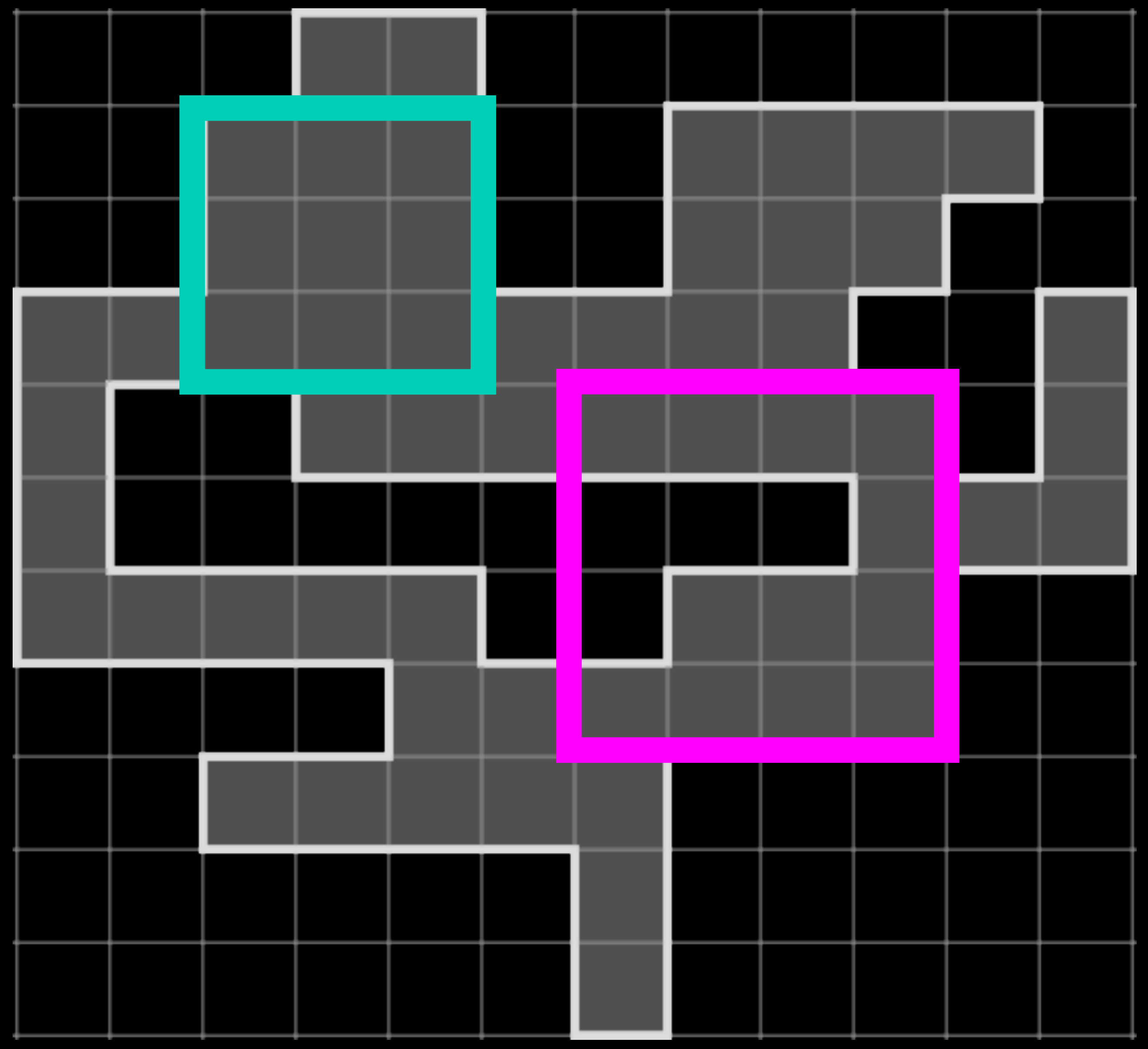

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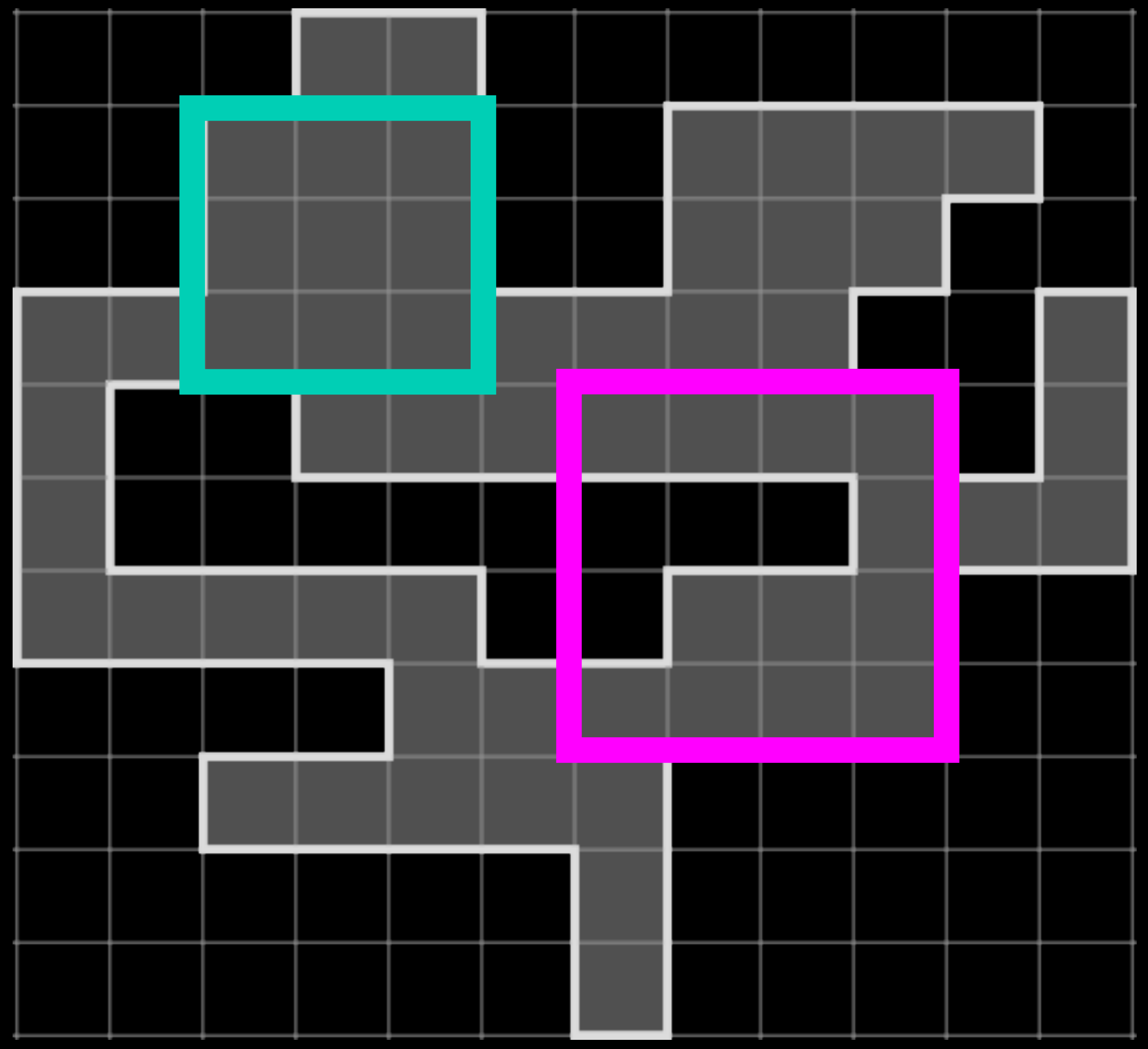

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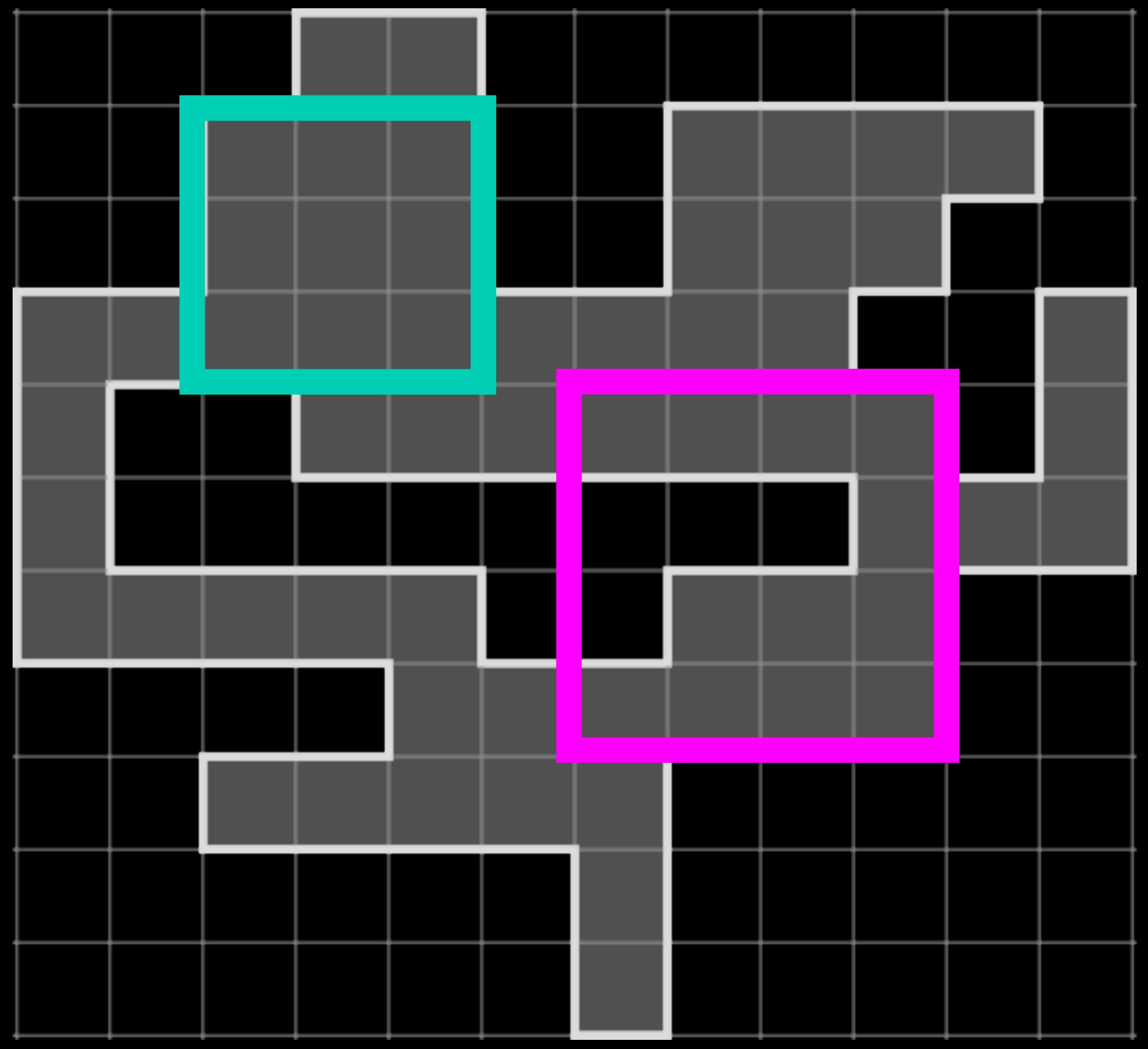

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- Polyomino is simple if it has no holes (every inner face in dual grid graph has unit area)

VC Dimension

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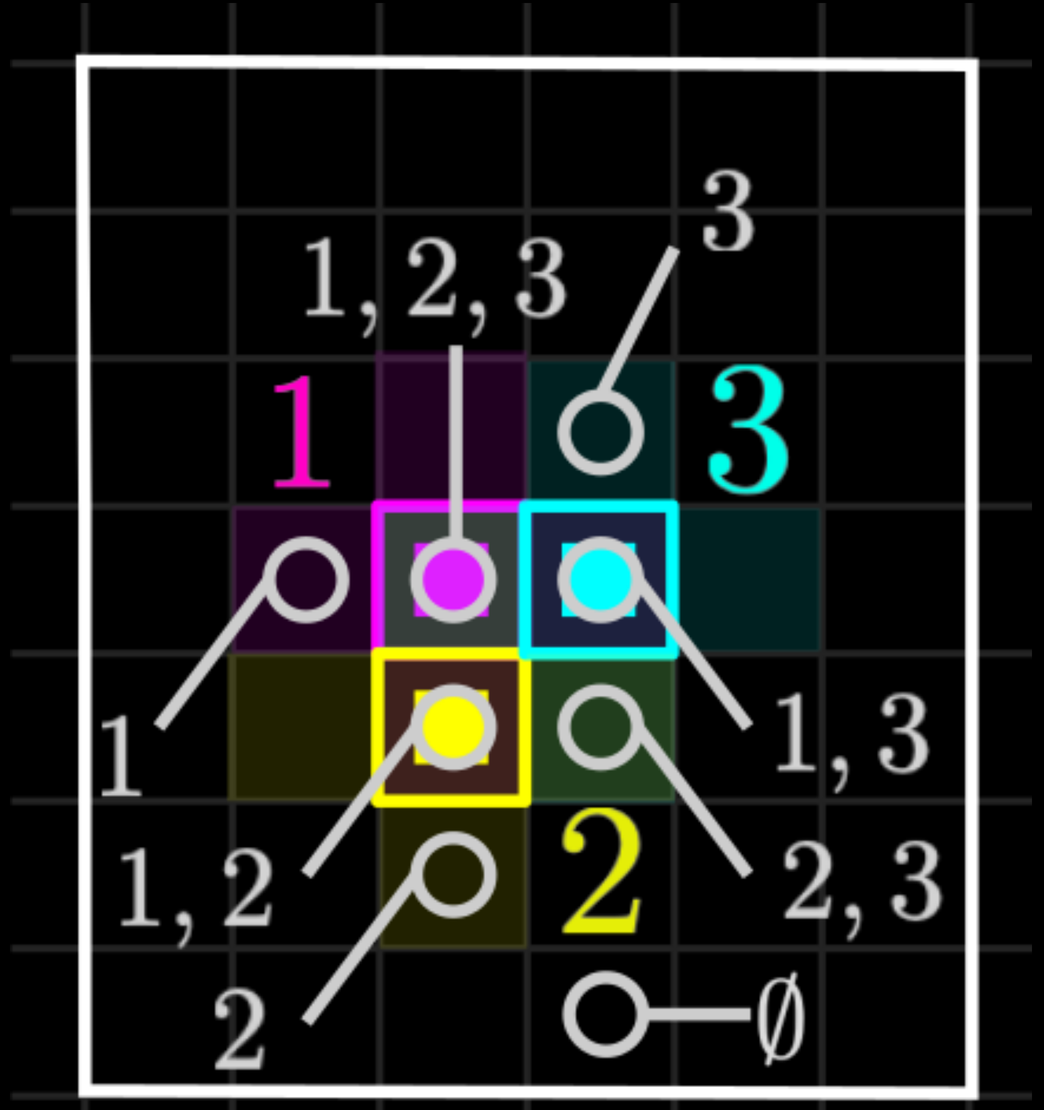
VC dimension: largest d, such that there exists a polyomino P and a set of d unit-square guards D that can be shattered.

What Is the VC Dimension?

3 guards (*d*=3)
k=1
2^3=8 viewpoints



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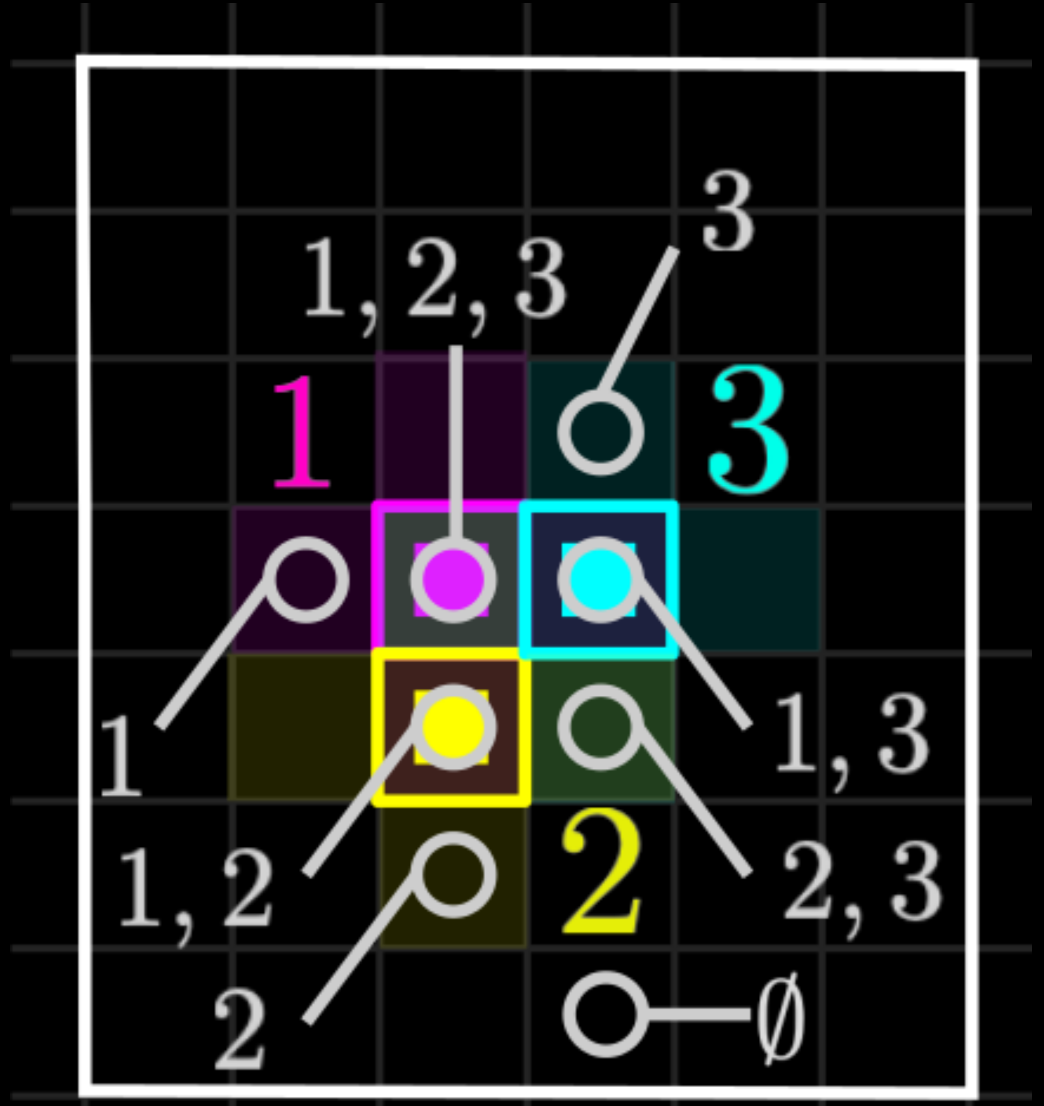

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  Has been studied for different guarding problems (e.g., for simple polygons, ordinary visibility VC dimension in [6,14])

VC Dimension

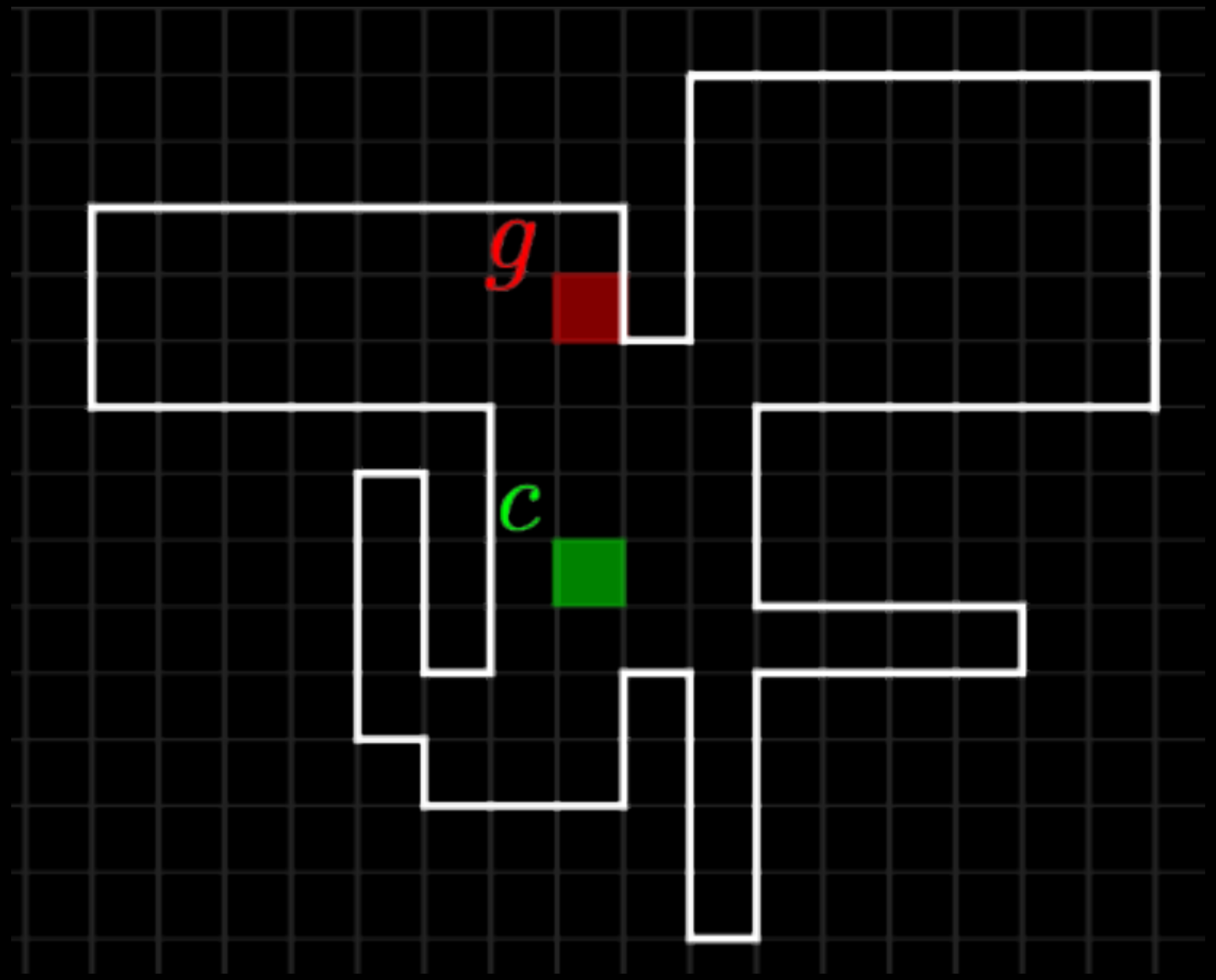
We show:

Theorem 1: For any $k \in \mathbb{N}$, the VC dimension of k-hop visibility in a simple polyomino is 3.

Theorem 2: For large enough $k \in \mathbb{N}$, the VC dimension of k-hop visibility in a polyomino with holes is 4.

For Upper Bounds: Rest Budget

- *Rest budget* of unit square $g \in P$ at a unit square $c \in P$: $\mathrm{rb}(g,c)=\max\{k-d(g,c),0\}$



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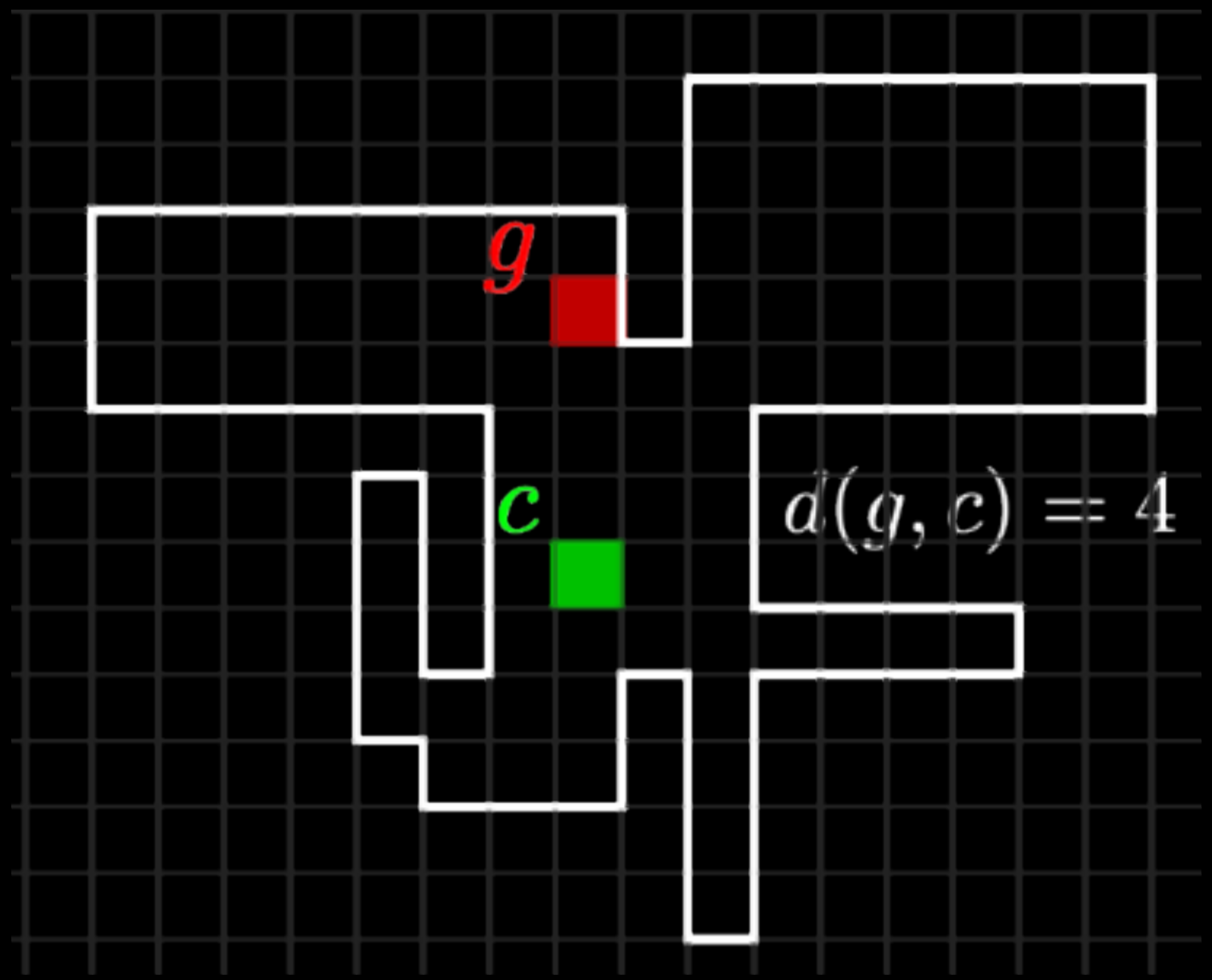

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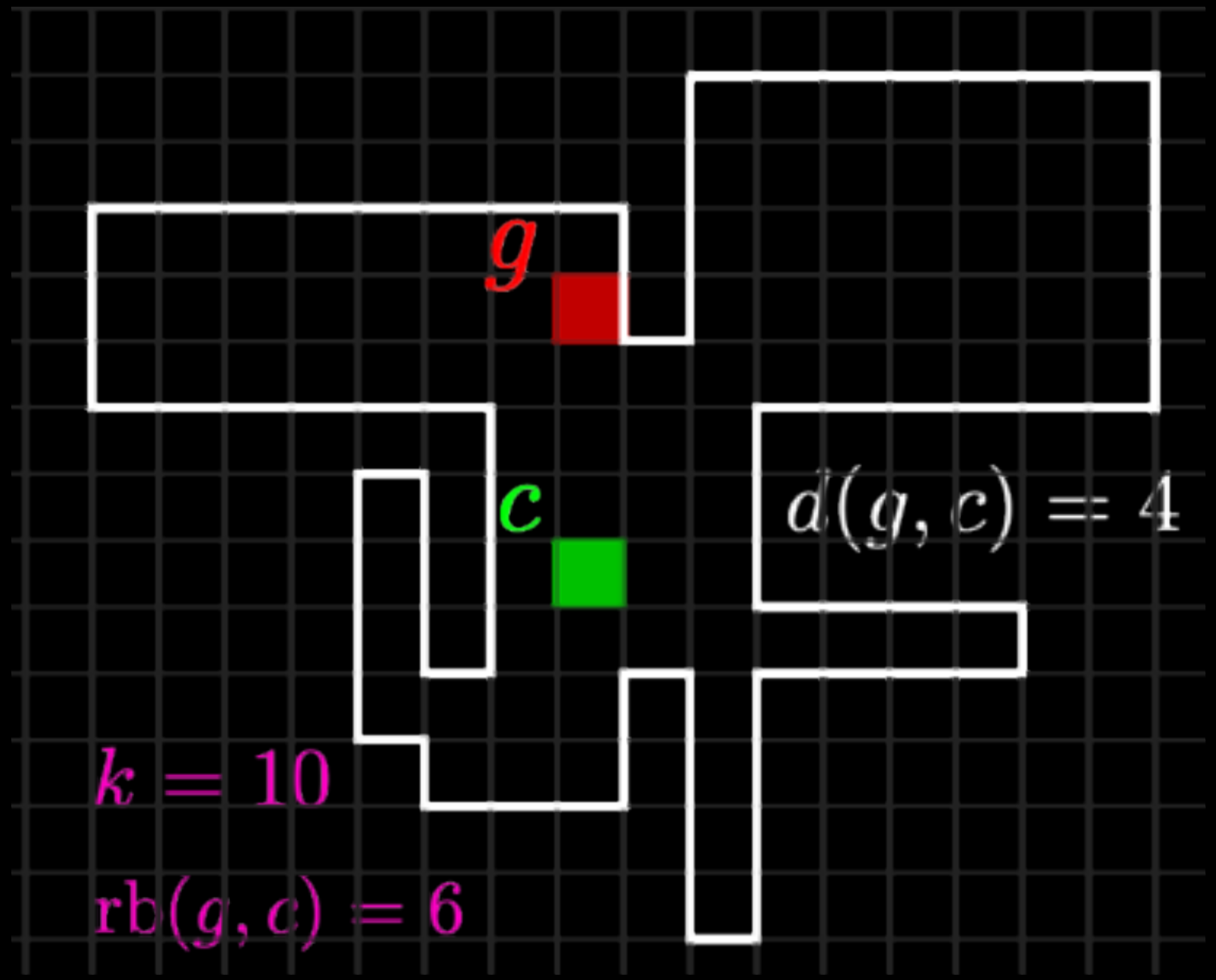

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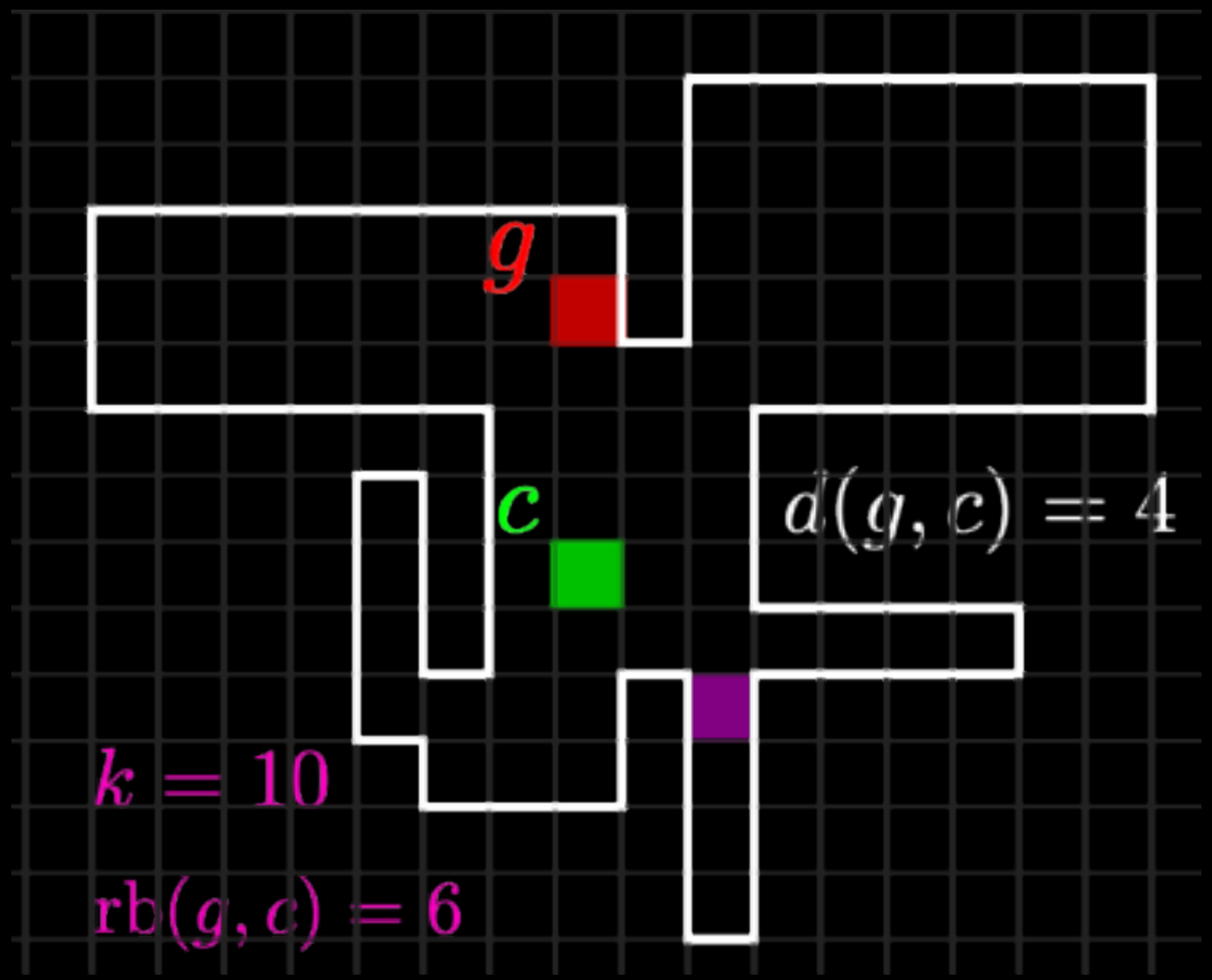

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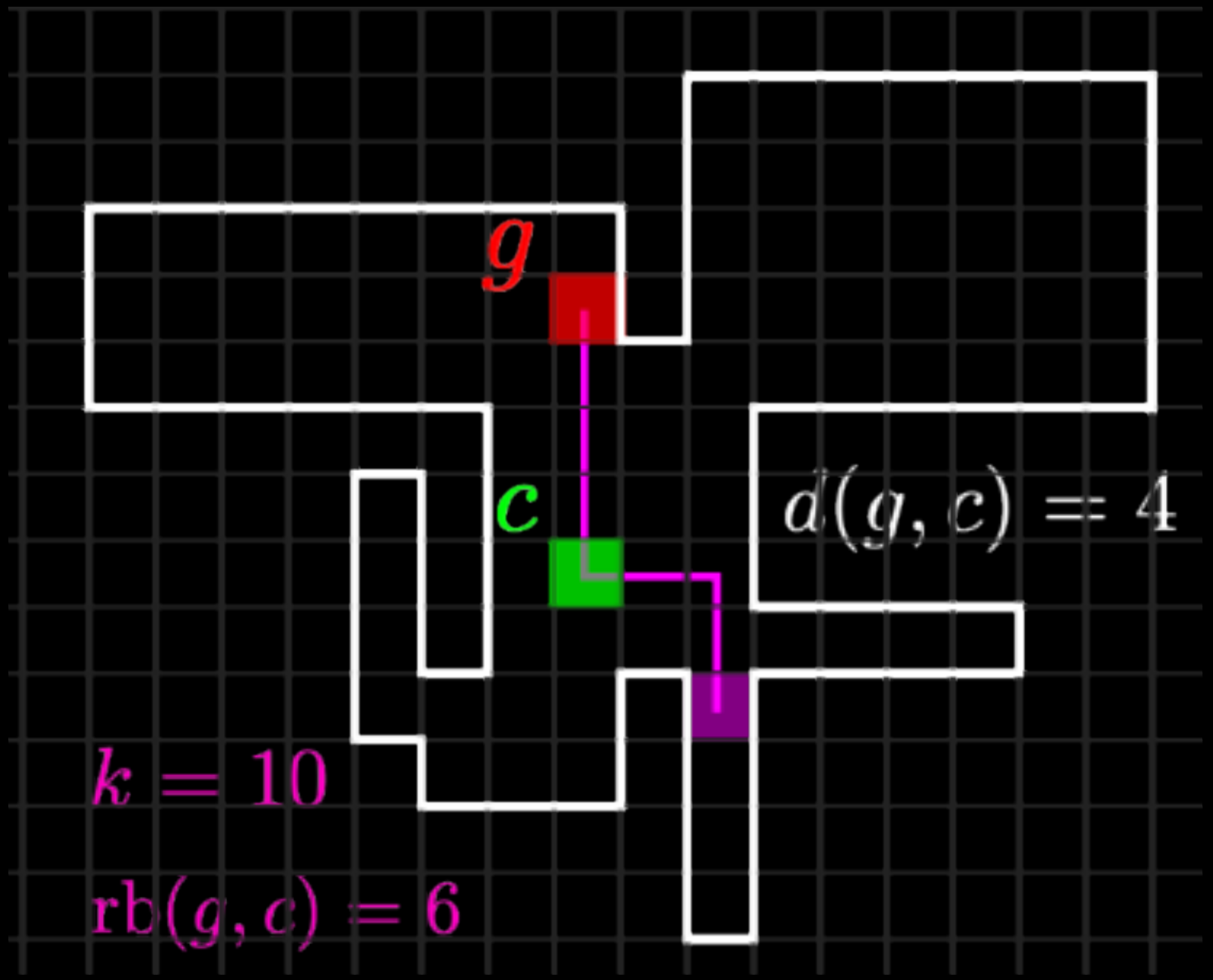

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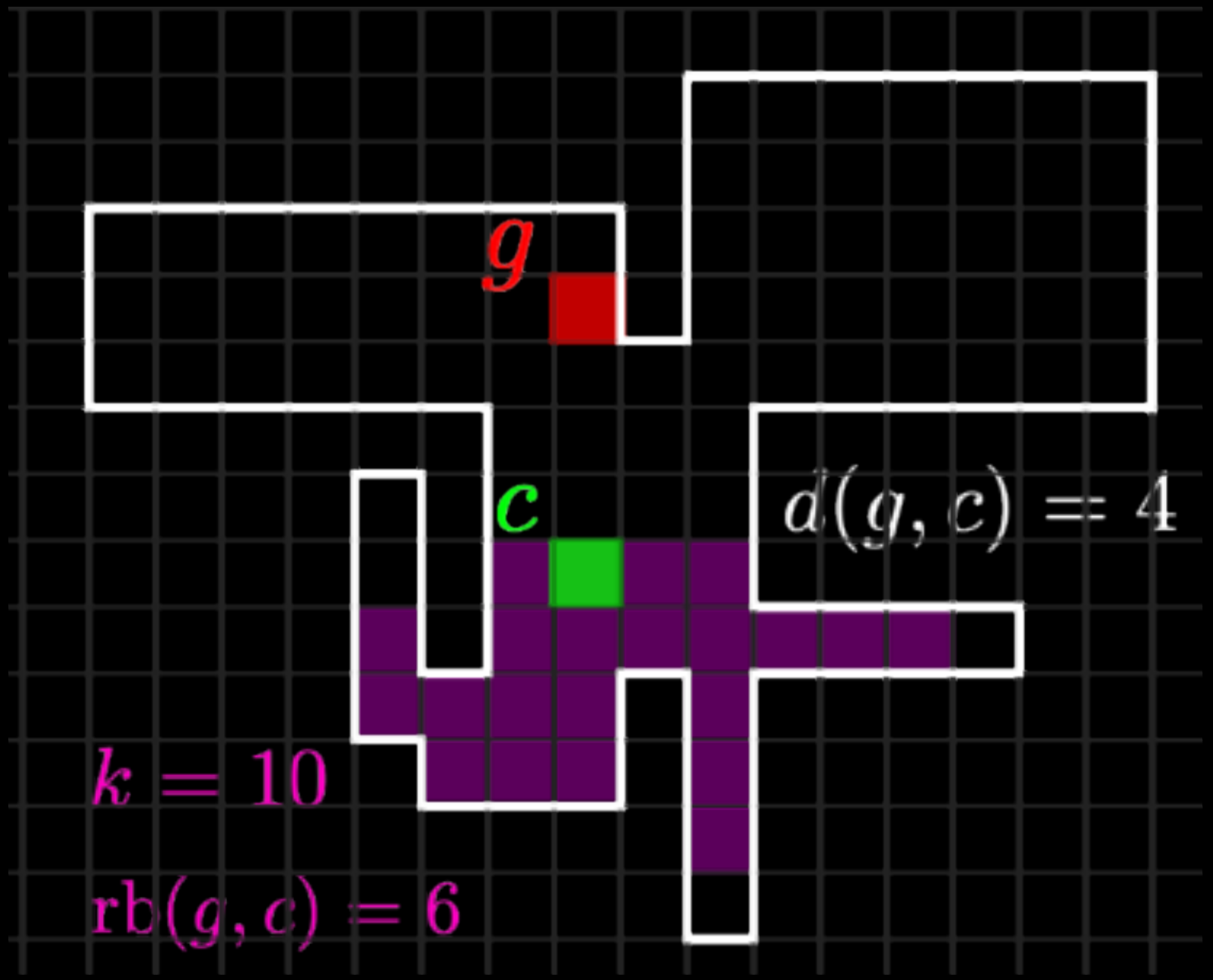

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Let *P* be a polyomino, and let *g* and *u* be two unit squares in *P* such that a shortest path between them contains a unit square *c*. Then the following holds:

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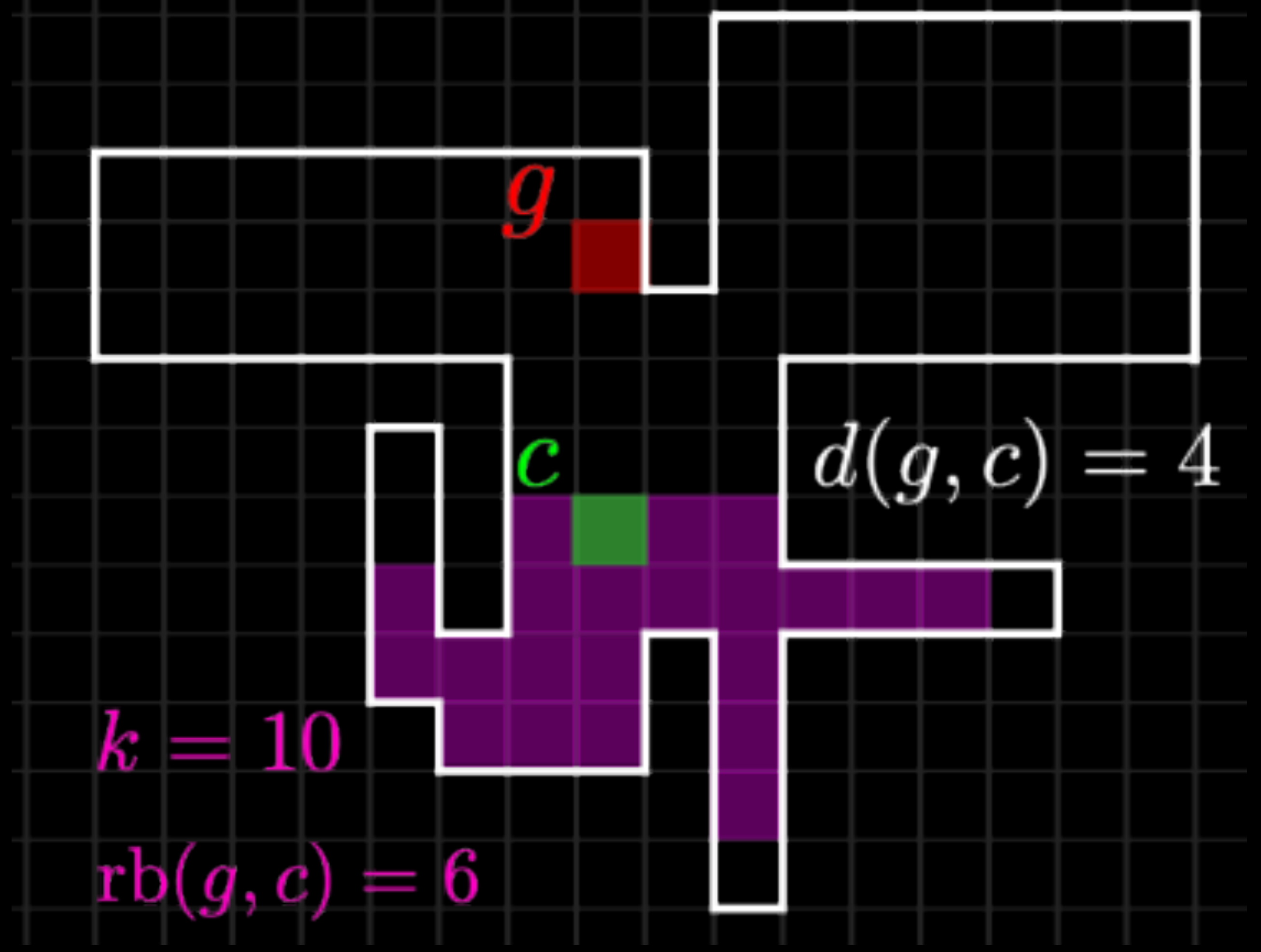

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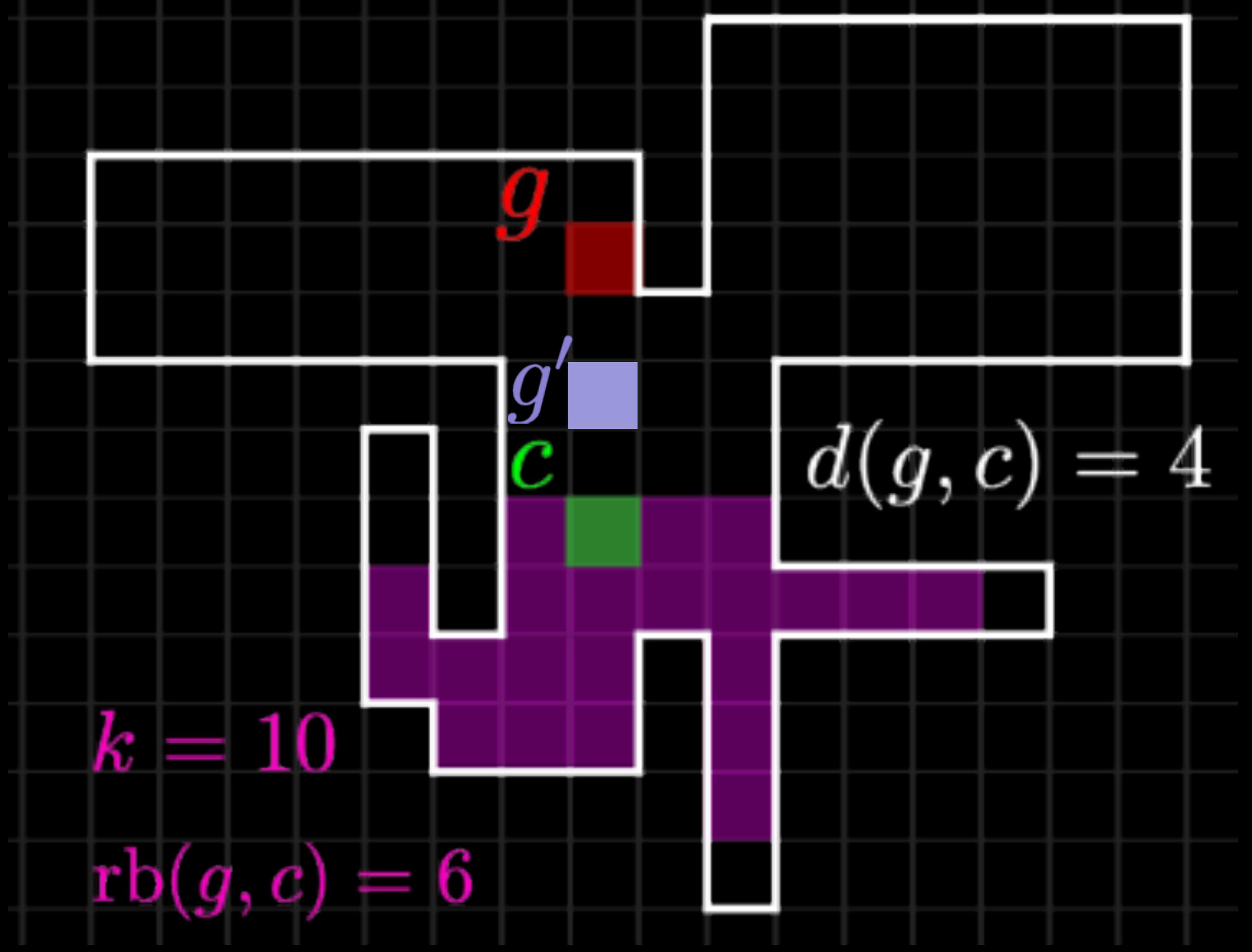

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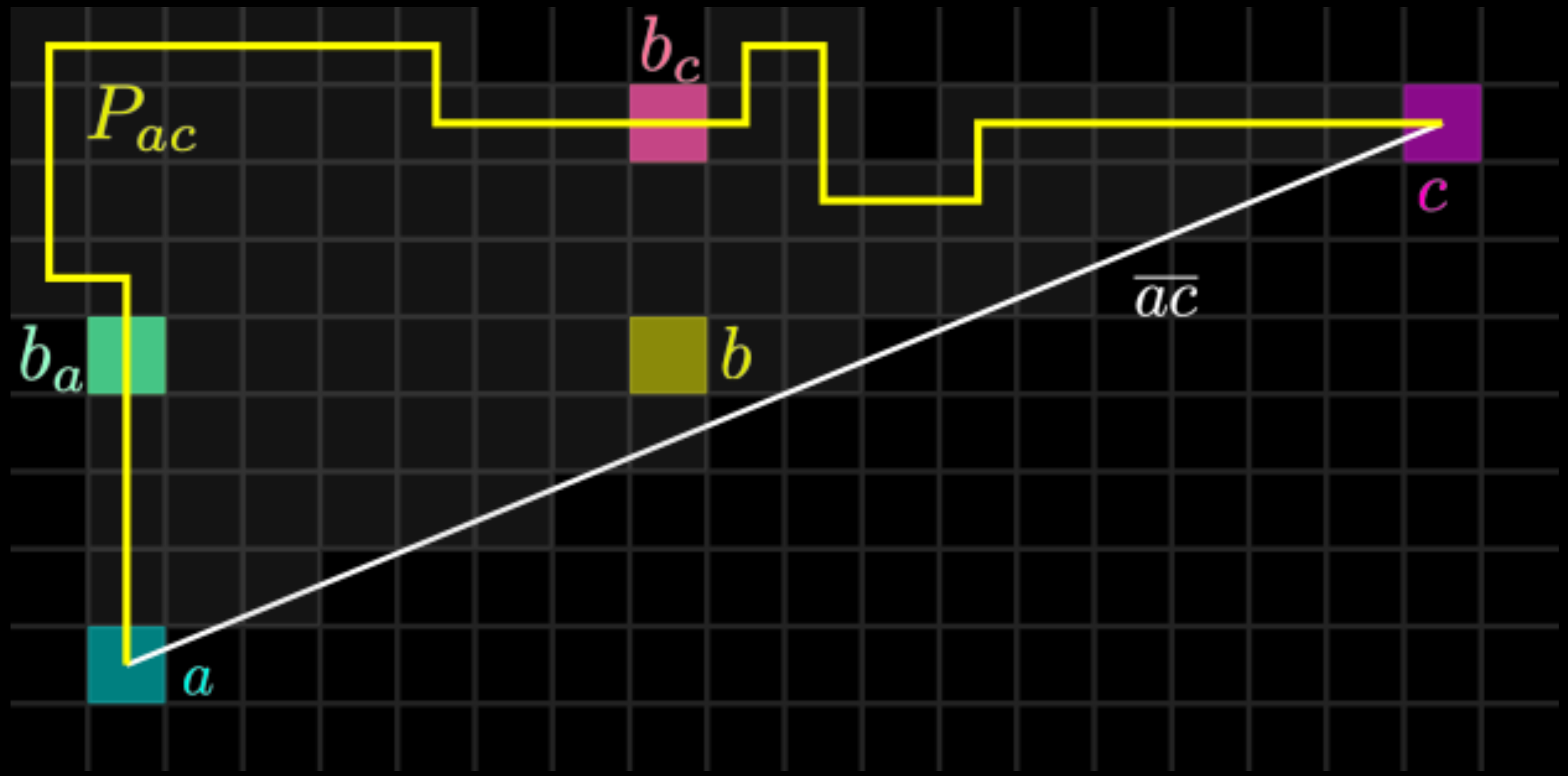

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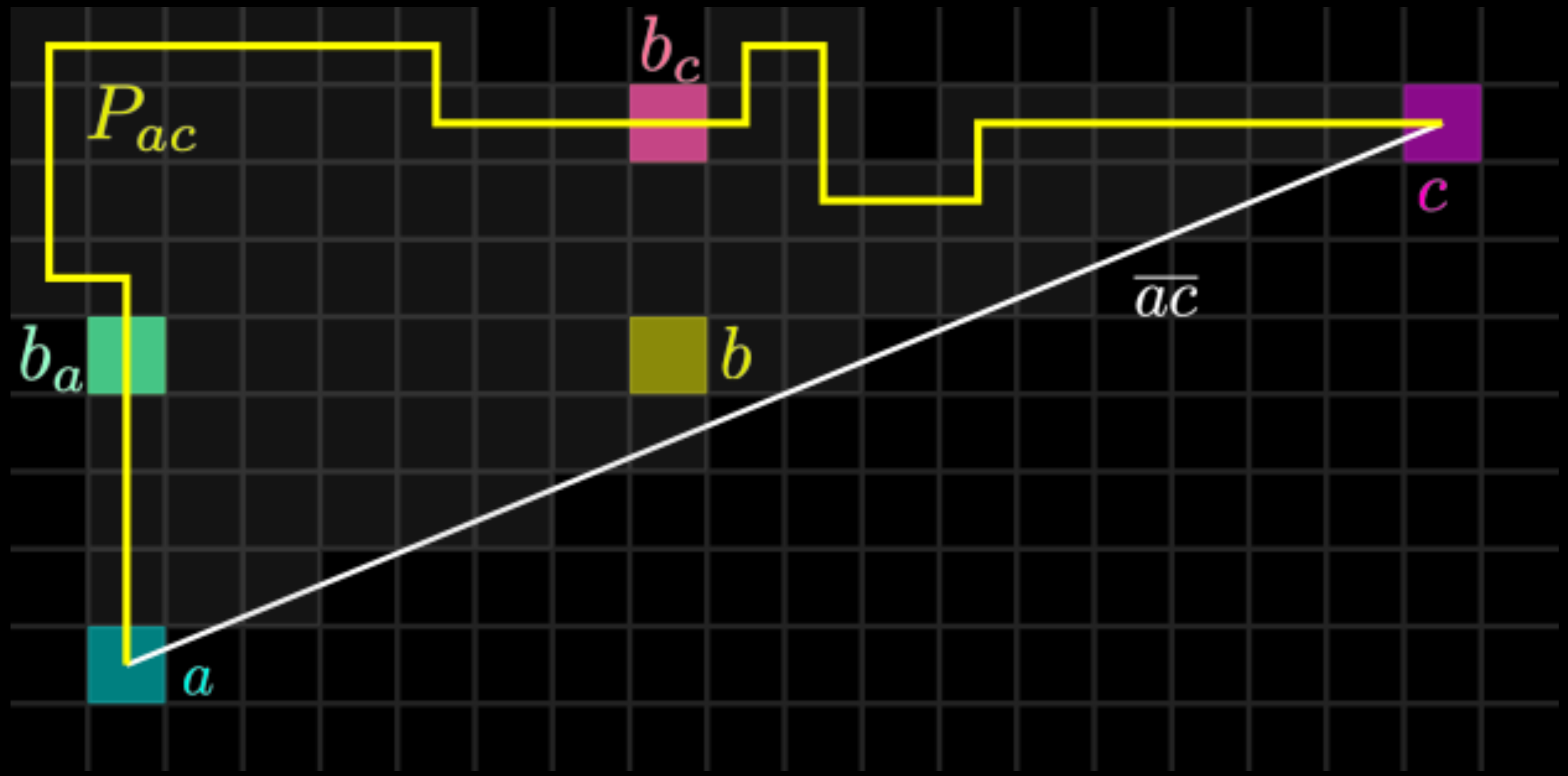

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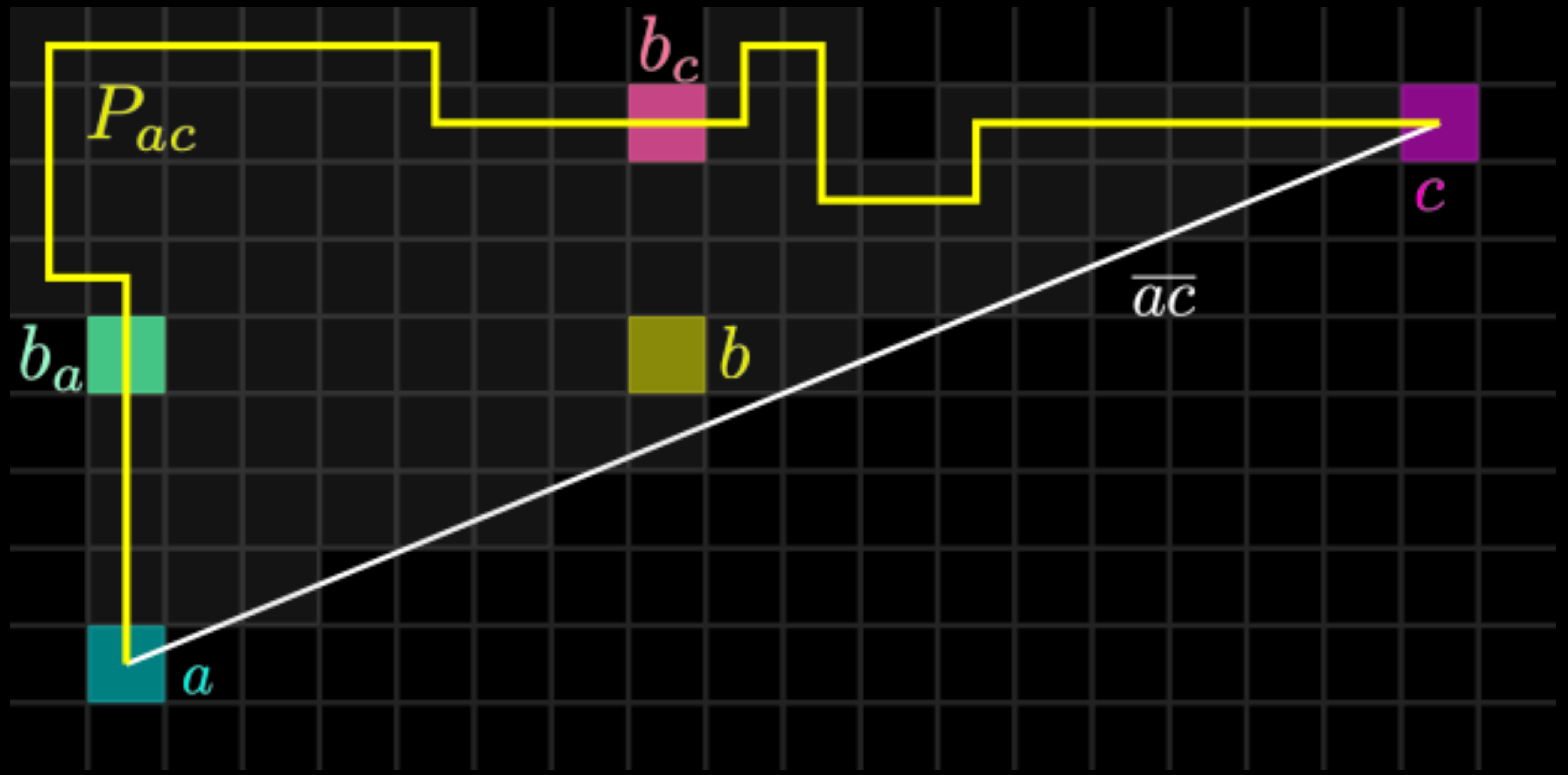

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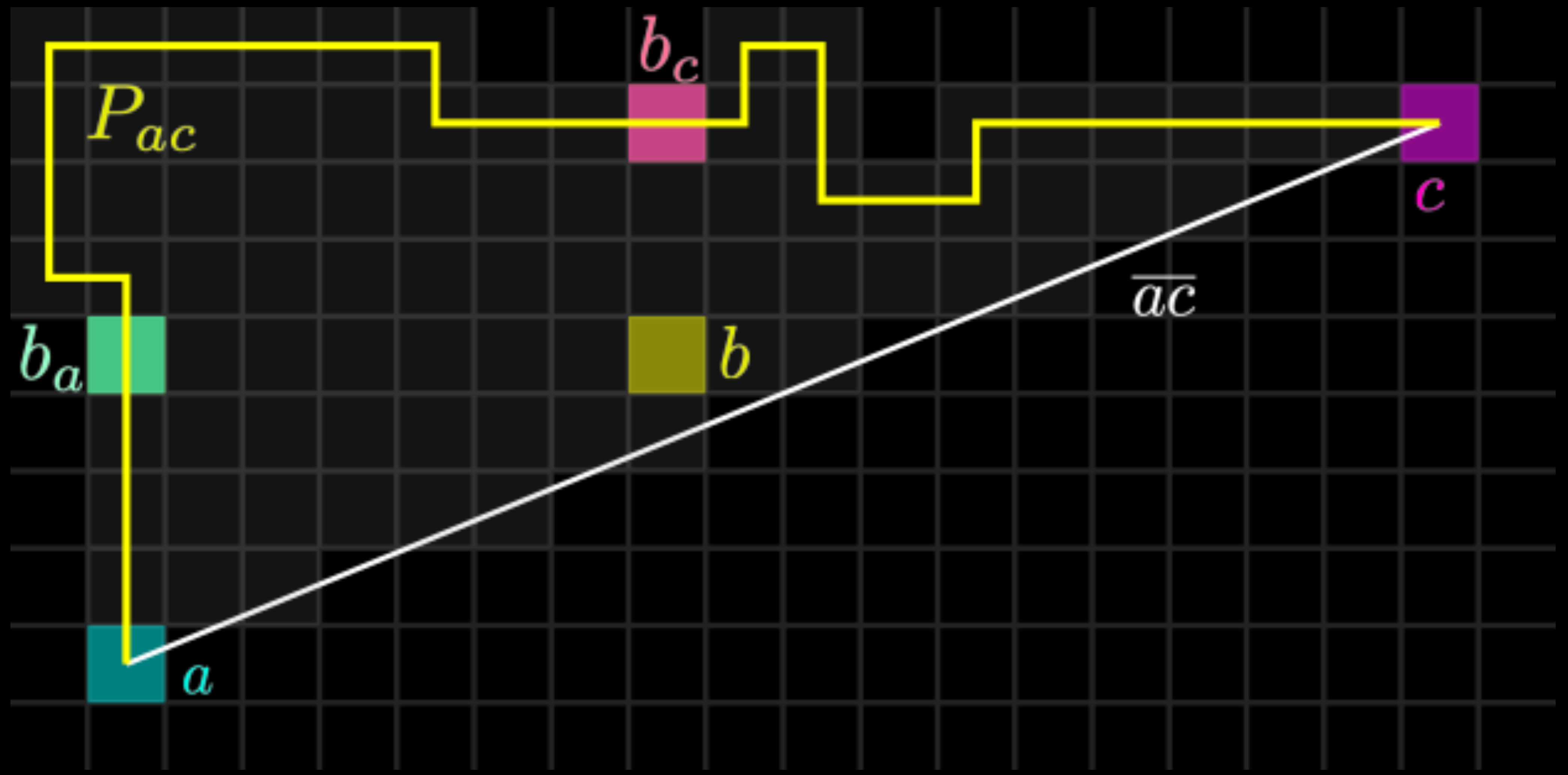

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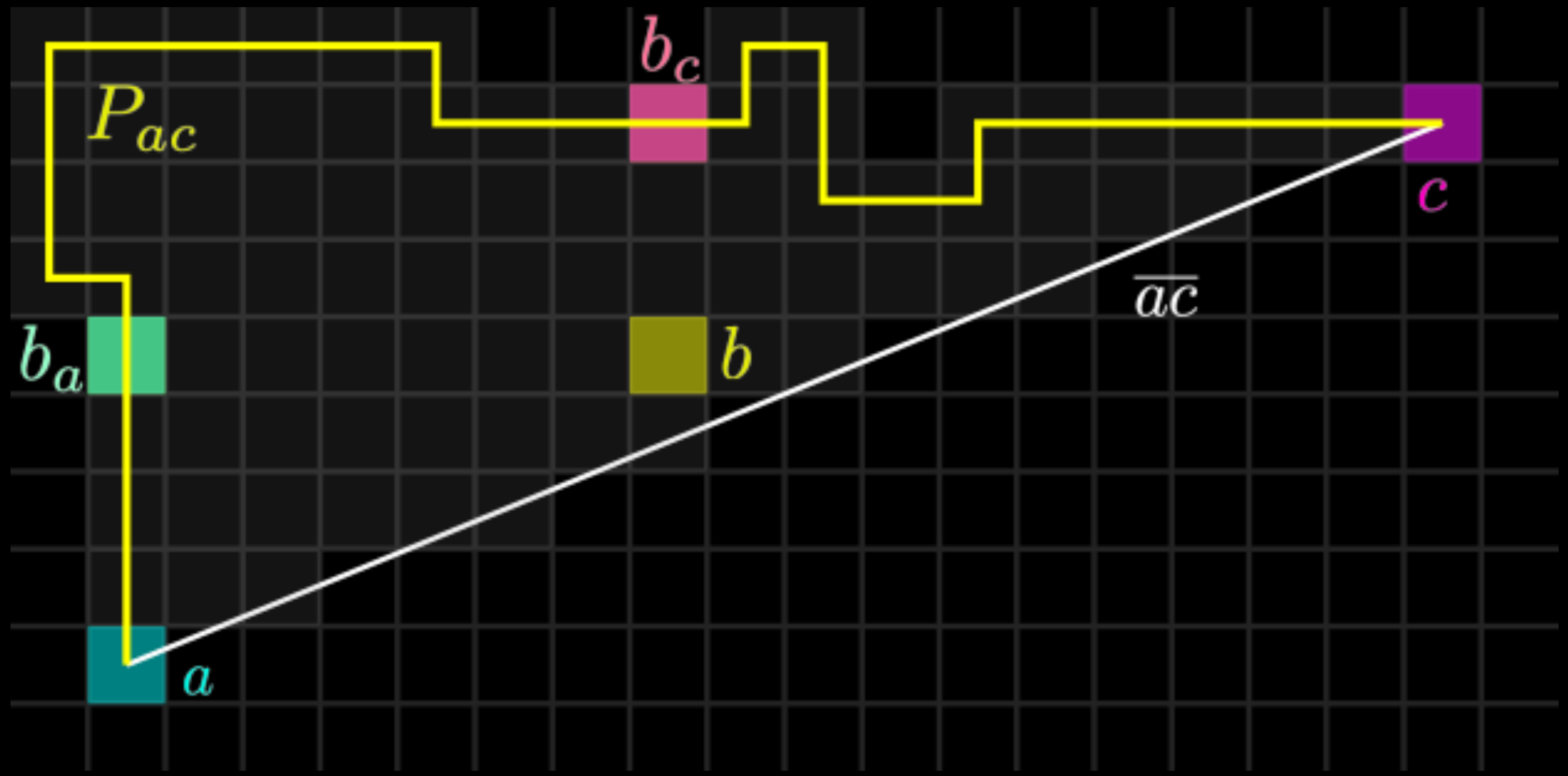

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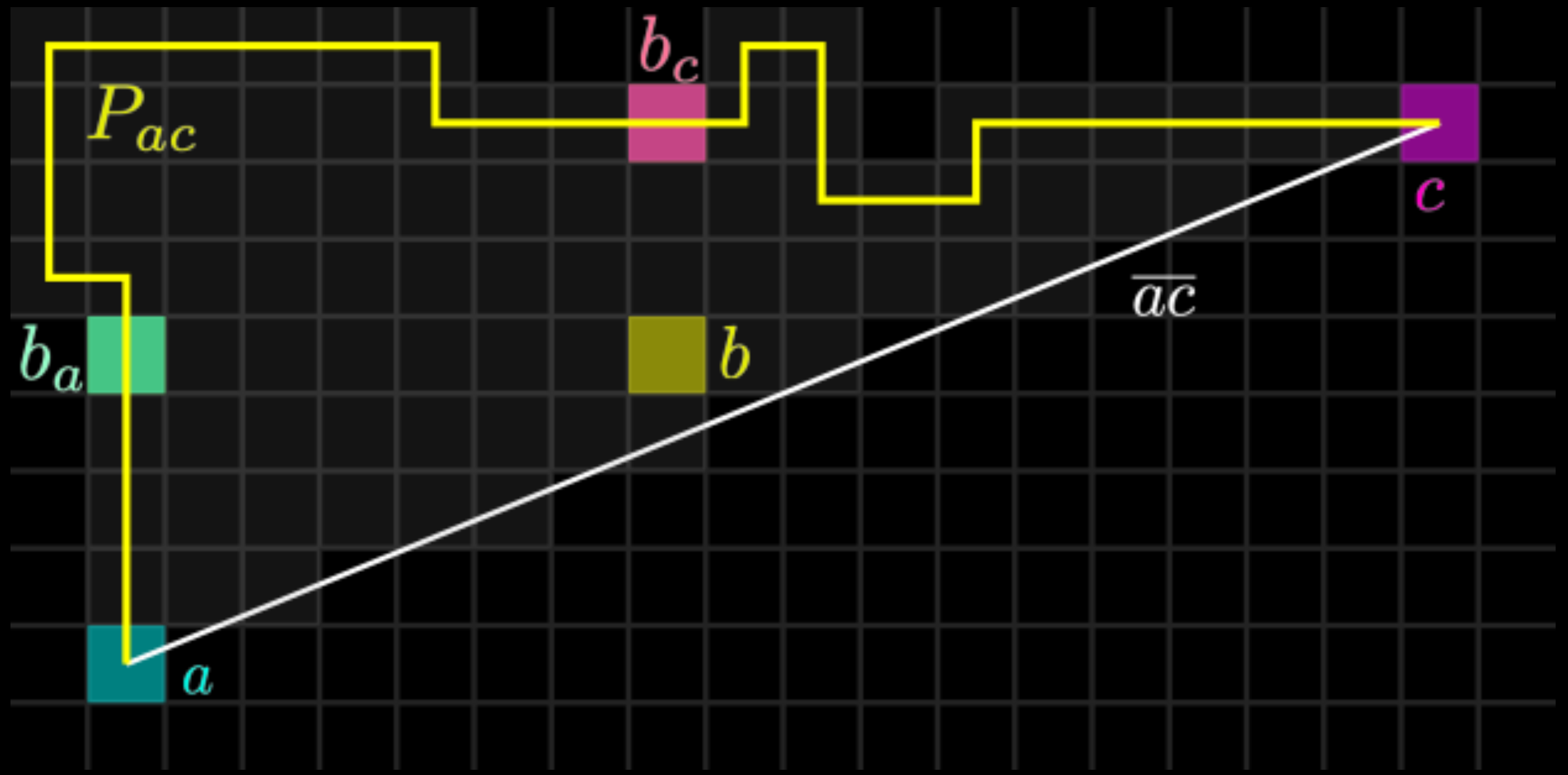

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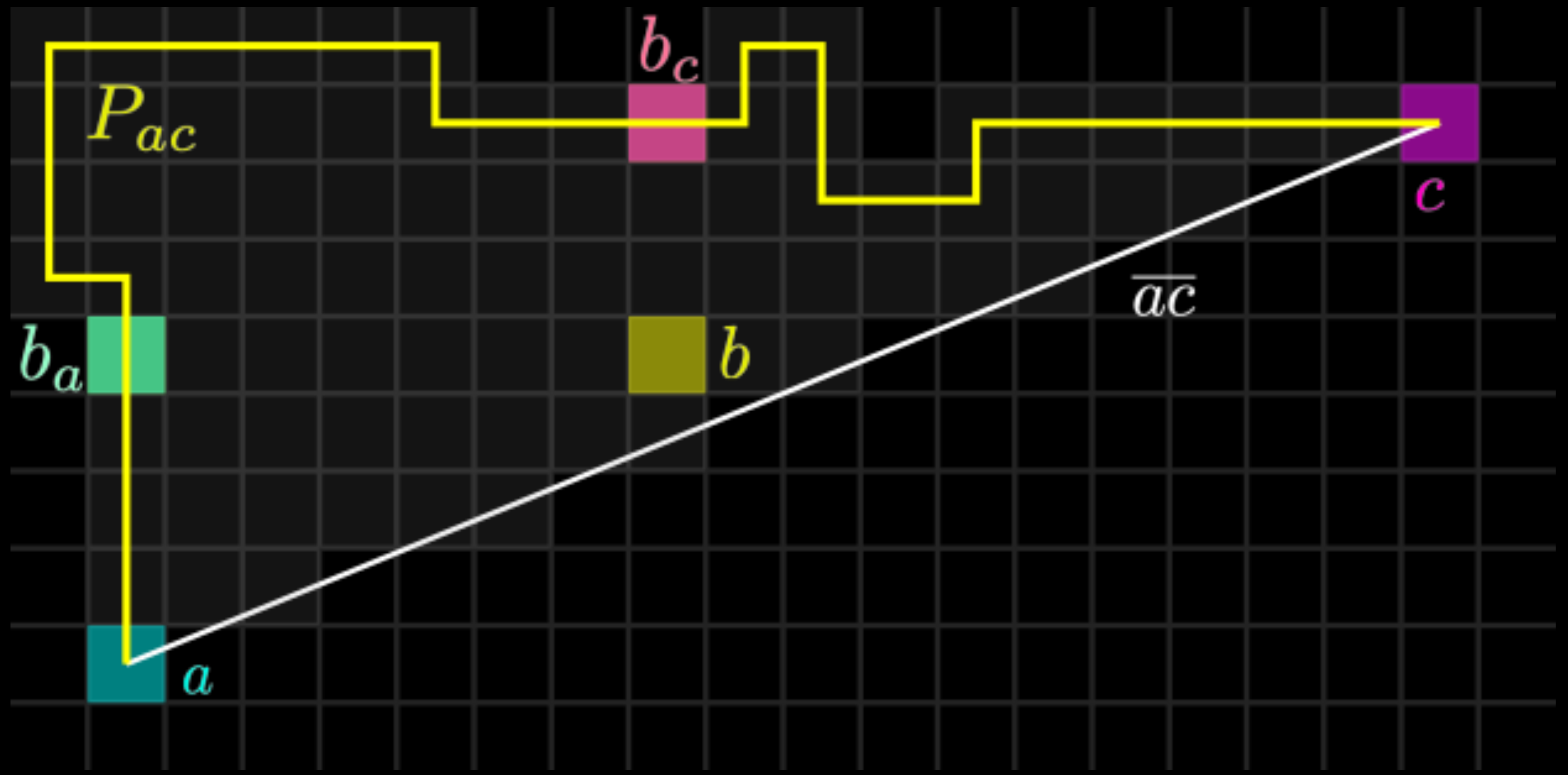
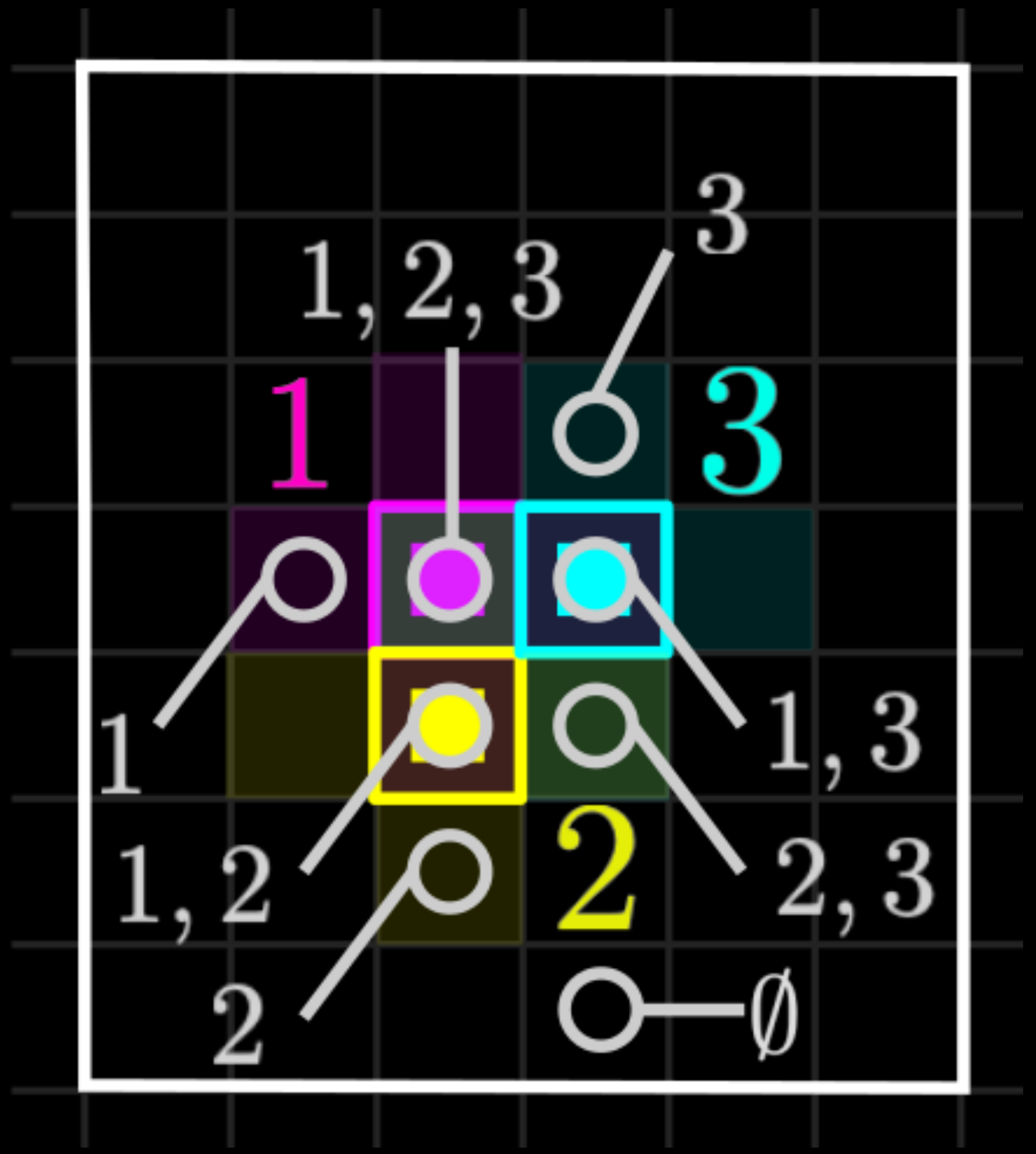


Simple Polyominoes: Lower Bound



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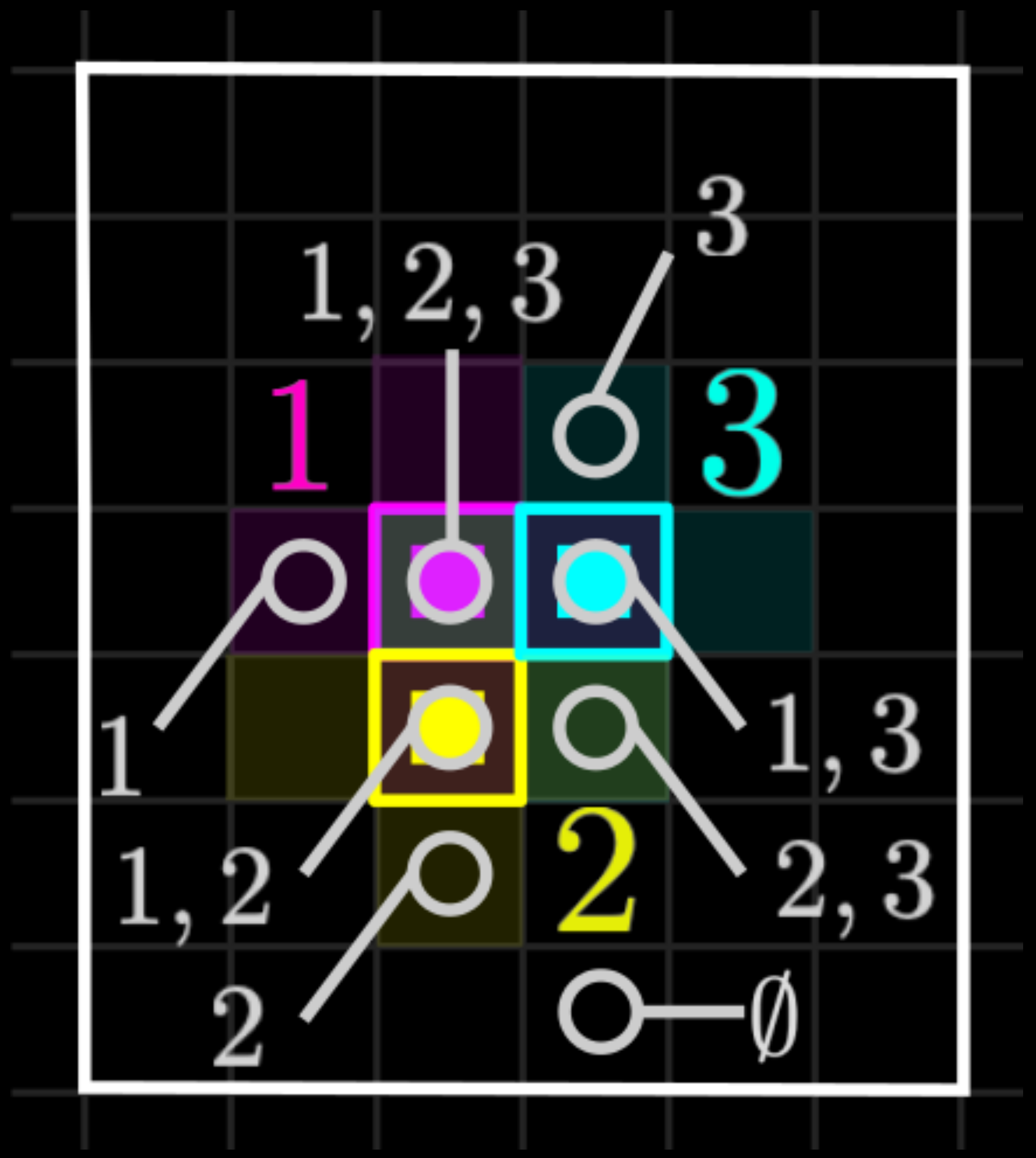
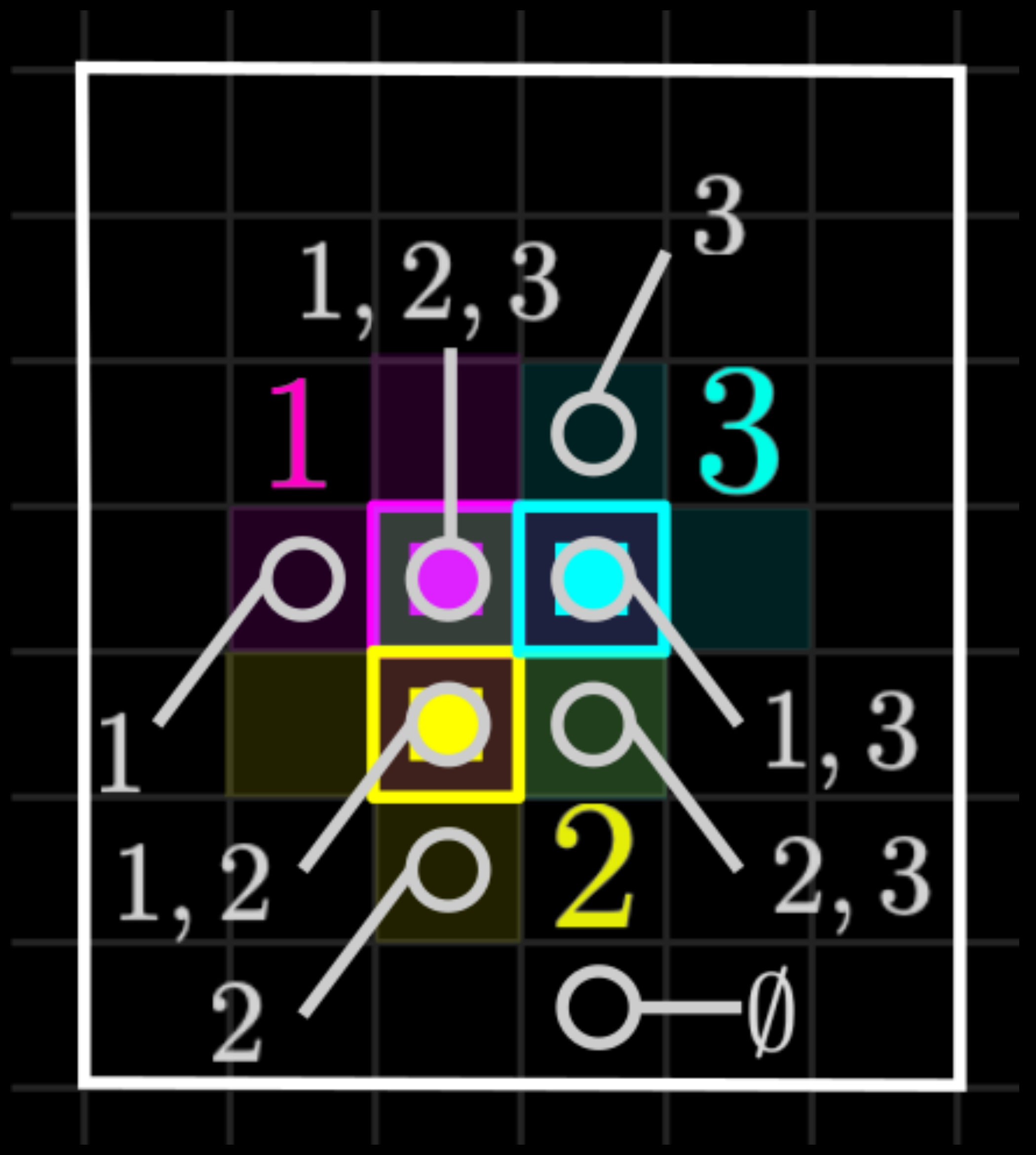


Simple Polyominoes: Lower Bound

3 guards (*d*=3)
k=1
2^3=8 viewpoints



For larger *k*: keep placement of guards, but polyomino is a large rectangle that contains all *k*-hop visibility regions. Because of the relative position of the guards, all guards are still shattered.

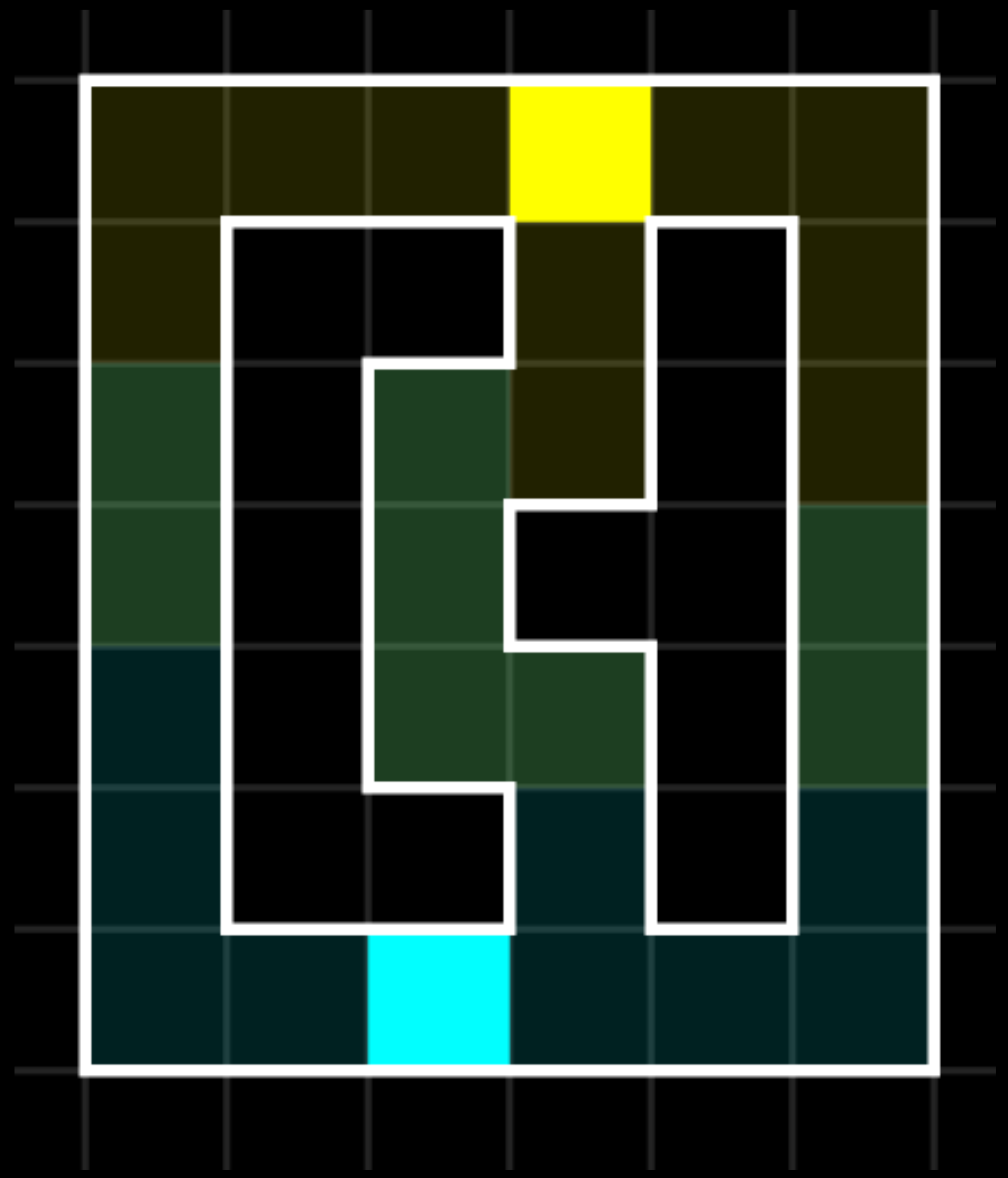
Polyominoes with Holes

- Upper bound of 4 for VC dimension in hypergraphs of pseudo disks [*]

[*] Boris Aronov, Anirudh Donakonda, Esther Ezra, and Rom Pinchasi. On pseudodisk hypergraphs. Computational Geometry, 92:101687, 2021

Polyominoes with Holes

- Upper bound of 4 for VC dimension in hypergraphs of pseudo disks [*]
- But: *k*-hop visibility regions of unit squares in polyominoes with holes are not pseudo disks



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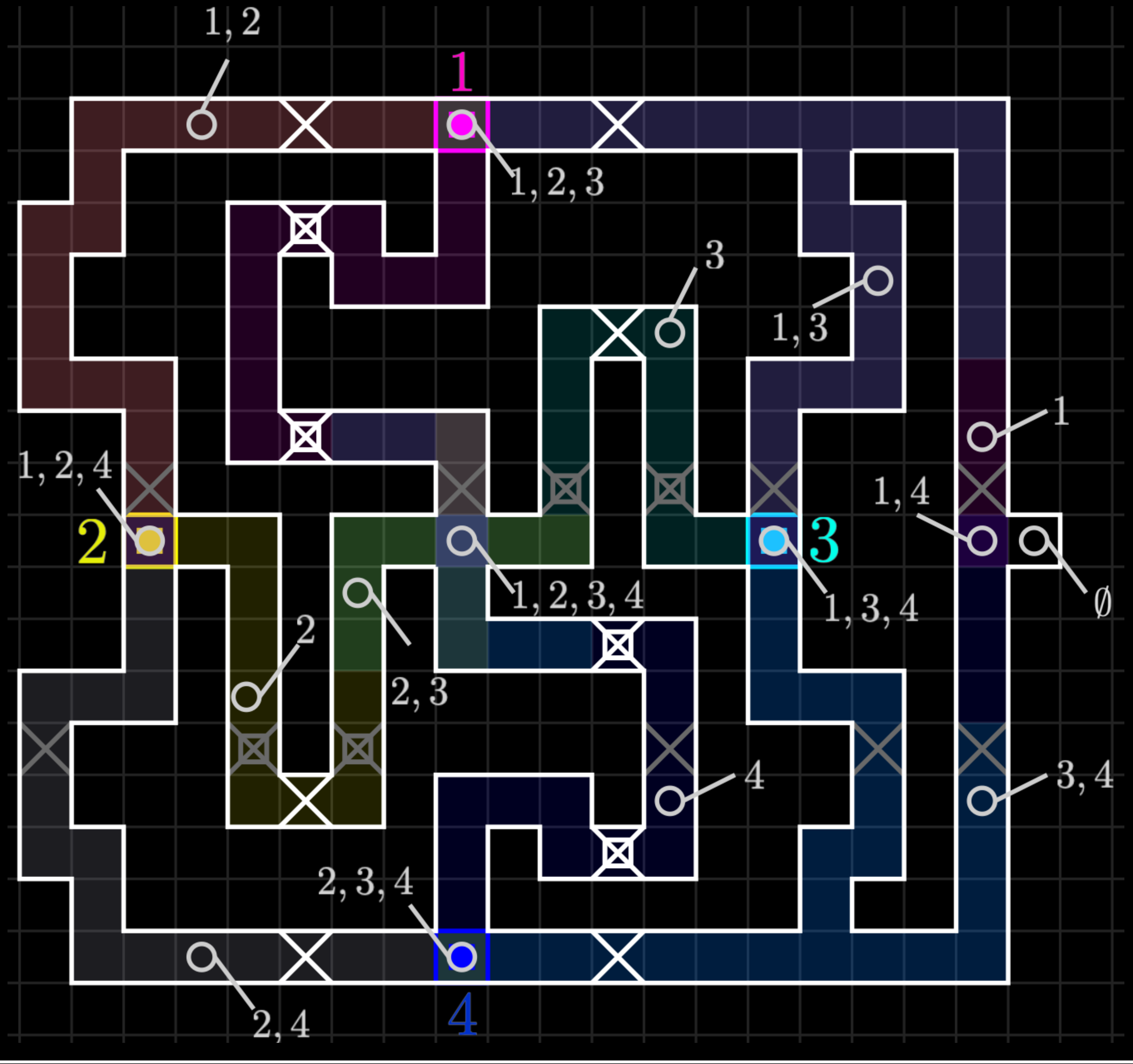
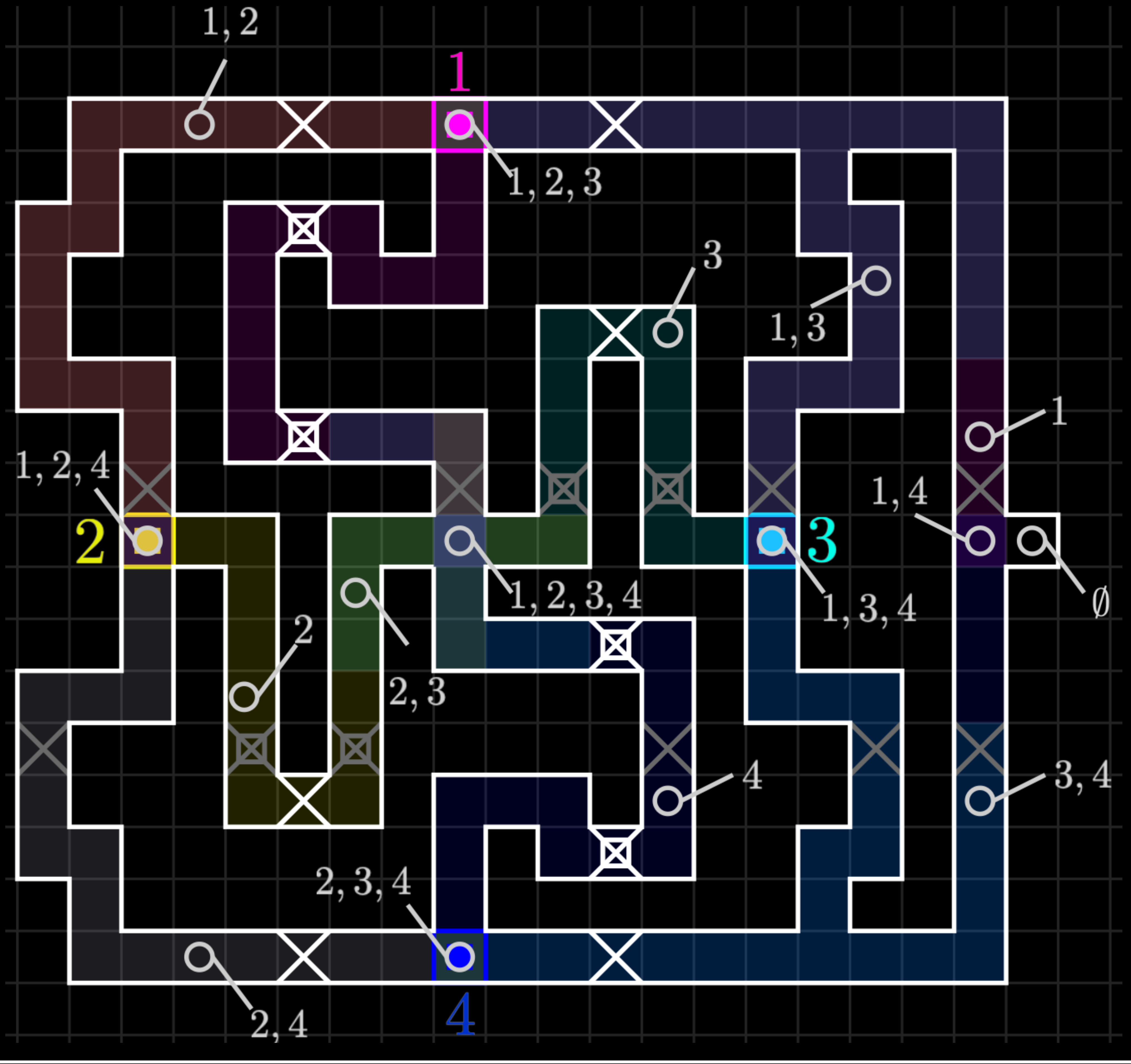


For k=18

d=4➔2^4=16 viewpoints (marked with the guard labels of guards that see them)

Polyominoes with Holes: Lower Bound



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Larger even k?

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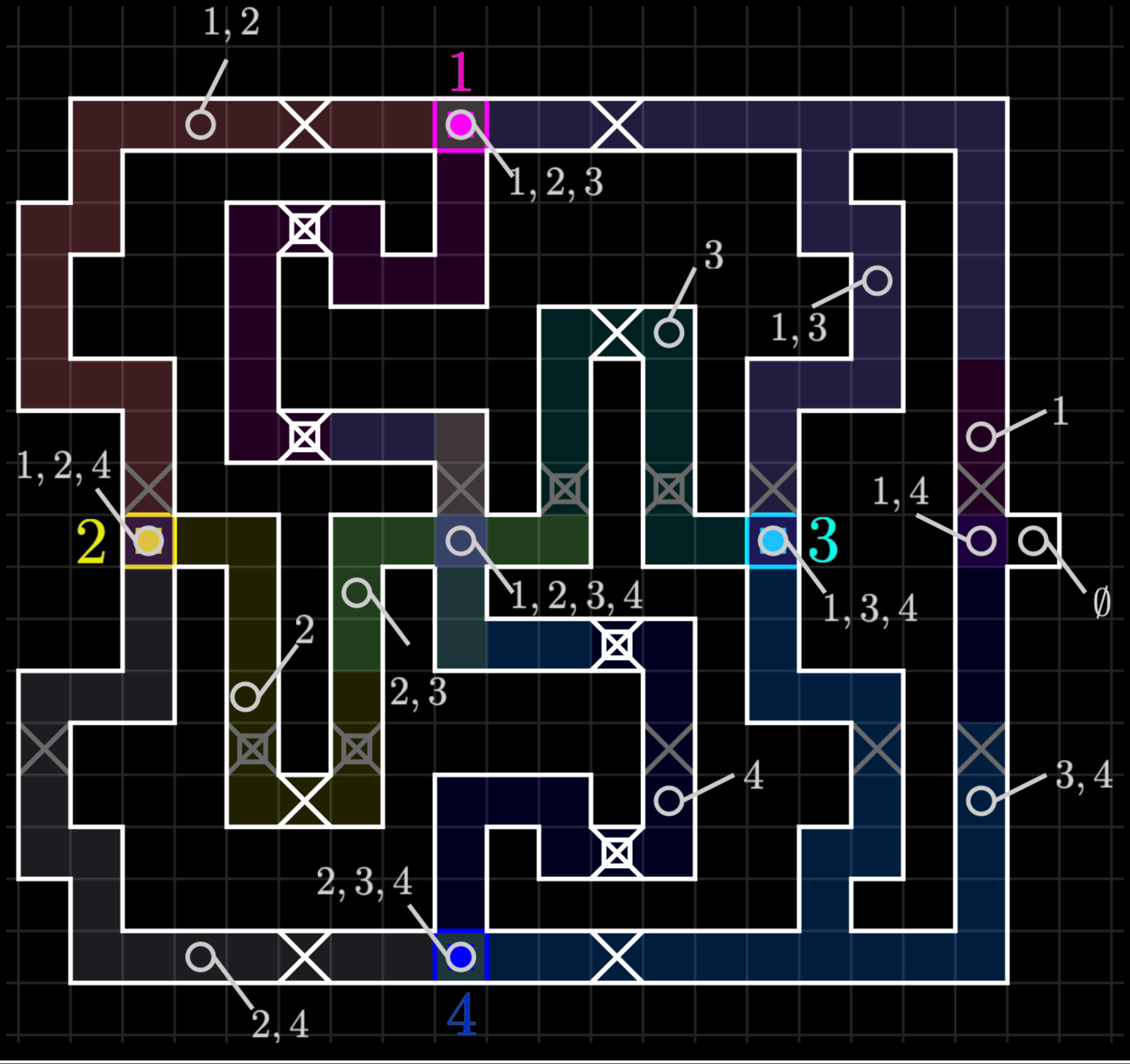

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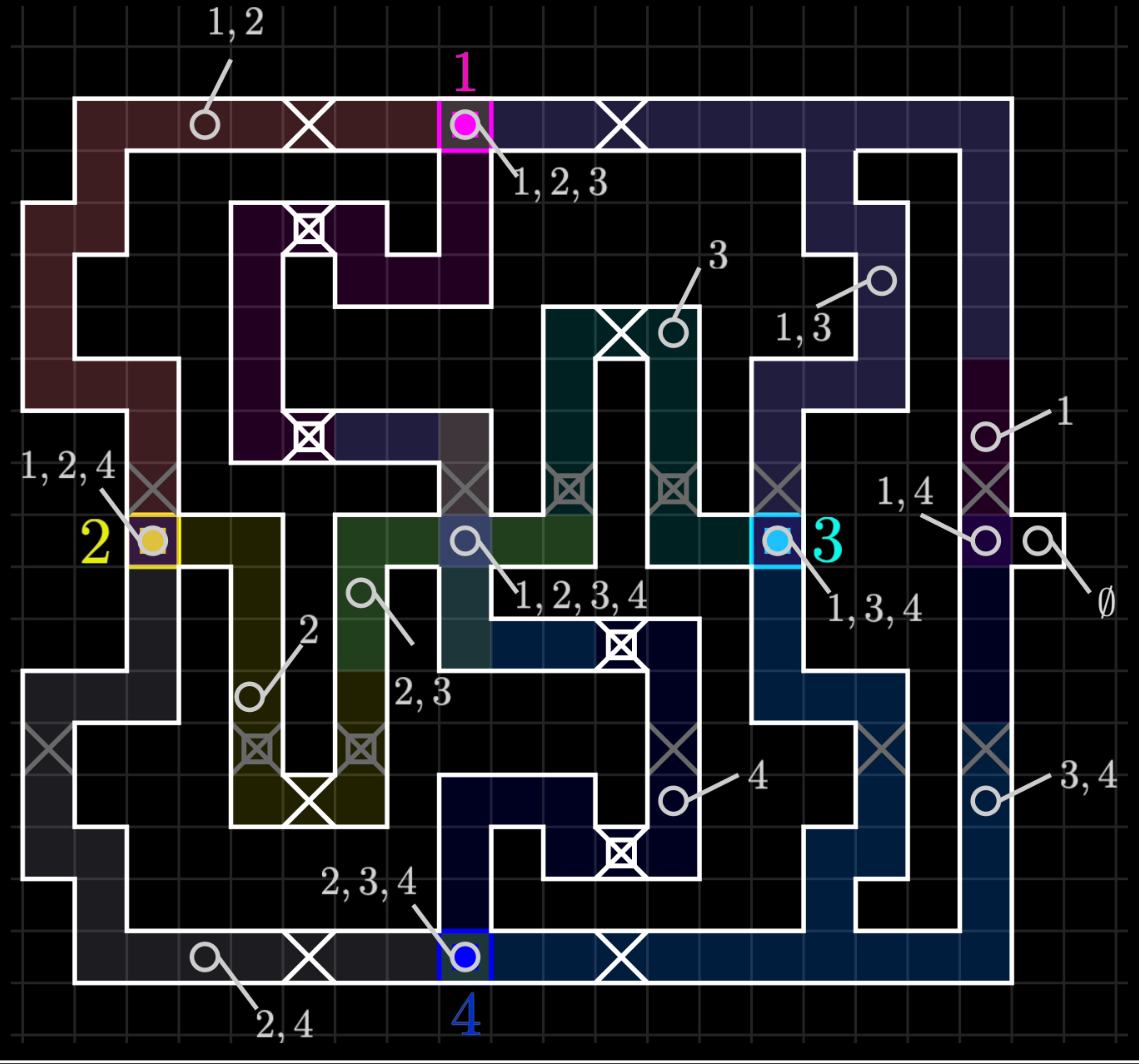

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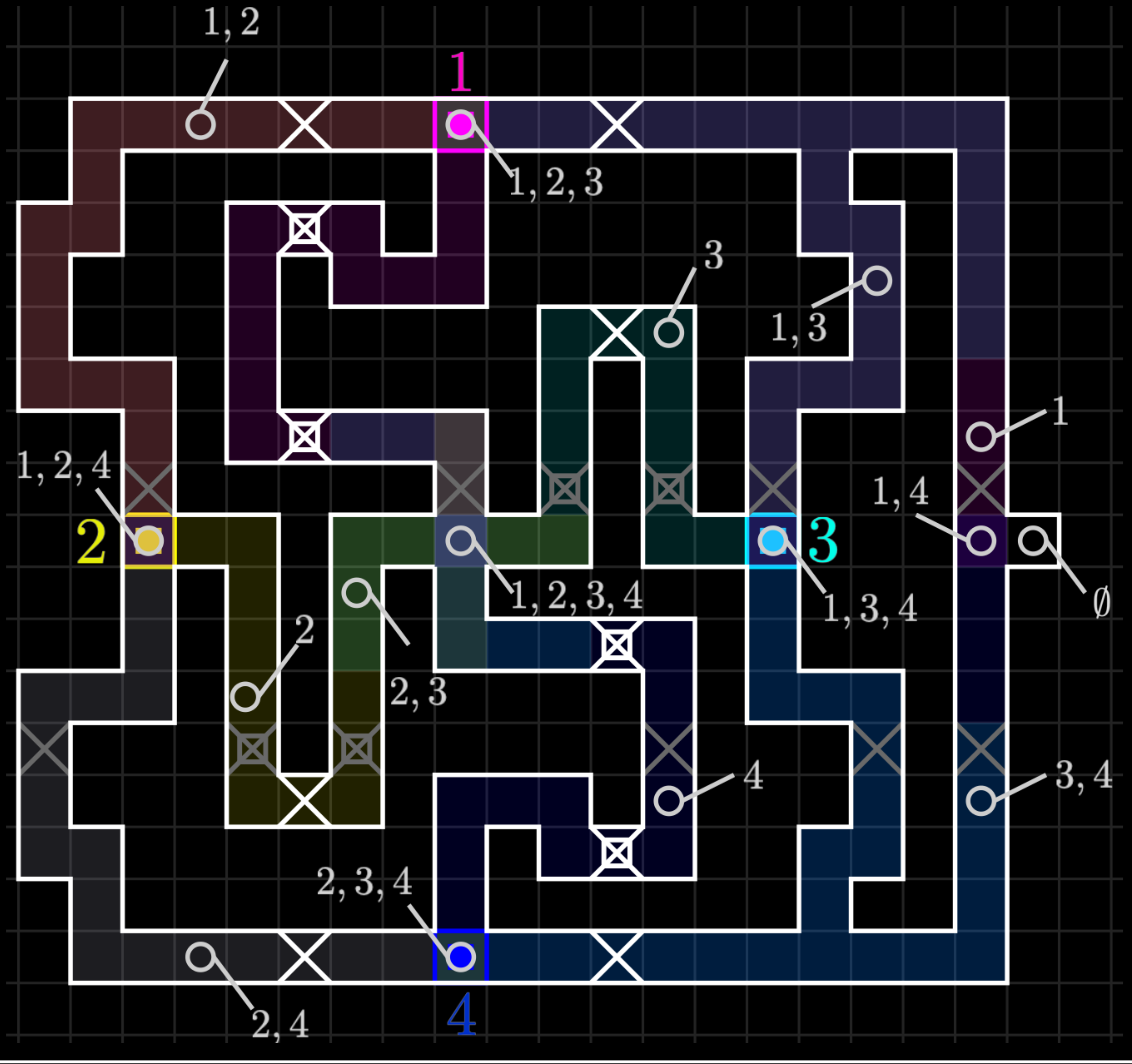

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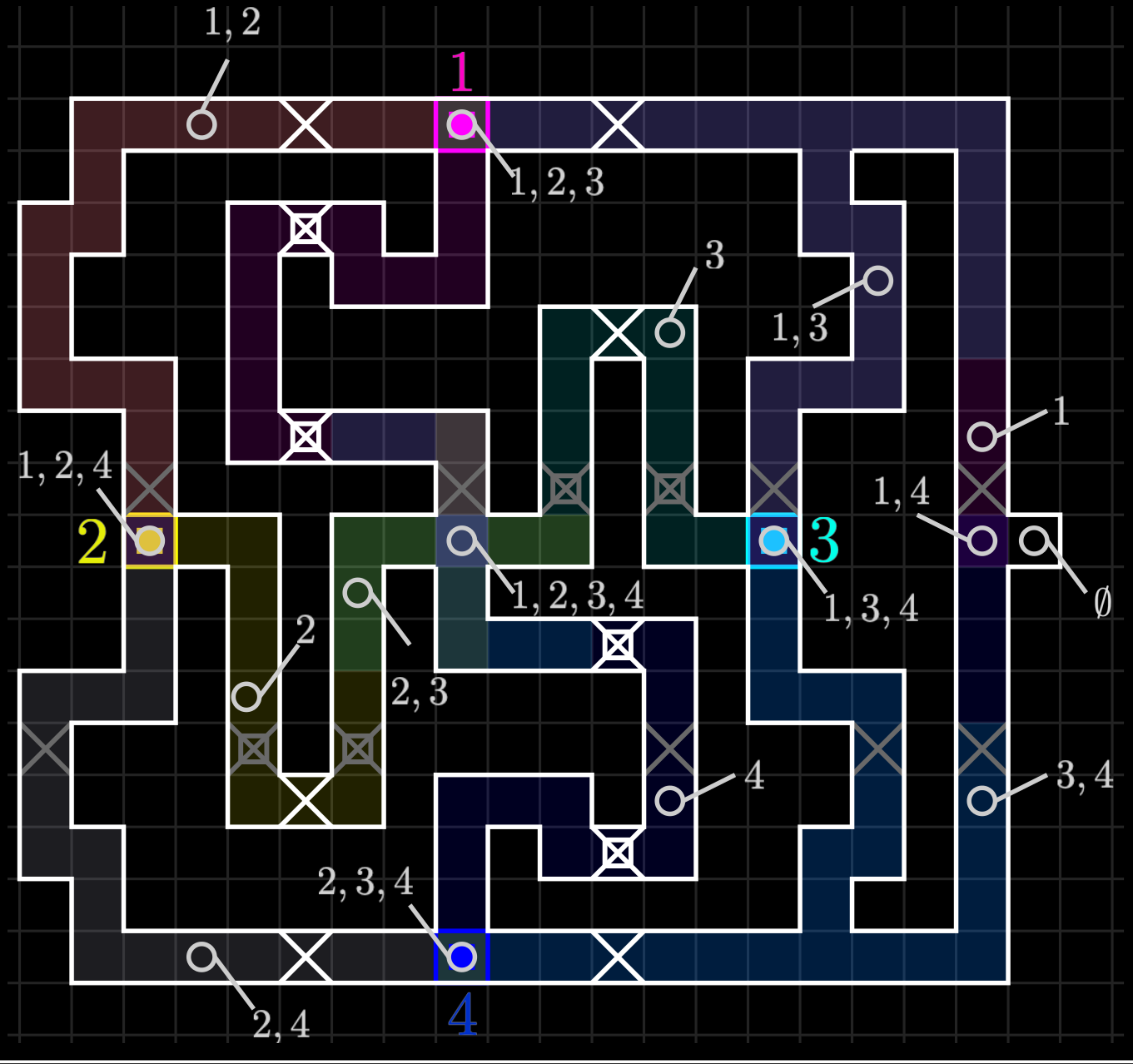

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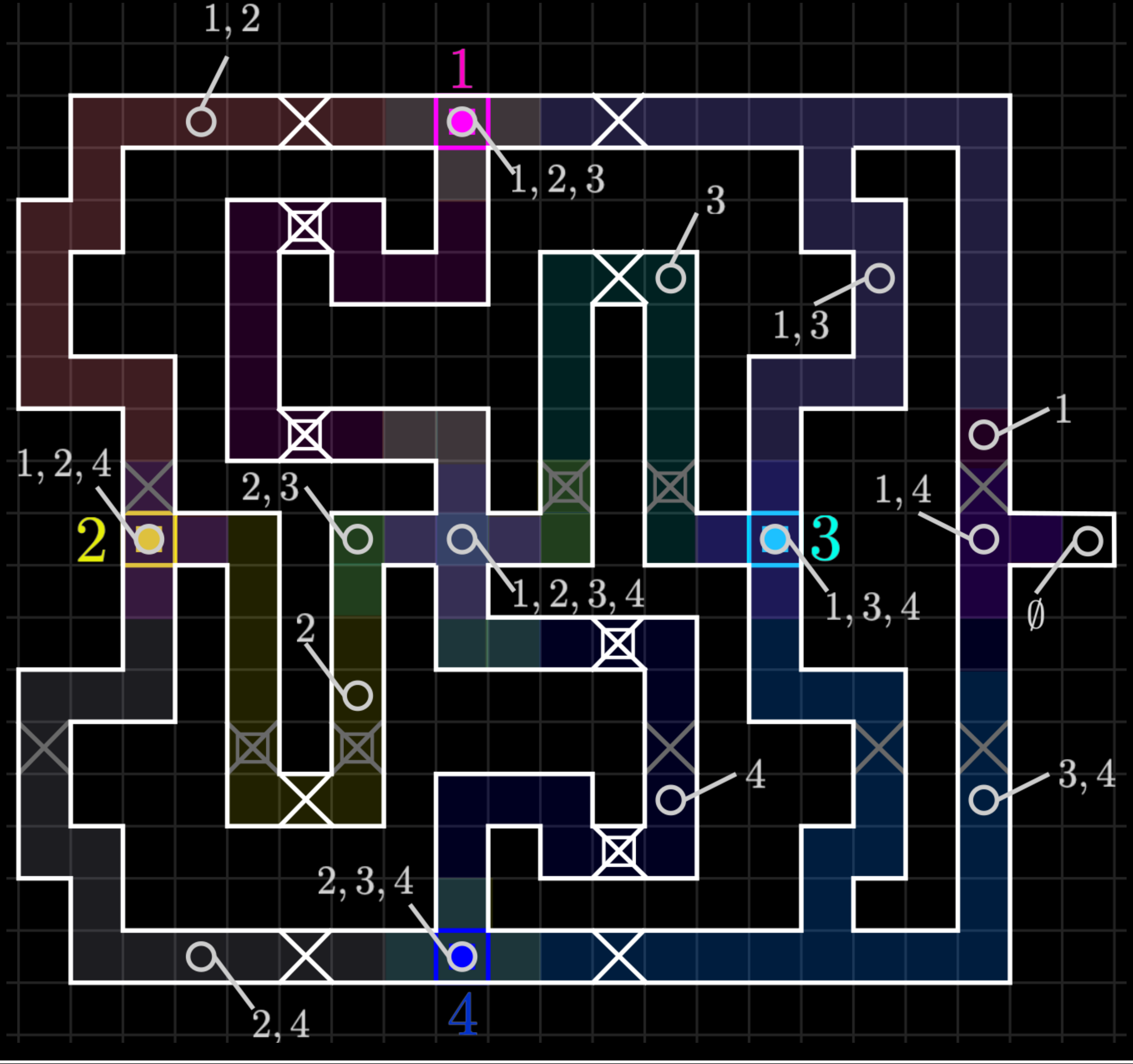

For k=19

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Larger **odd** k?

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Polyominoes with Holes

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Polyominoes with Holes

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➜ $P_{i,j}$ includes the shortest path from g_i to $v_{\{i,j\}}$ and the shortest path from g_j to $v_{\{i,j\}}$

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Let's start with placing four guards g_1, g_2, g_3, g_4

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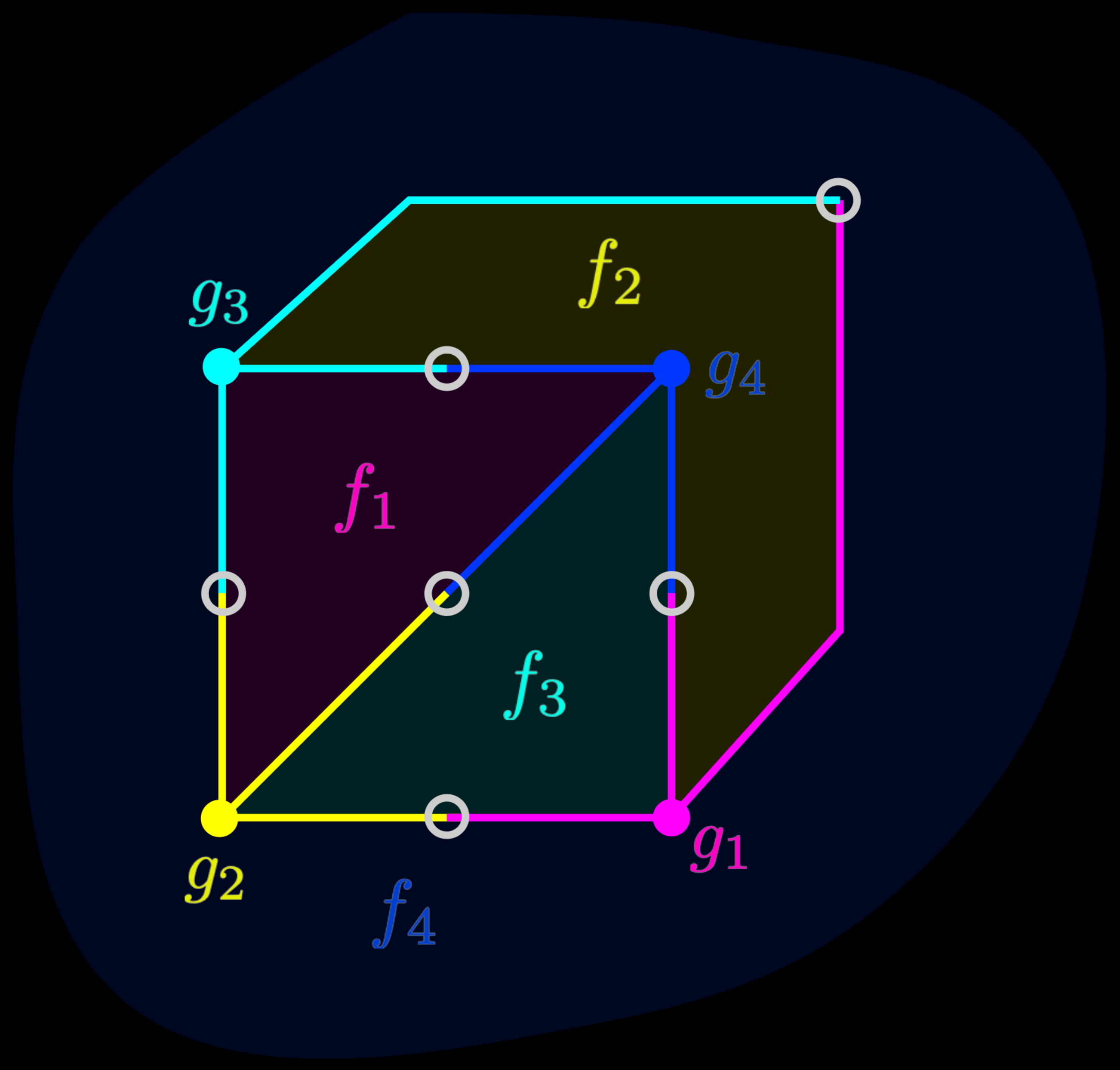

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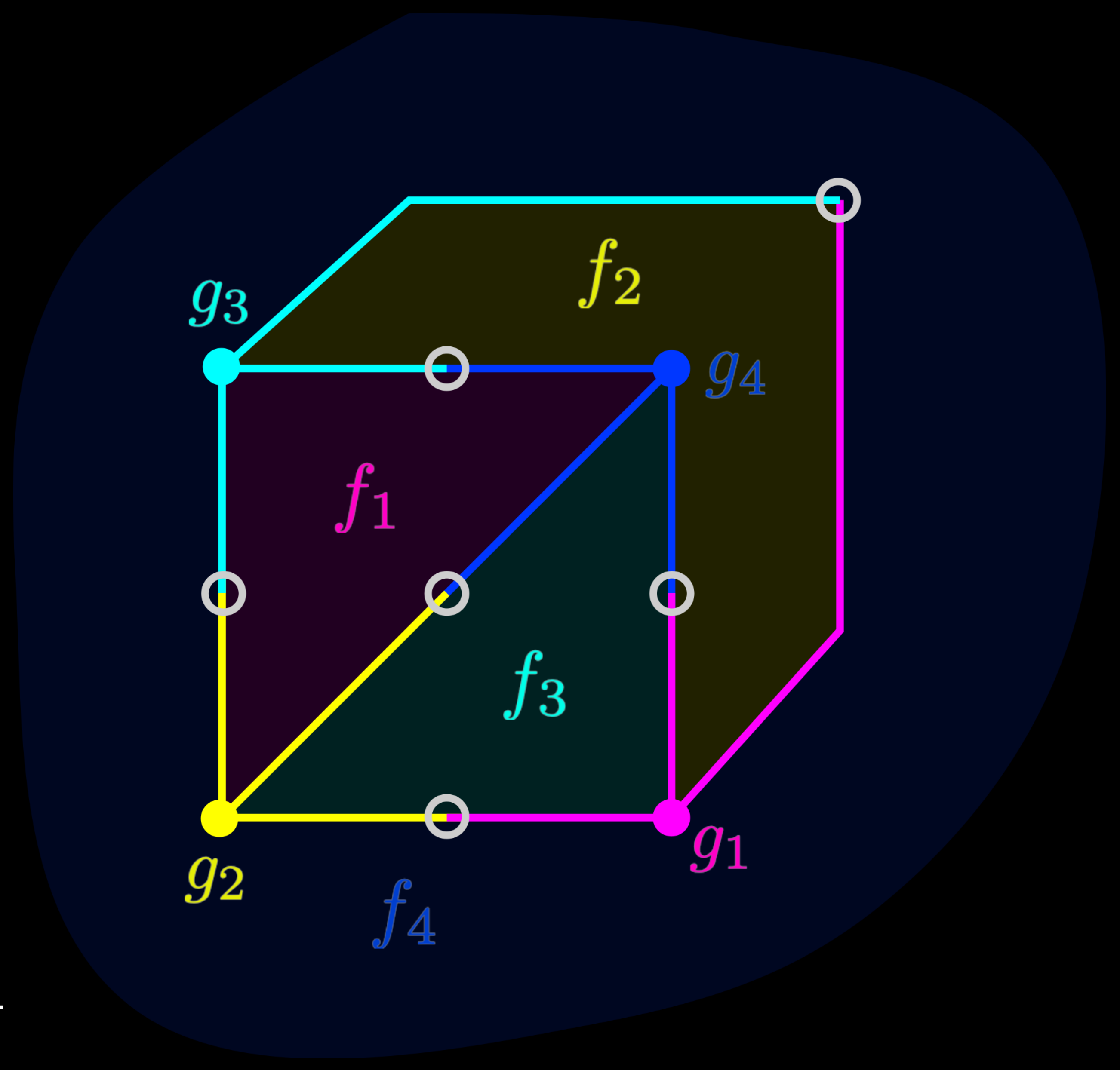

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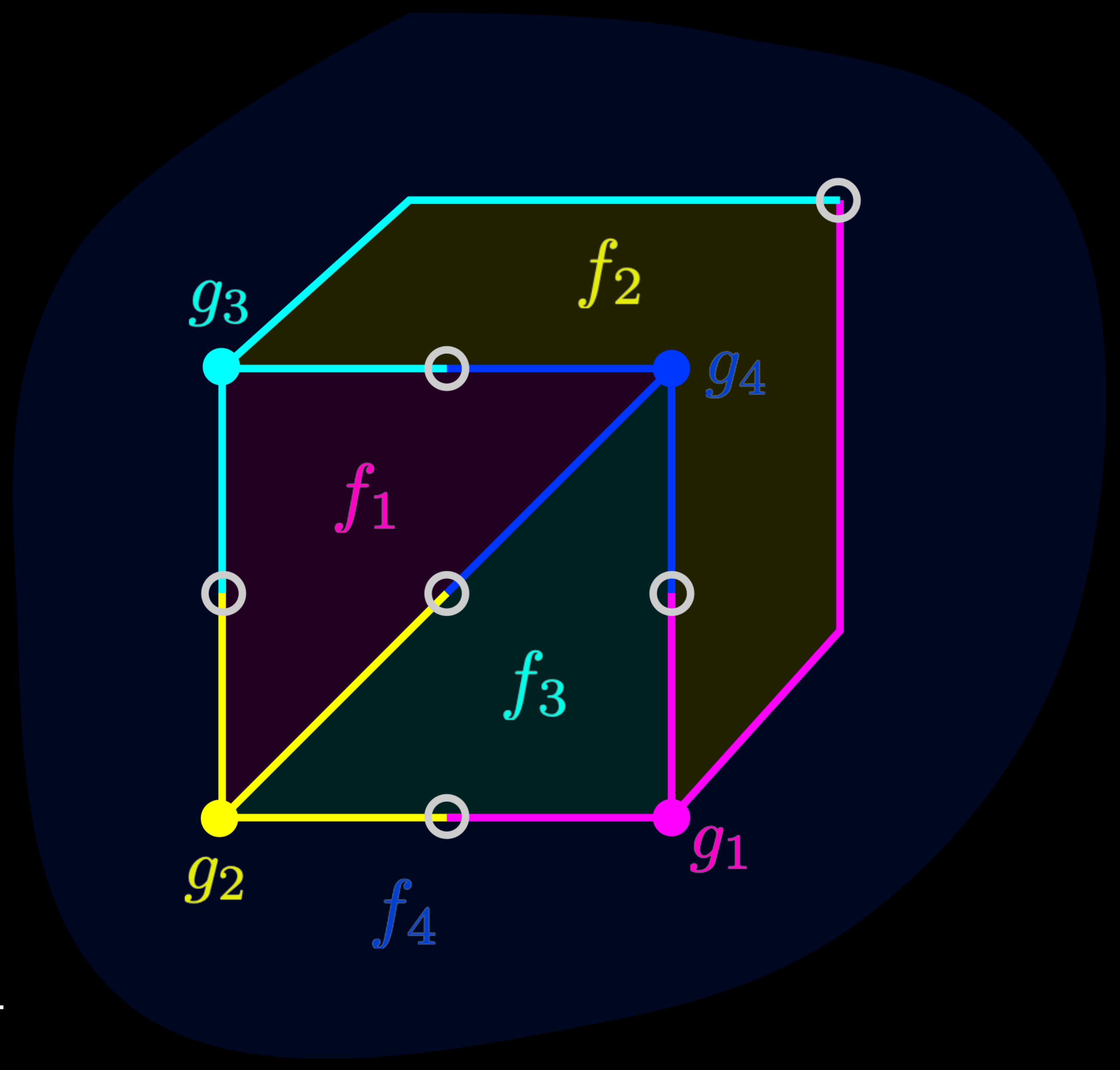

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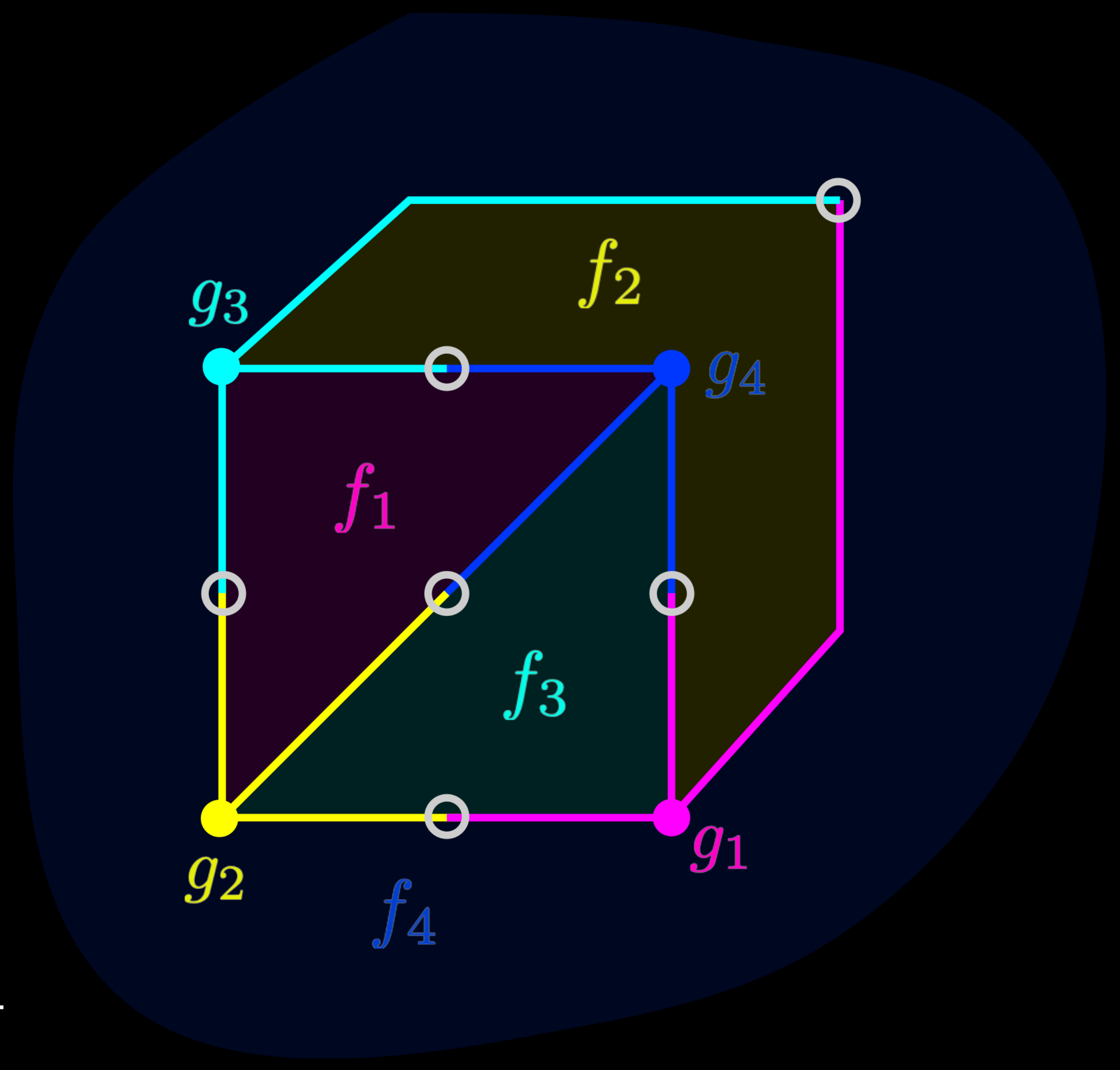

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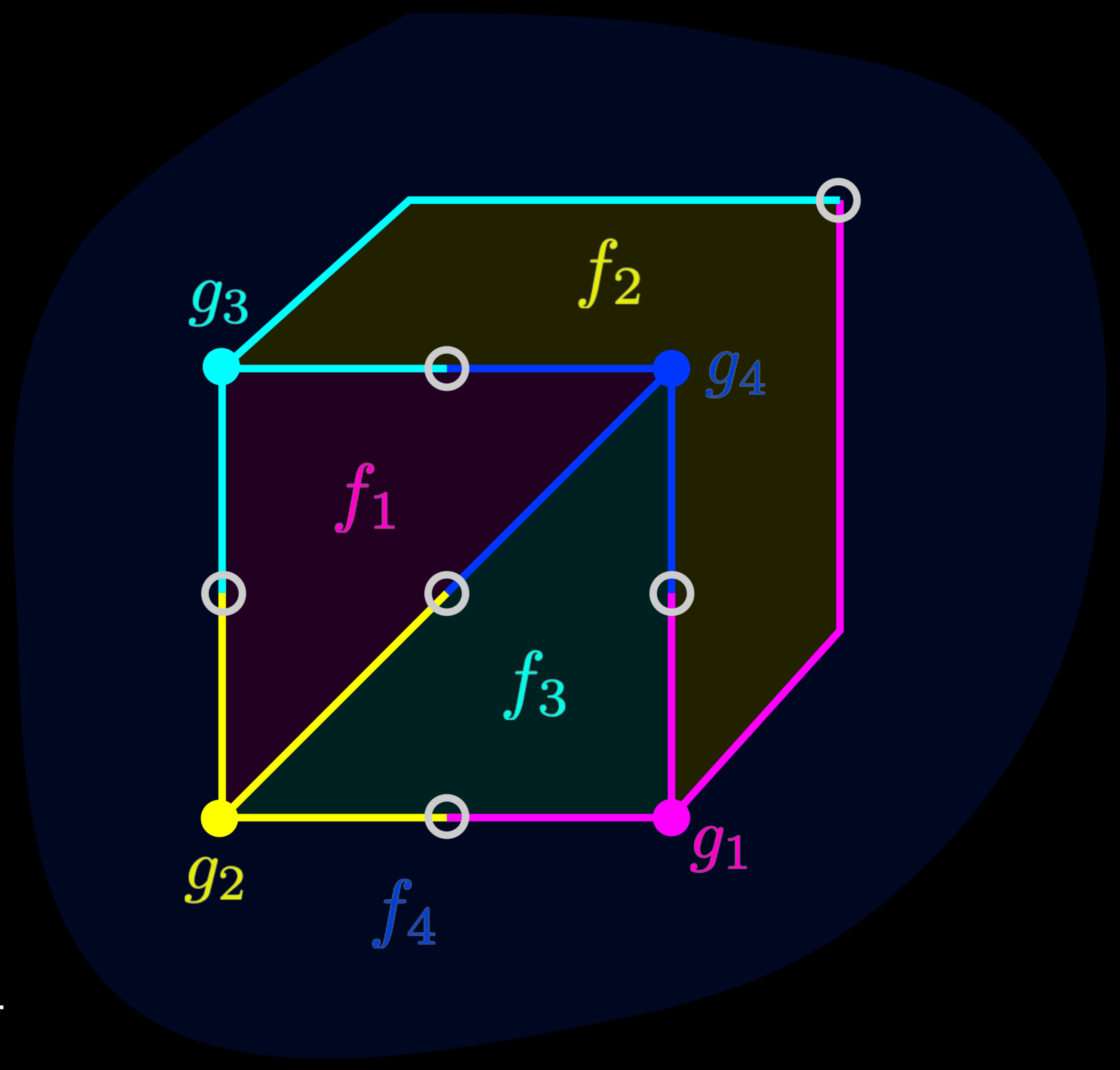

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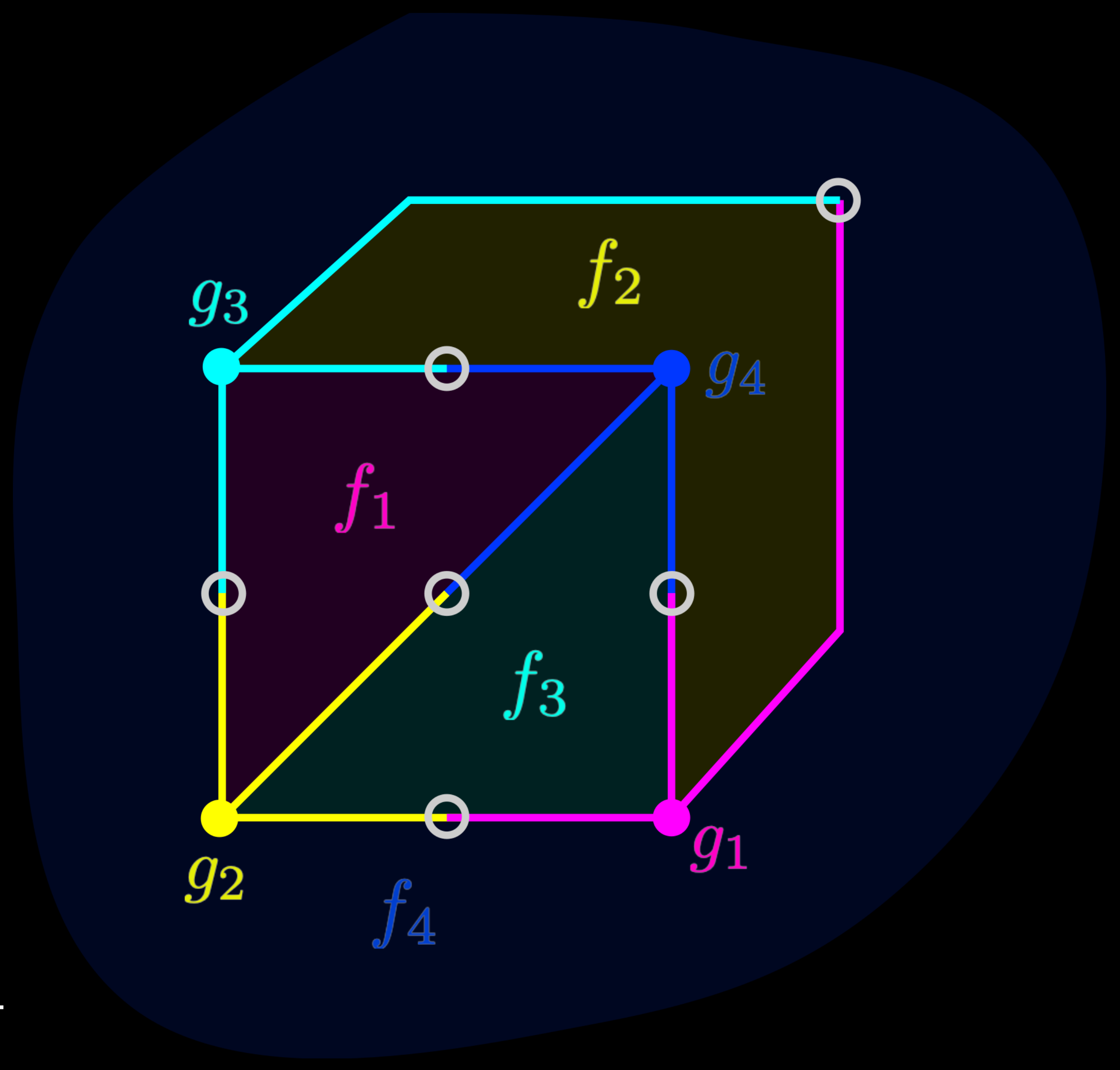

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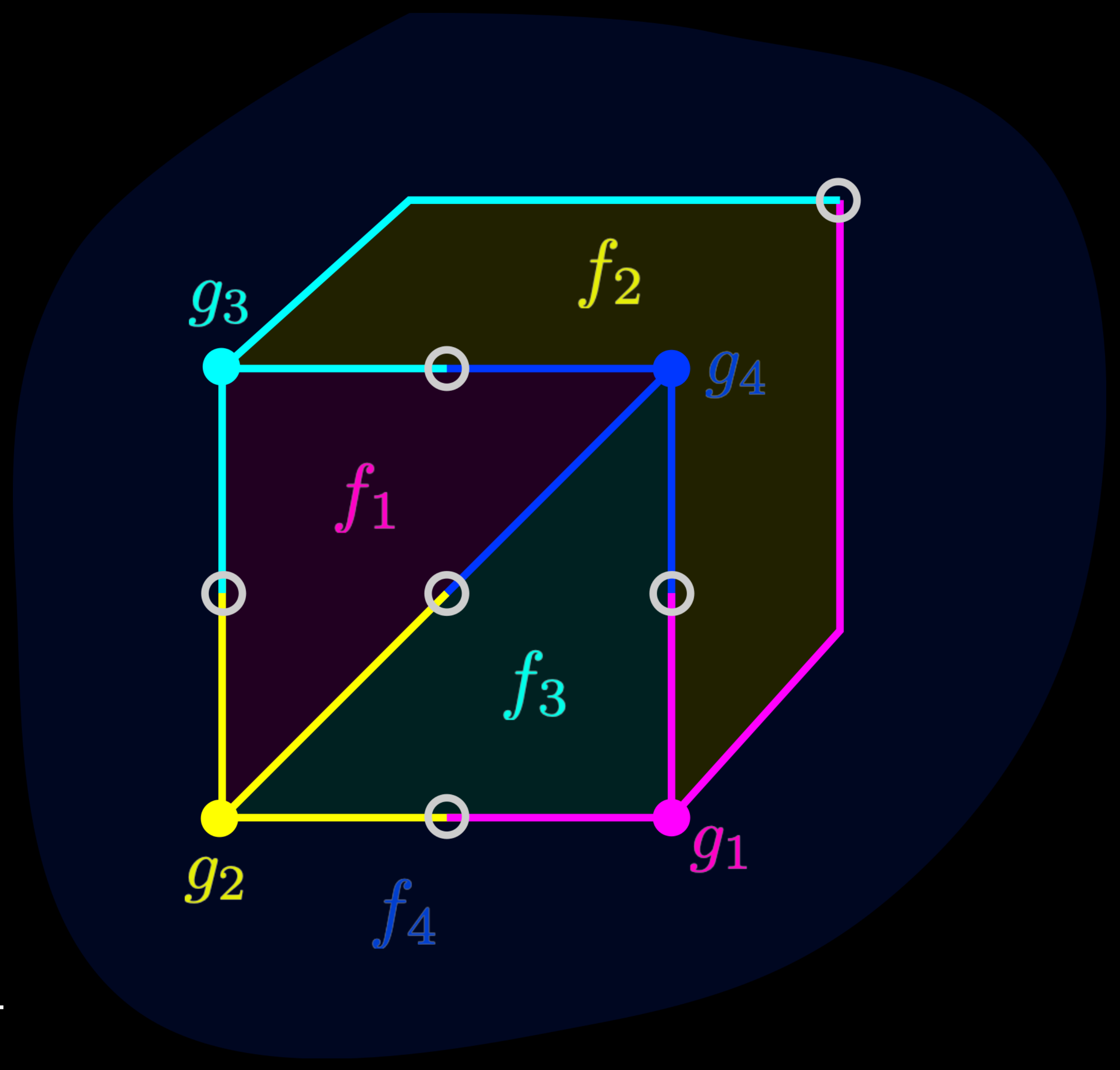

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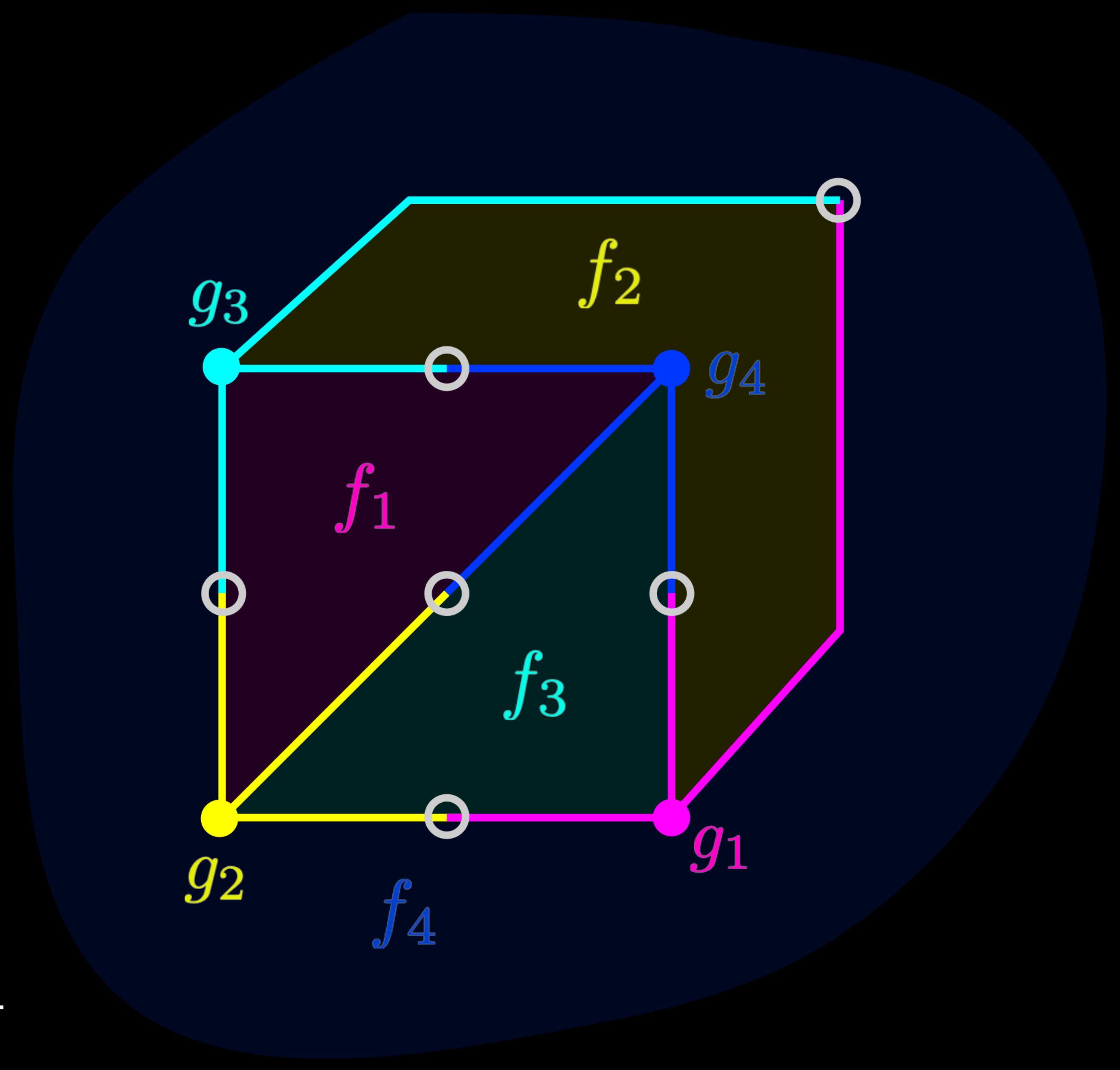

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Rest-budget observation

➜ One of the viewpoints $v_{\{i,5\}}$ and $v_{\{j,\ell\}}$ cannot be realized (a guard from the other pair will always see such a viewpoint) ↯

Computational Complexity: NP-Completeness for 1-Thin Polyominoes with Holes*

* Without holes: the dual is a tree, and we can solve it in linear time!

NP-Completeness for 1-Thin Polyominoes with Holes

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- We reduce from PLANAR MONOTONE 3SAT

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$$\varphi = (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_1 \vee x_2 \vee x_4) \wedge (\overline{x_1} \vee \overline{x_3} \vee \overline{x_4})$$

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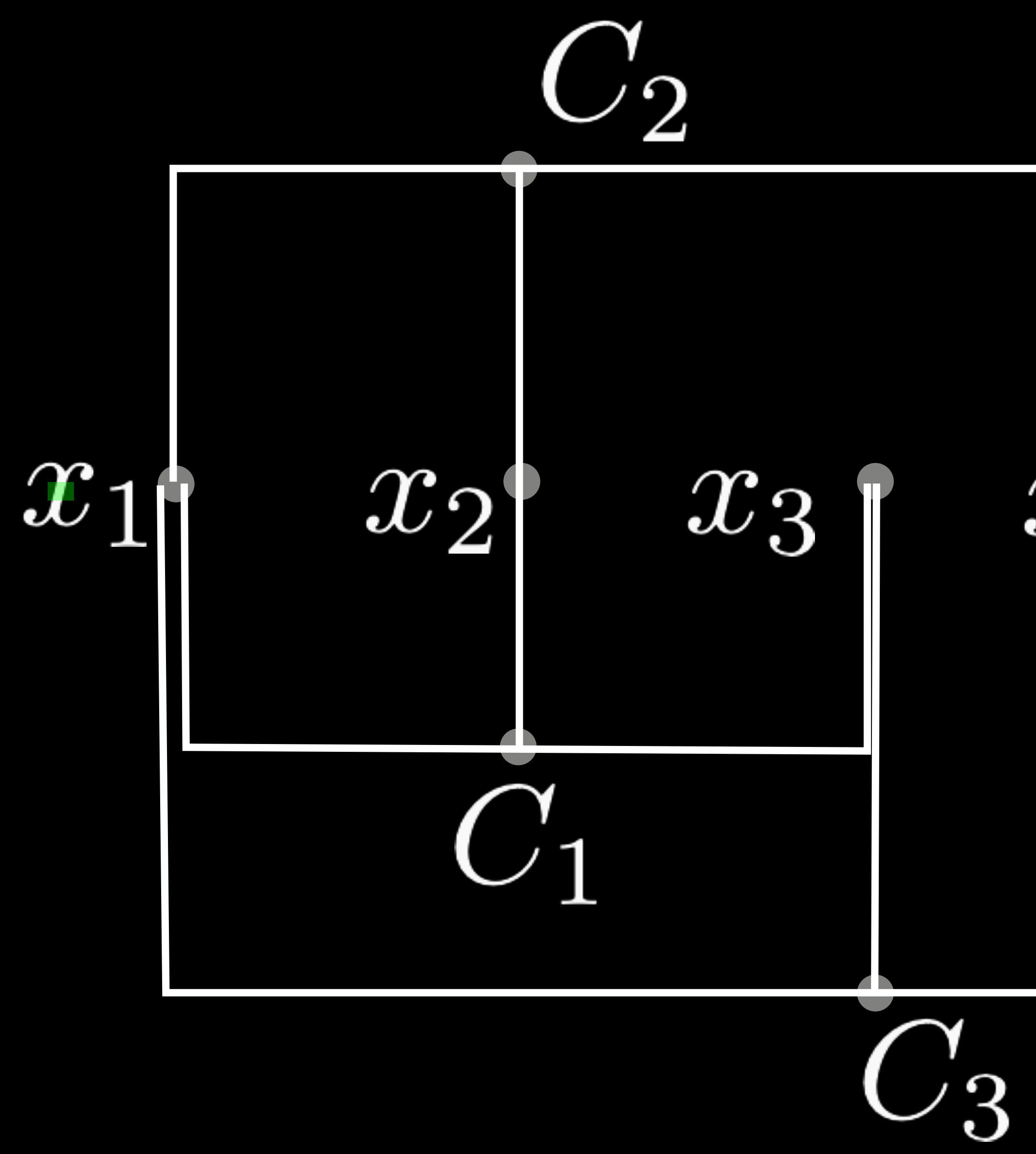

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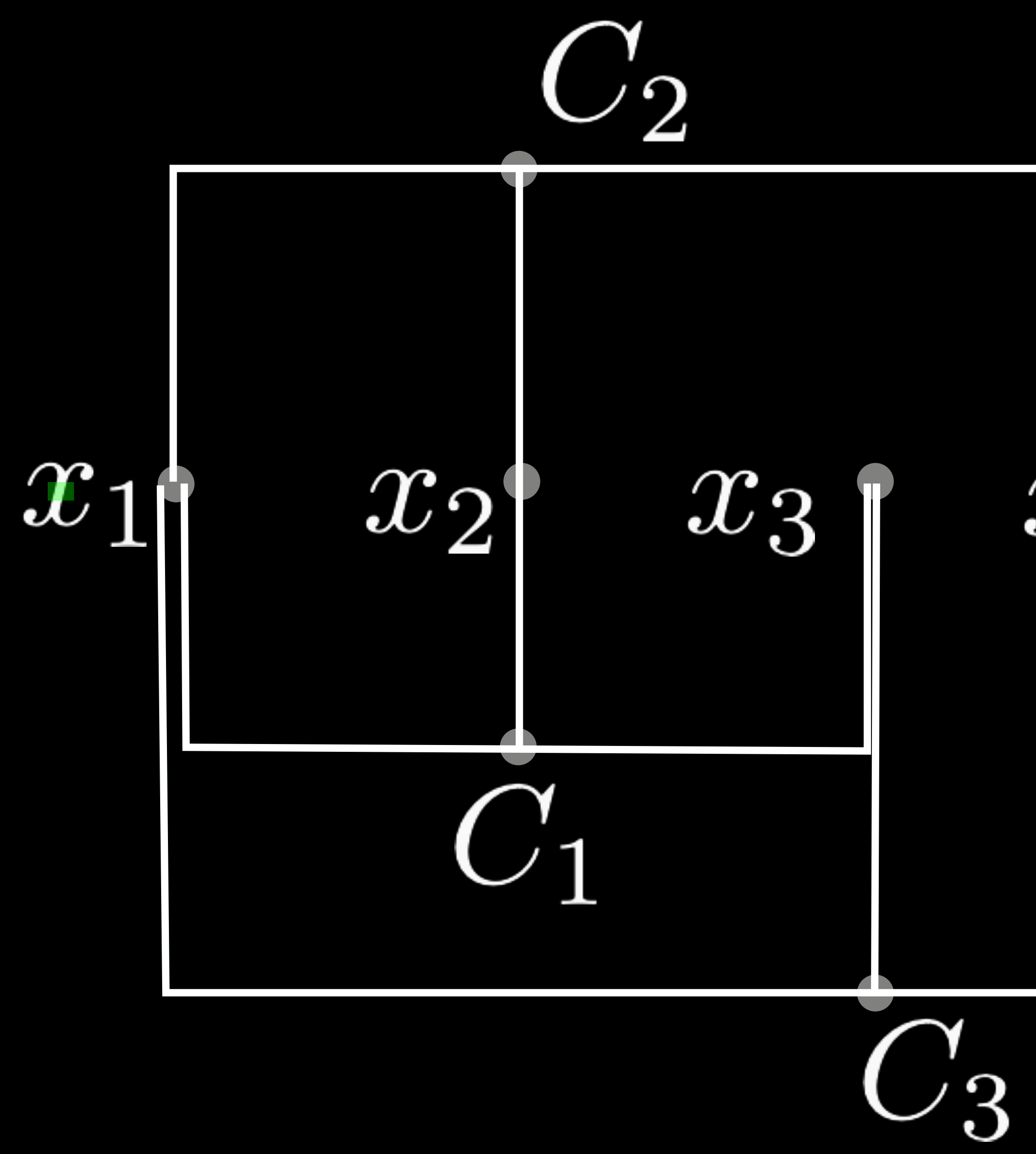

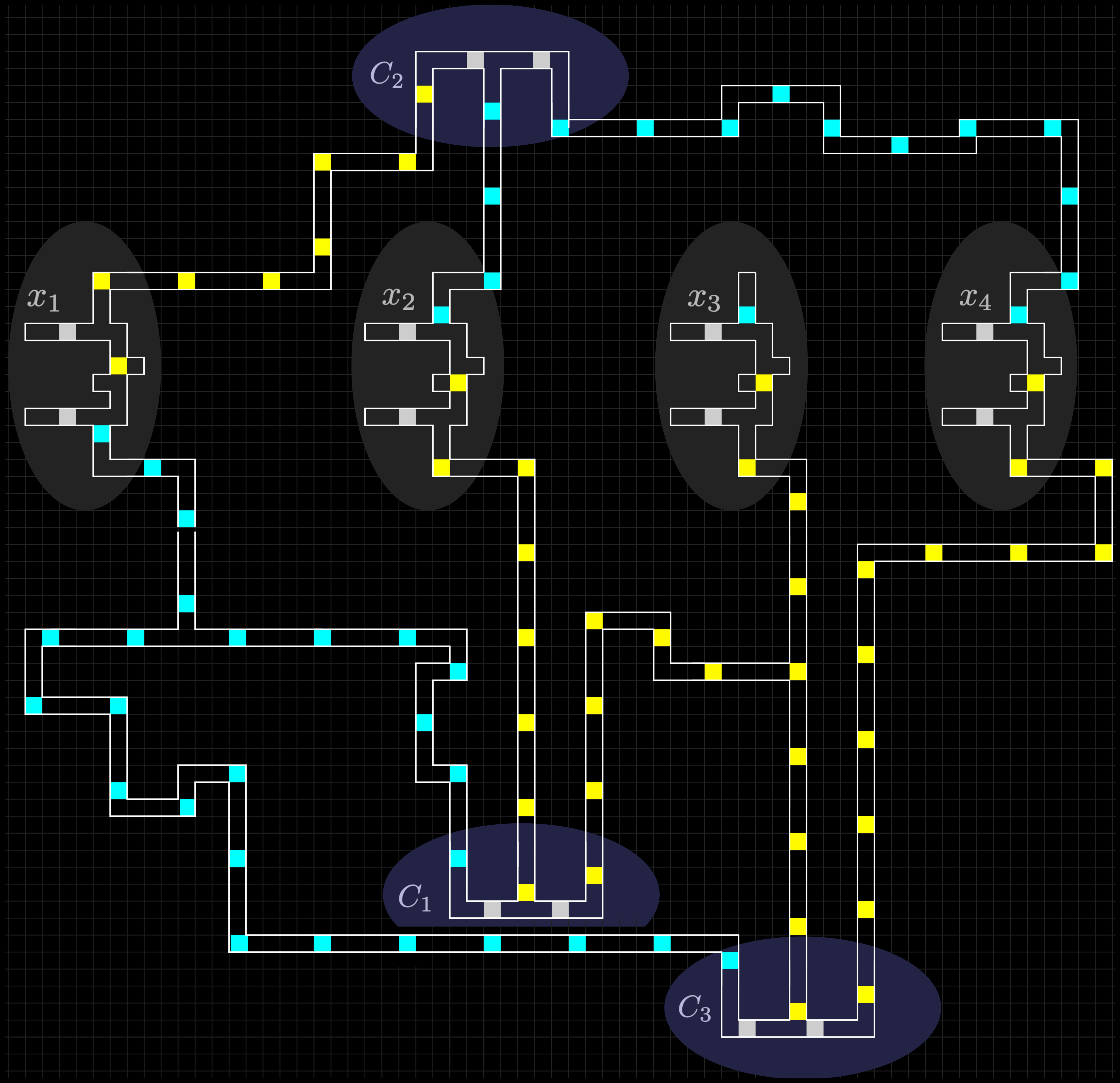

A Linear-Time 4-Approximation for Simple 2-Thin Polyominoes

Linear-Time 4-Appx for Simple 2-Thin Polyominoes

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- In each iteration, we place 1, 2, or 4 guards and 1 witness
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[*] Arnold Filtser and Hung Le. Clan embeddings into trees, and low treewidth graphs. In Symposium on Theory of Computing, pages 342–355, 2021.
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Linear-Time 4-Appx for Simple 2-Thin Polyominoes

- There is a PTAS for k-hop domination in H-minor free graphs based on local search [*], but runtime may be infeasible for extremely large n
- Exact algorithm for graphs with treewidth tw has runtime $O((2k+1)^{tw}\, n)$
  - 2-thin polyominoes have $tw=2$, as K_4 is not a minor
  - But runtime may be too high if $k=\Omega(n)$
- Runtime of our algorithm does not depend on k
- Idea: construct a tree T on the polyomino P and let T guide us in placing guards
- Inspired by the linear-time algorithm for trees [+]
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➡ 4-approximation

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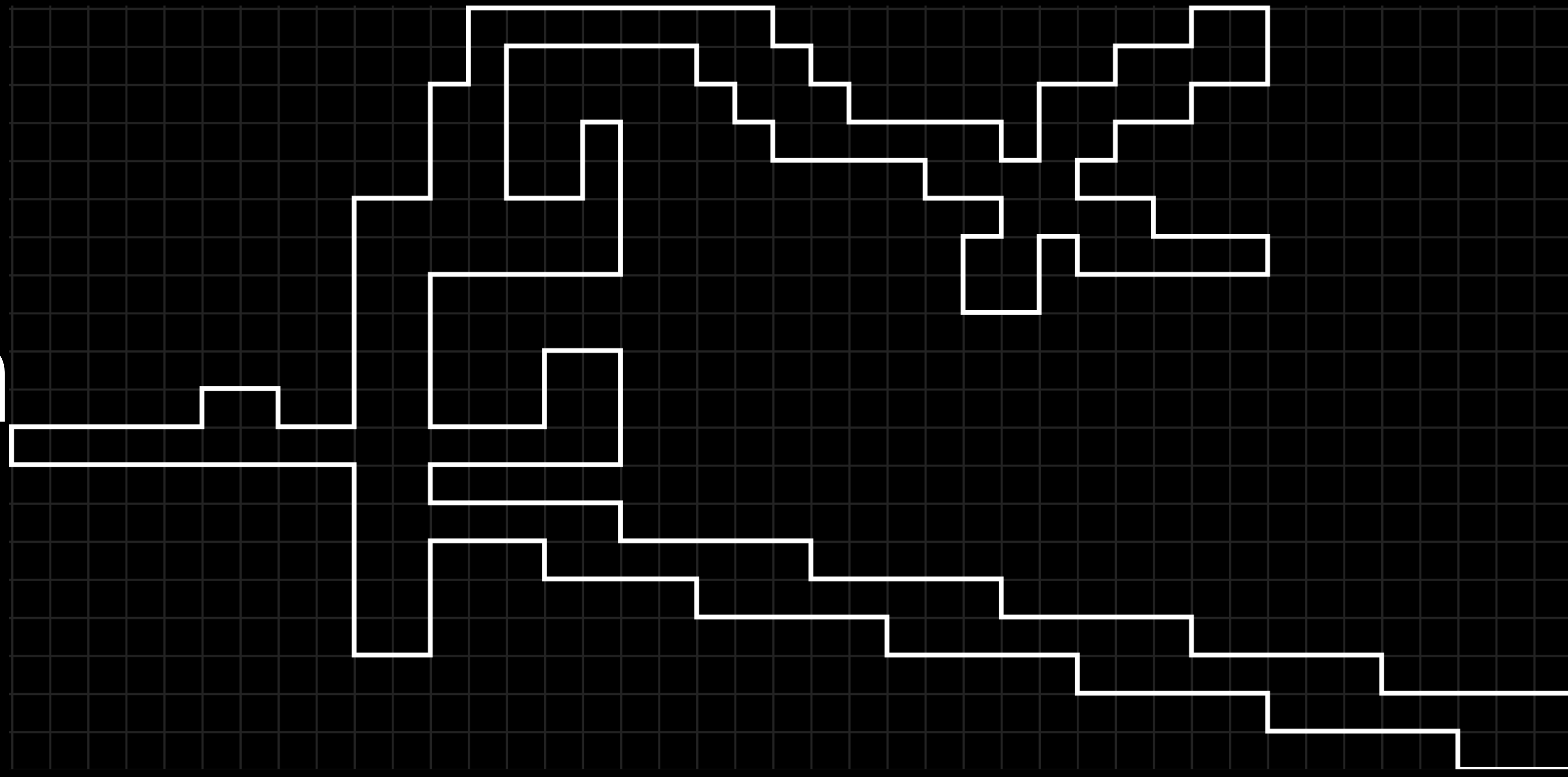
Constructing a Skeleton Graph



- P: 2-thin polyomino

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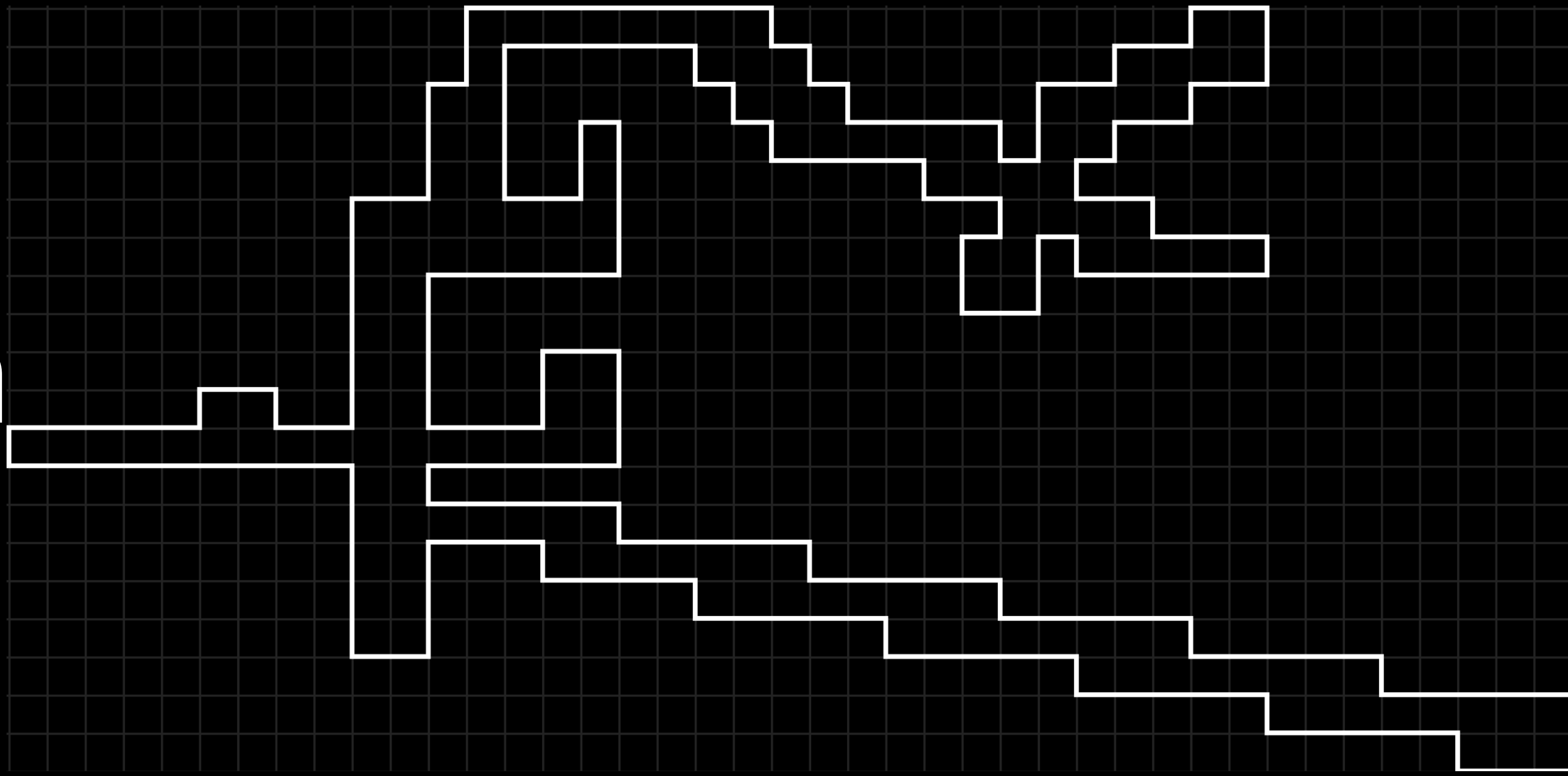
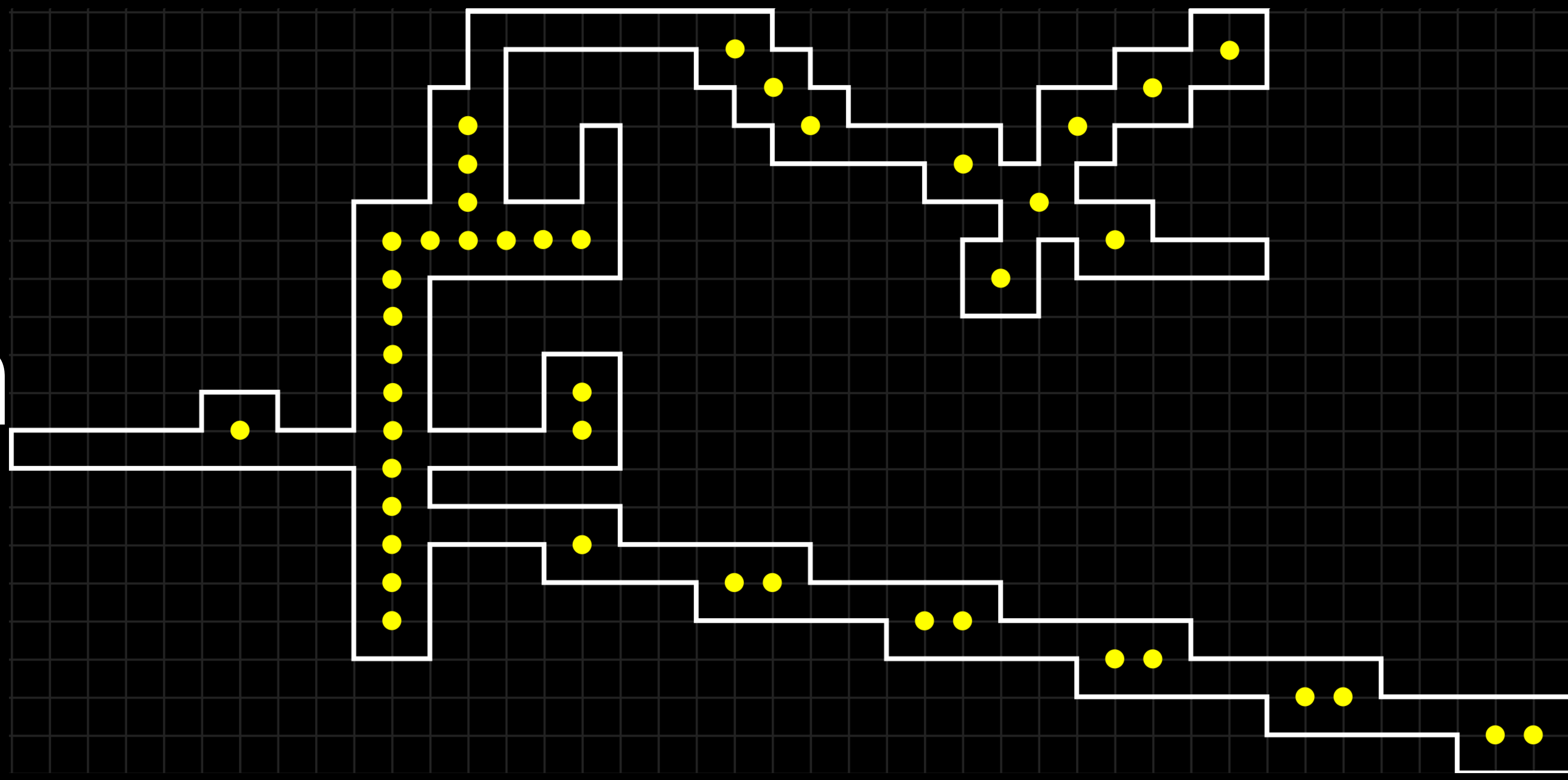


- P: 2-thin polyomino
- Each unit square/cell is incident to four vertices



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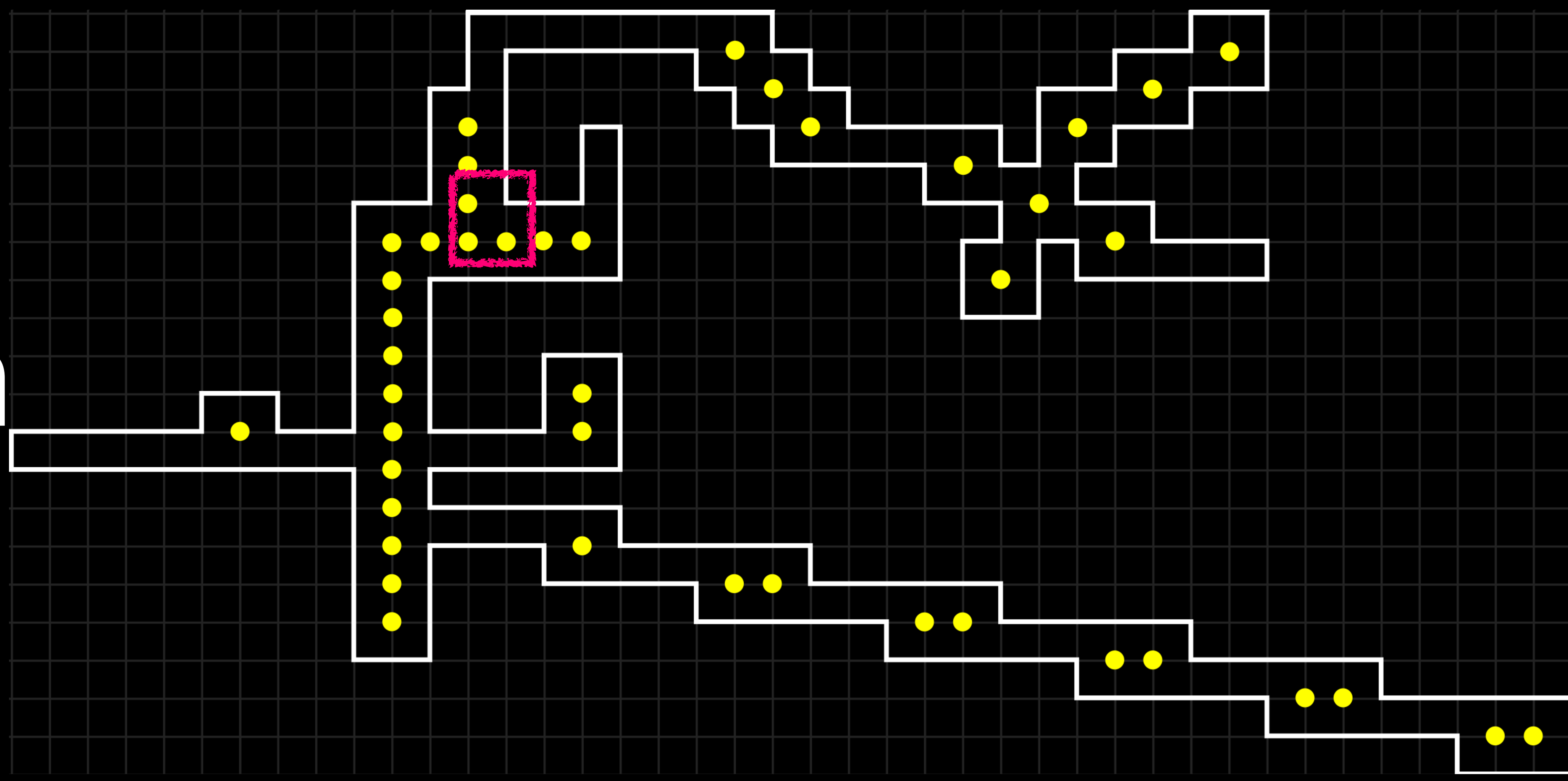

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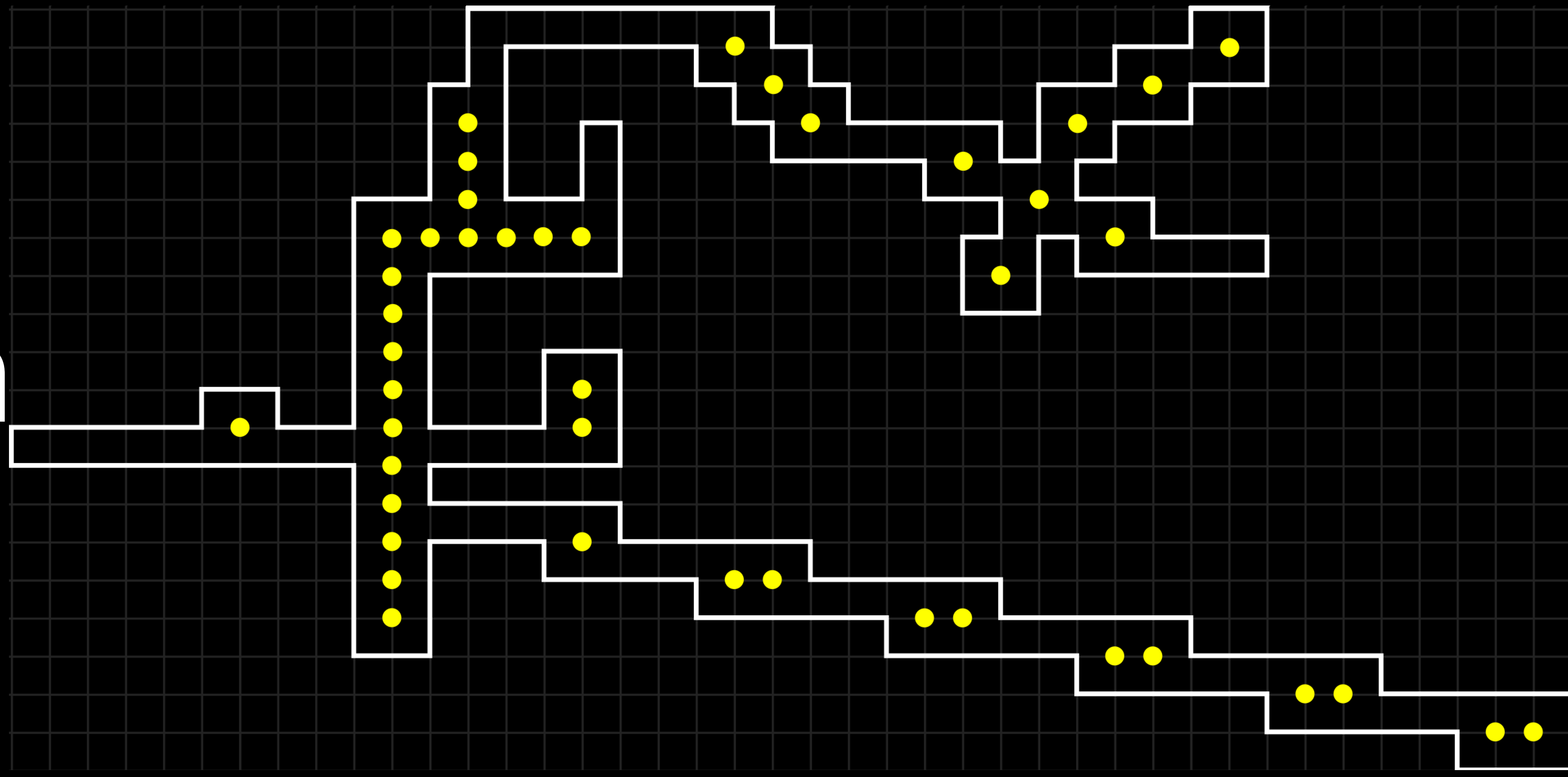

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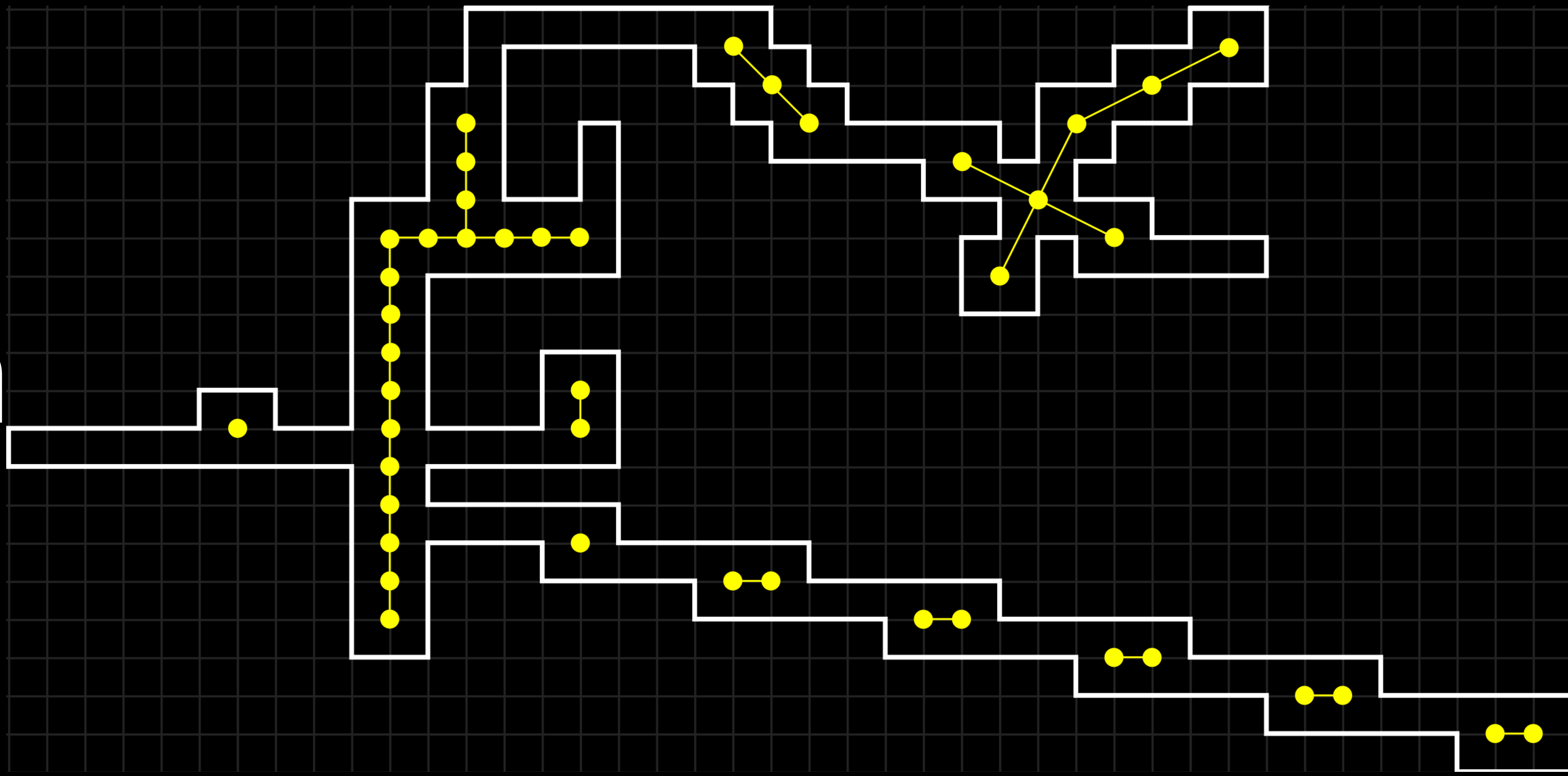

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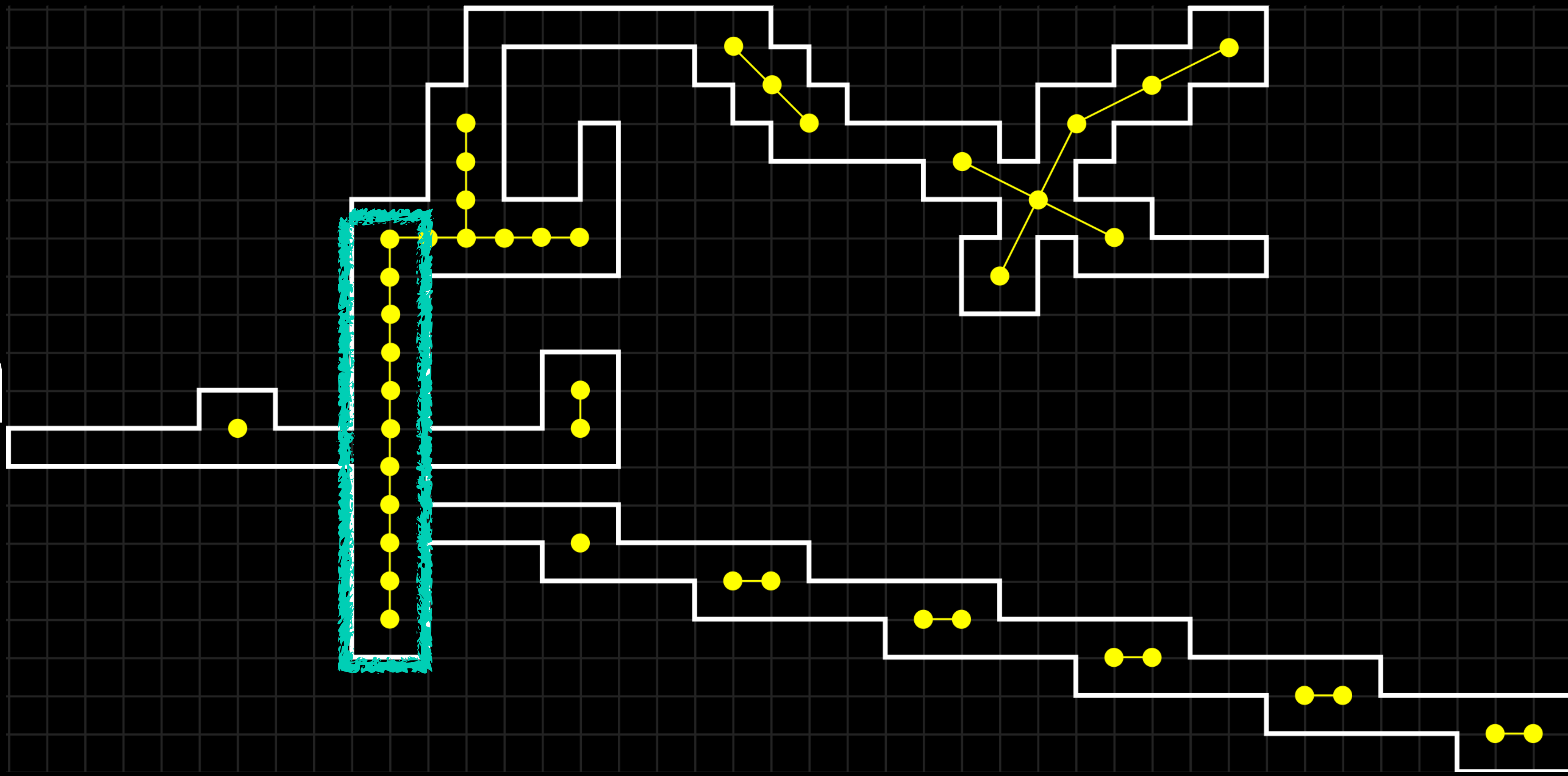

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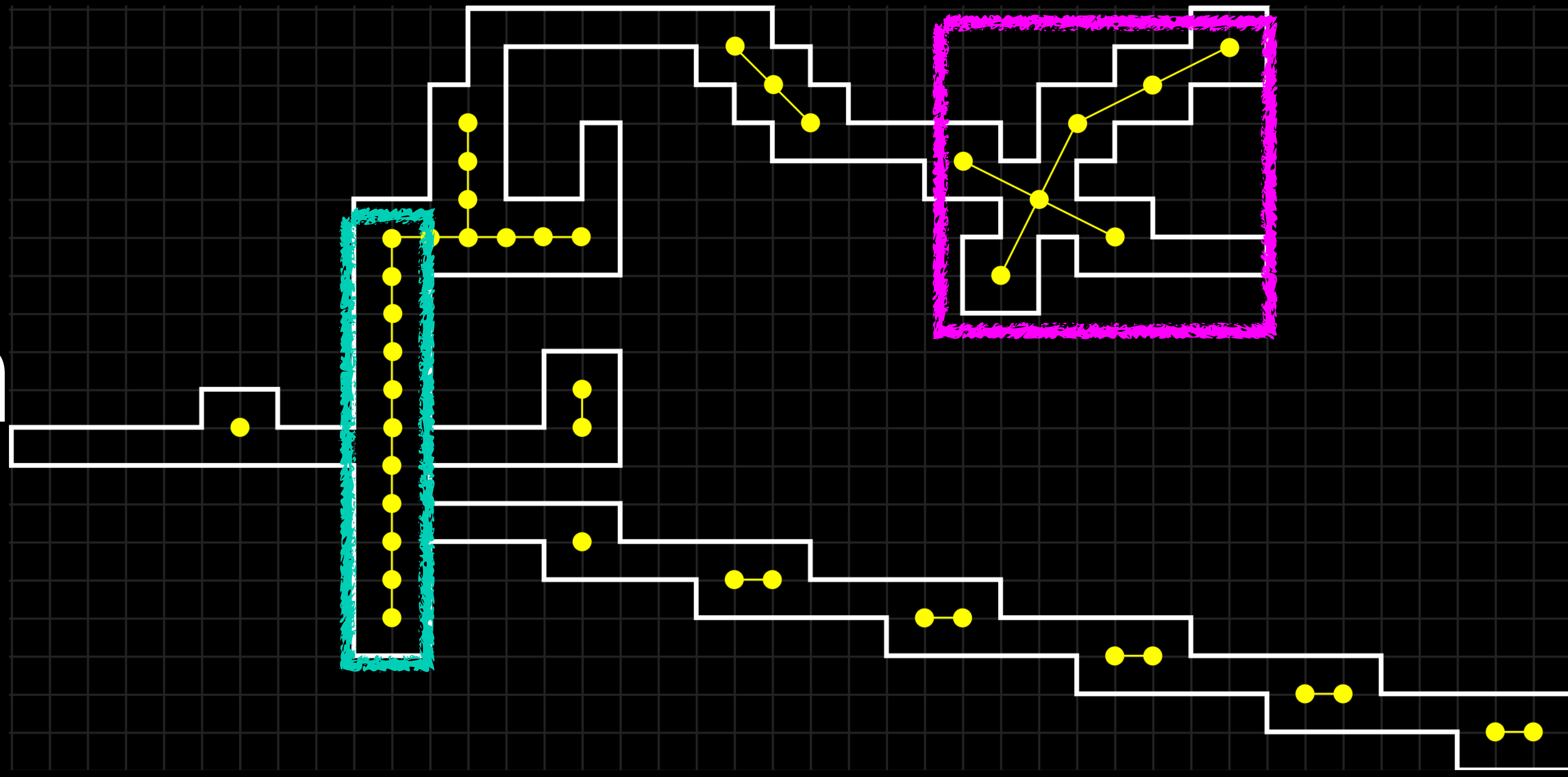

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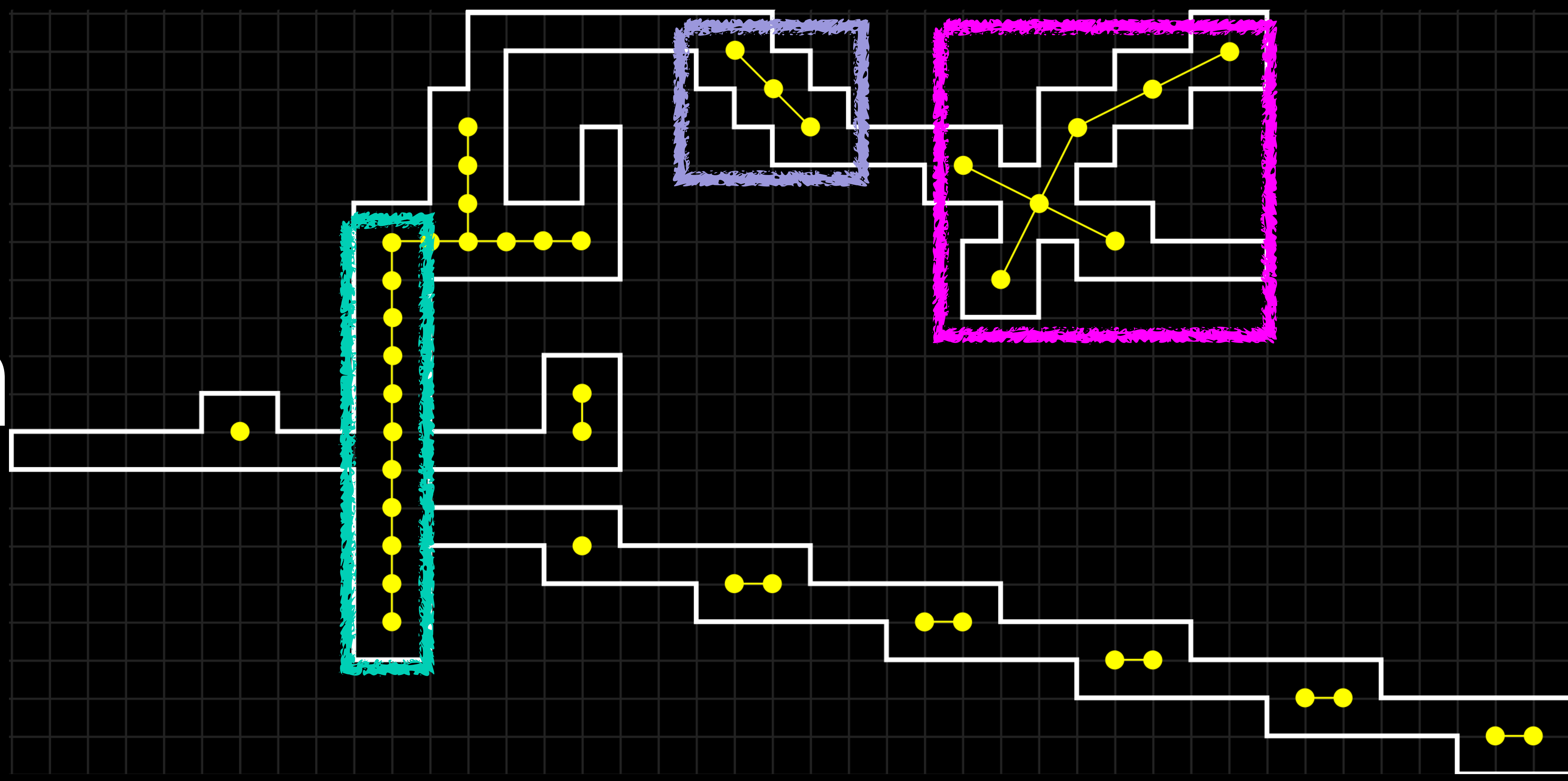

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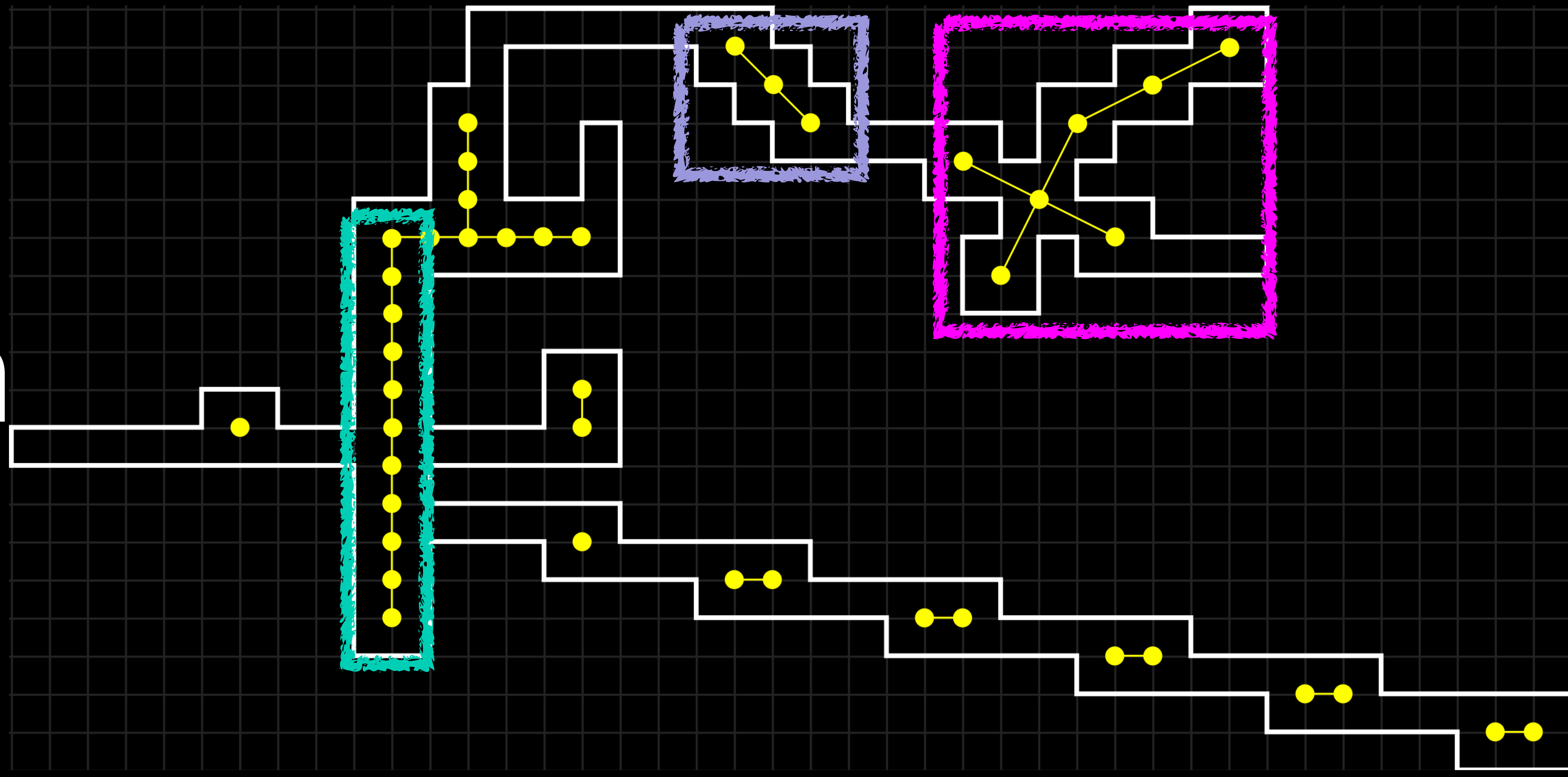

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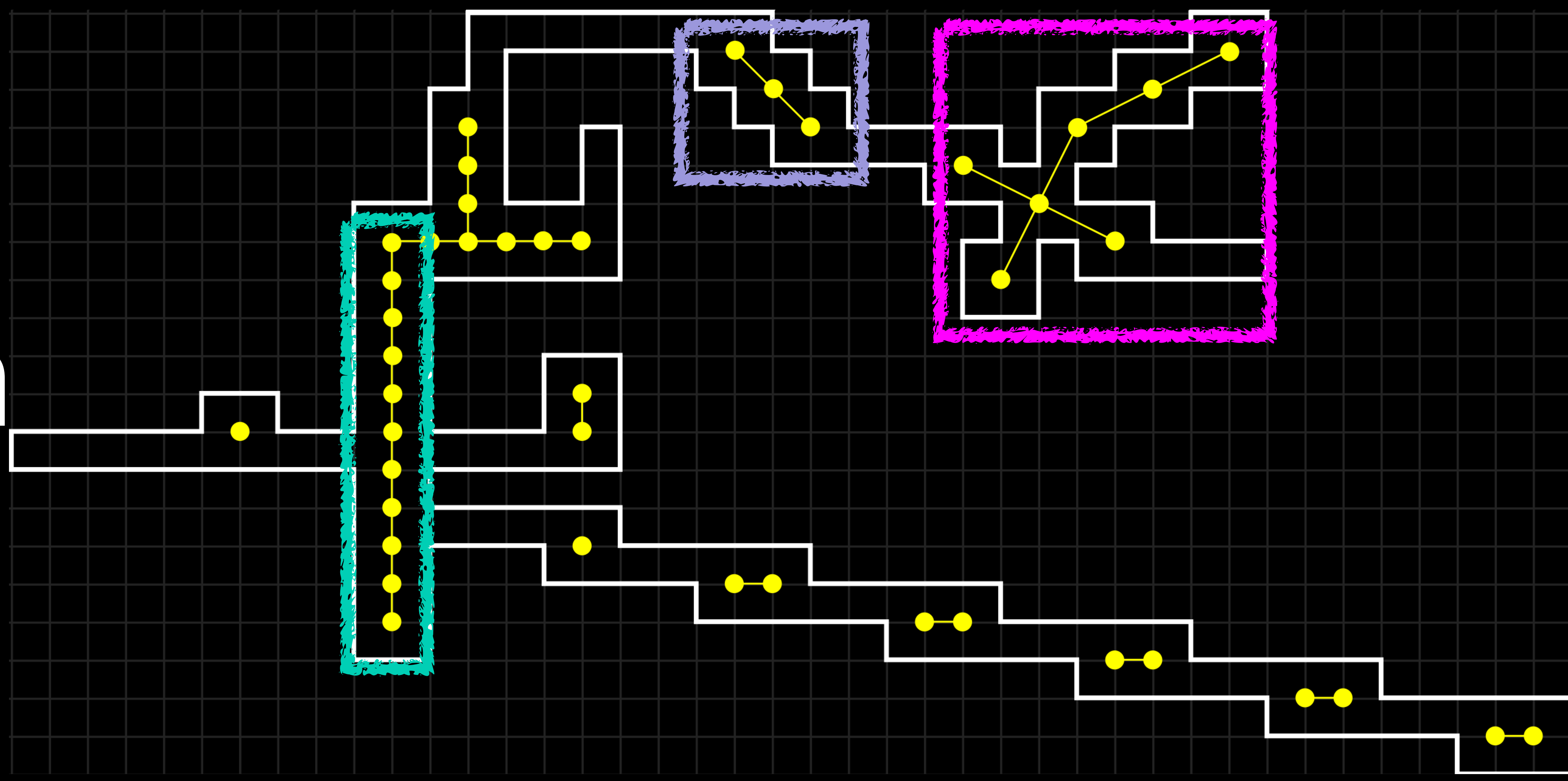

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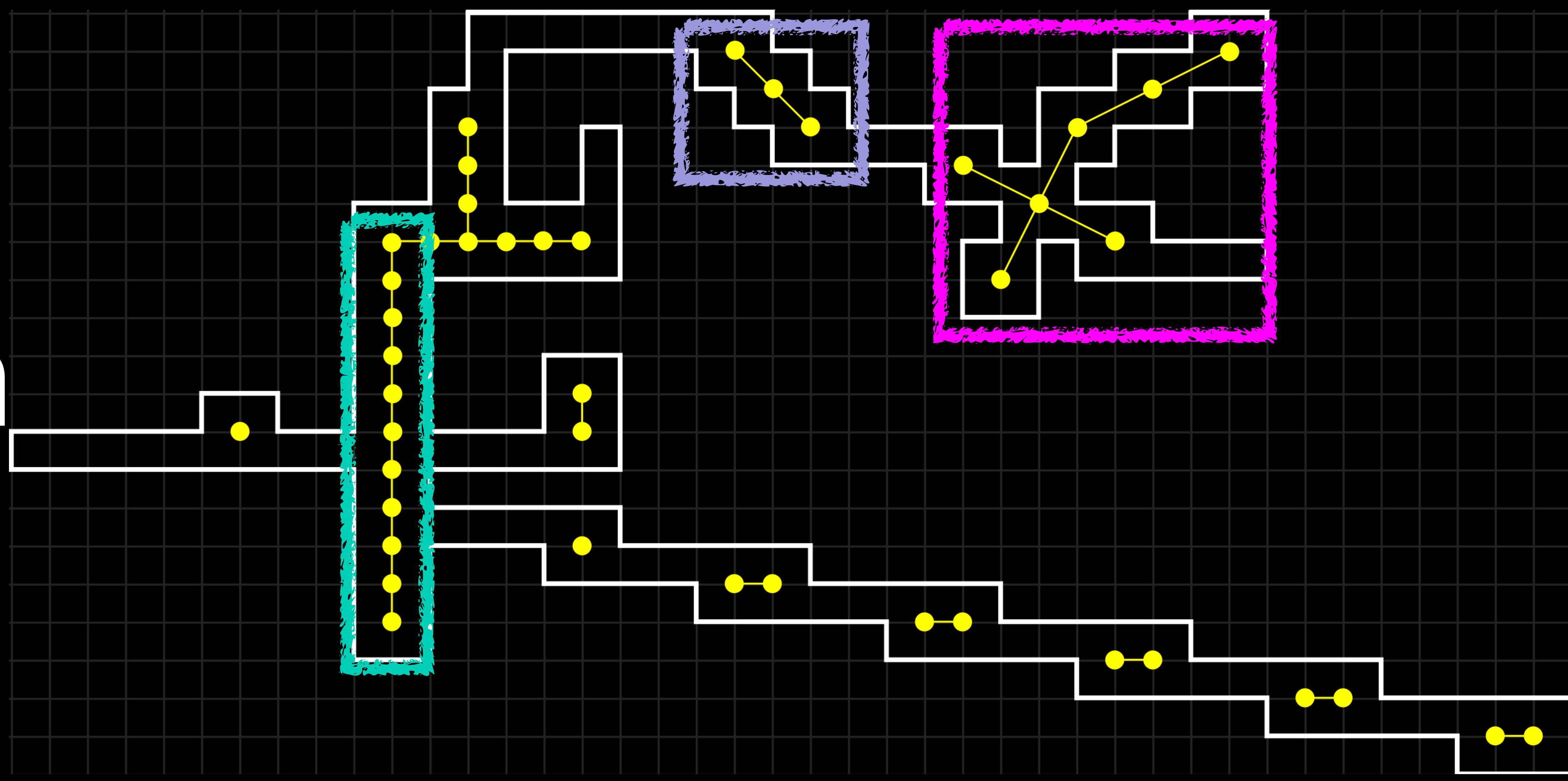

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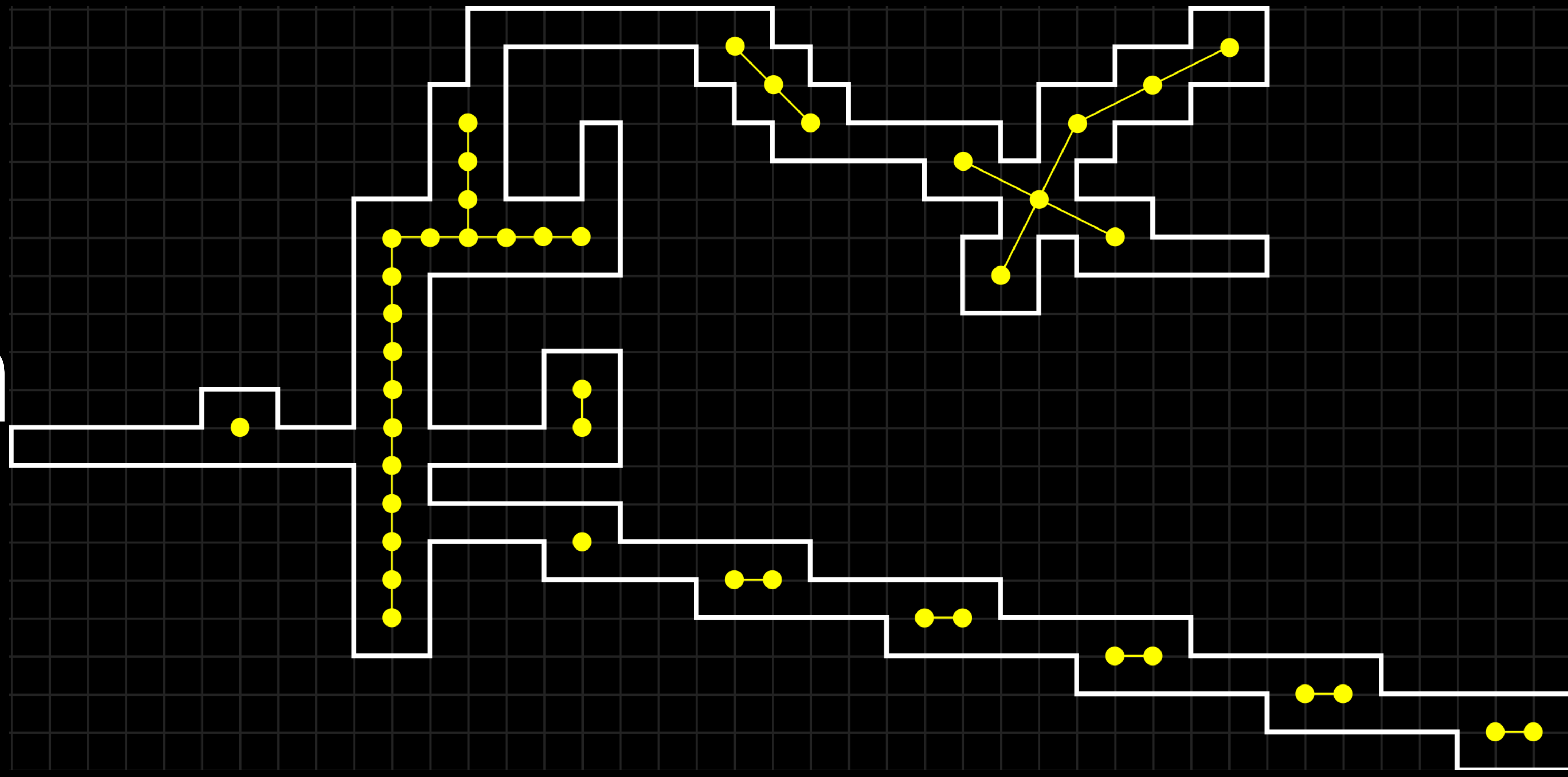
➡ Edges from E_I form a forest T_I on I

Constructing a Skeleton Graph



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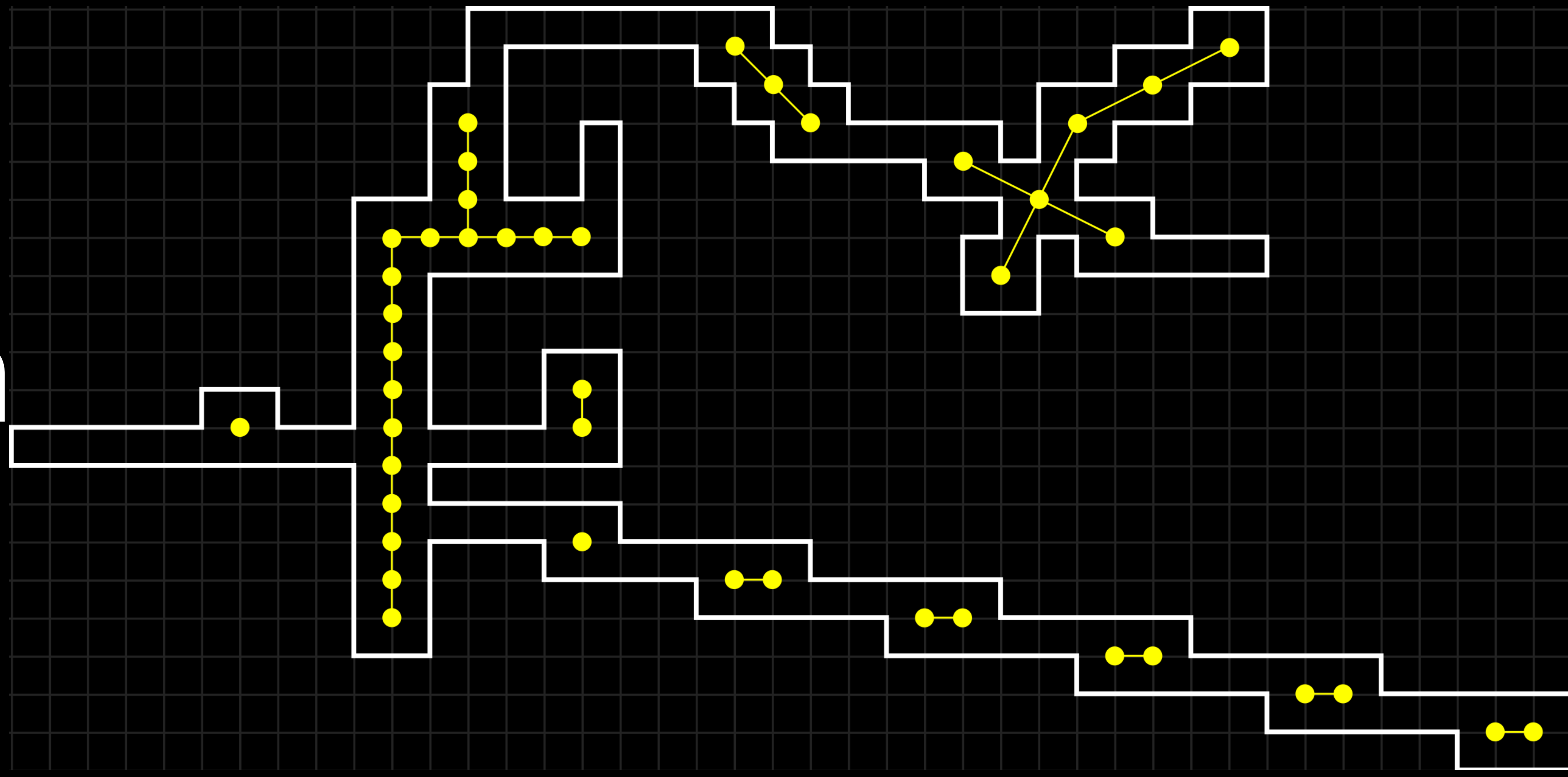
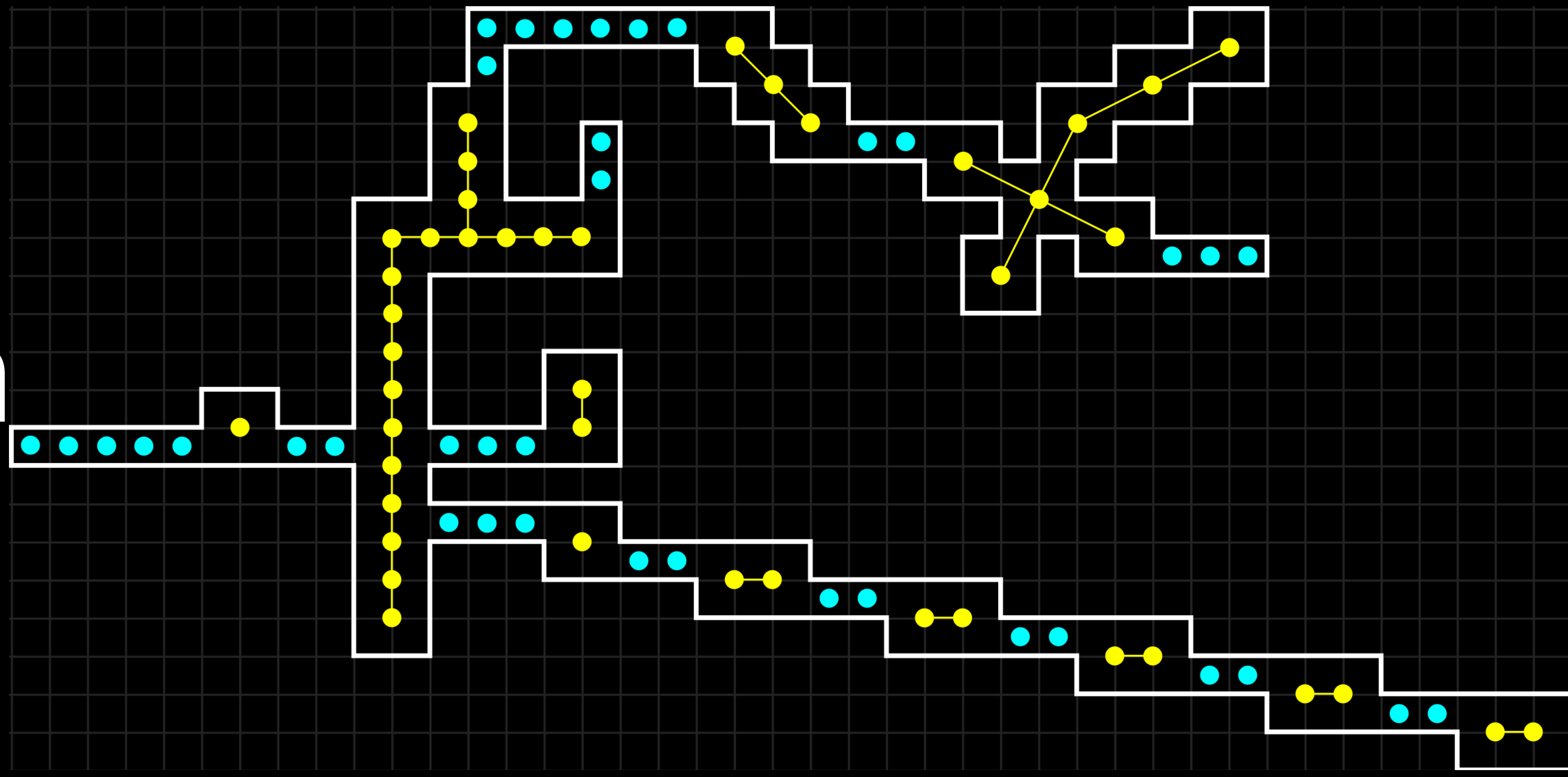


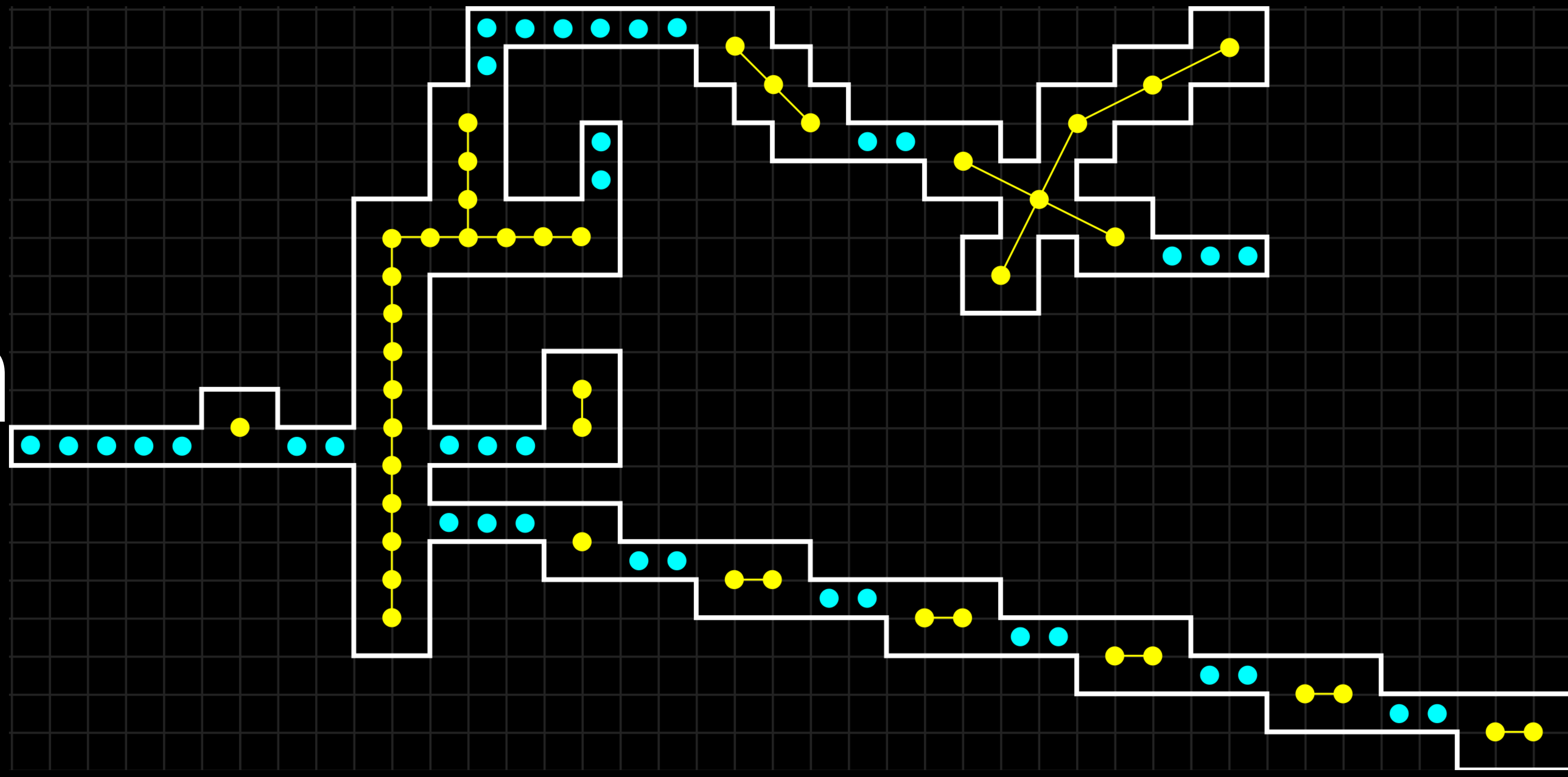
- Unit square s without internal vertex:



Constructing a Skeleton Graph

- Unit square s without internal vertex:
  - Vertex b_s in center of s

Constructing a Skeleton Graph



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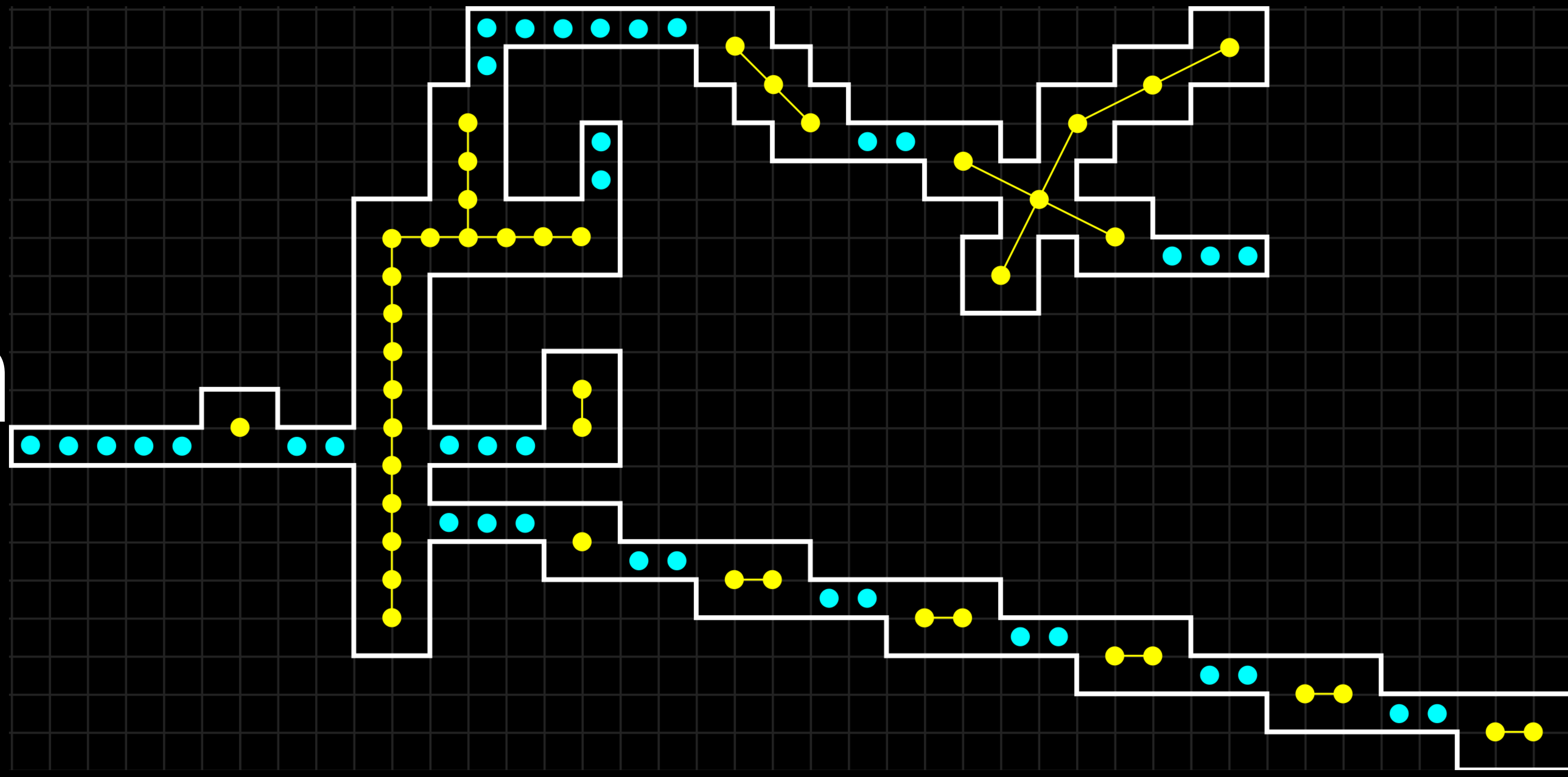
Constructing a Skeleton Graph



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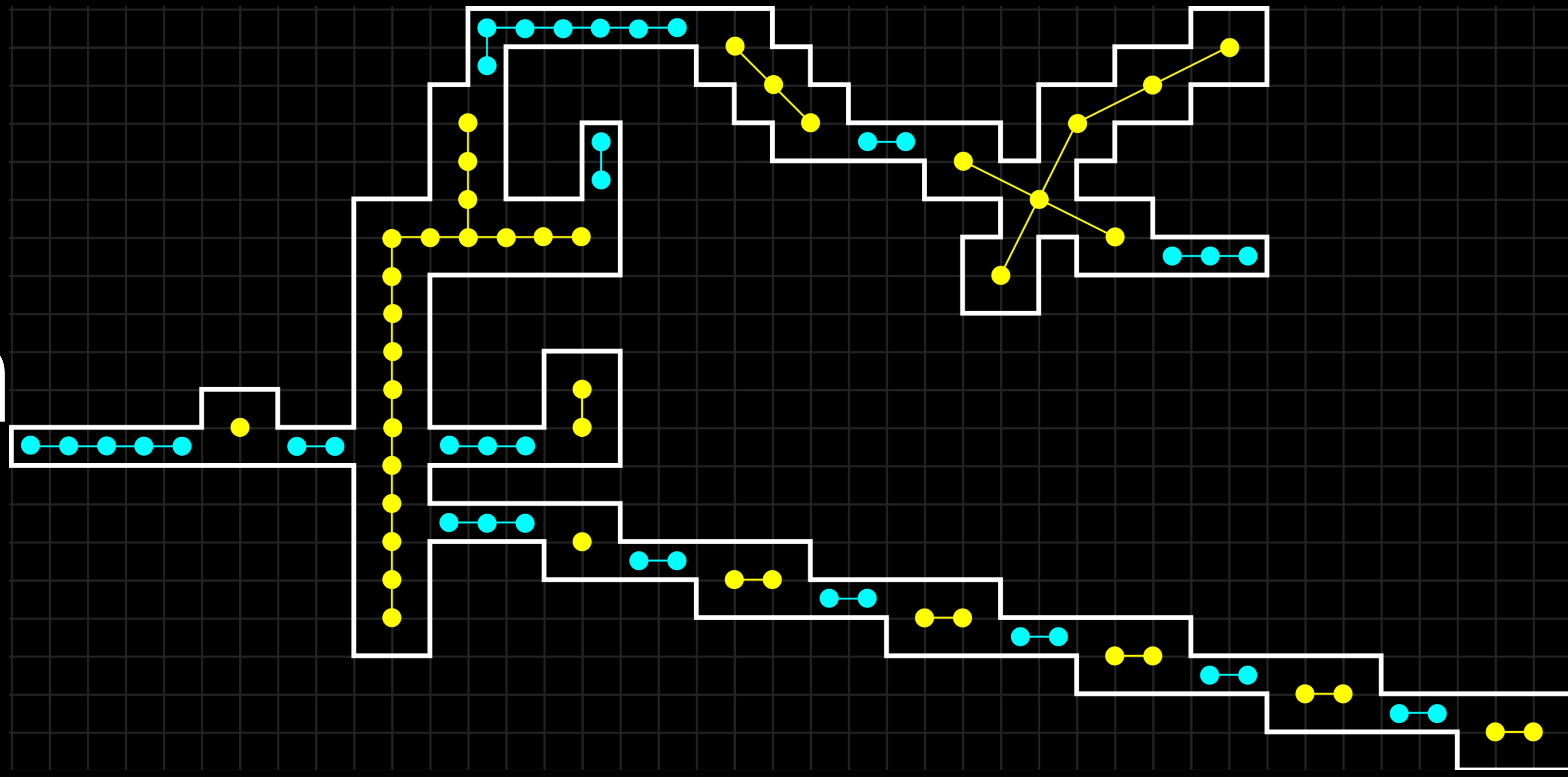

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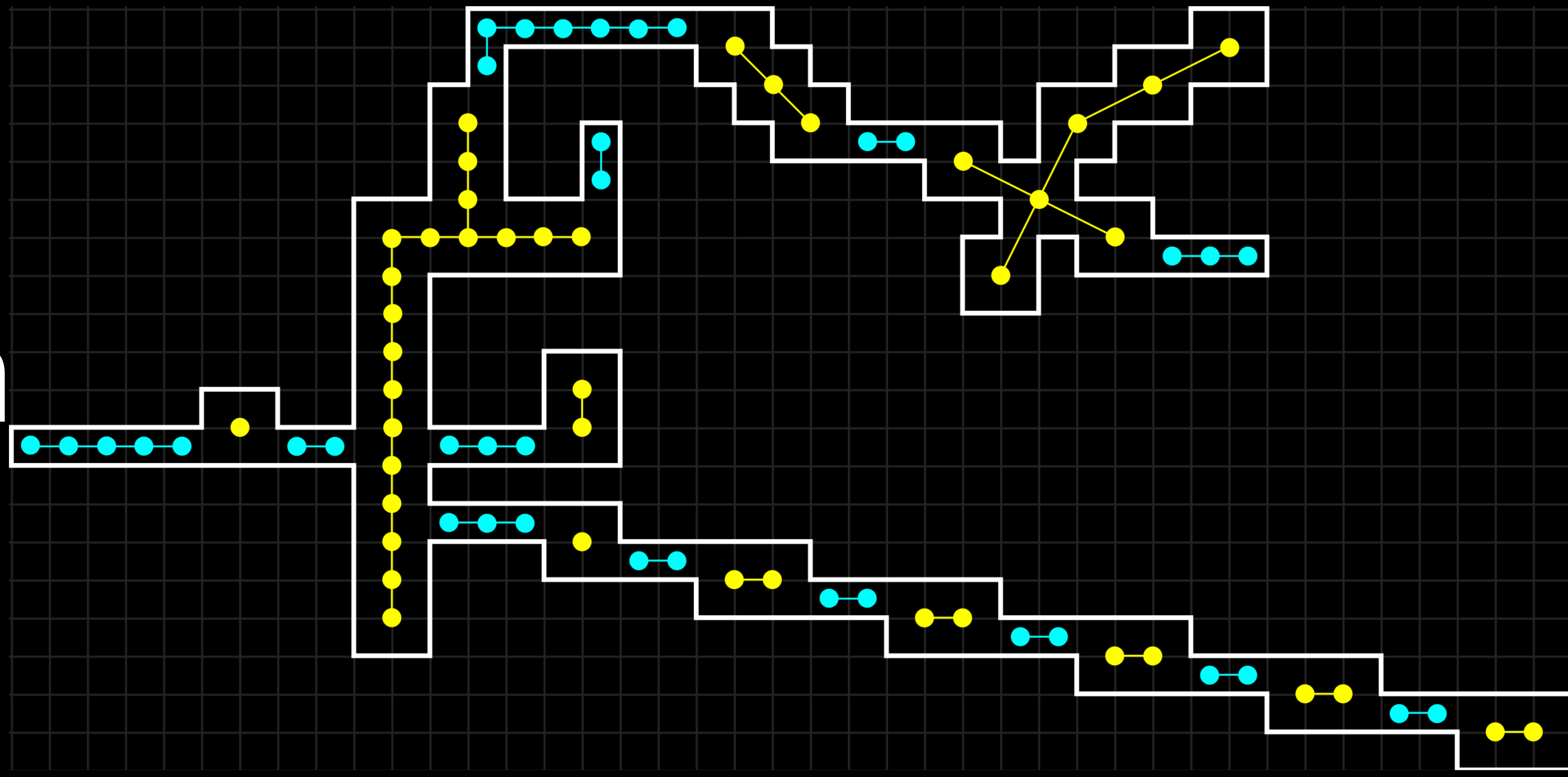

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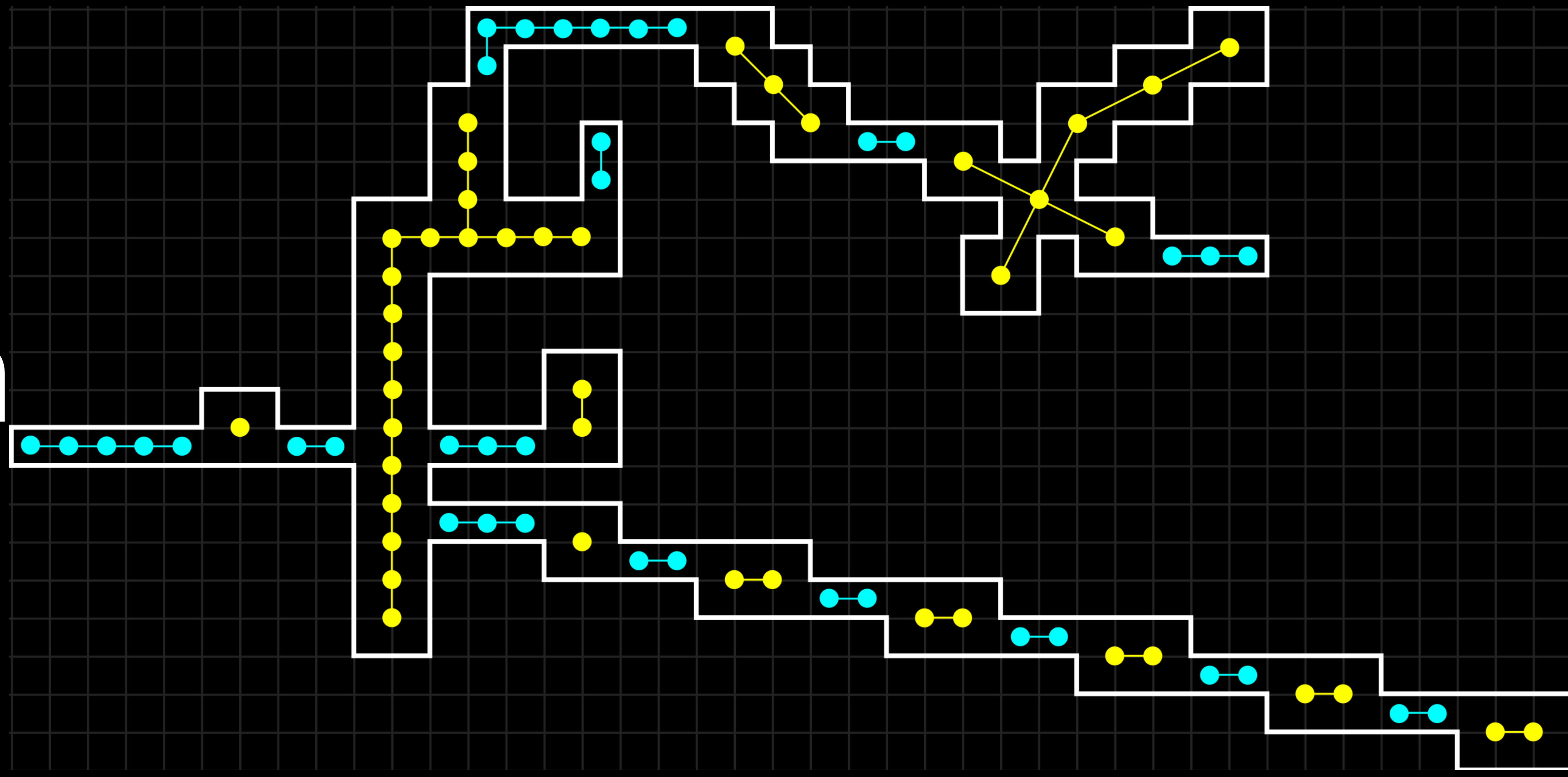

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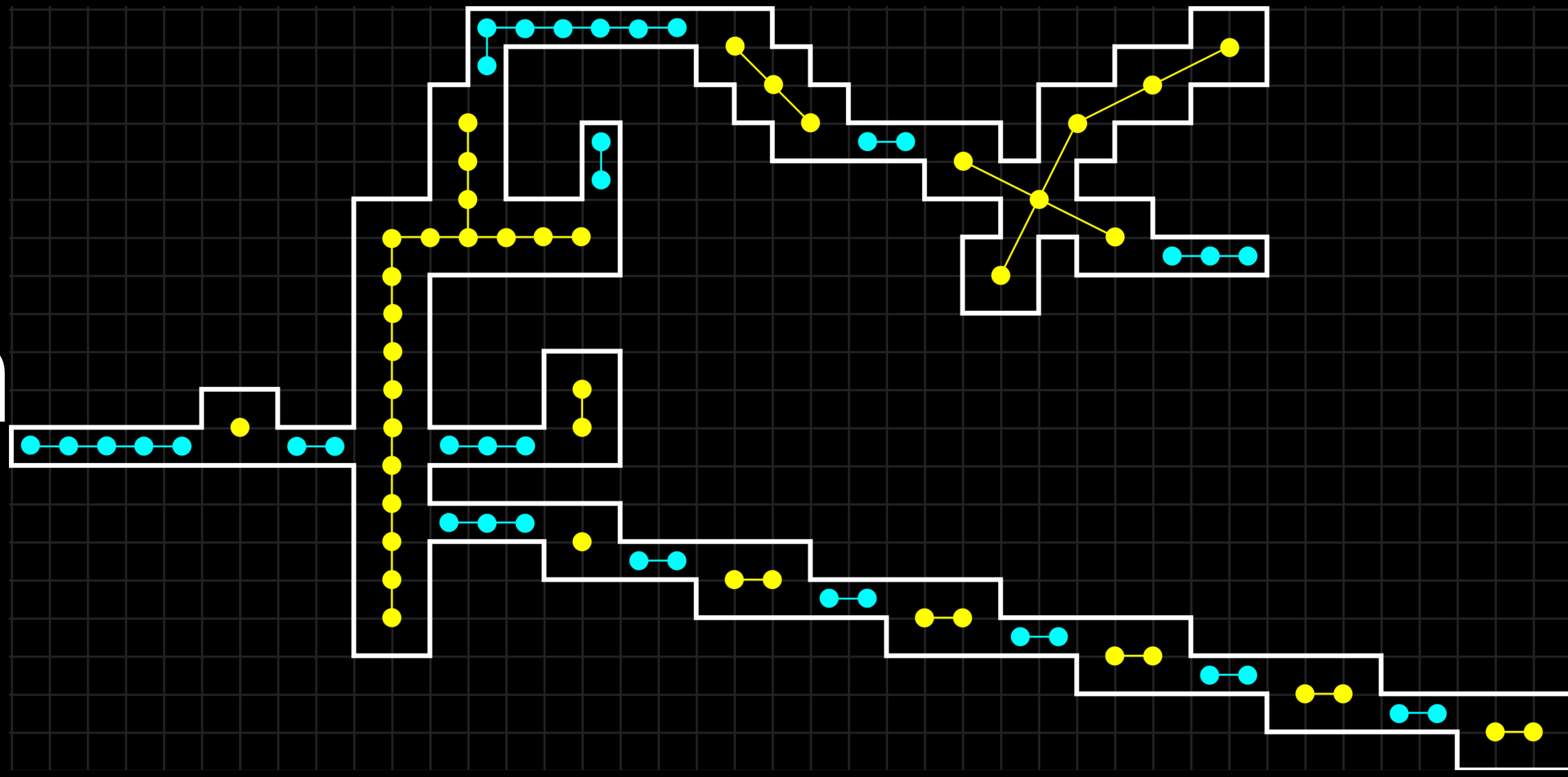
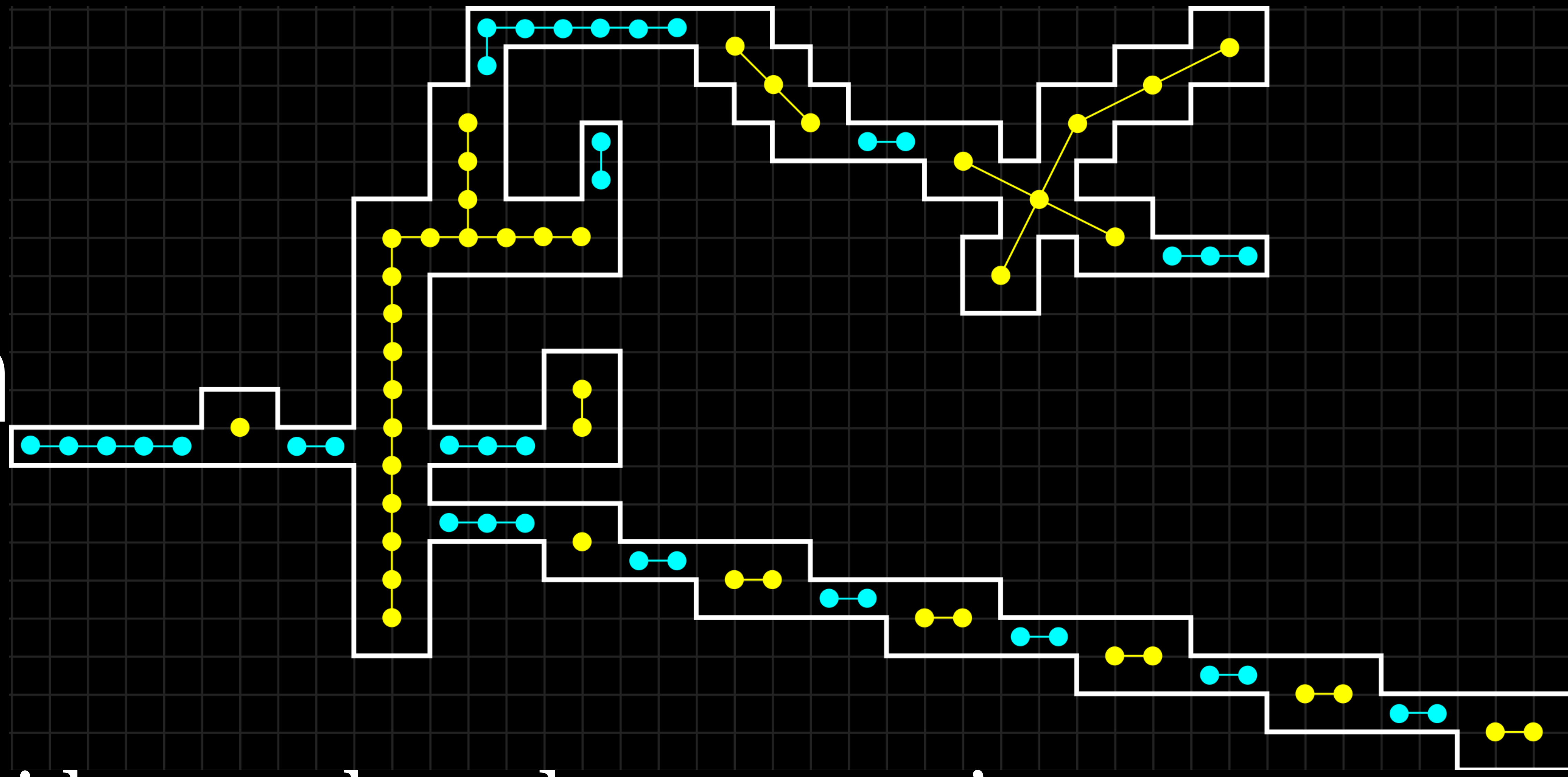
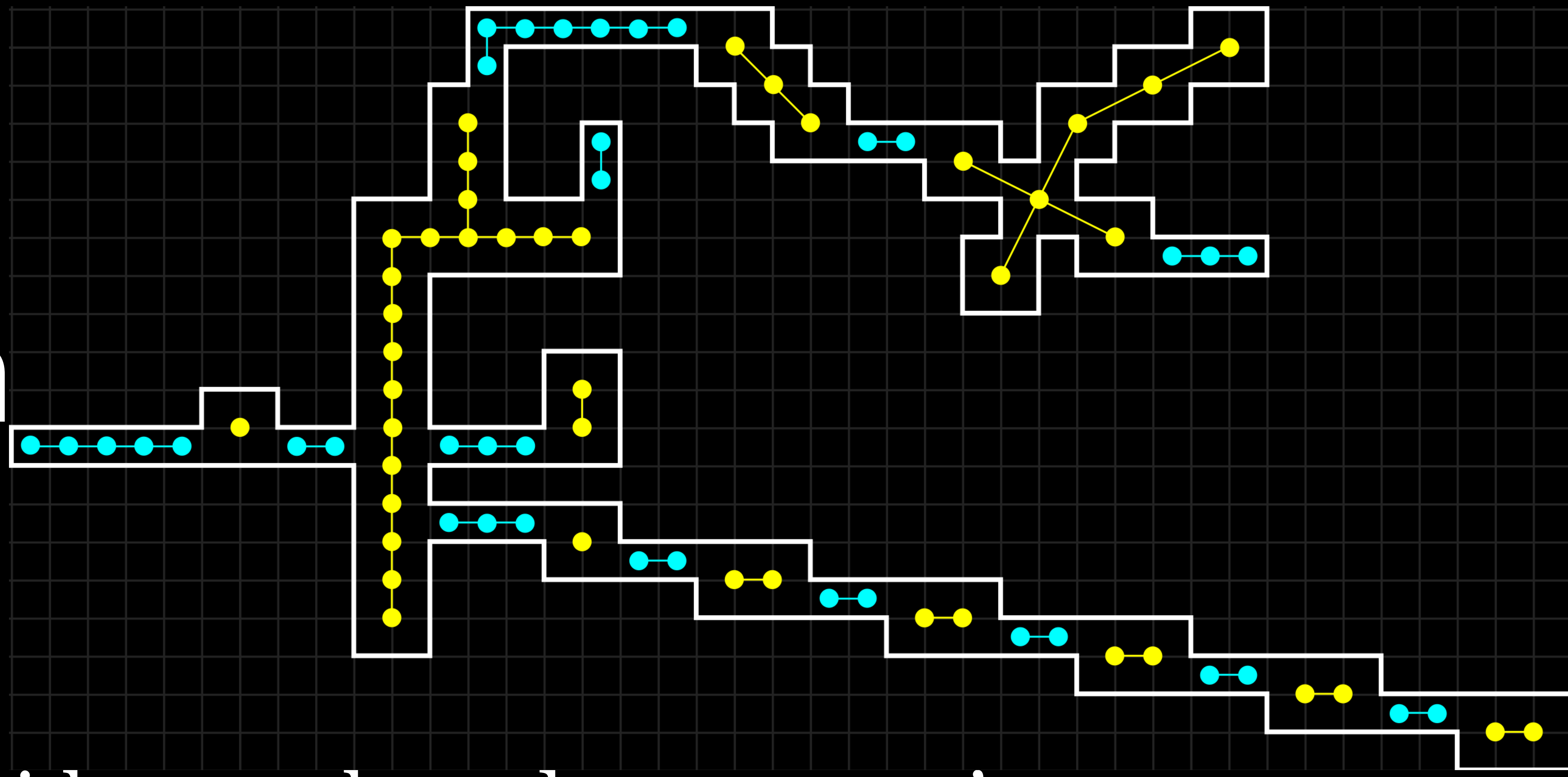


- Connect T_I and T_B

Constructing a Skeleton Graph



- Connect T_I and T_B
- Let s be a boundary square that shares an edge with a non-boundary square s'



Constructing a Skeleton Graph

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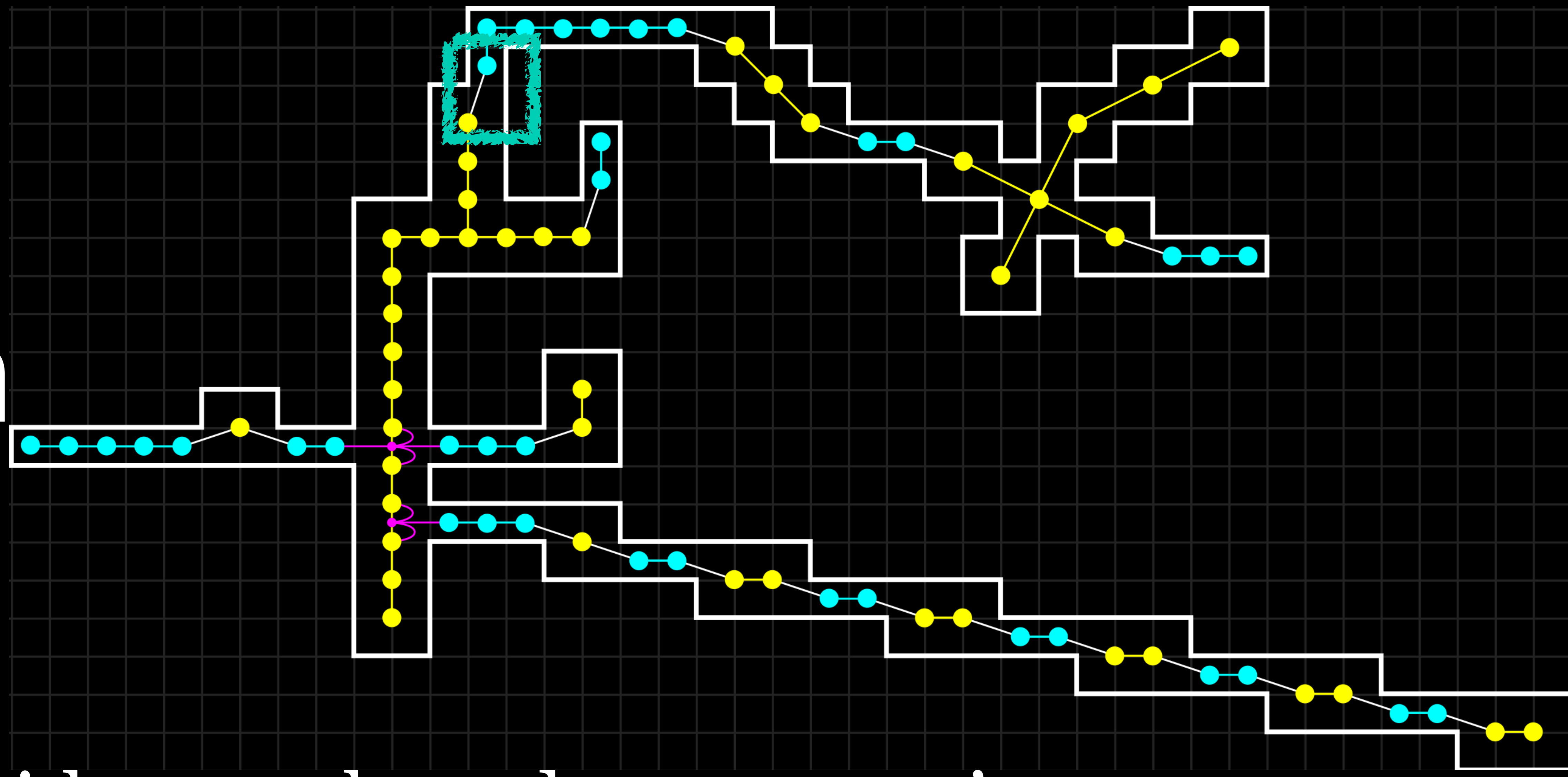
Constructing a Skeleton Graph



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- Let s be a boundary square that shares an edge with a non-boundary square s'

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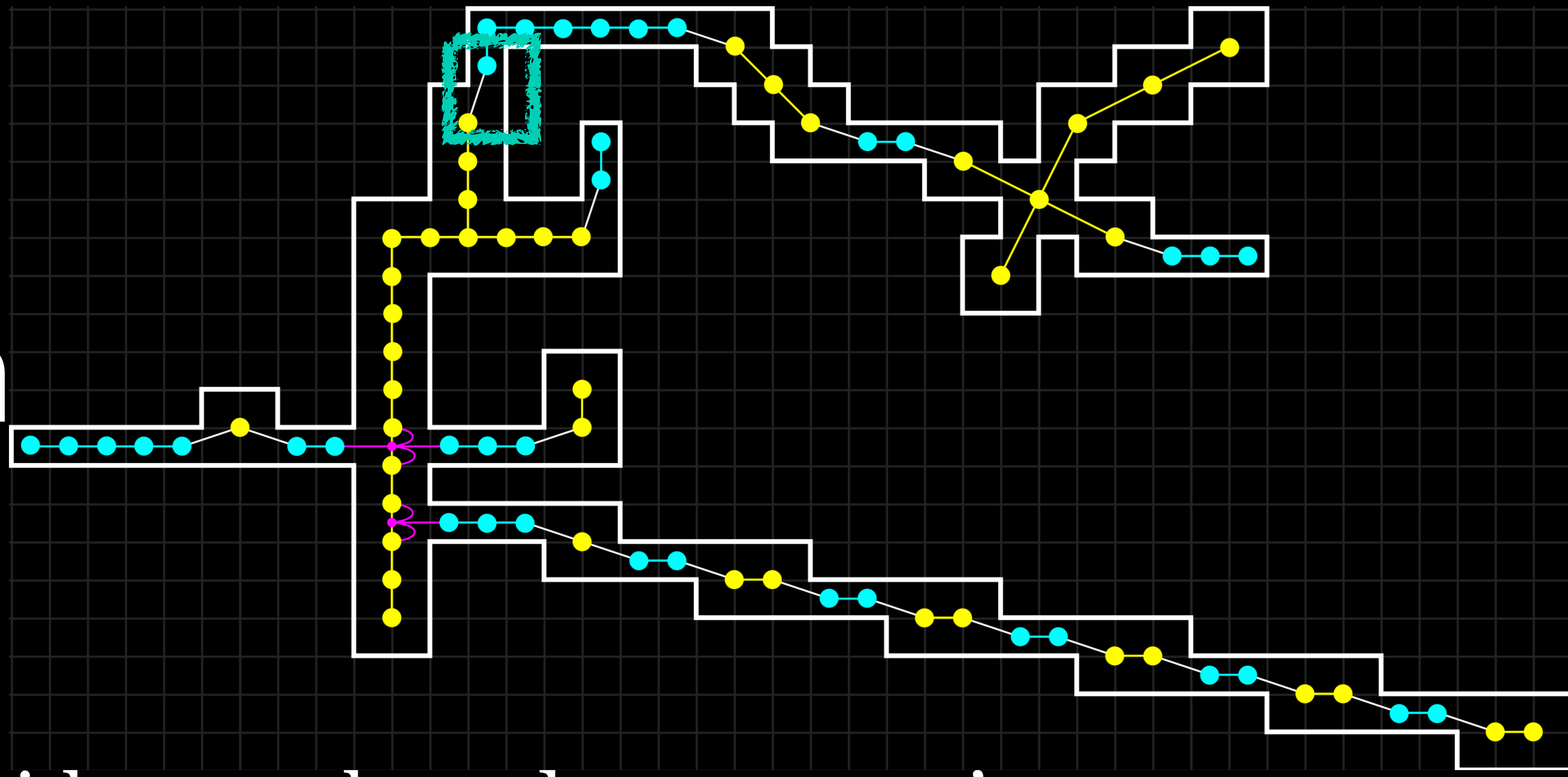
Constructing a Skeleton Graph



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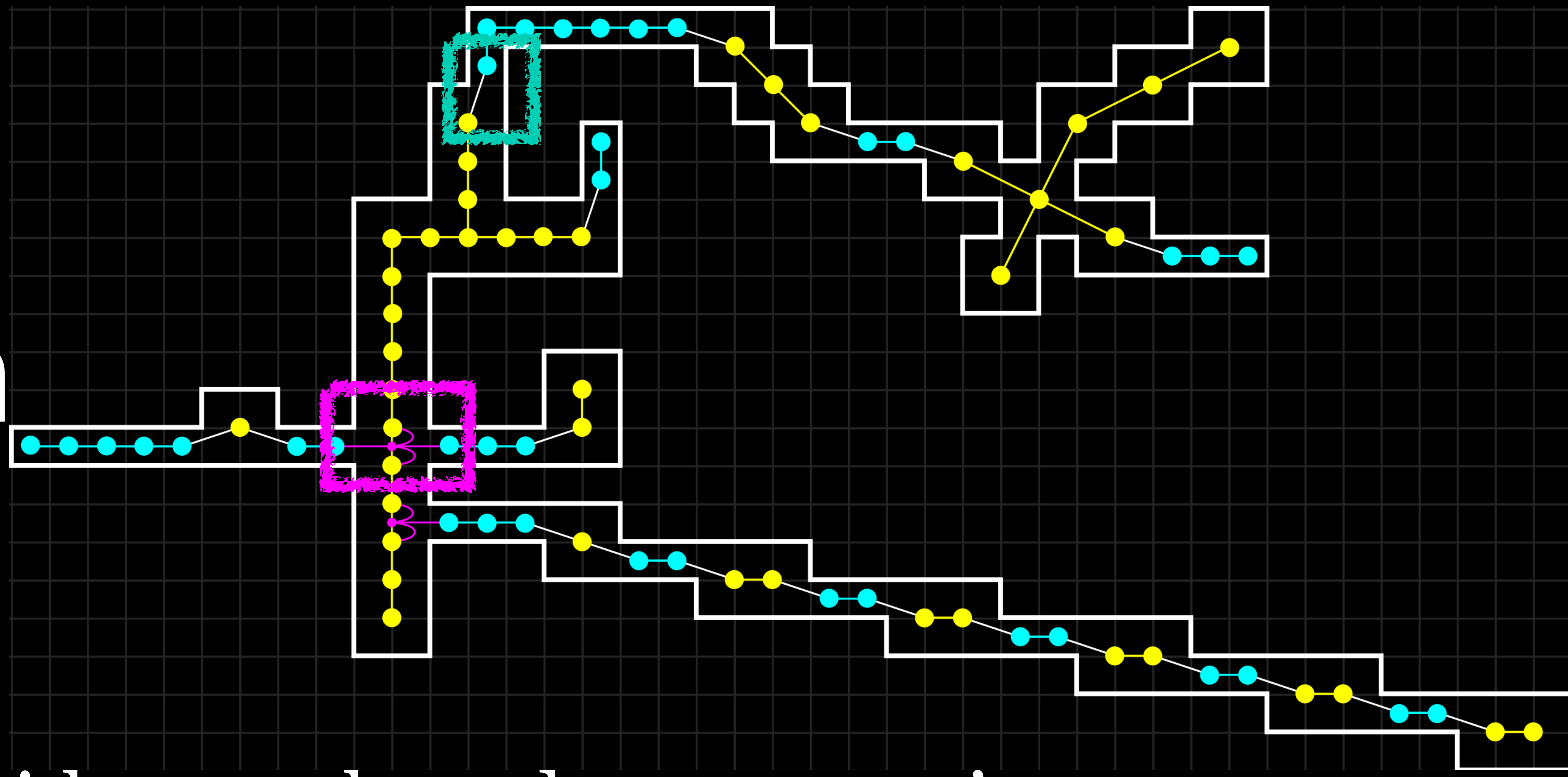

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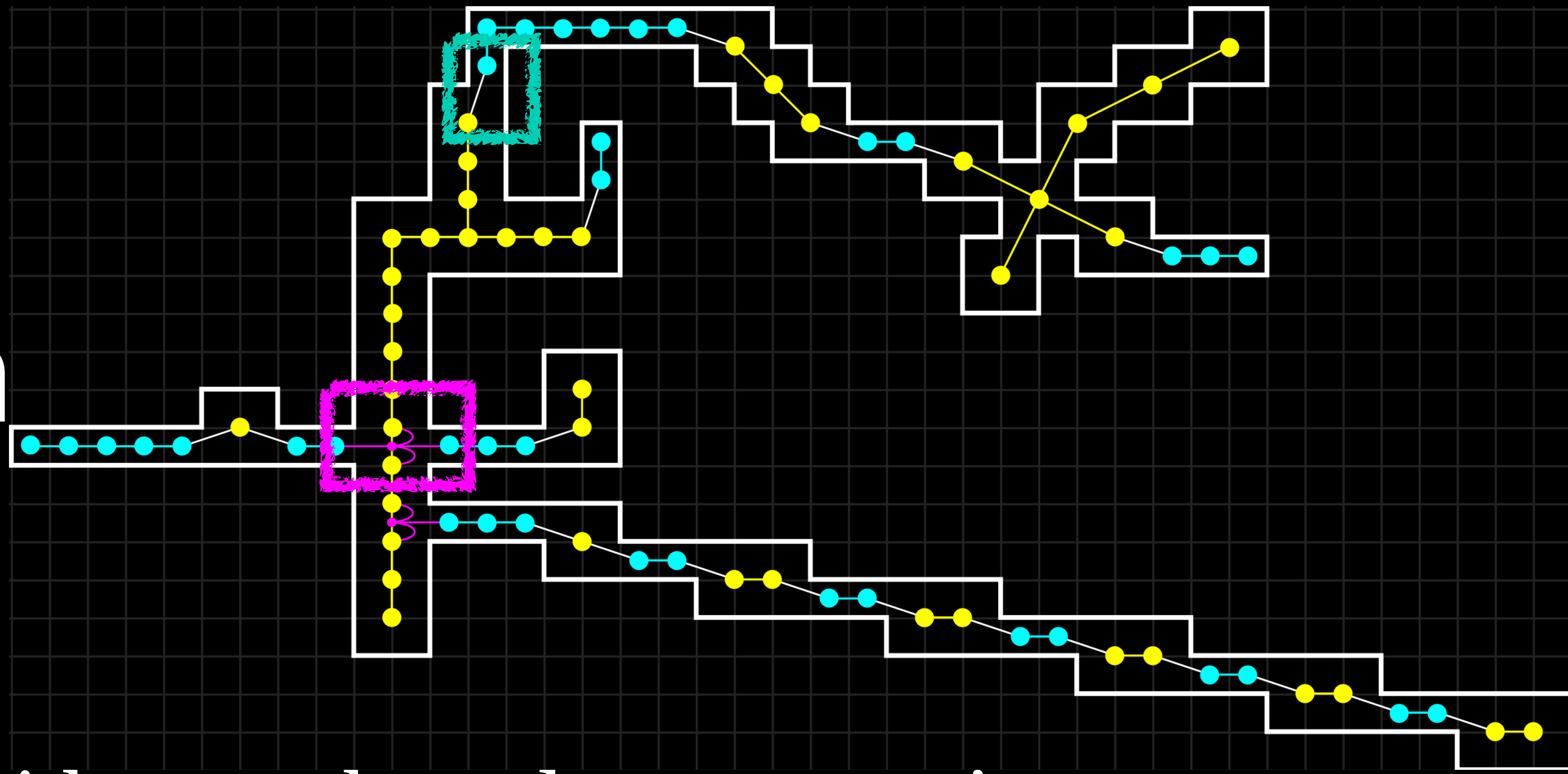
Constructing a Skeleton Graph



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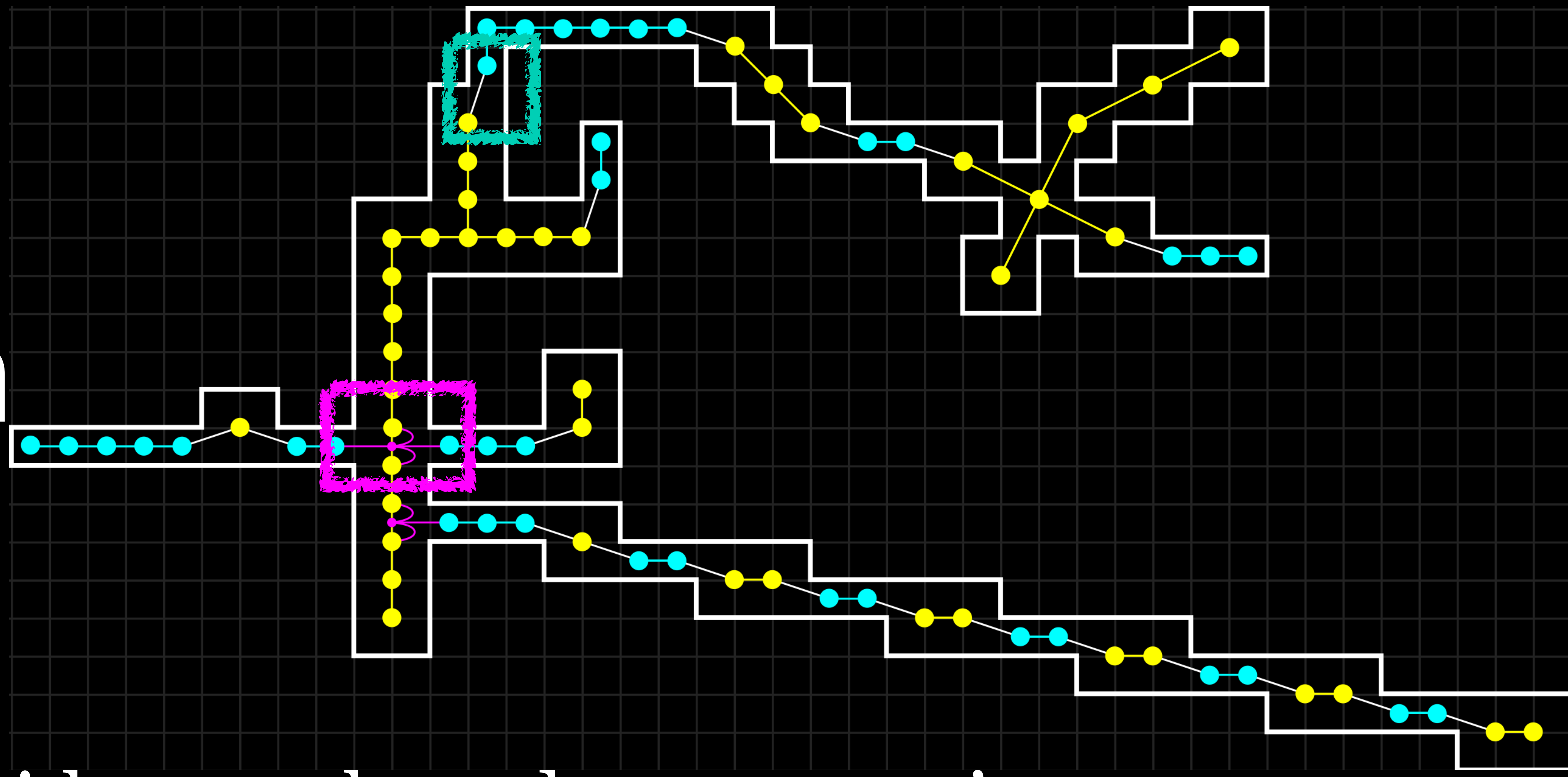

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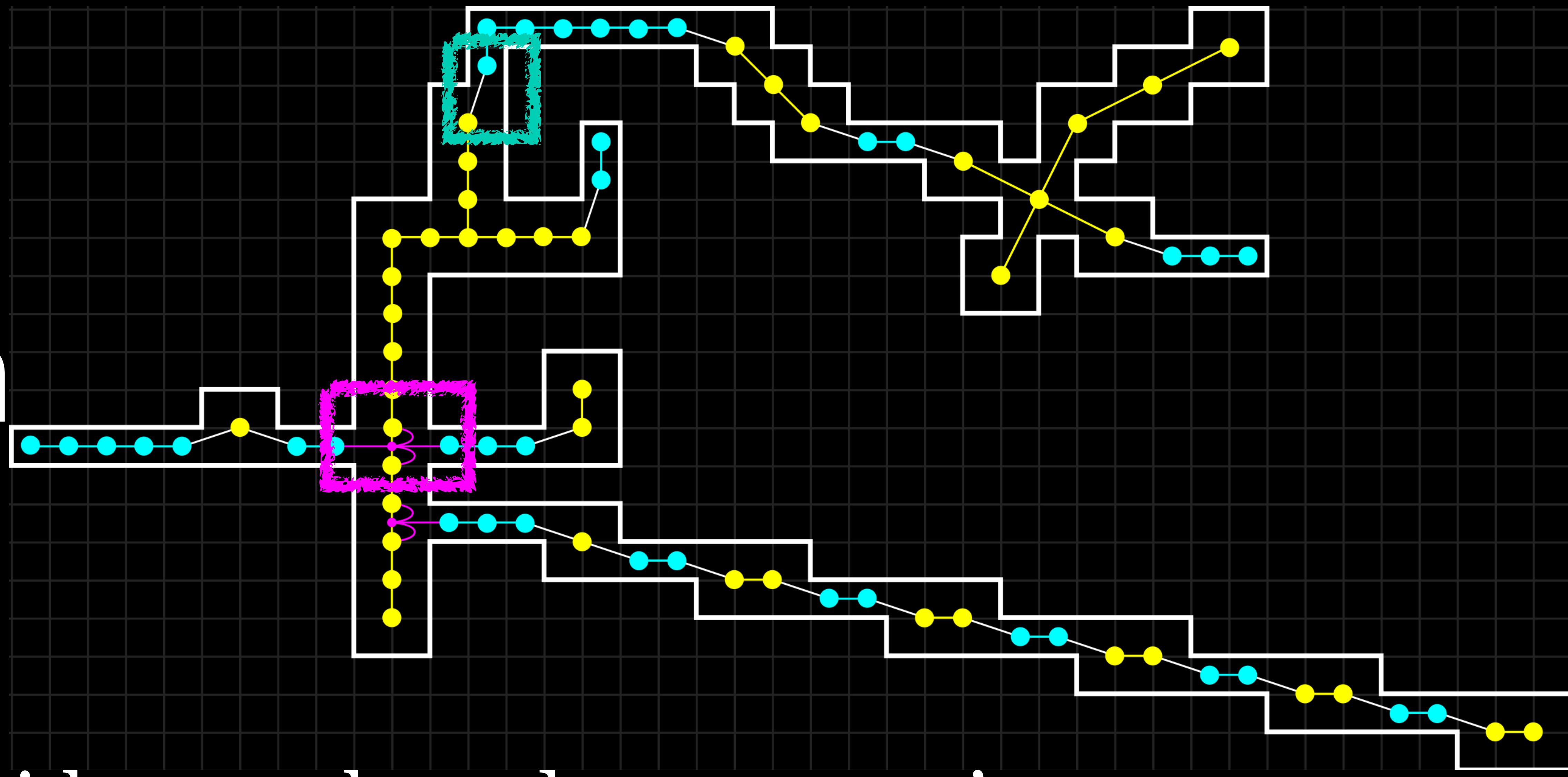

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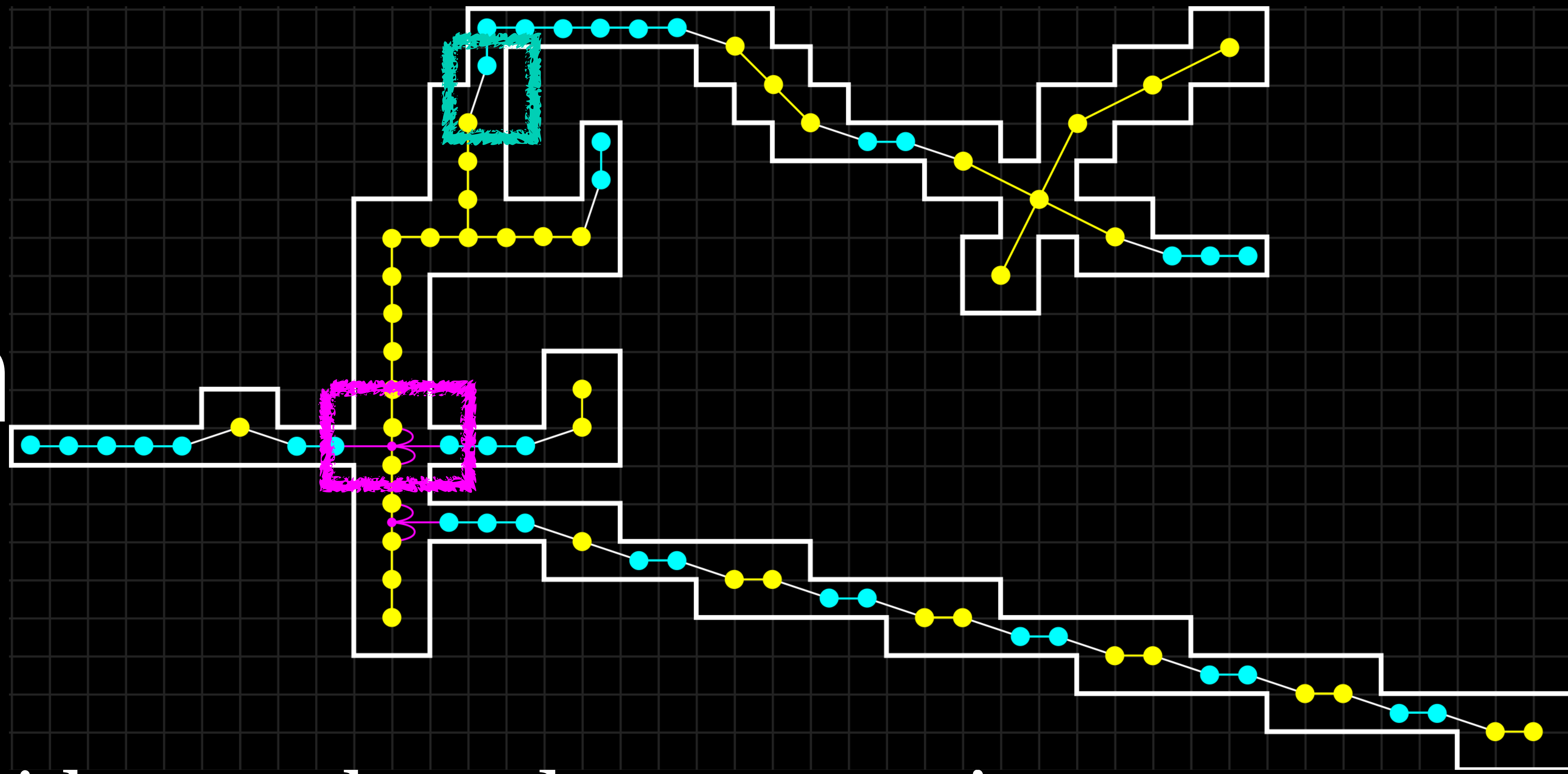

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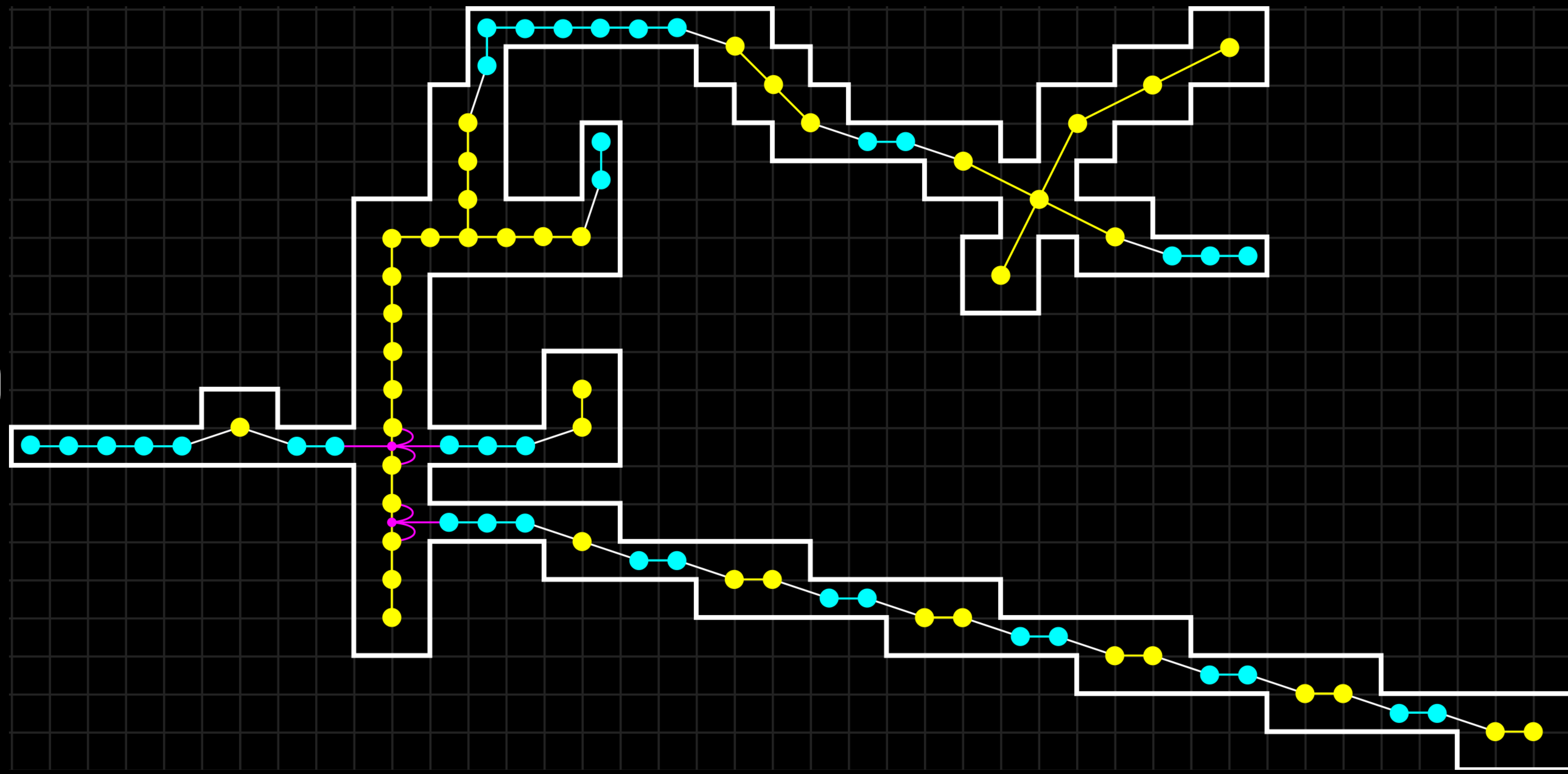
- T graph on vertex set $V=I \cup B \cup X$ and edge set $E=E_I \cup E_B \cup E_{con}$
- When we connect T_I and T_B no cyles are created ➜ T is a tree and max degree is 4

Associated Squares for the No



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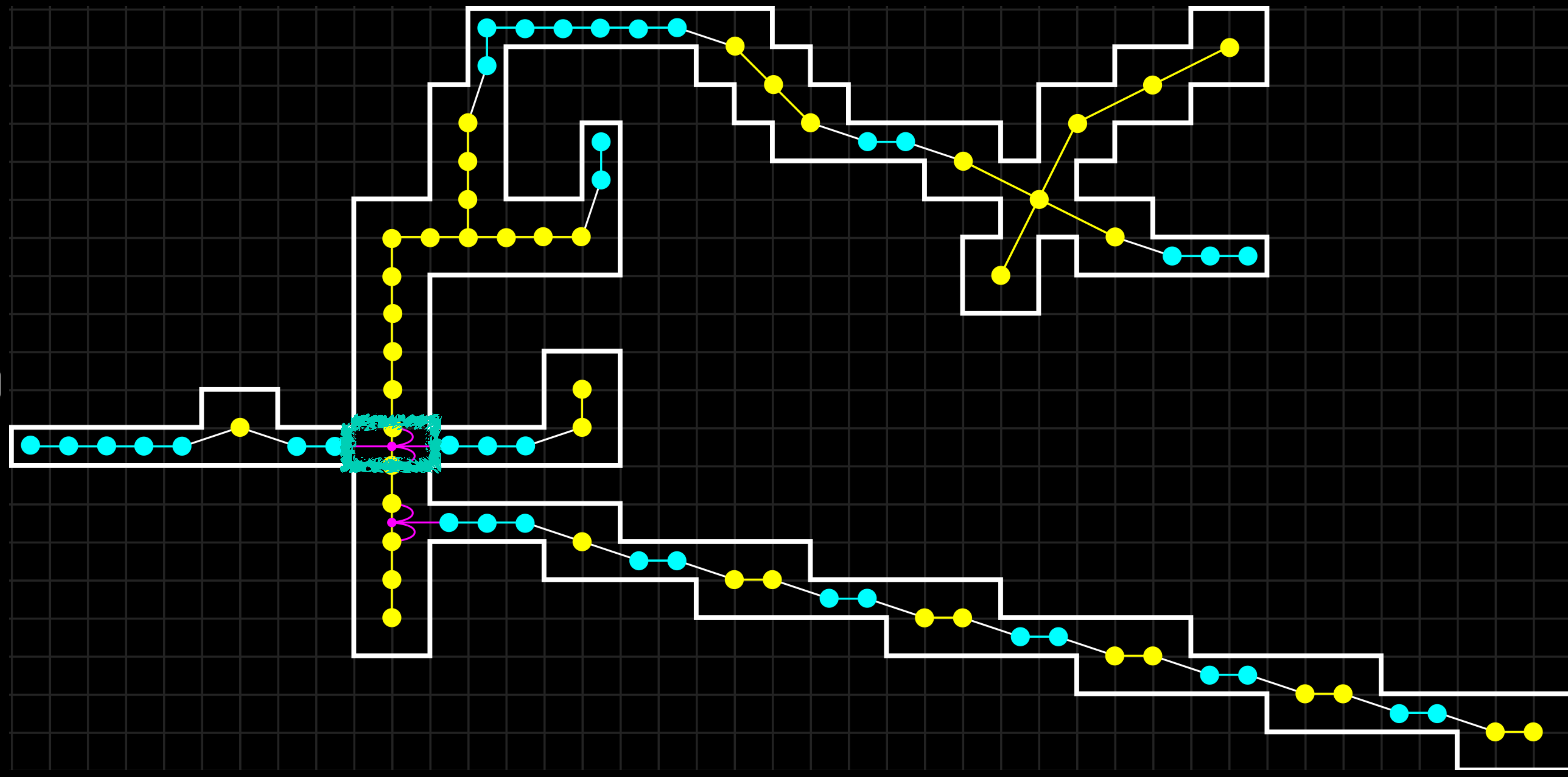

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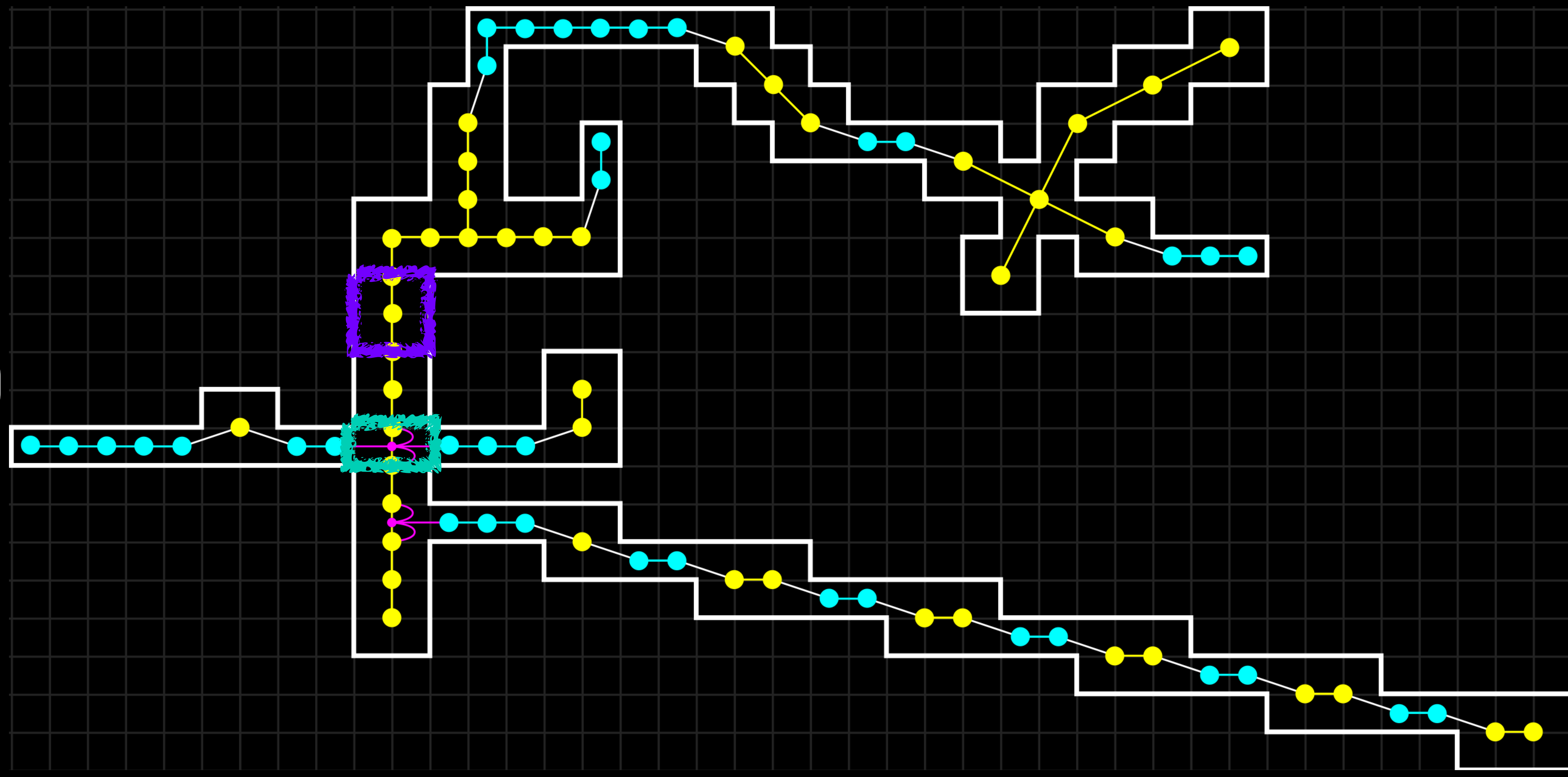

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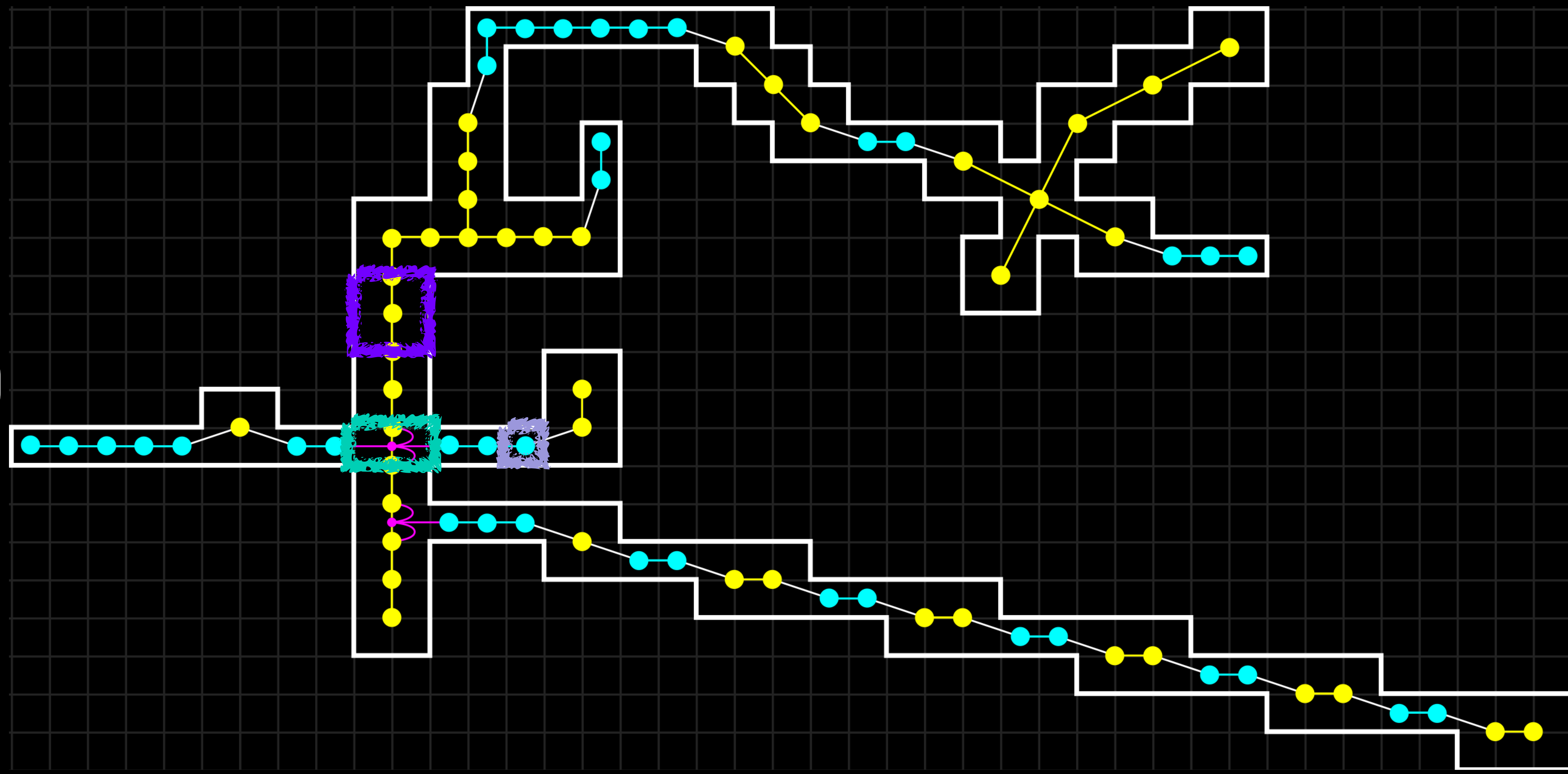

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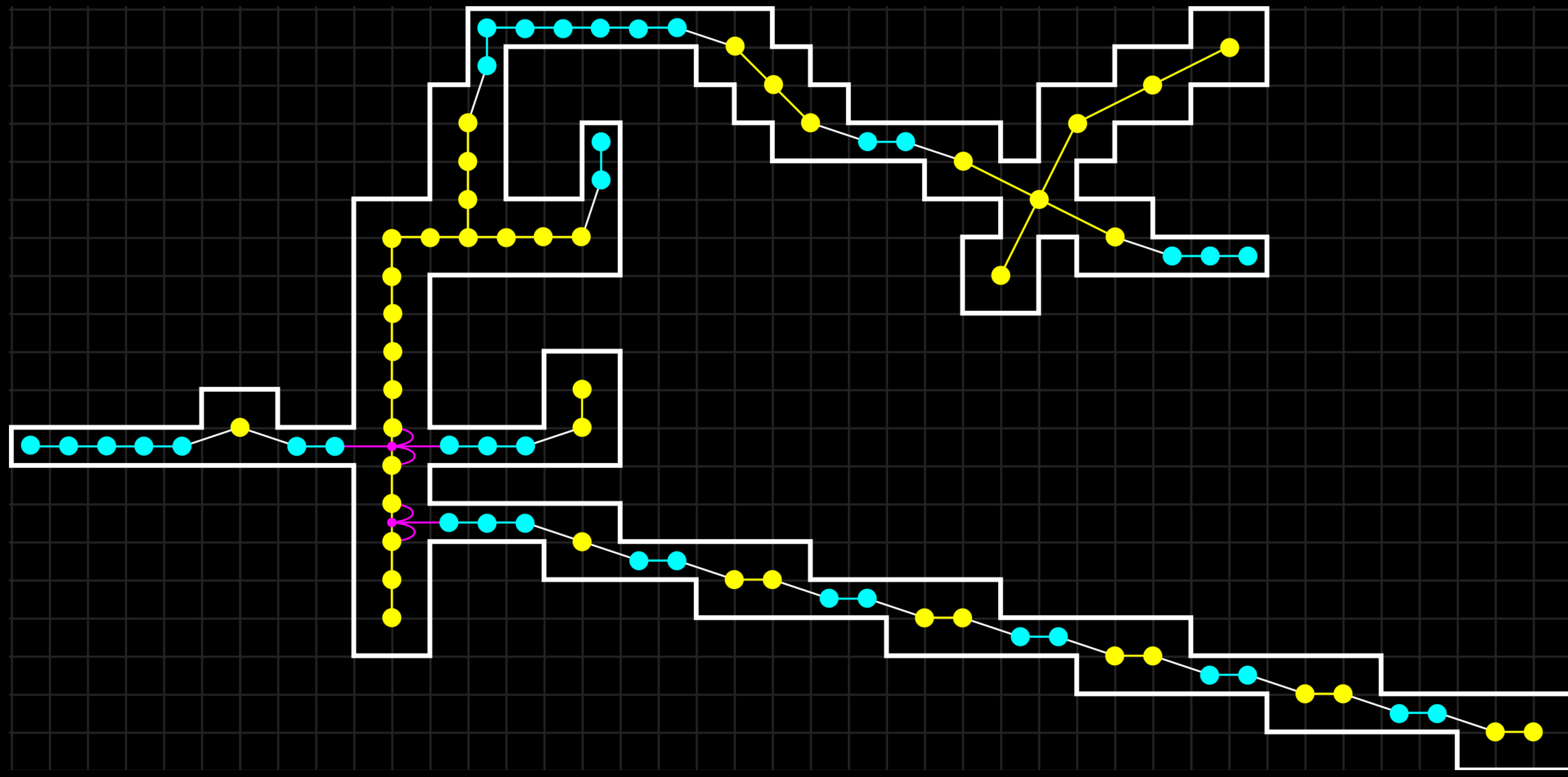
Our Approximation Algorithm



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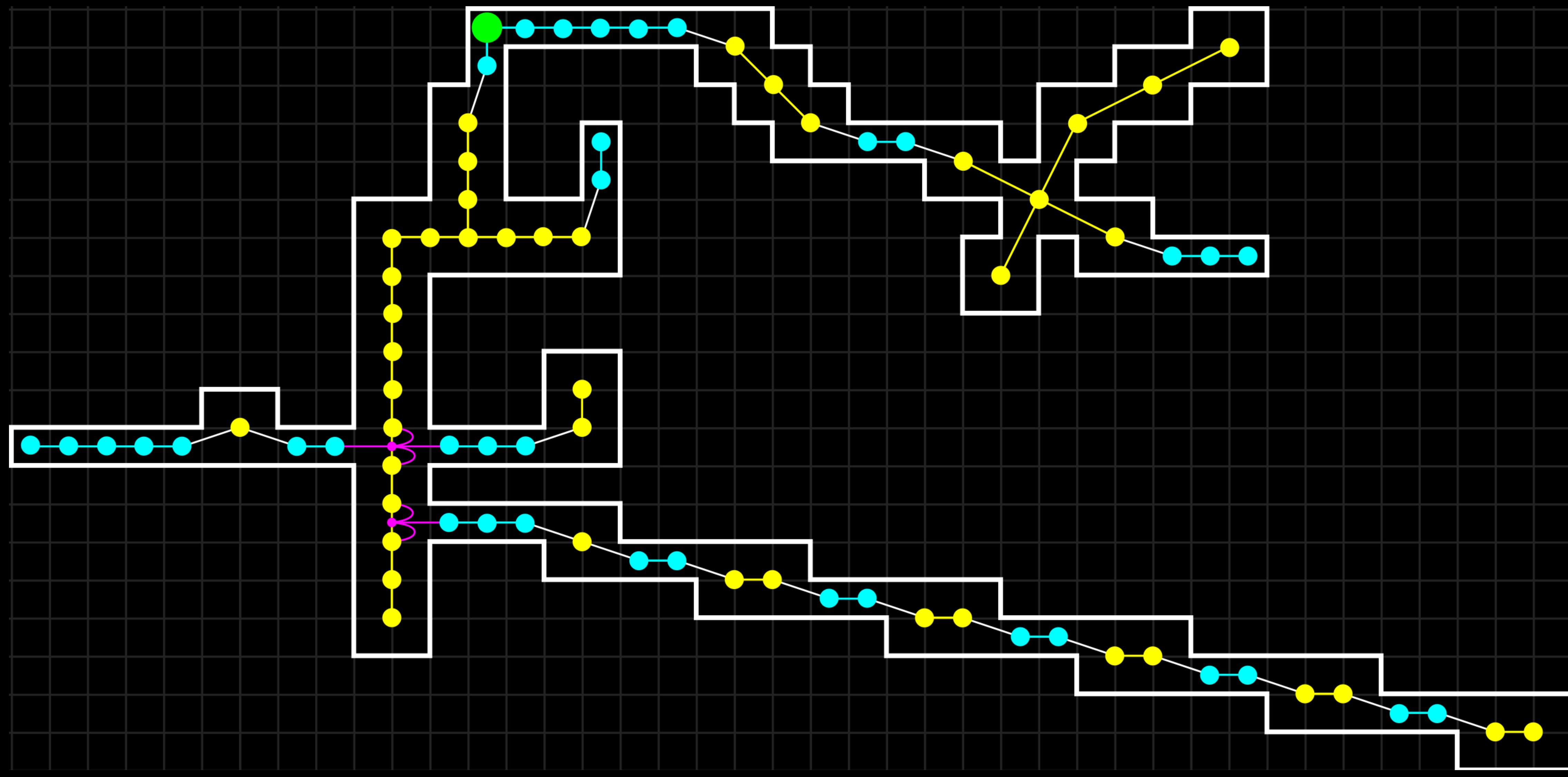

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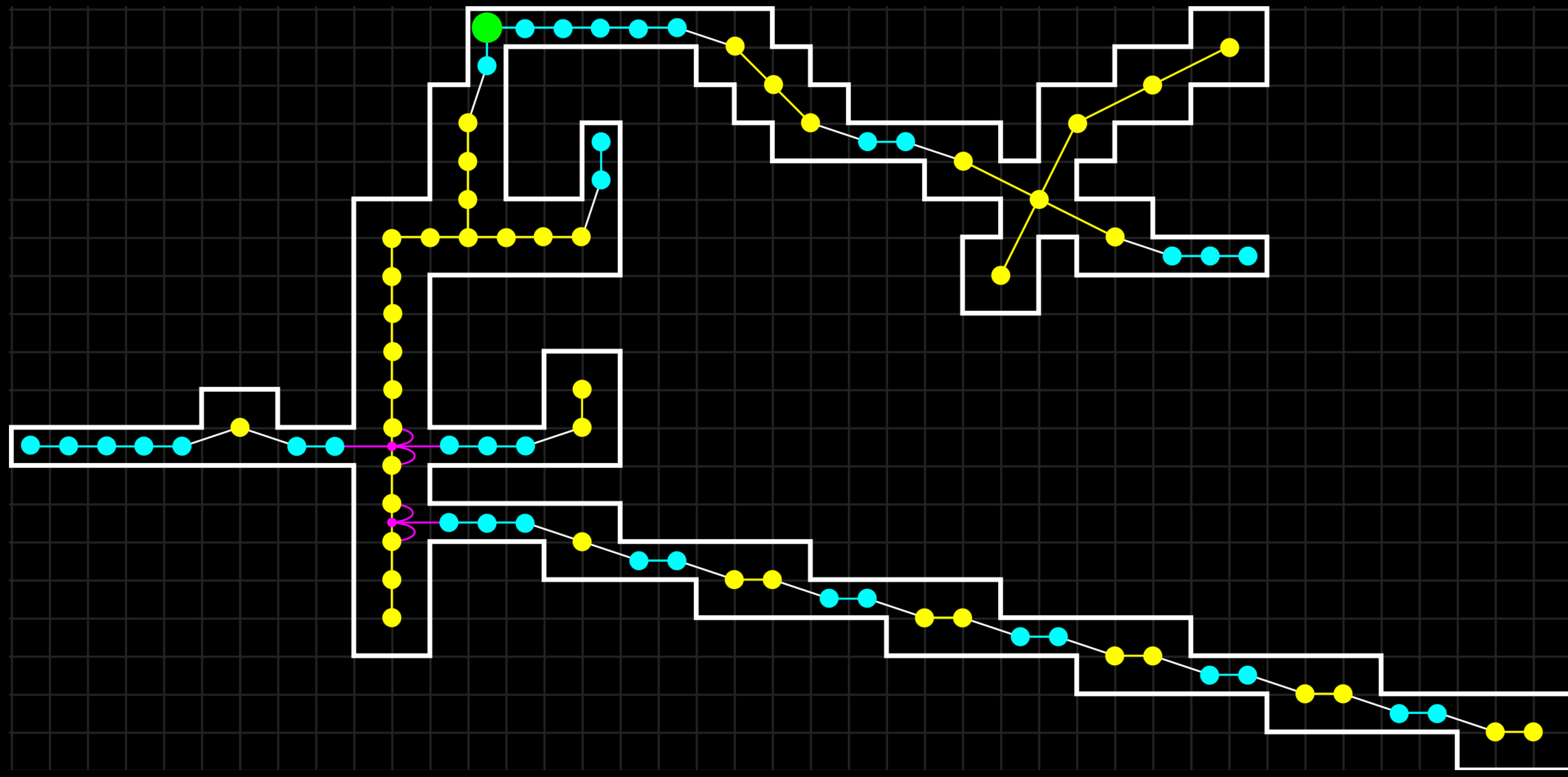

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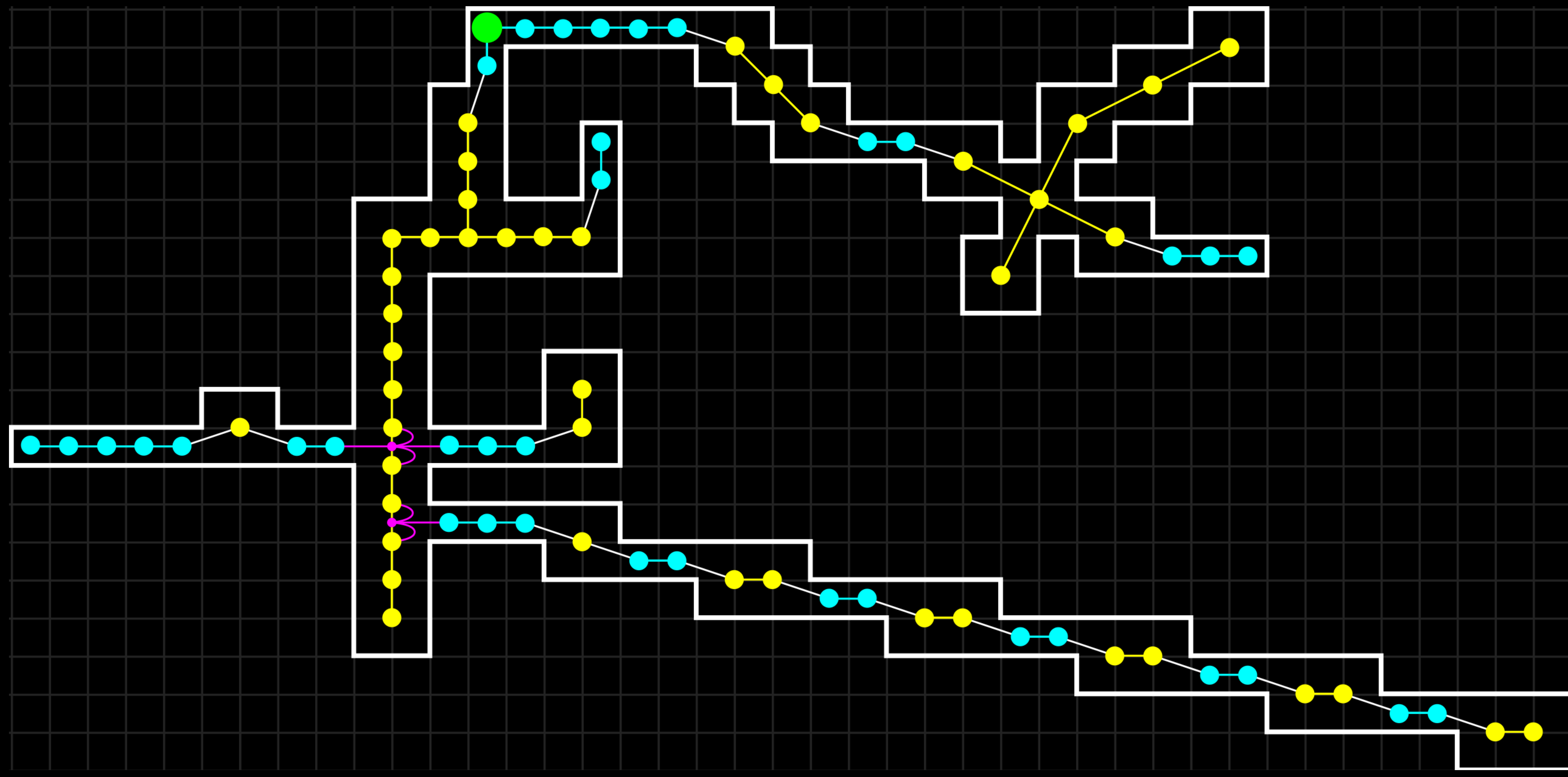

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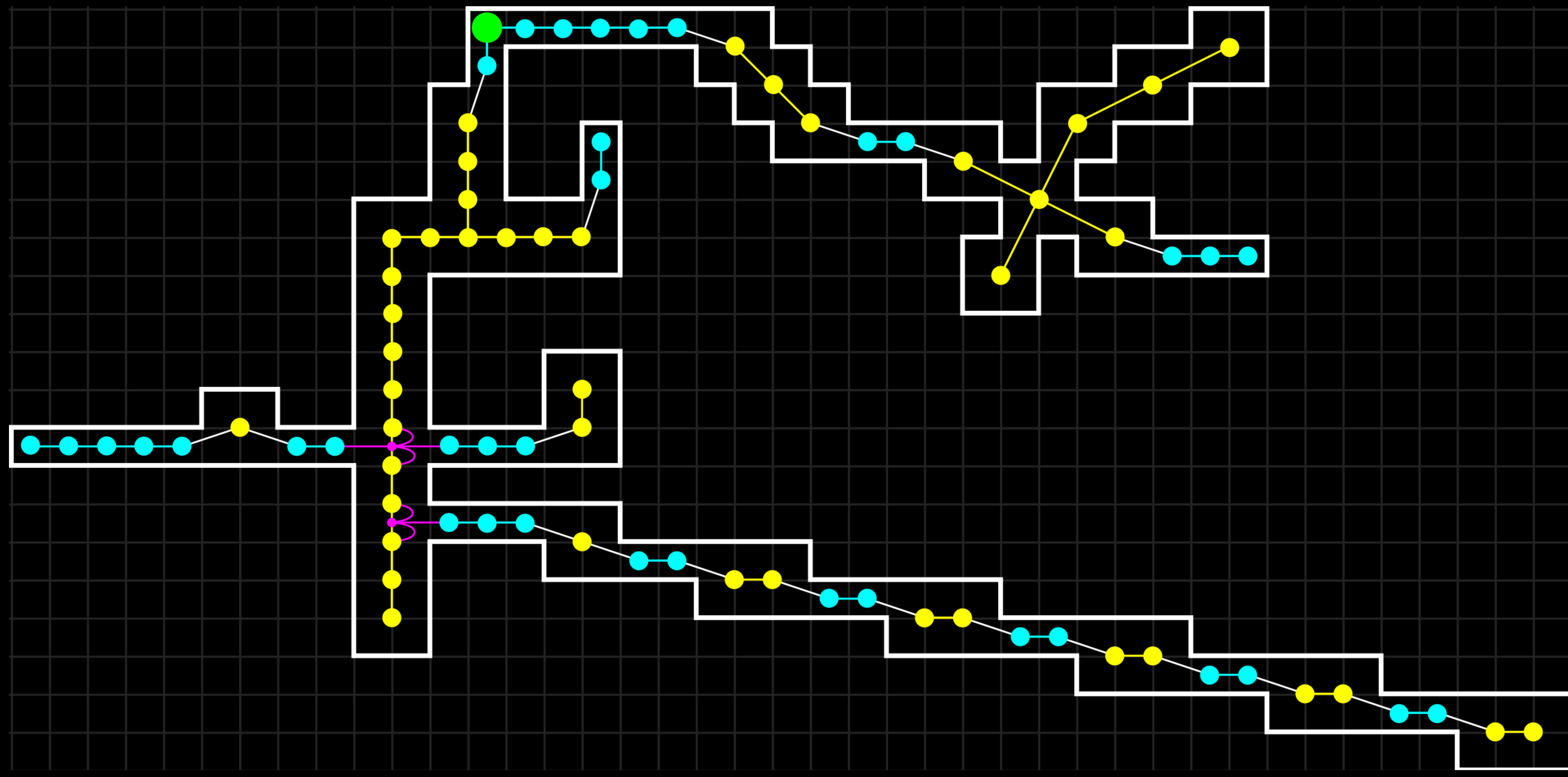

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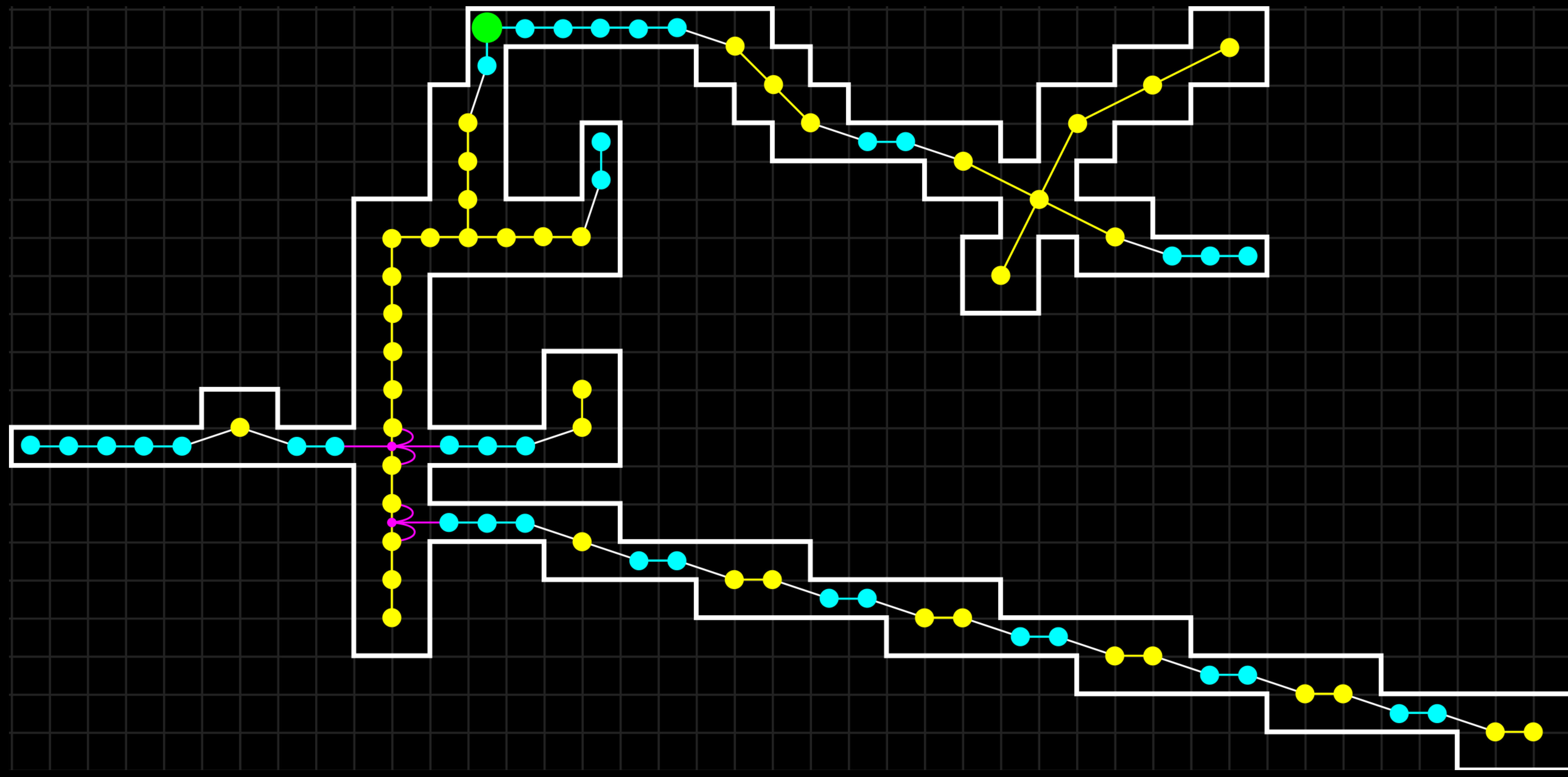

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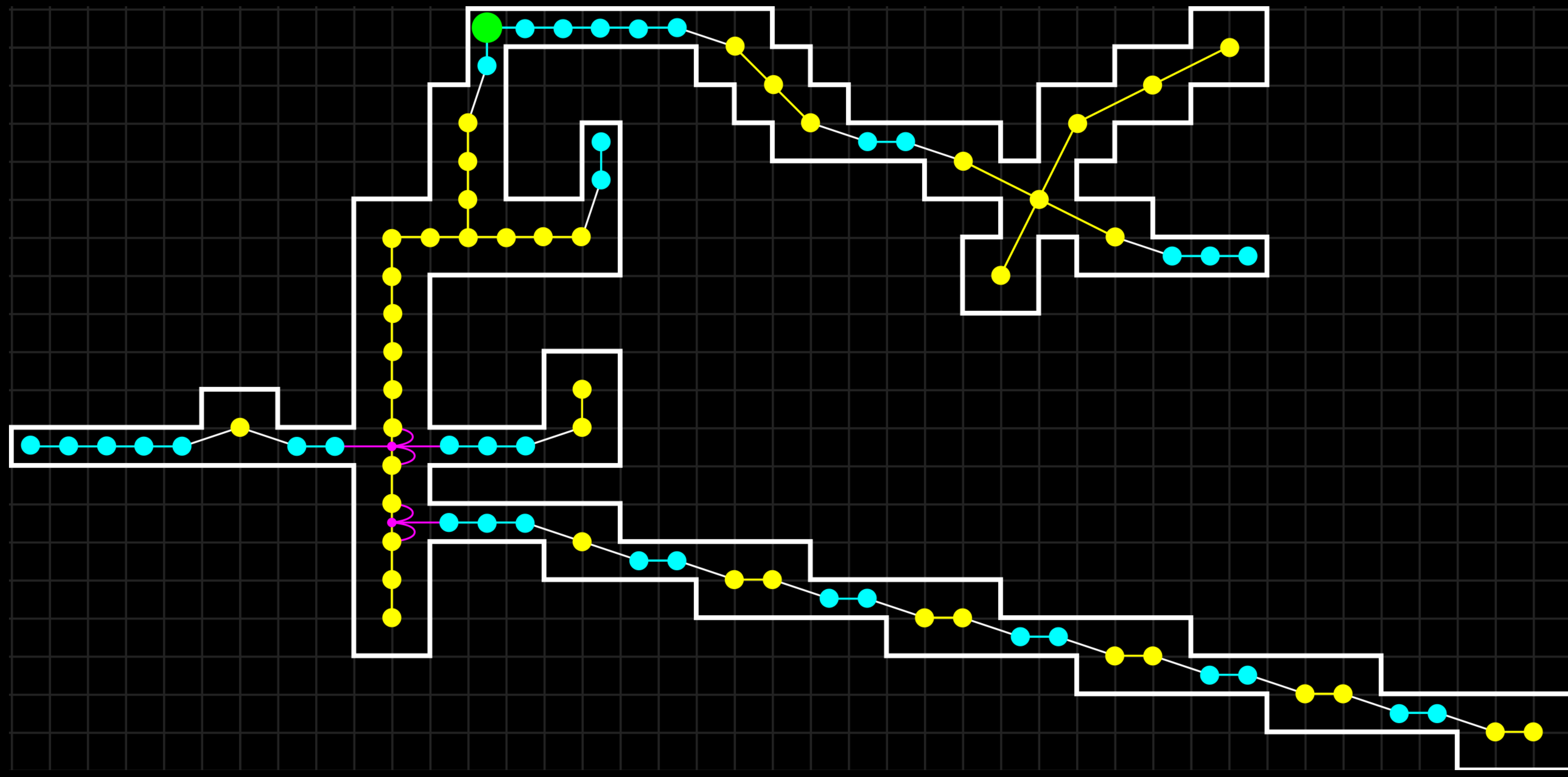

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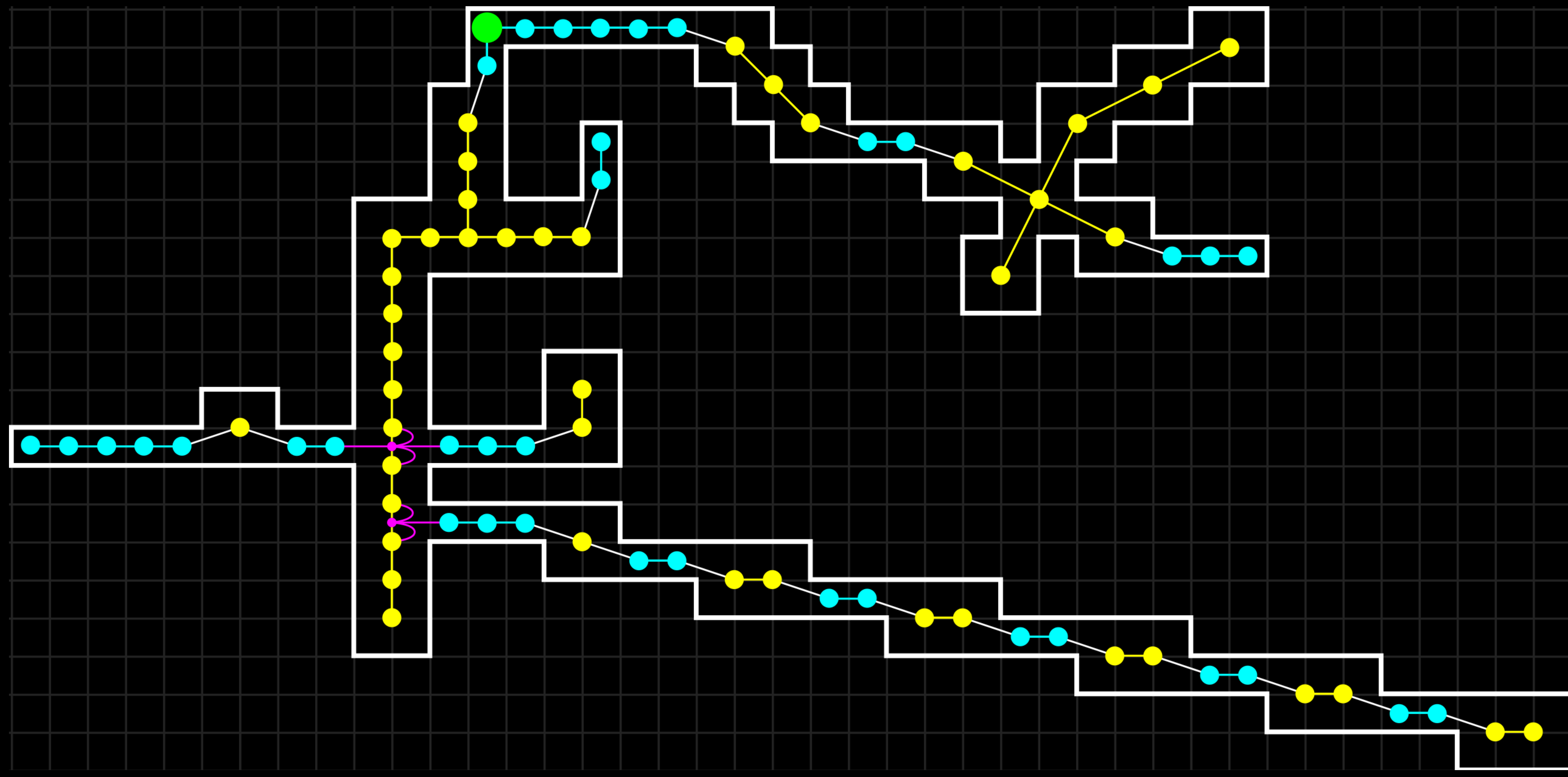

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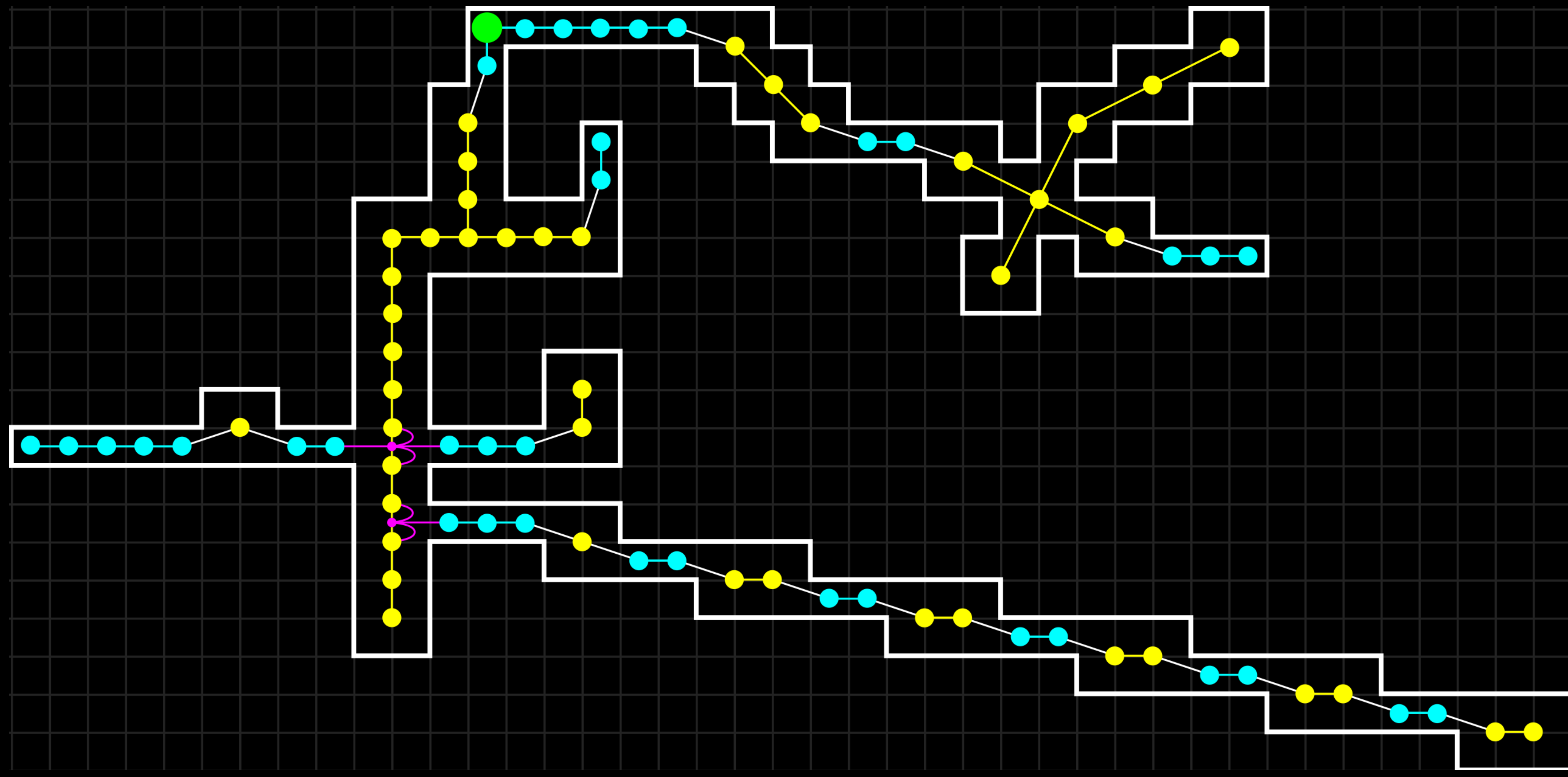

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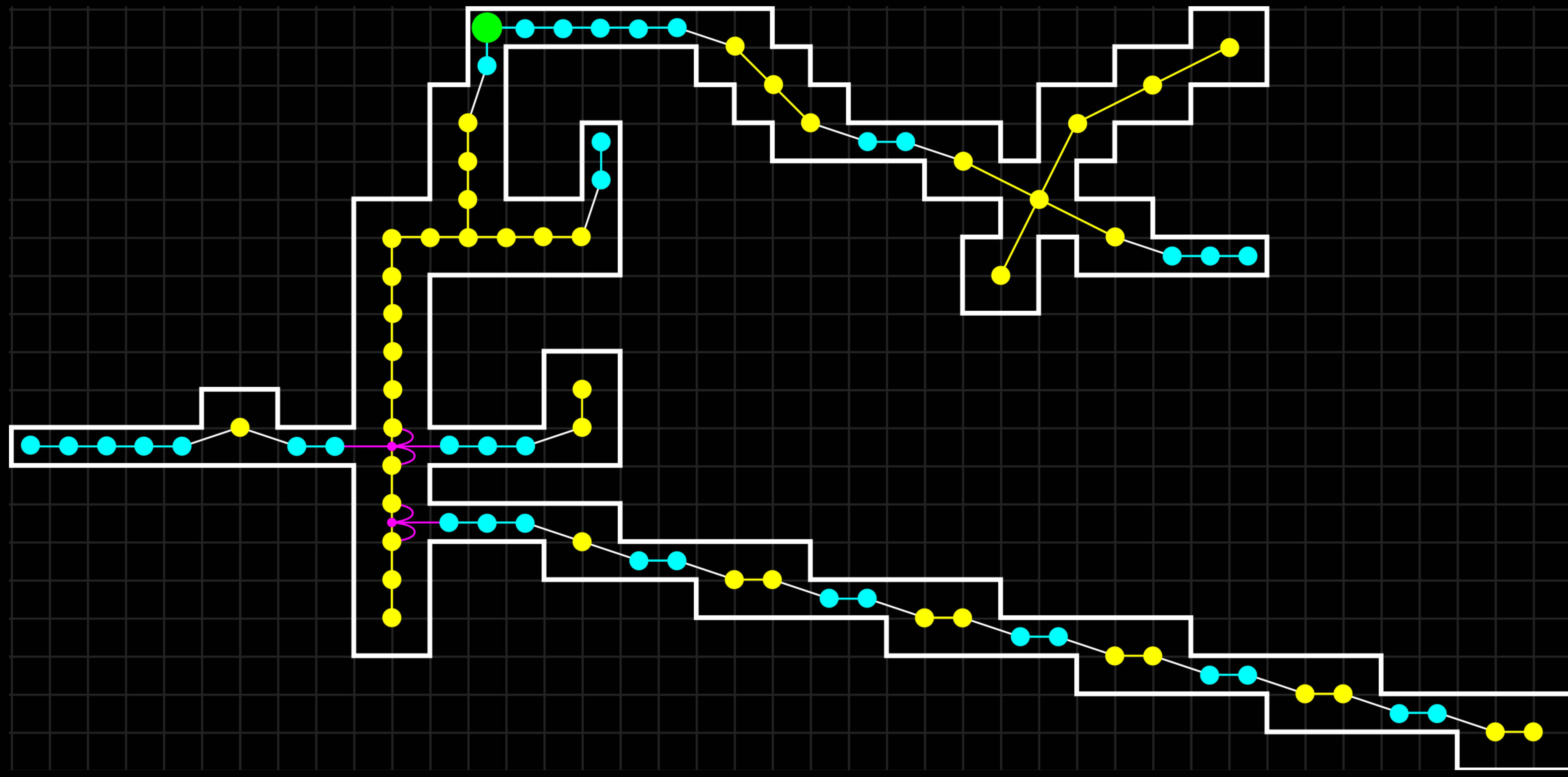

Our Approximation Algorithm

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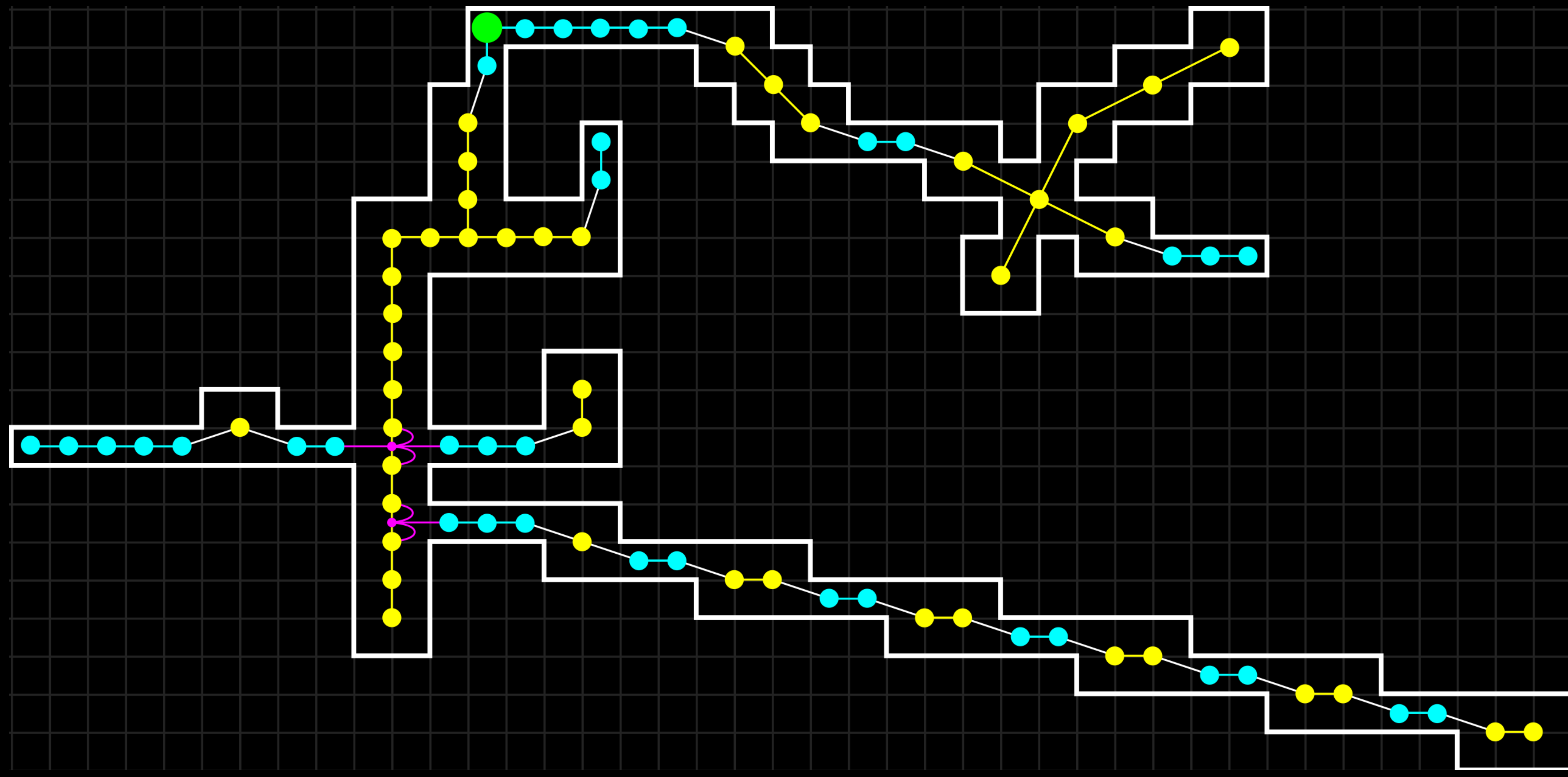

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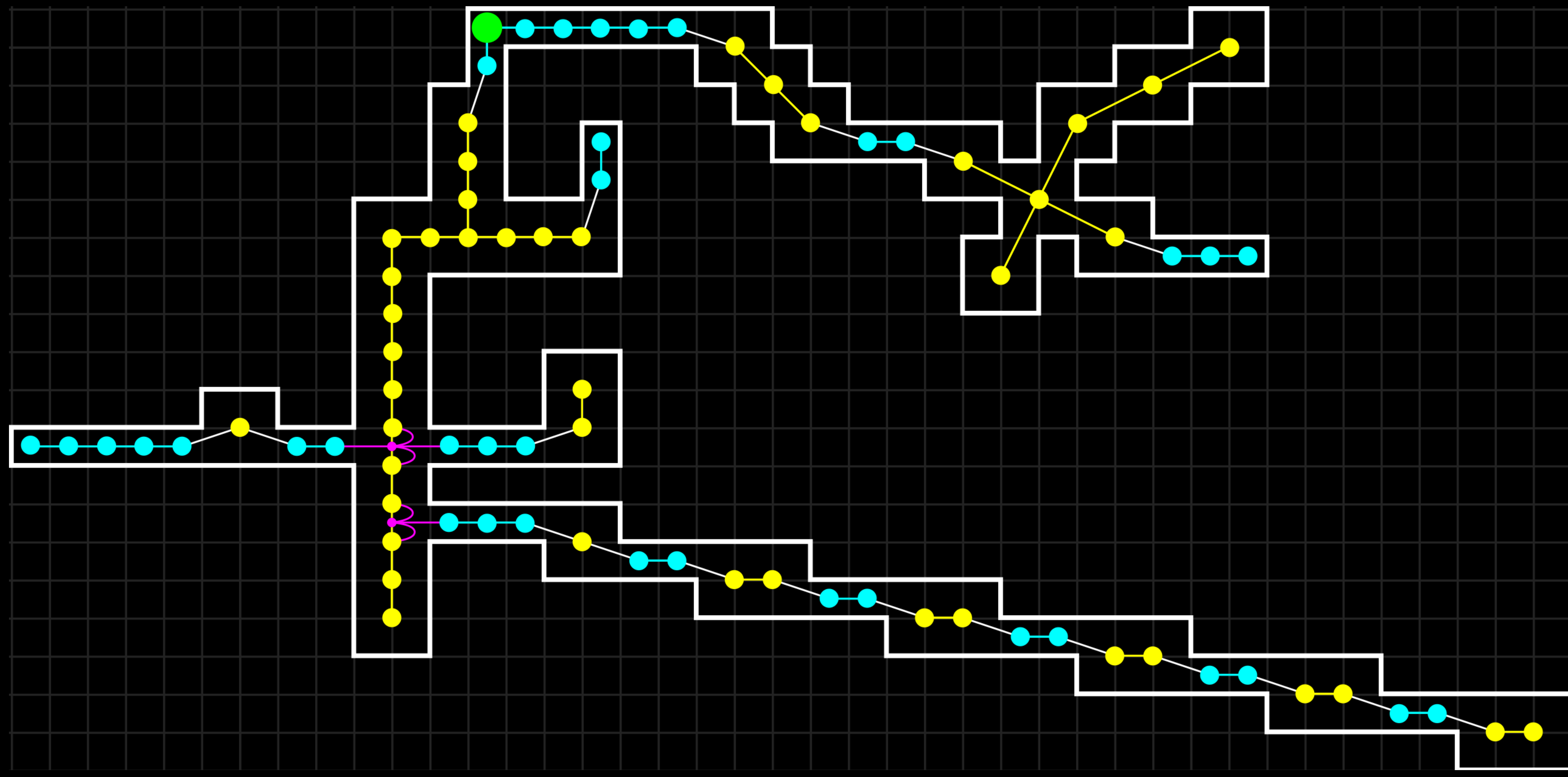

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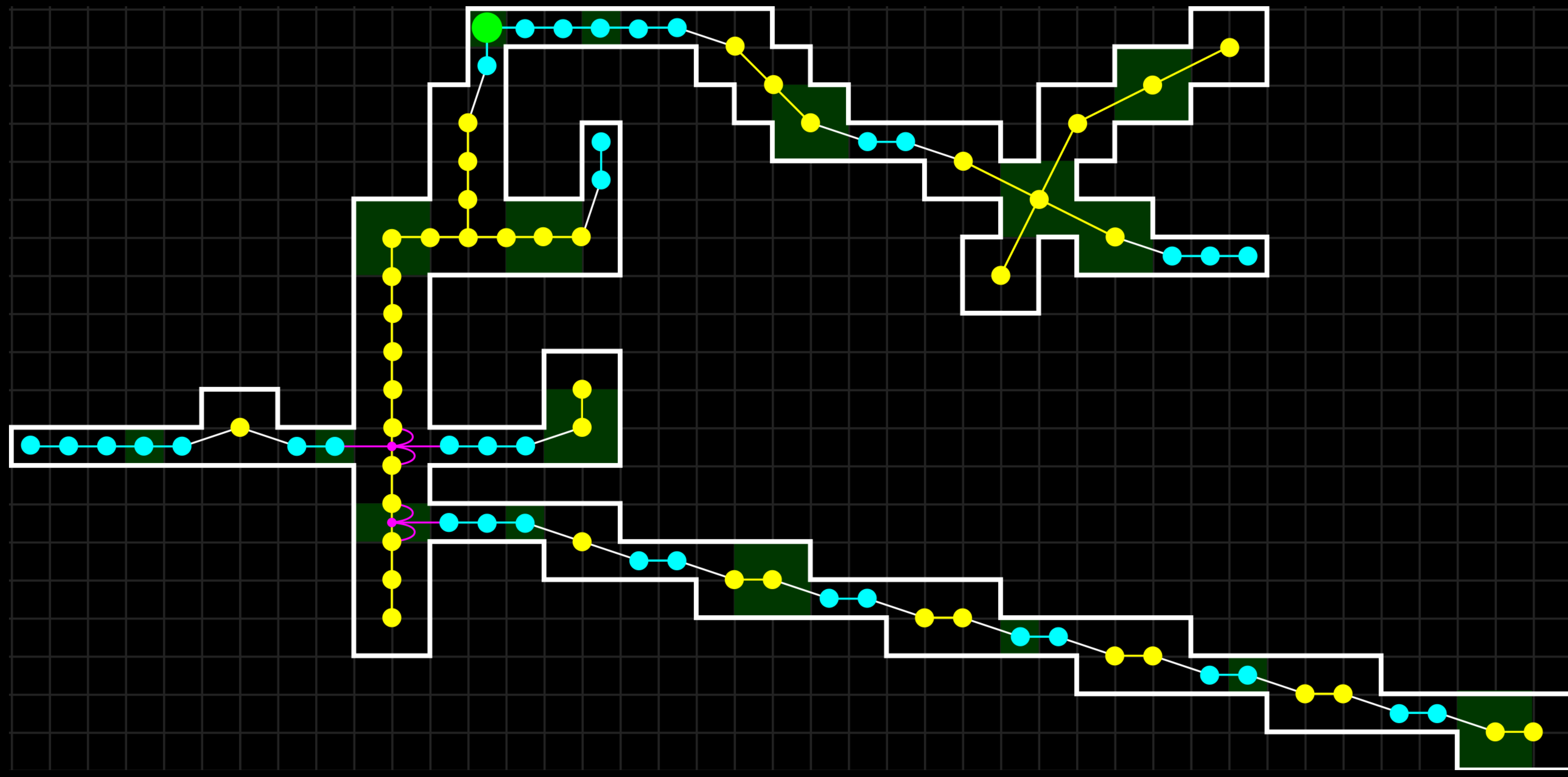
D for k=3



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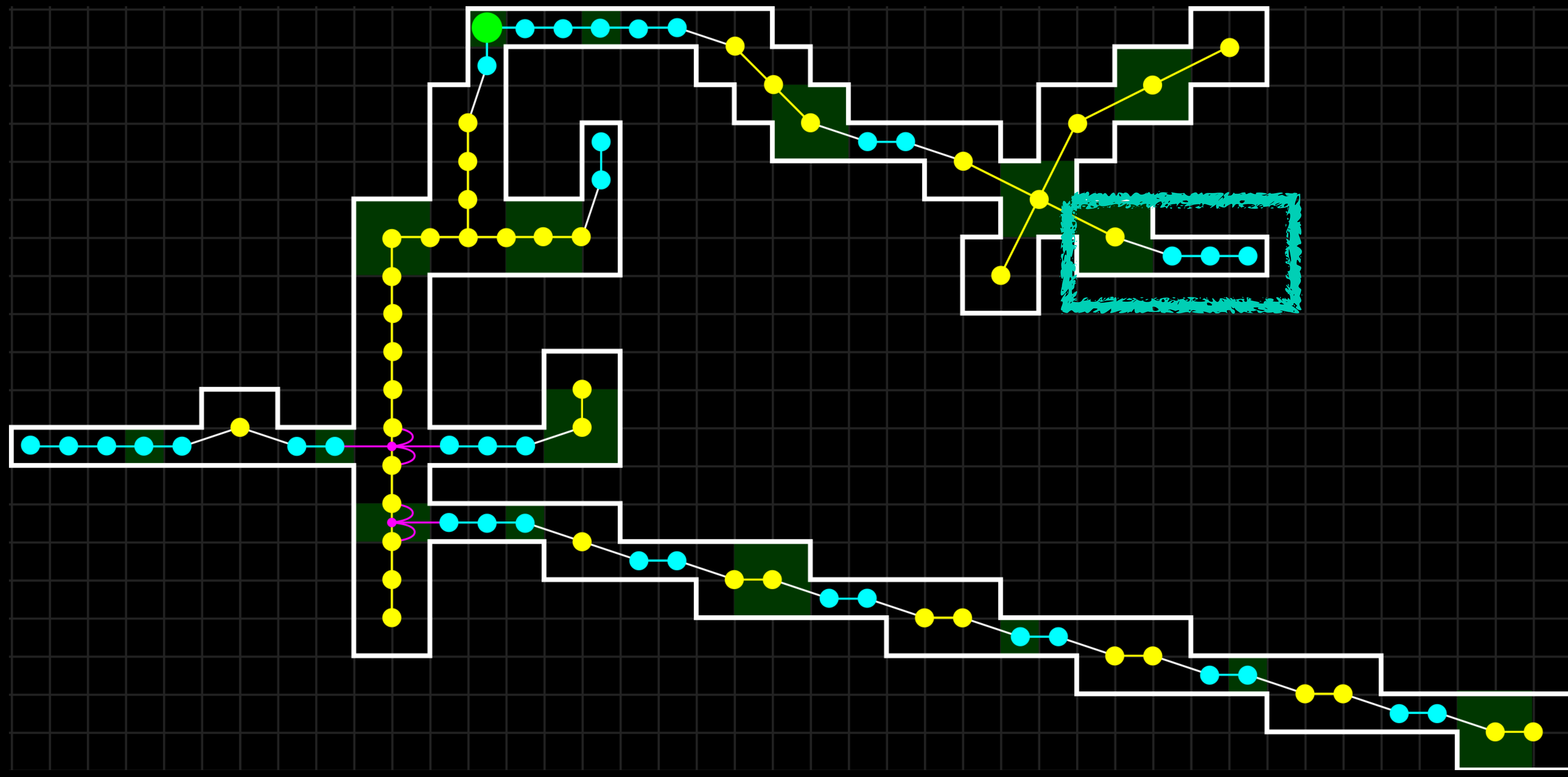

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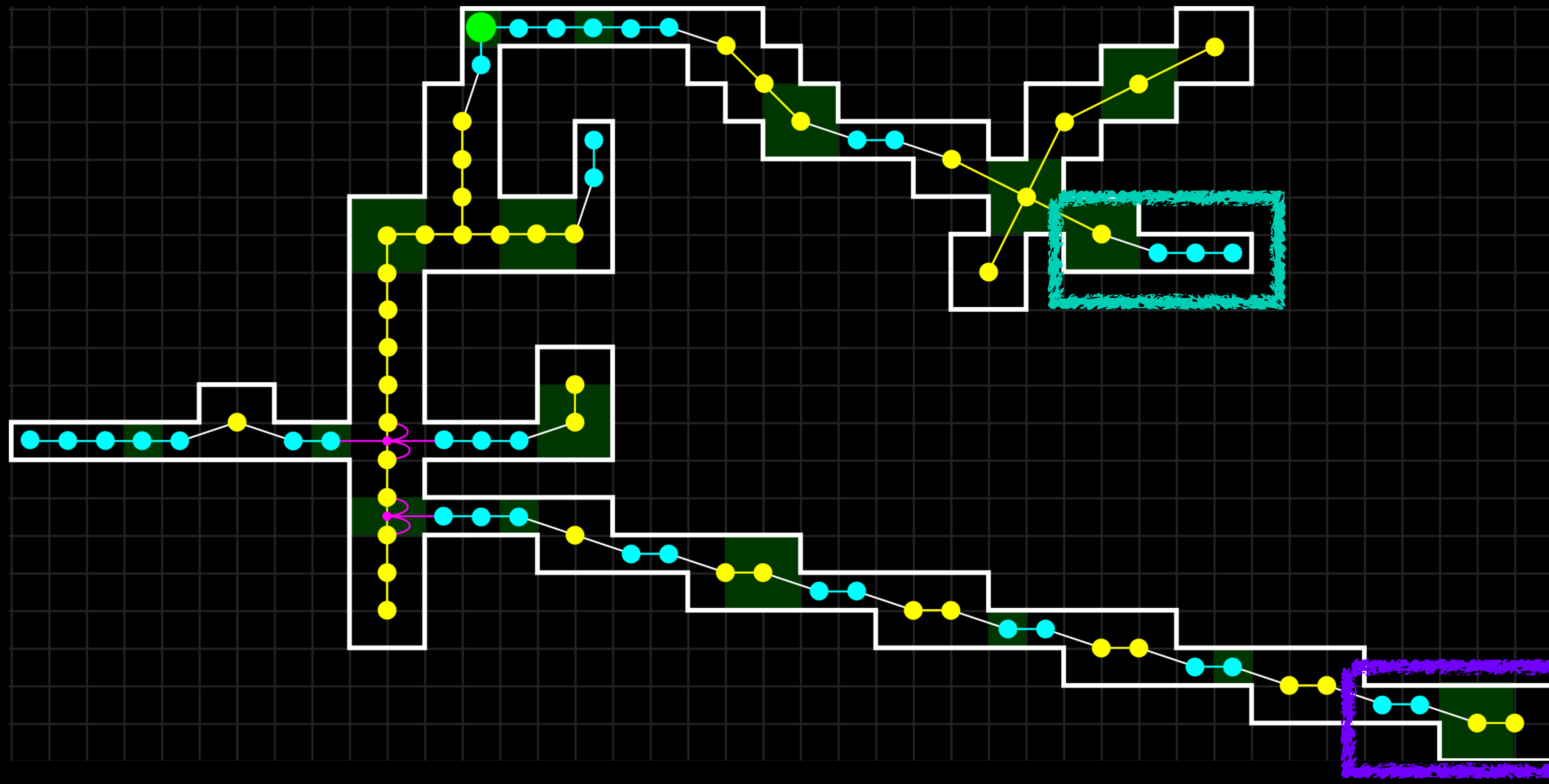

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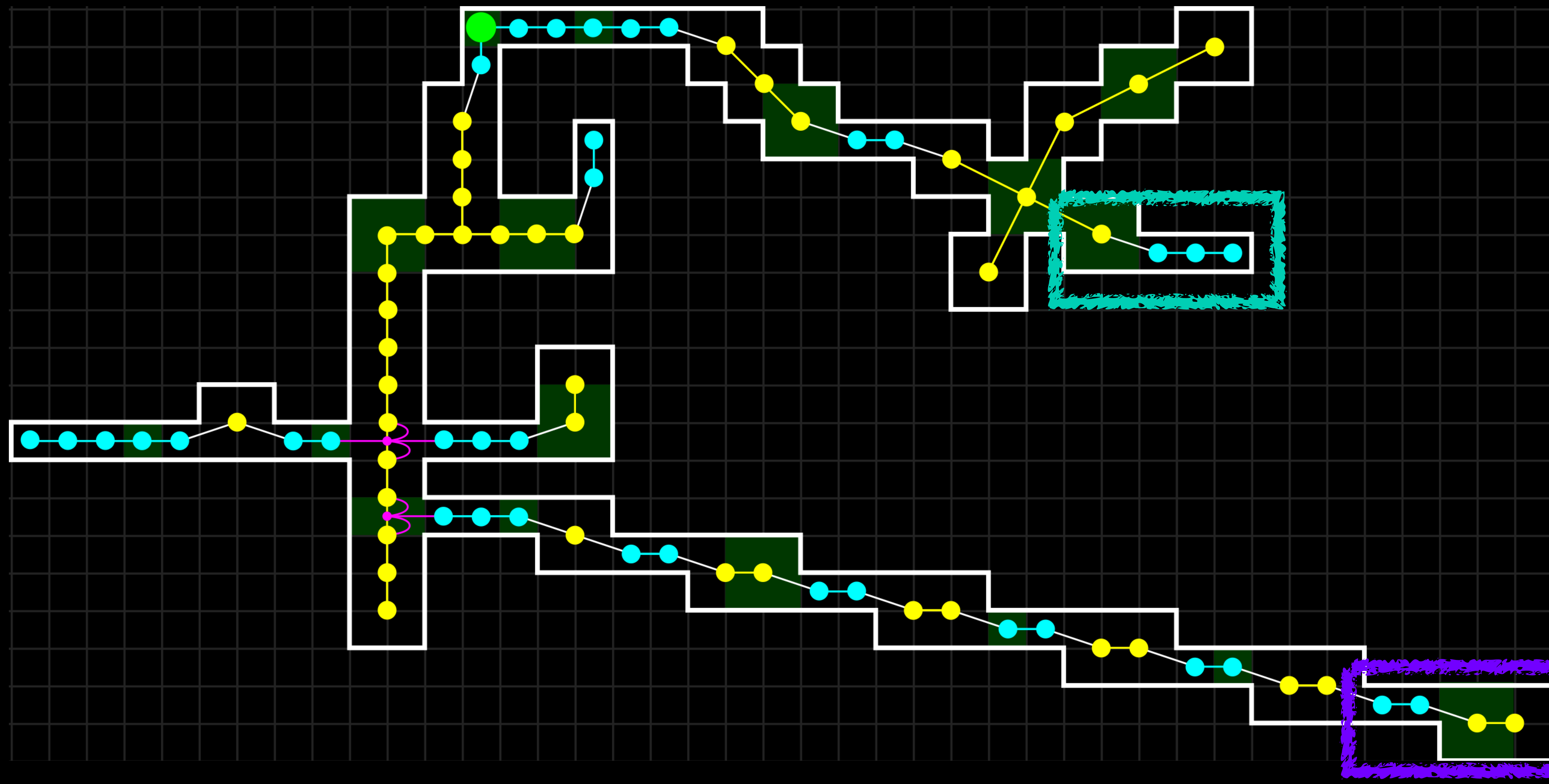

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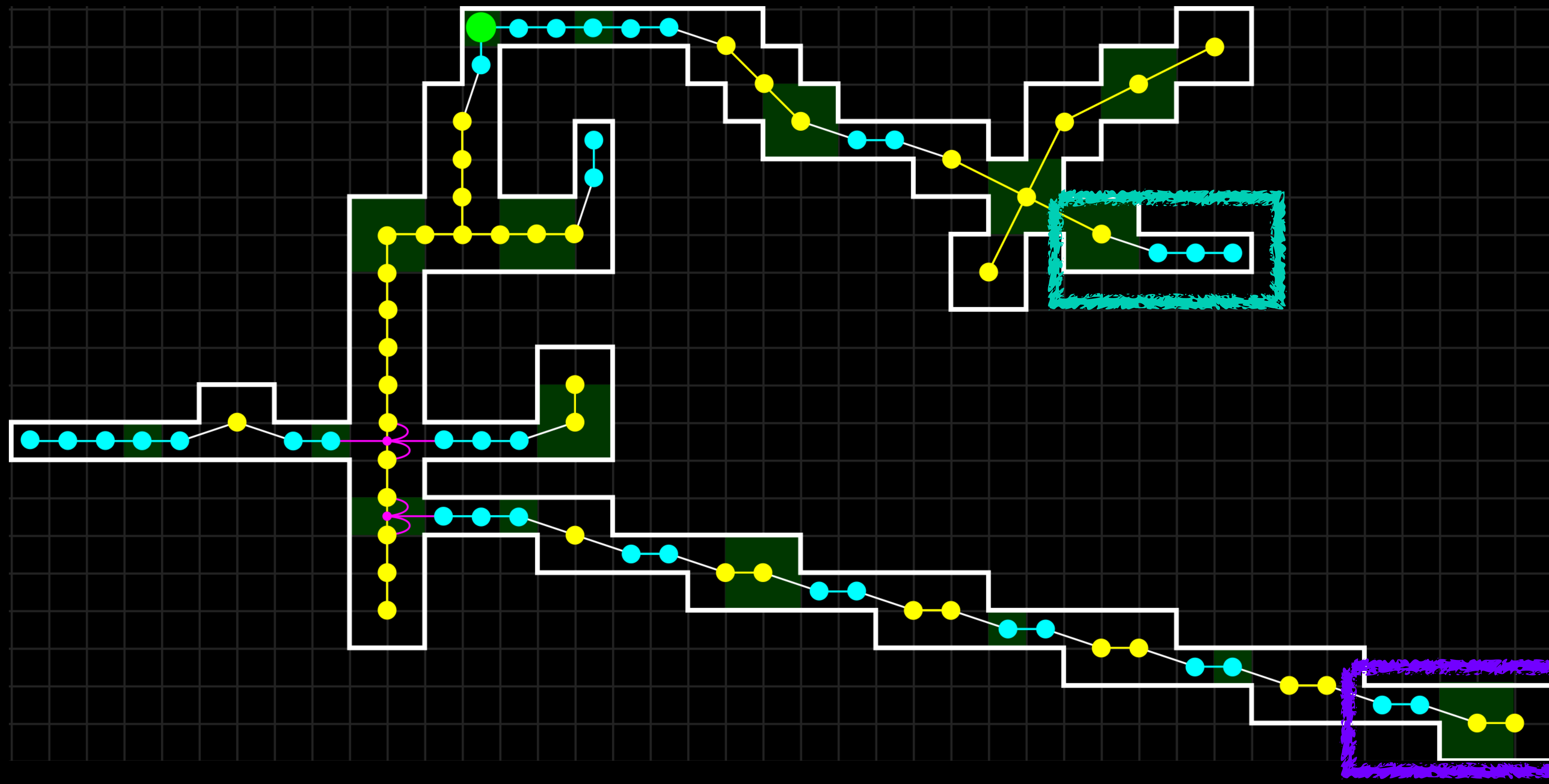

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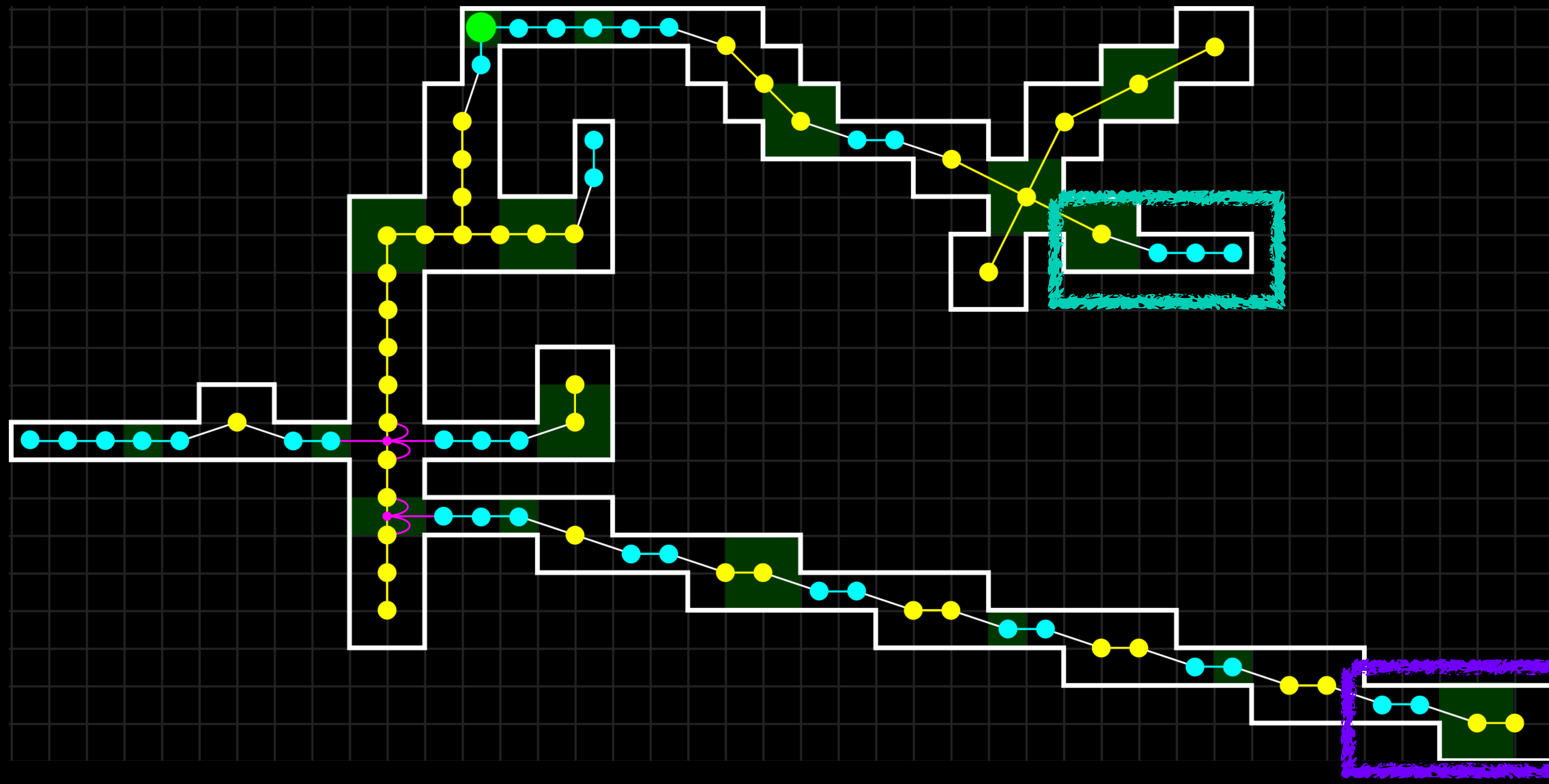

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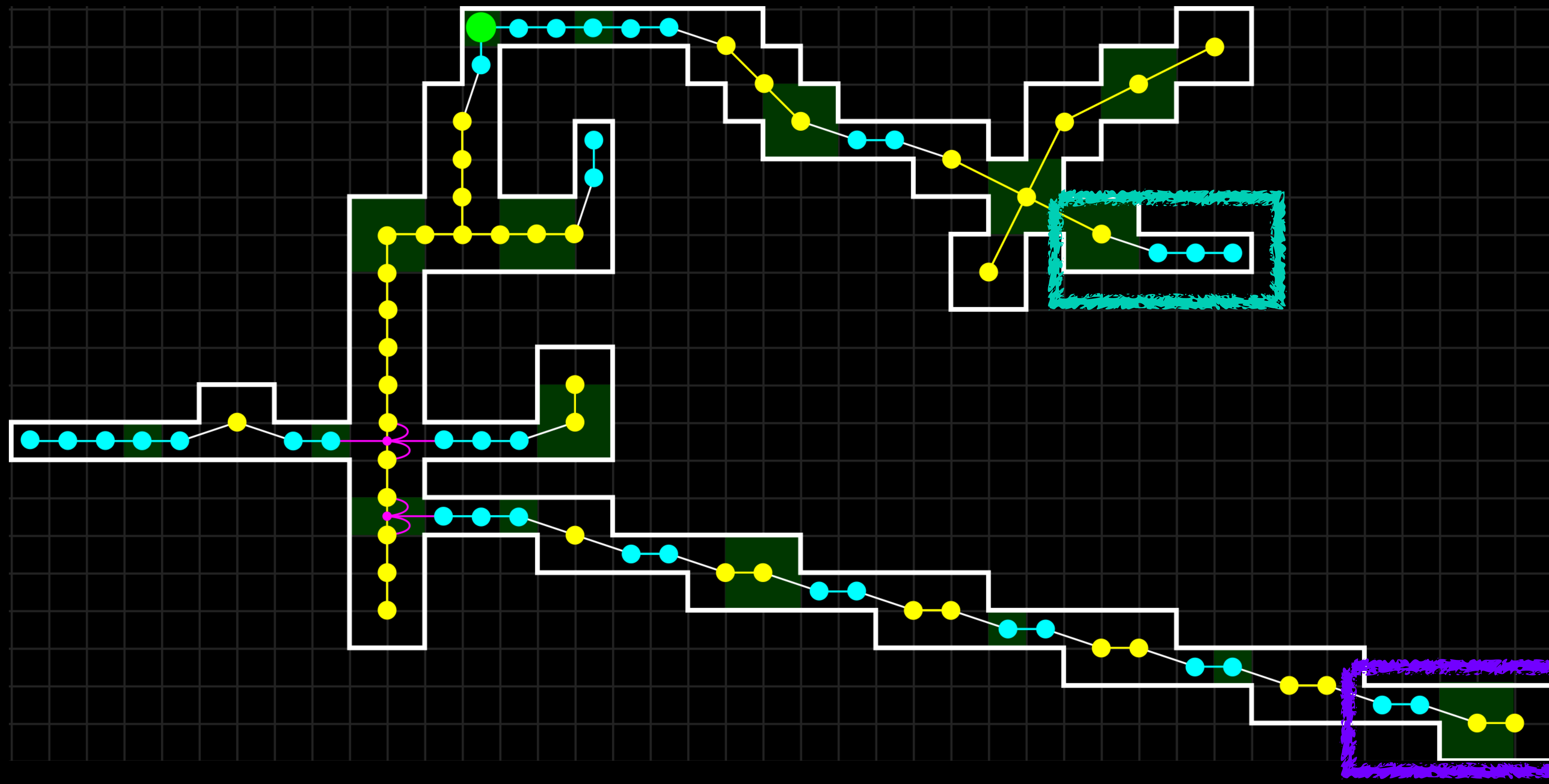

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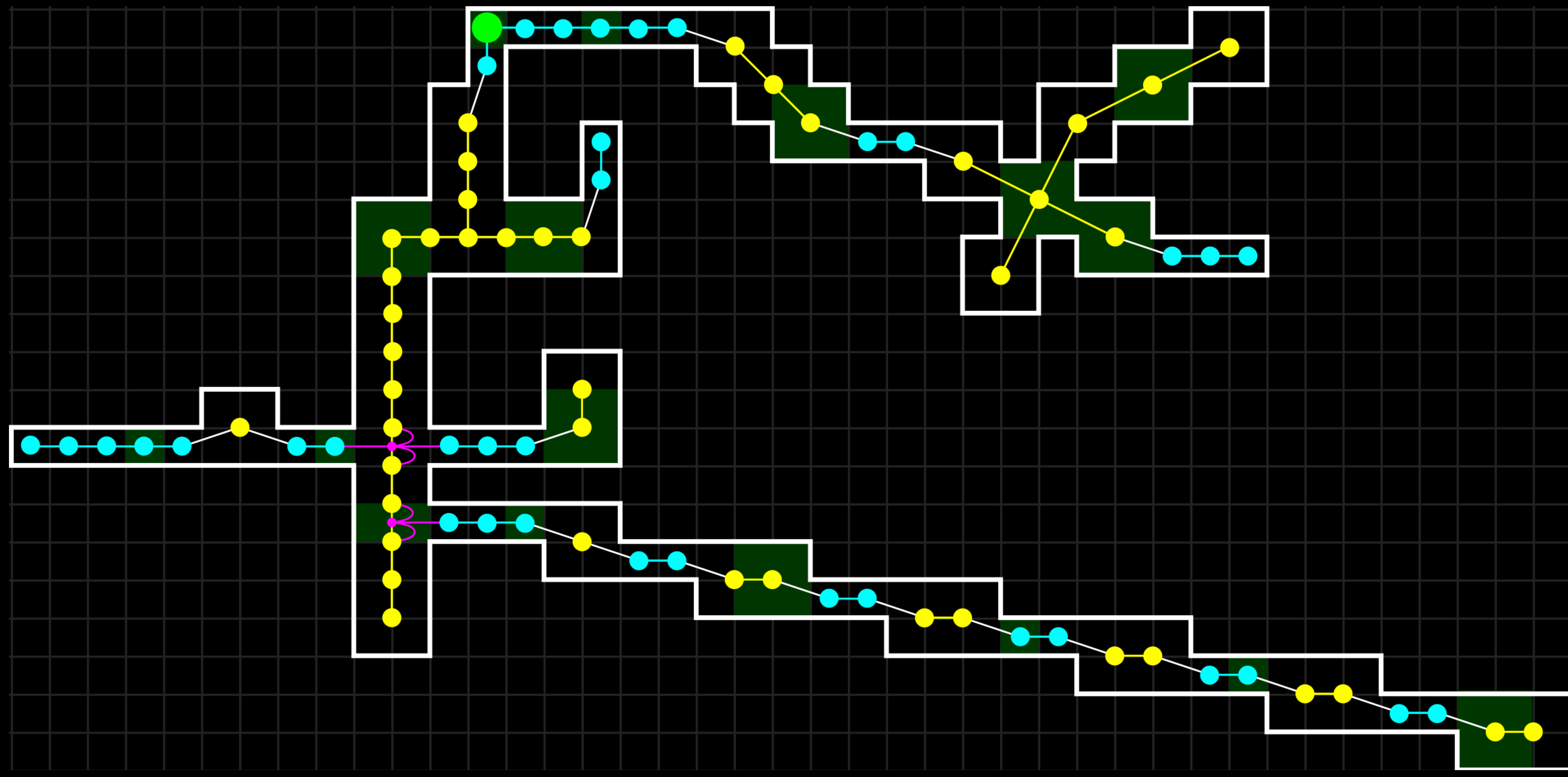
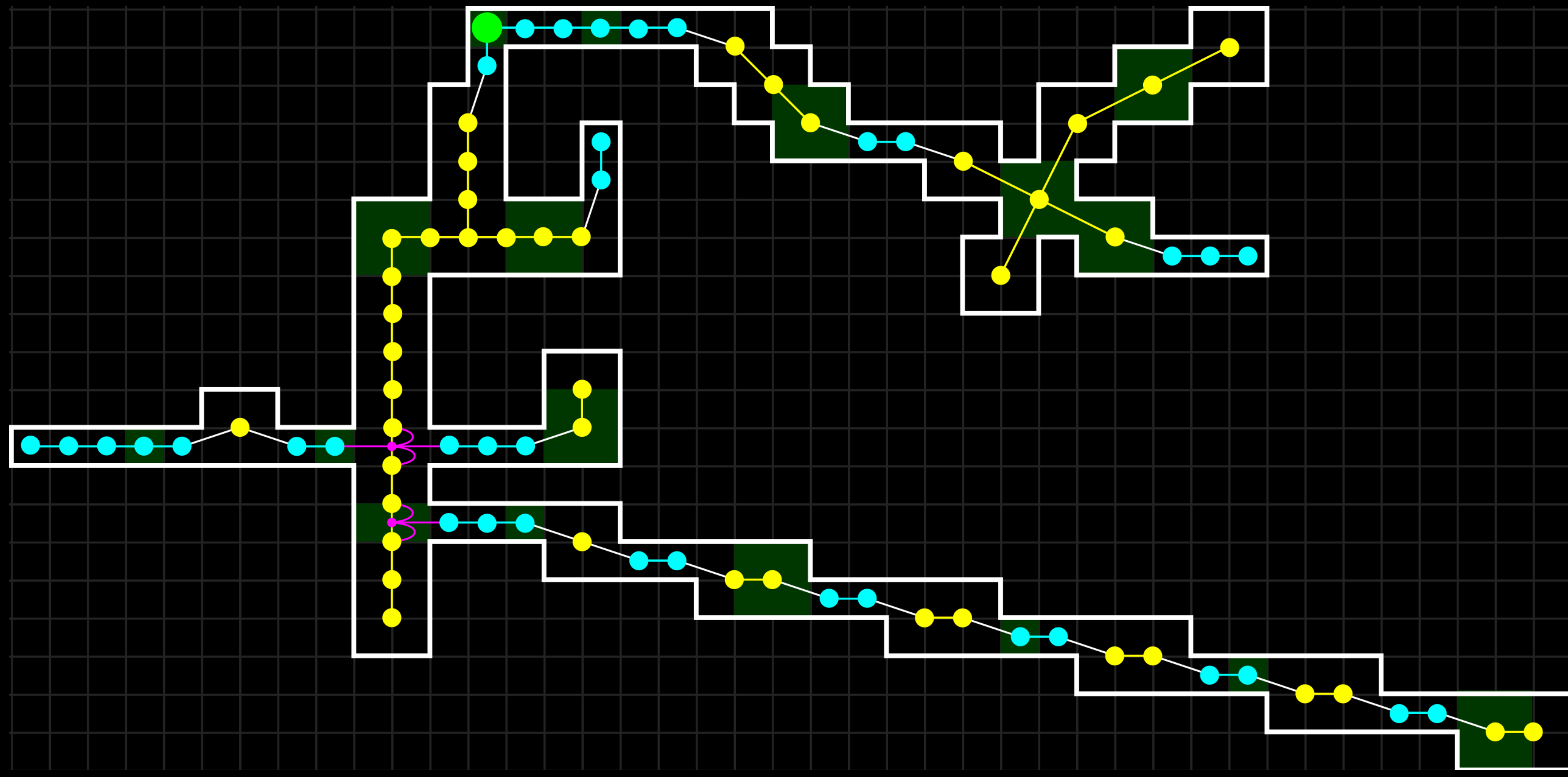


Yields Approx Factor of 4!

Theorem 4: There is a linear-time algorithm computing a 4-approximation for the M*k*GP in simple 2-thin polyominoes.



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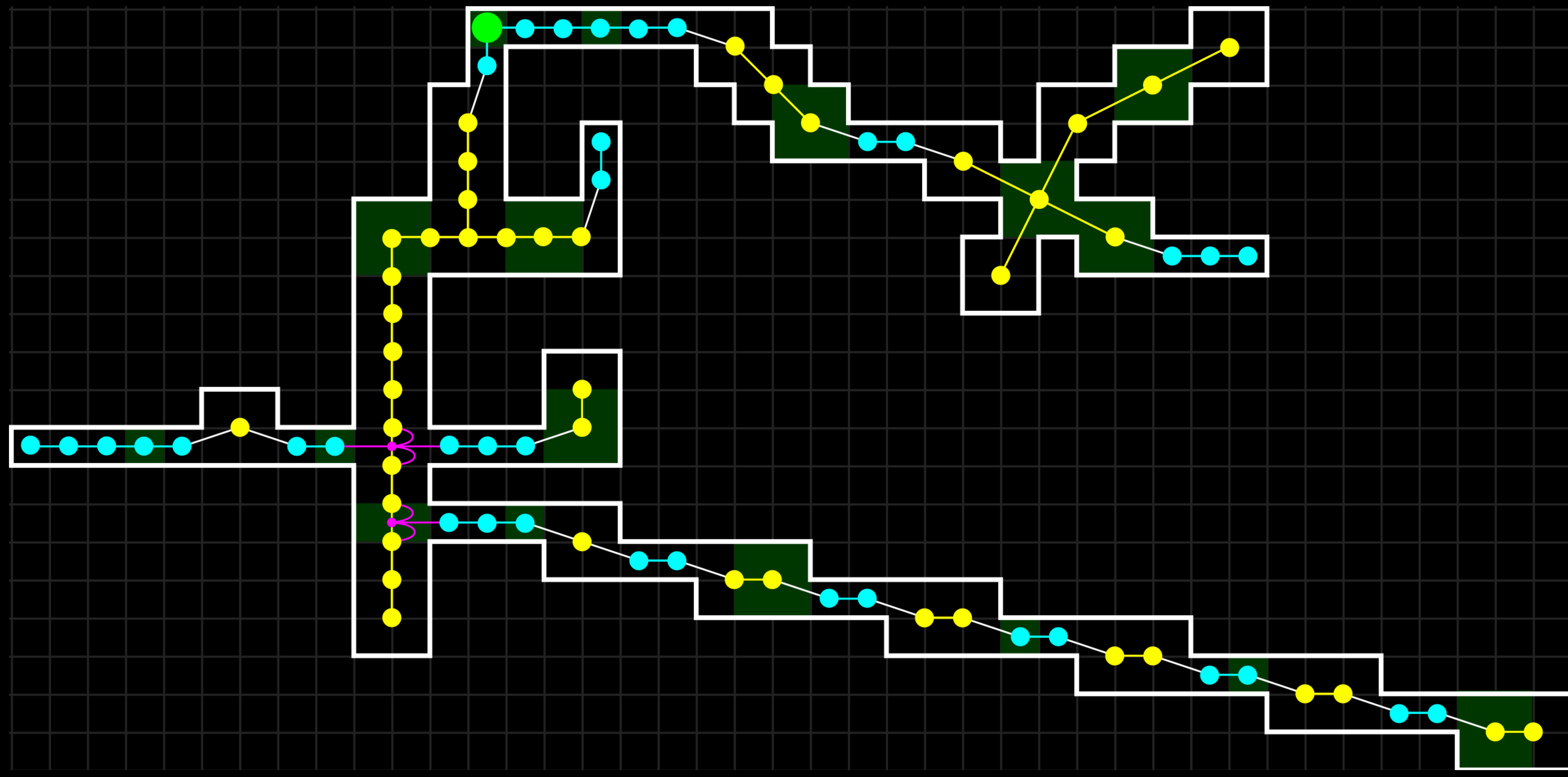
We only remove a node v from T if S(v) is covered by cells in D.



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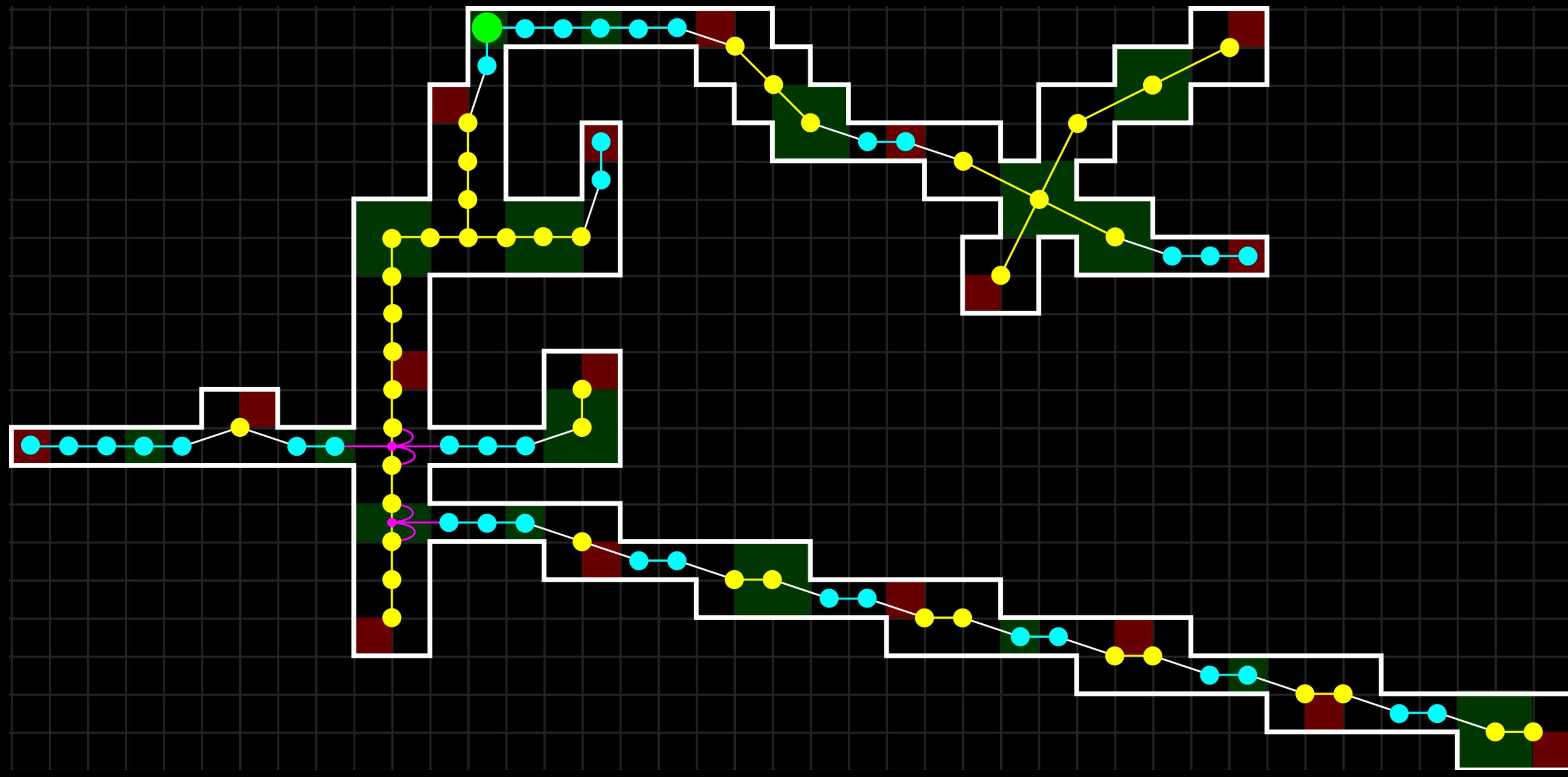

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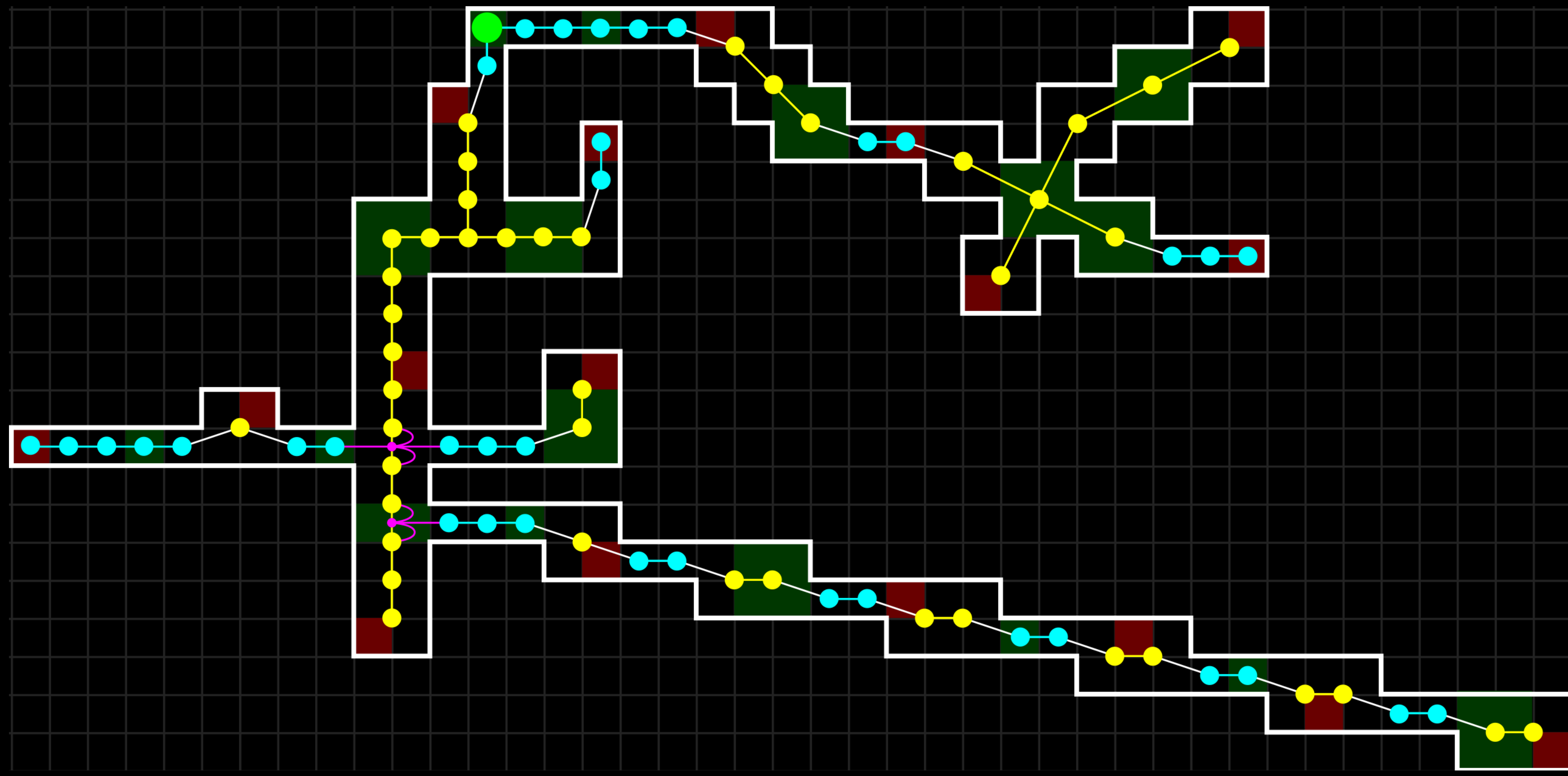

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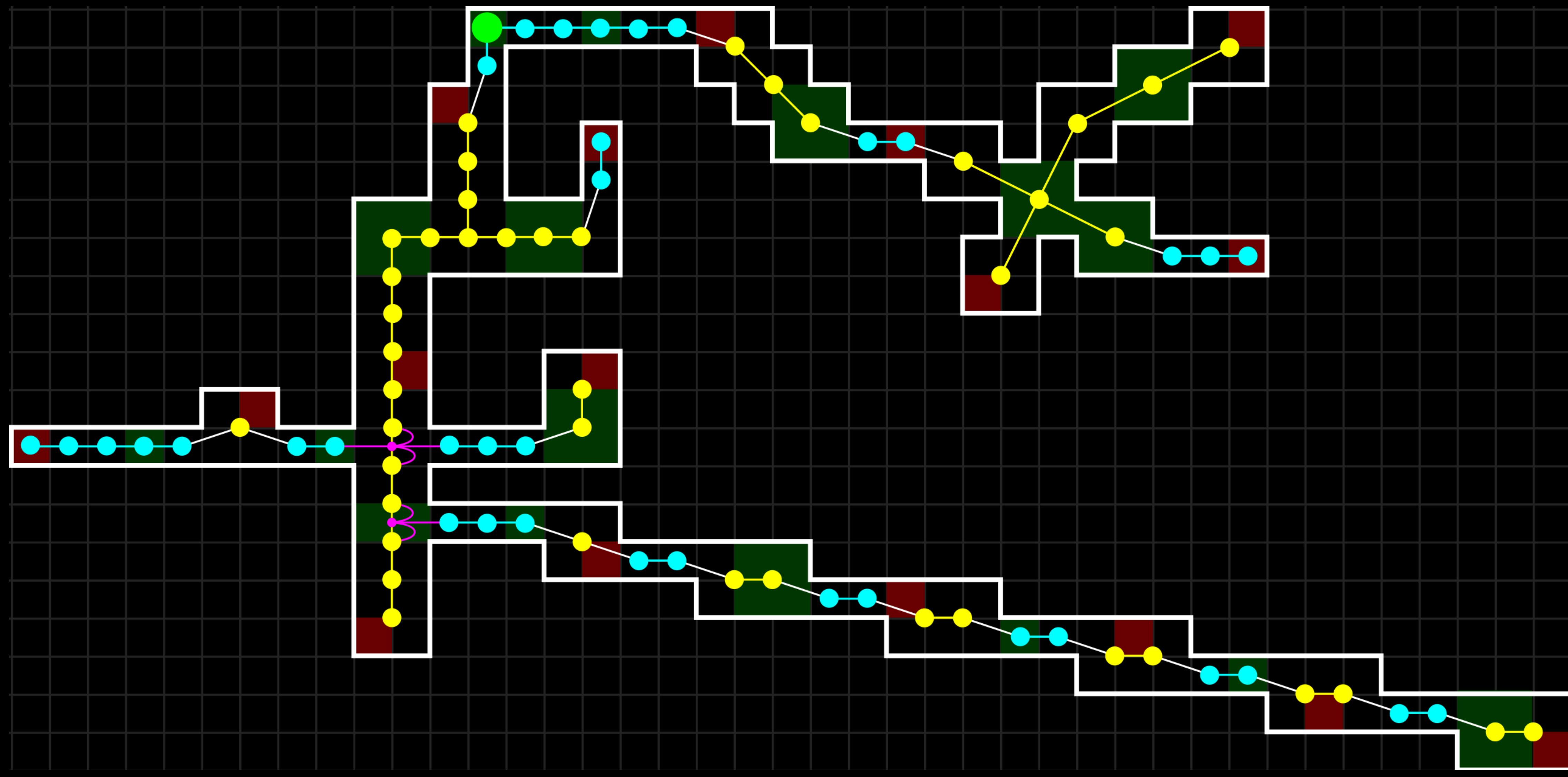

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We add at most 4 unit squares to D in each step: $|D| \leq 4\ell$

Outlook

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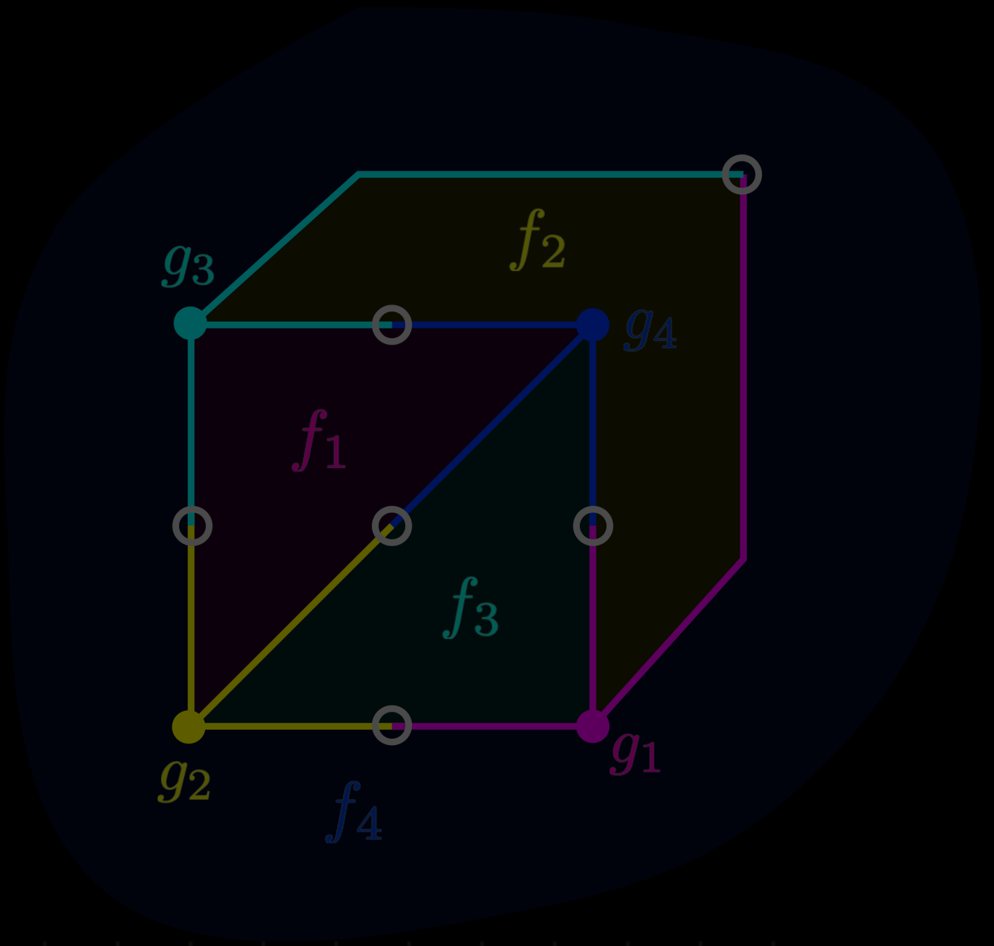
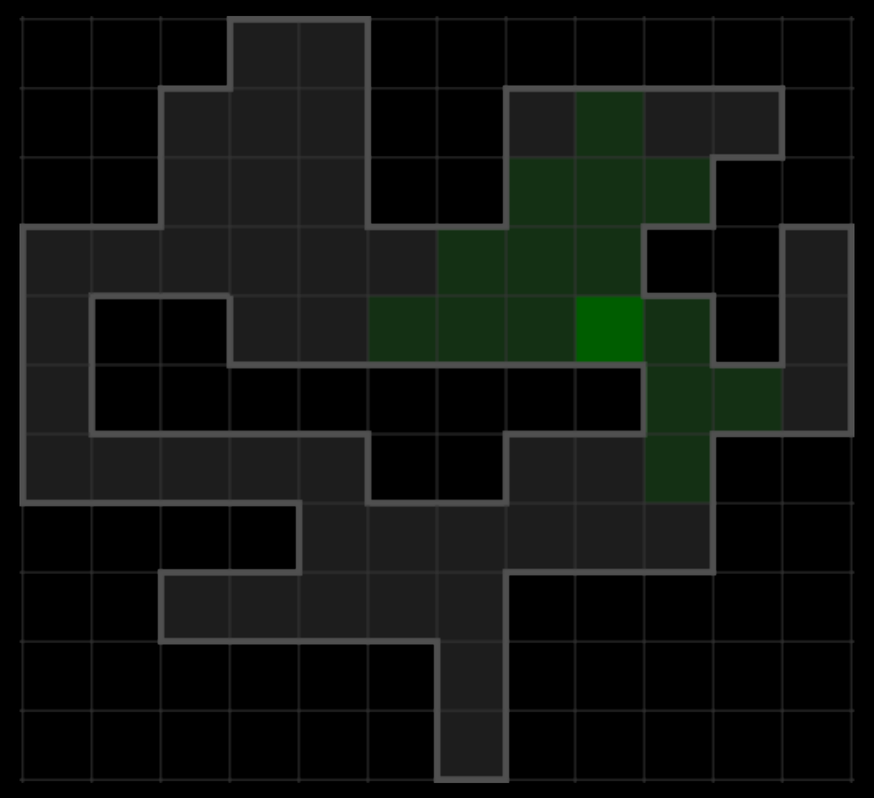
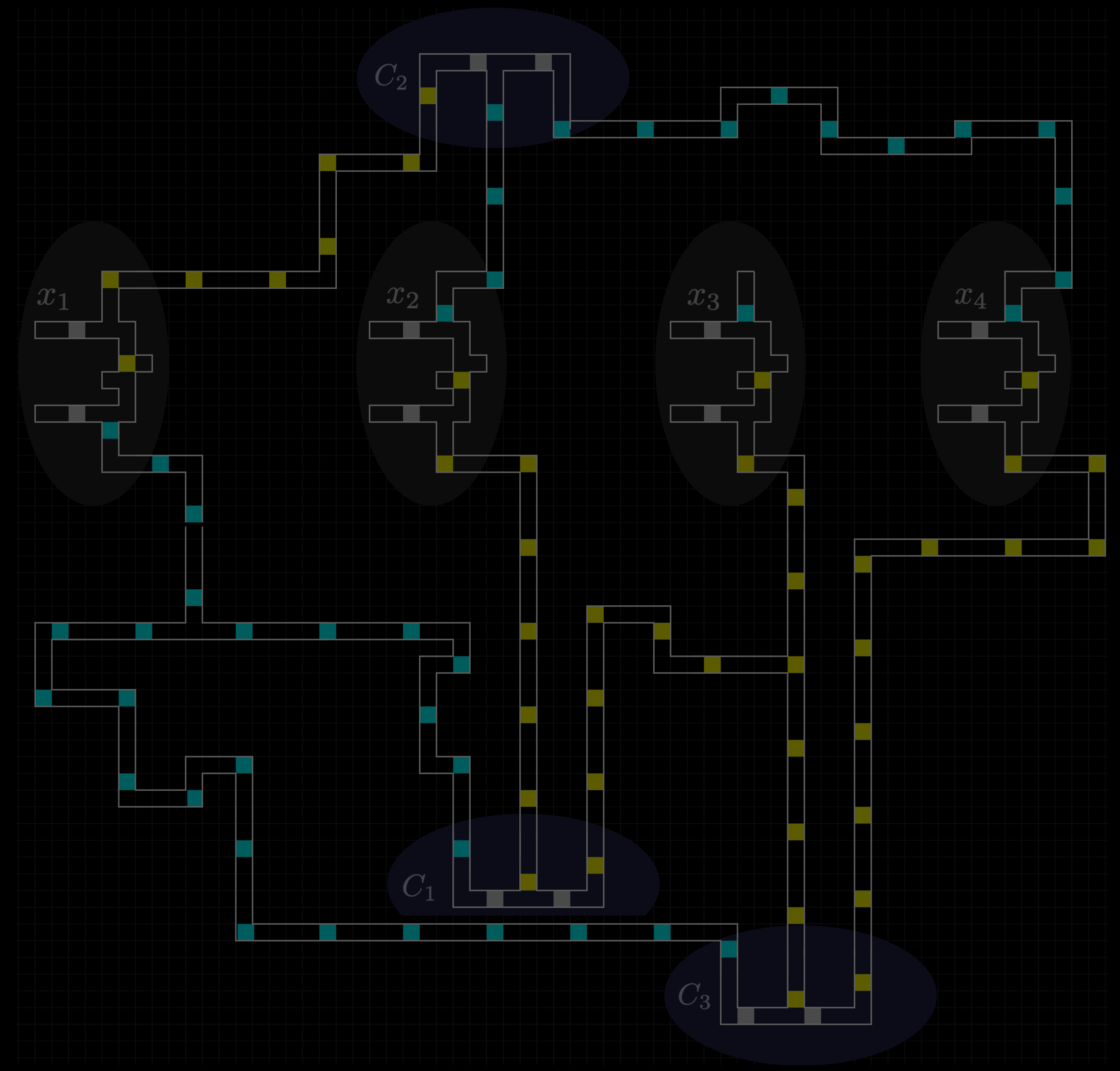
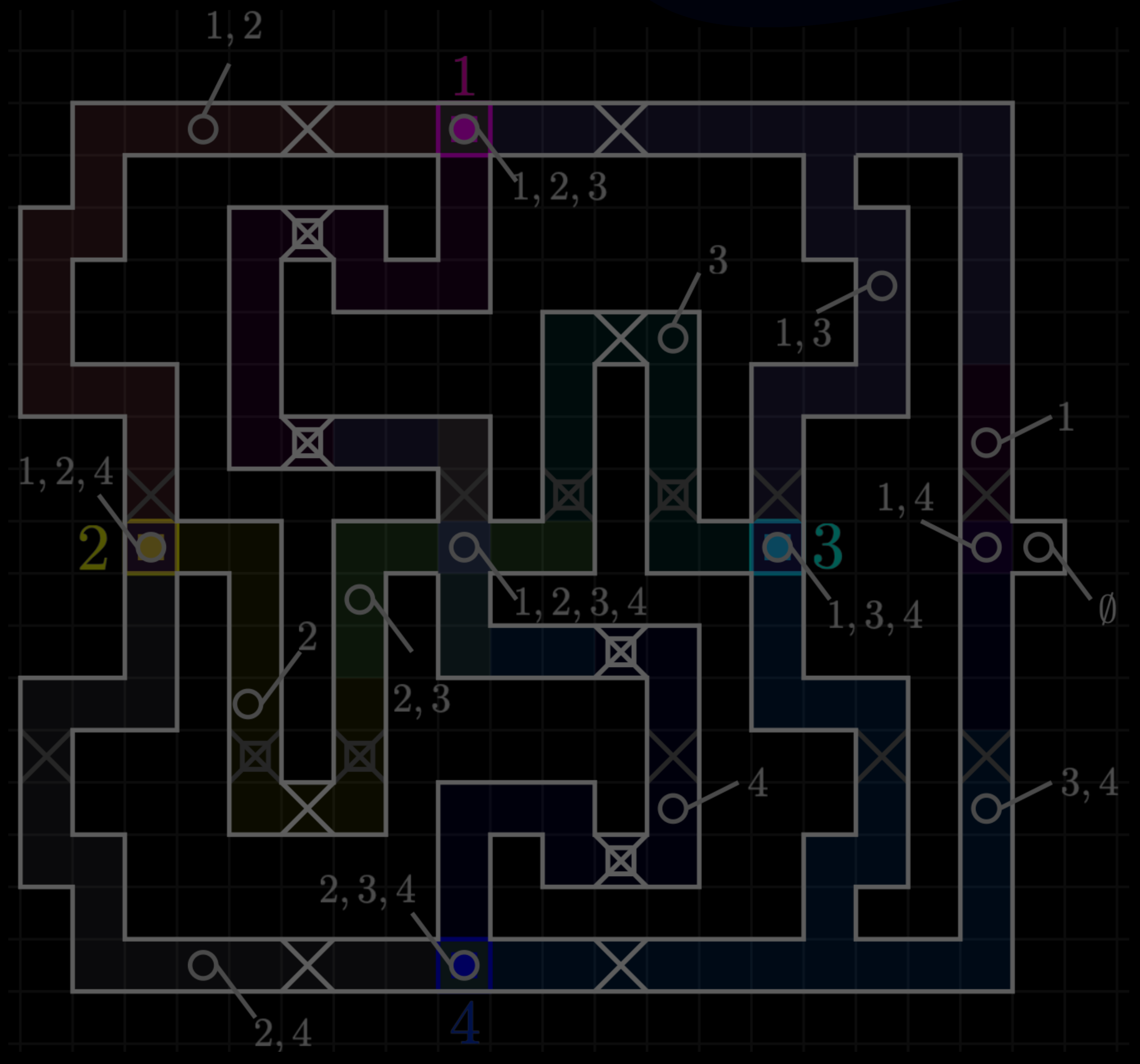
- Complexity in arbitrary simple polyominoes?
- More general approximation framework for arbitrary *k* in arbitrary simple polyominoes? (Or polytime algorithm??)
- Constant-factor approximation for polyominoes with holes?
- CS: city center requires higher density of carsharing stations, lower density in outskirts—what if we have various classes of guards with different values of *k*?

OUR PAPER

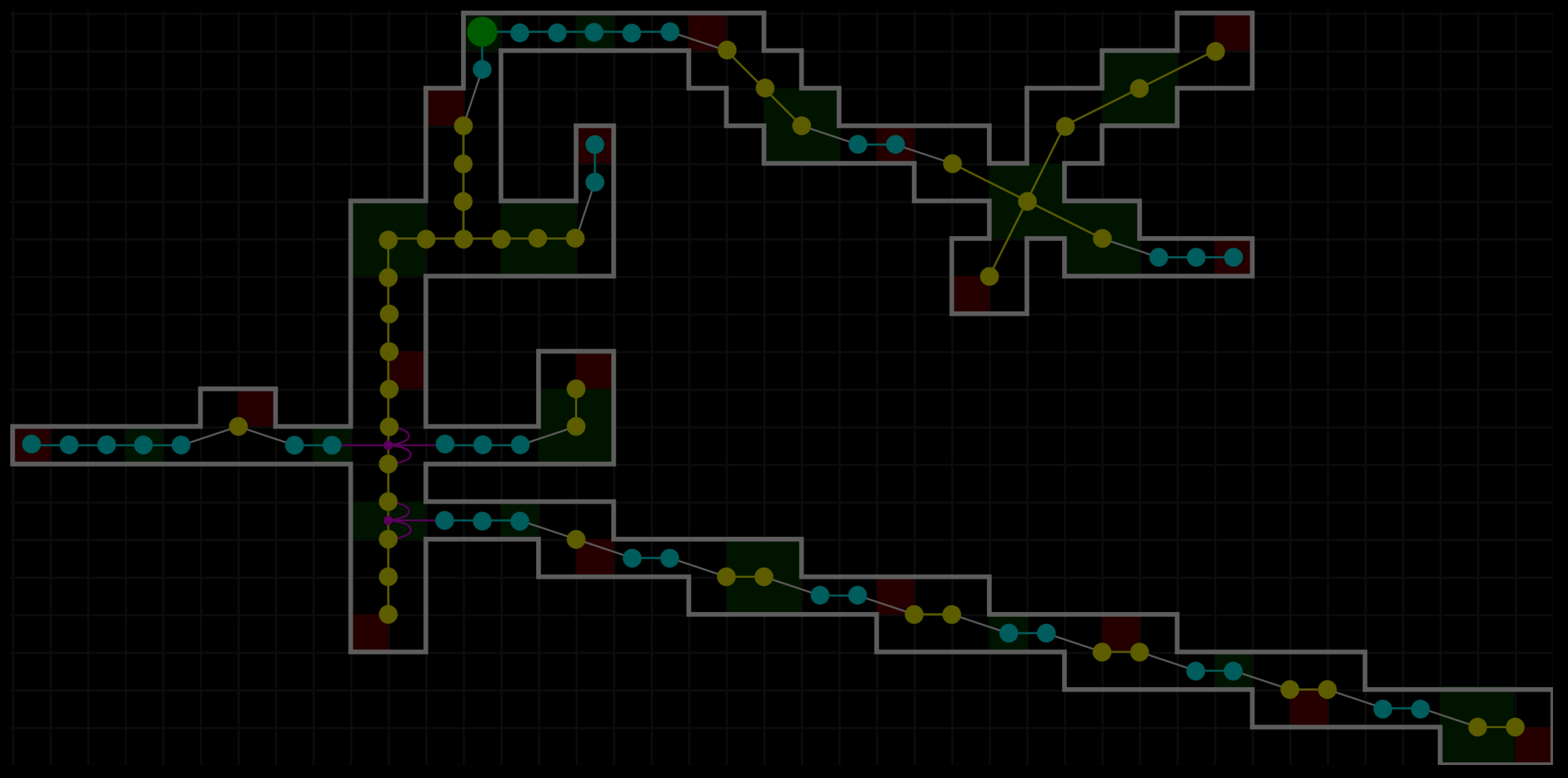








Thank you.



SCAN ME
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