Segment Watchman Routes

Anna Brötzner, Omrit Filtser, Bengt J. Nilsson, Christian Rieck, **Christiane Schmidt**

MALMÖ UNIVERSITET
האוניברסיטה הפתוחה THE OPEN UNIVERSITY OF ISRAEL الجامعة المفتوحة
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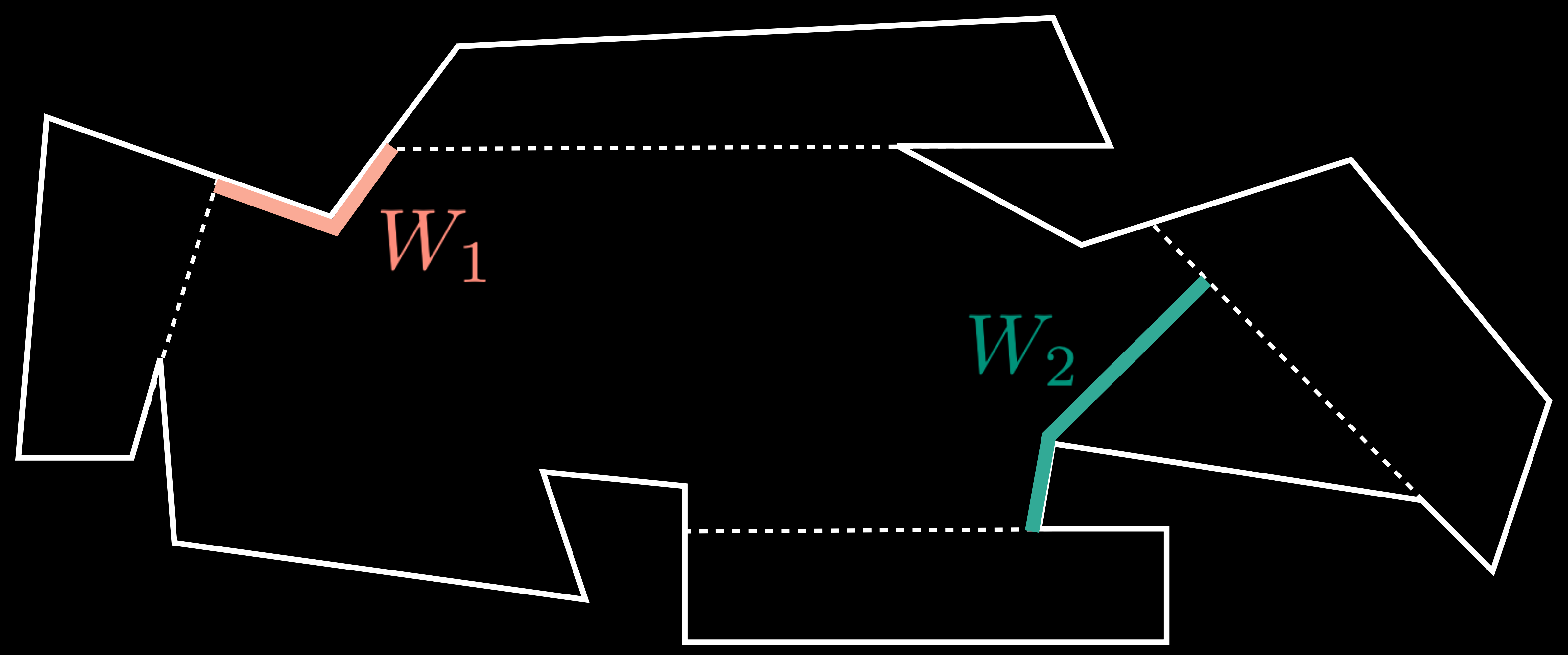
- Given: Polygon *P*, *m* watchmen with or without starting points
- Find: *m* routes, such that all points in *P* are visible from at least one point on one of the routes—usual objectives: min-max or min-sum of the *m* routes



m=2

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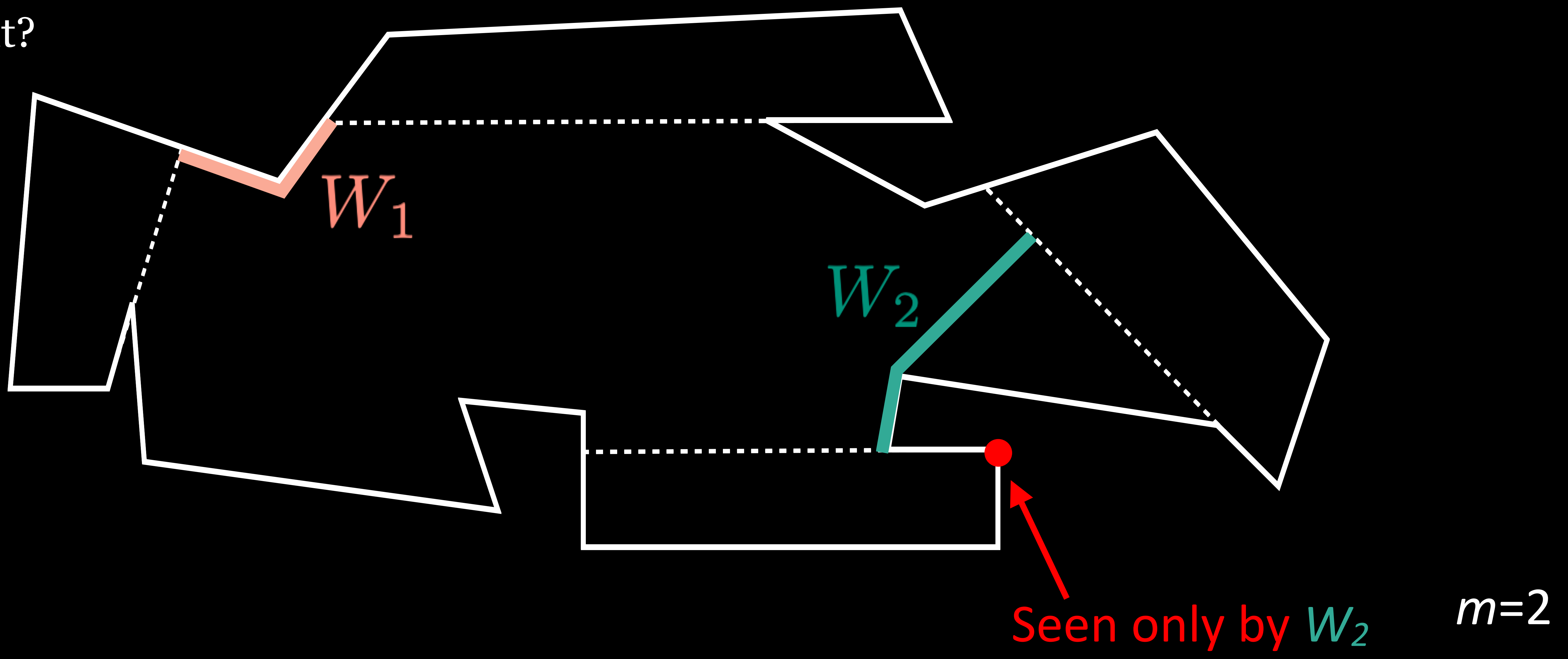

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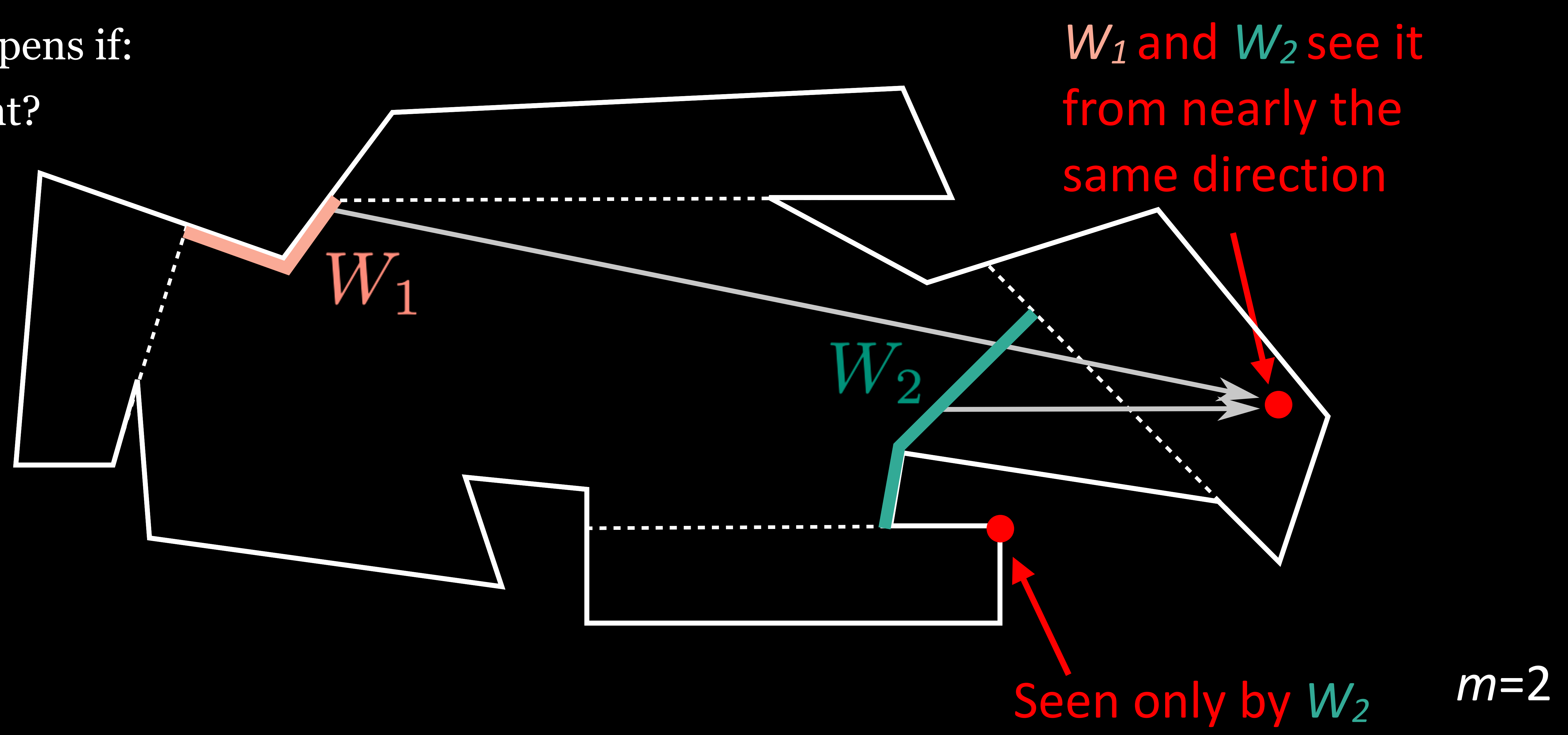

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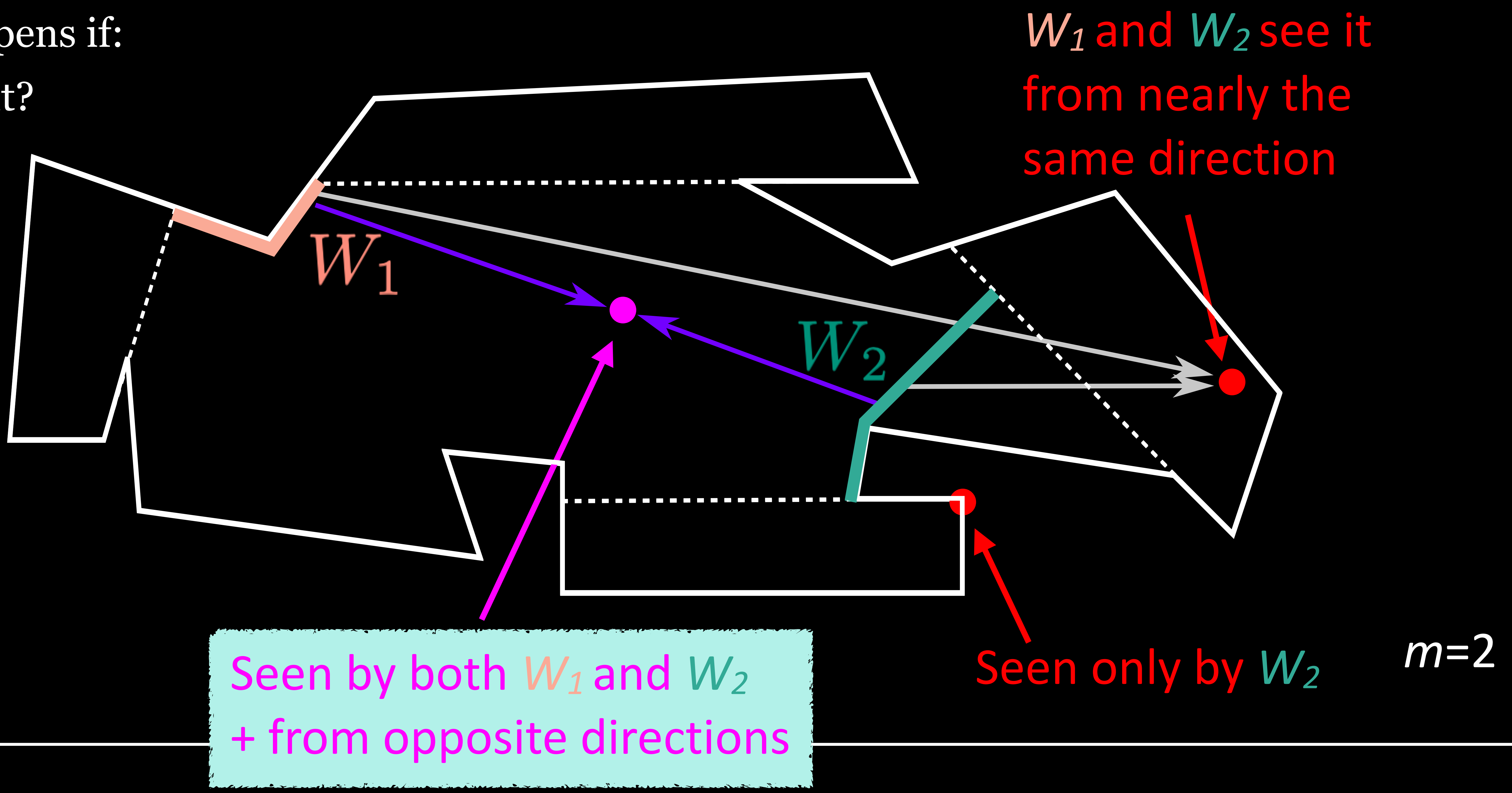

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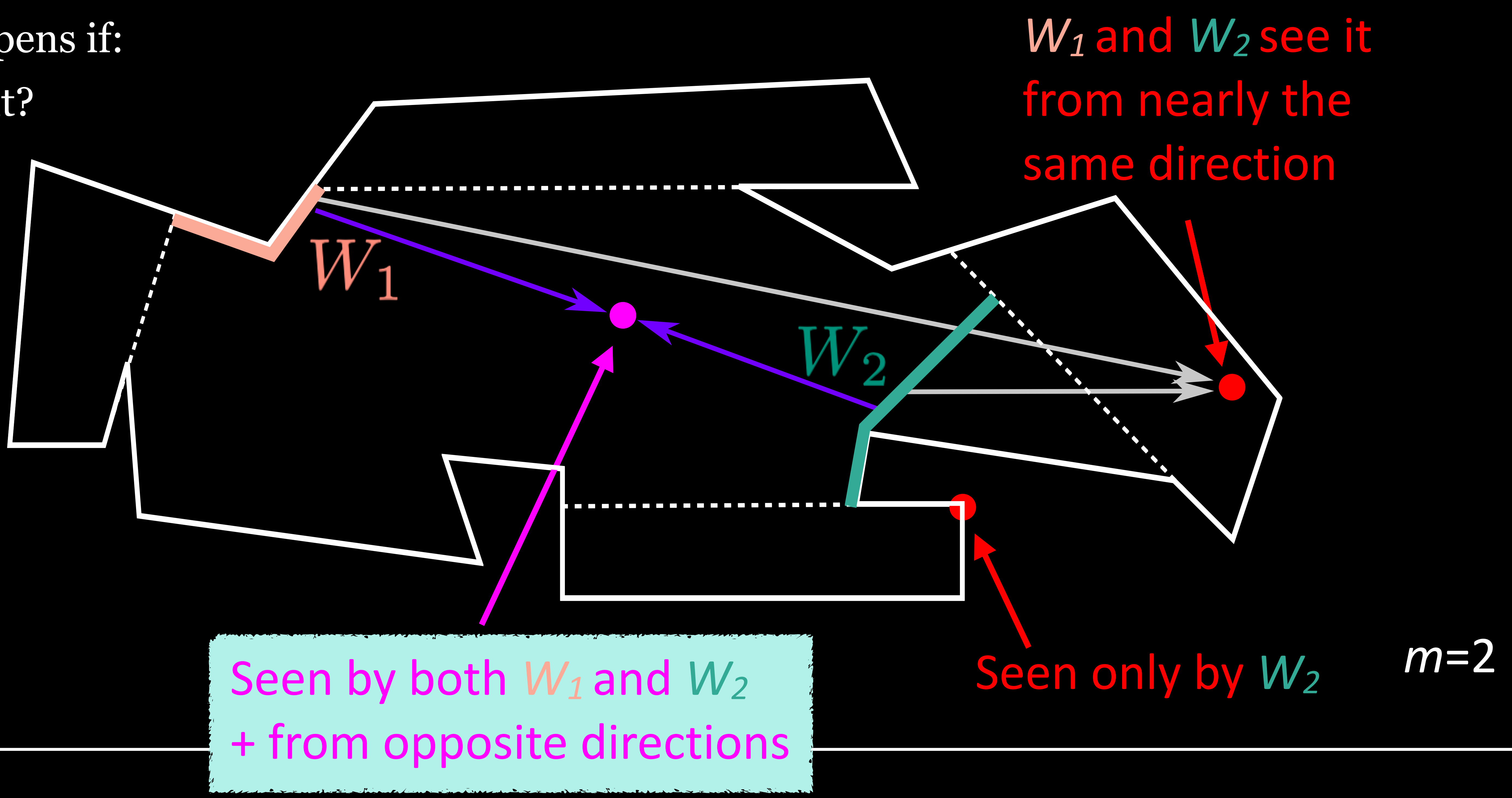
➡ We want to make our routes robust against some of these aspects!



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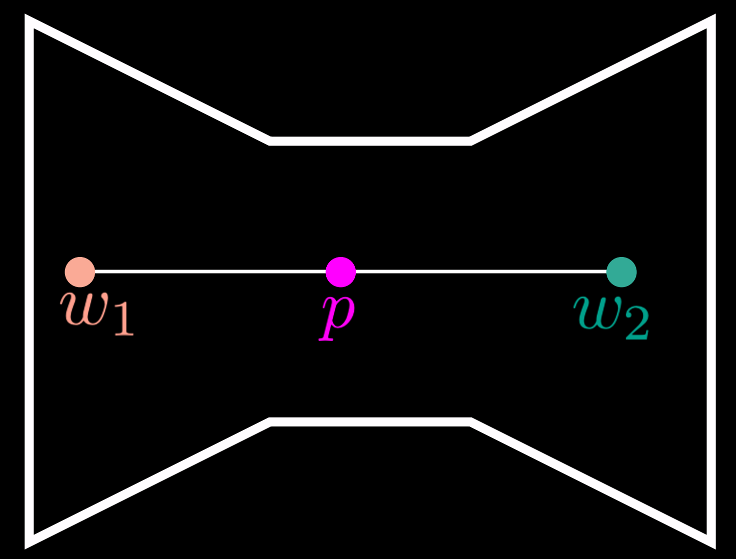

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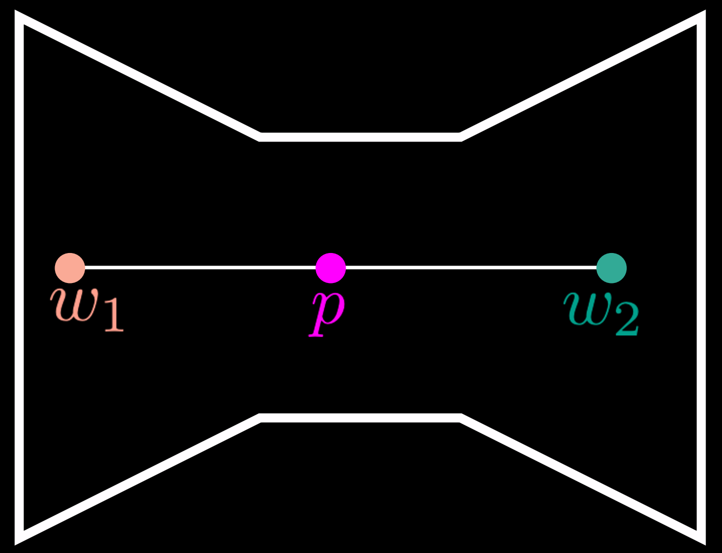

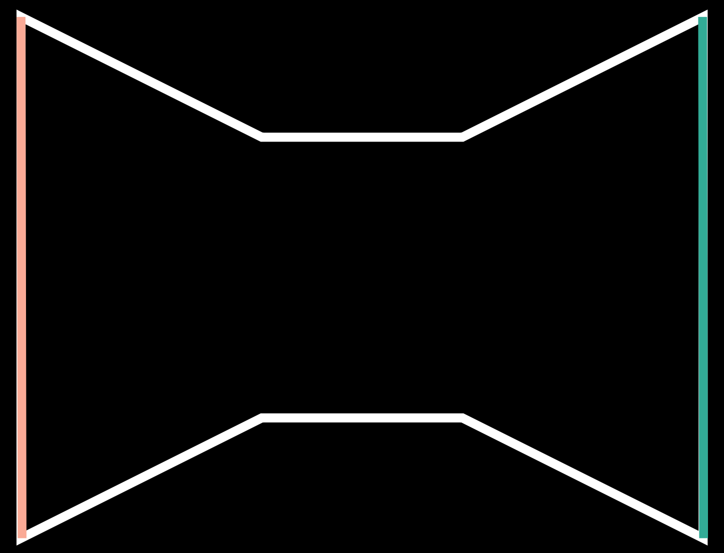

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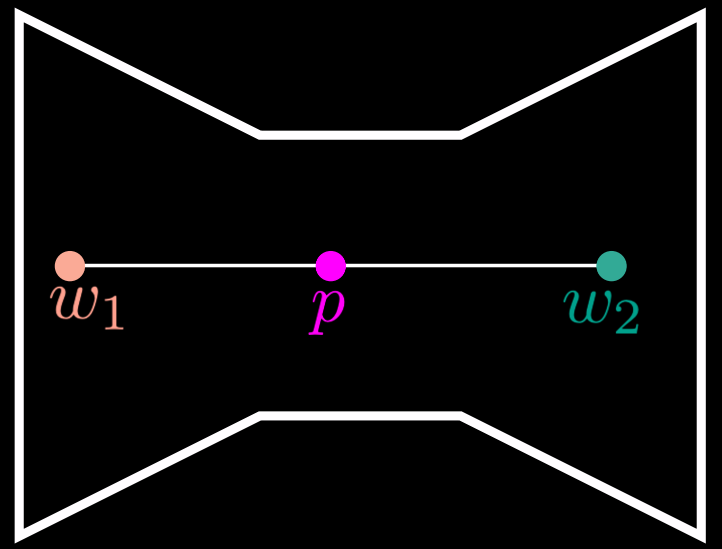

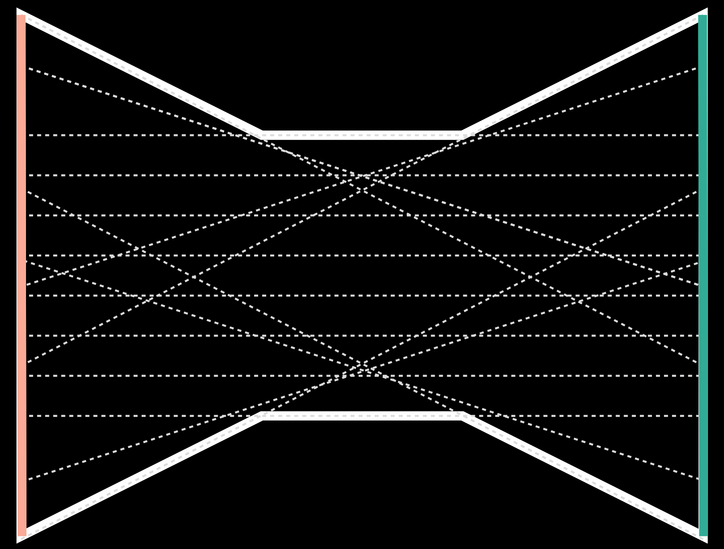

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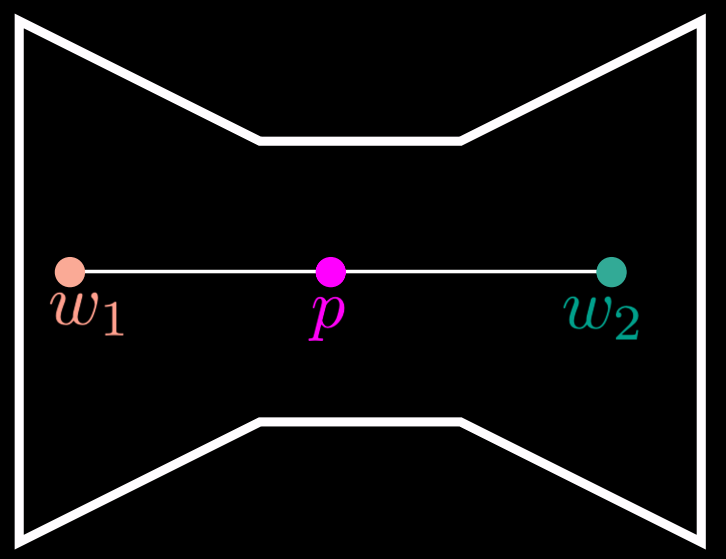

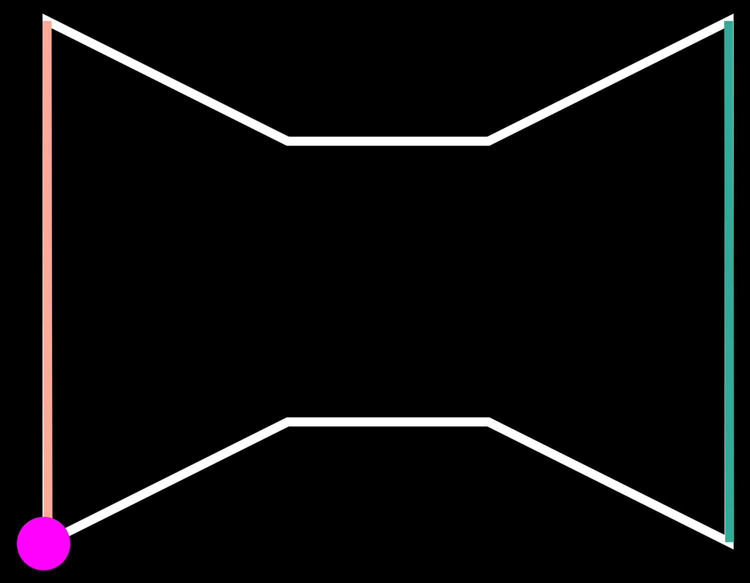

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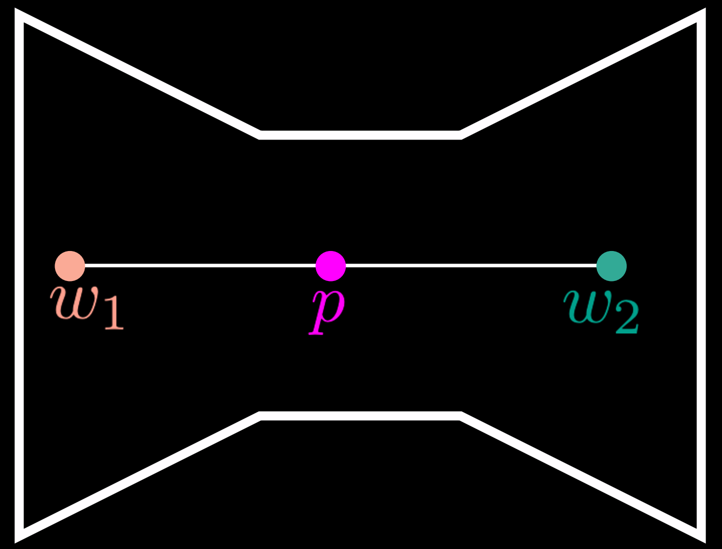

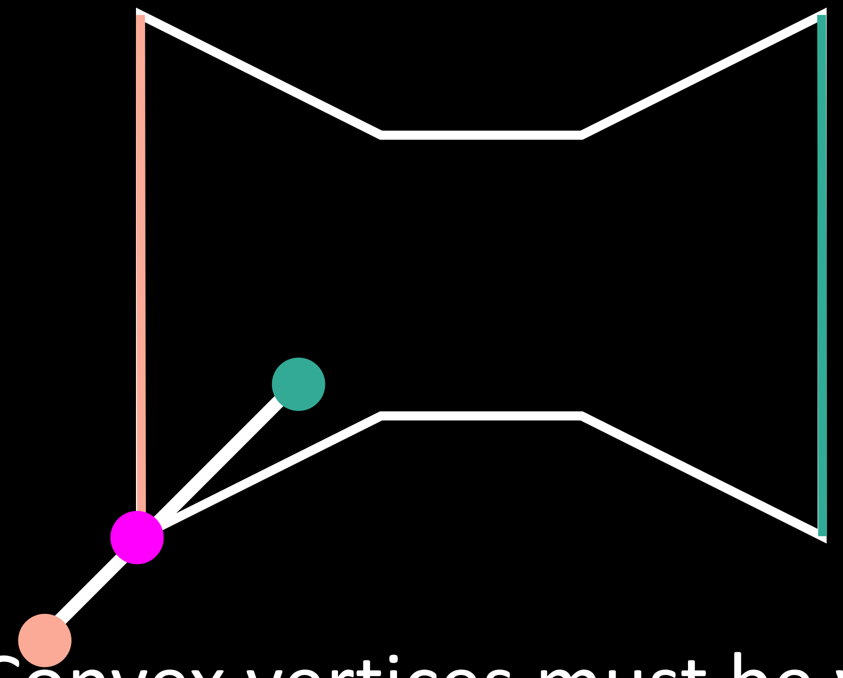

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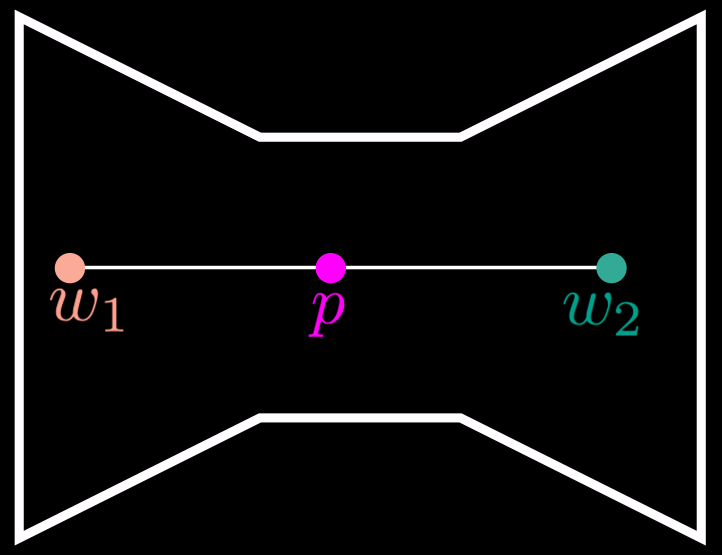

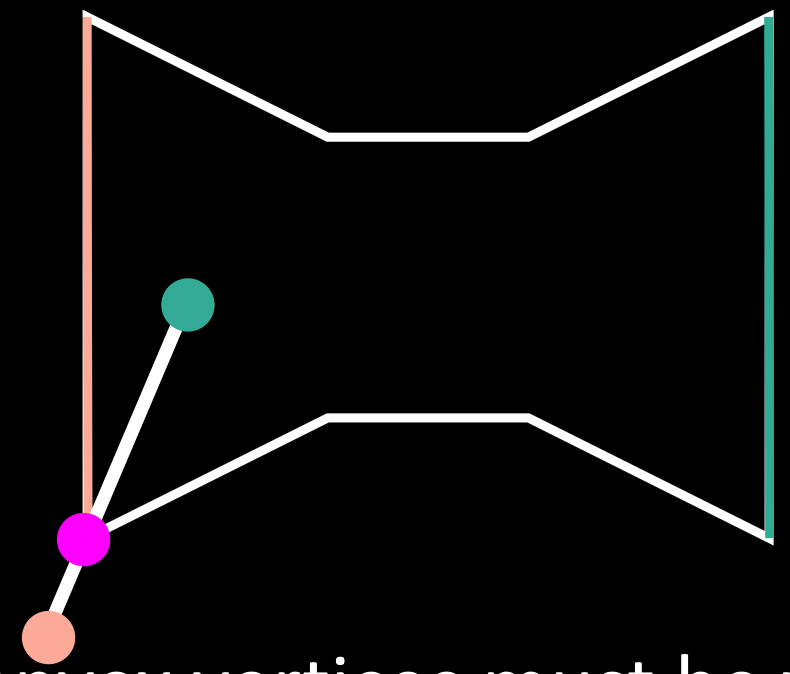

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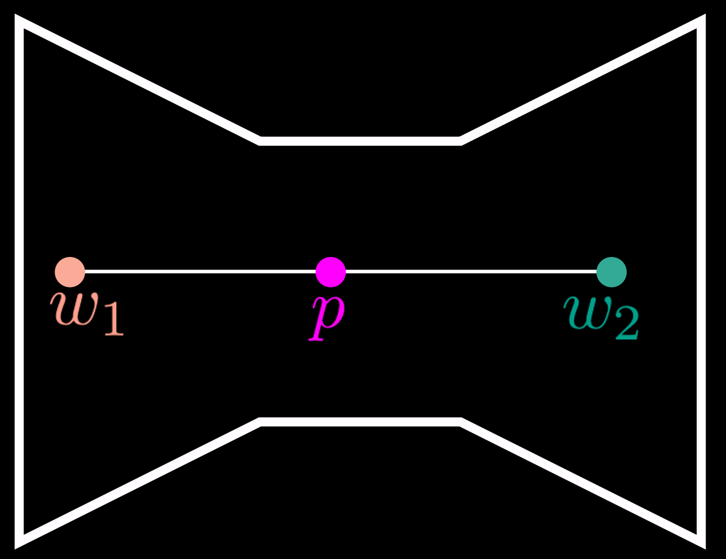

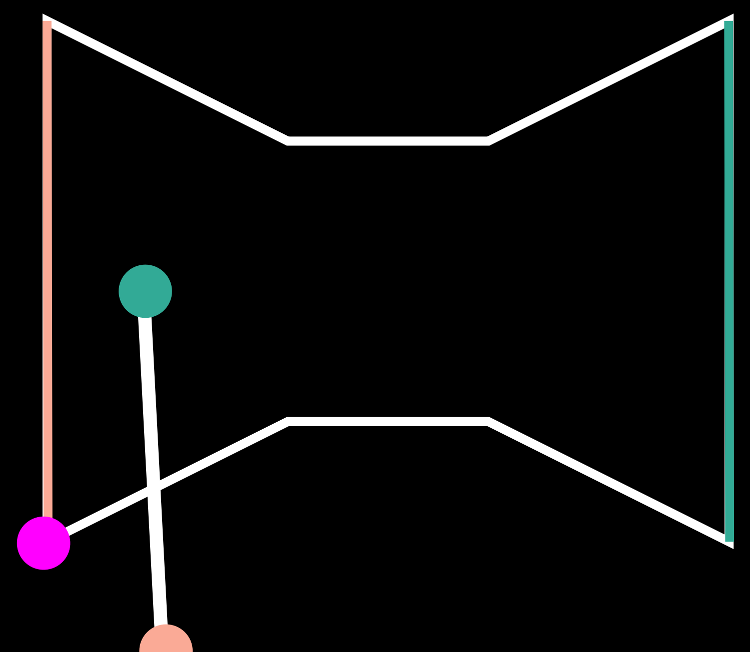

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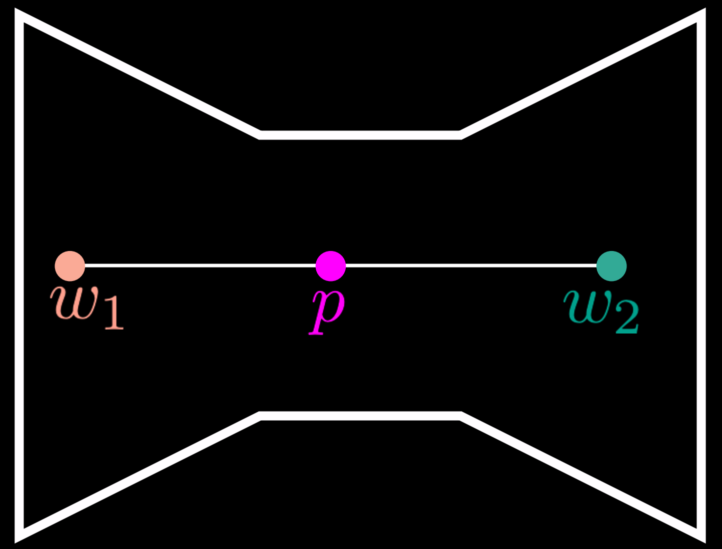

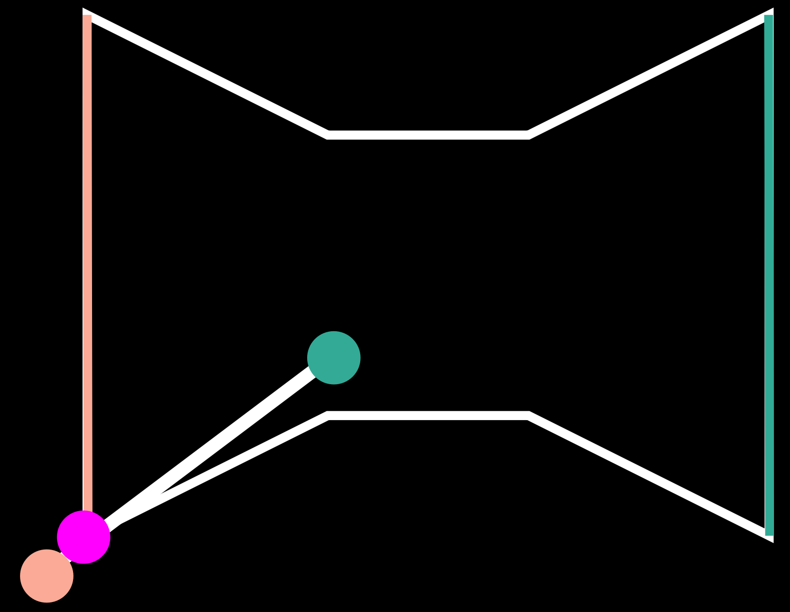

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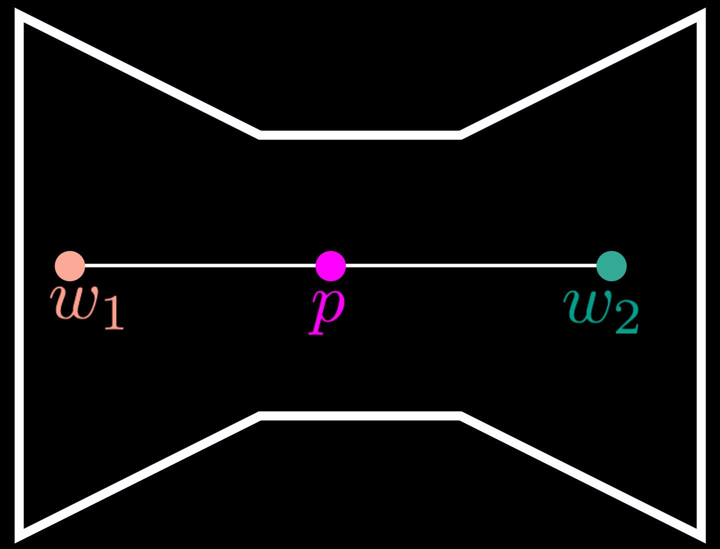

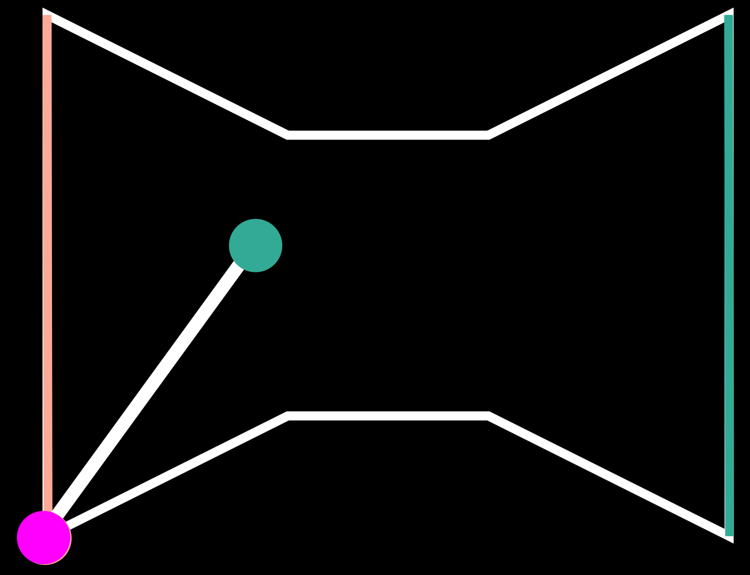

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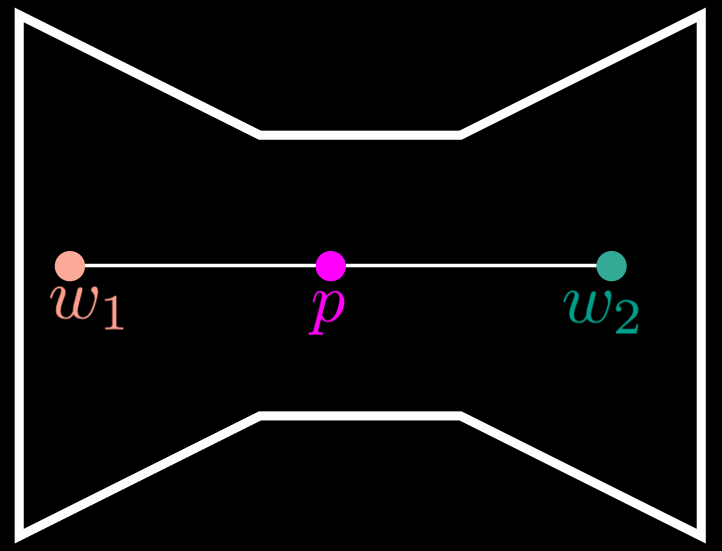

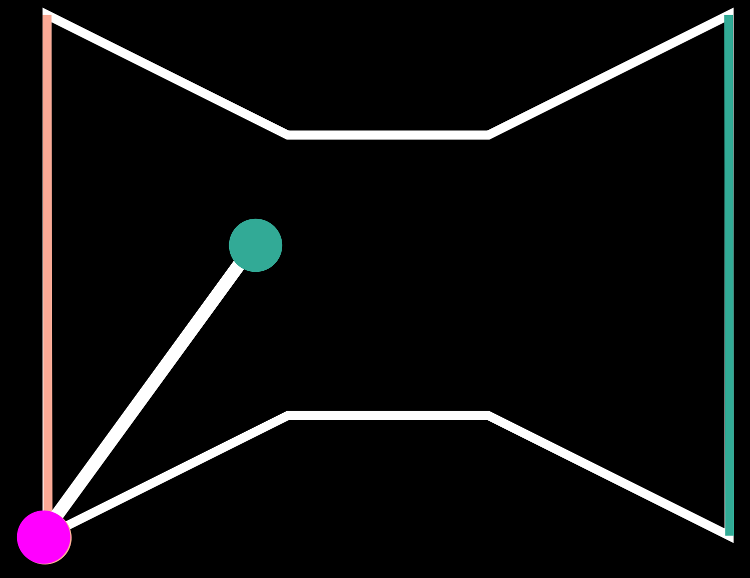

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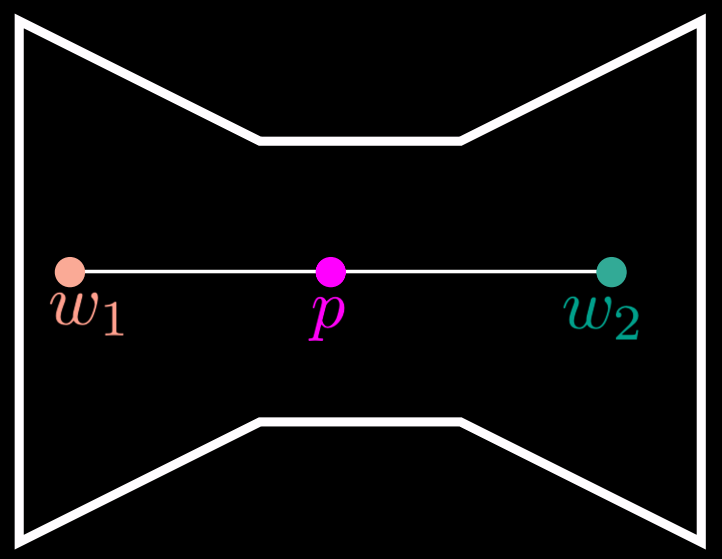

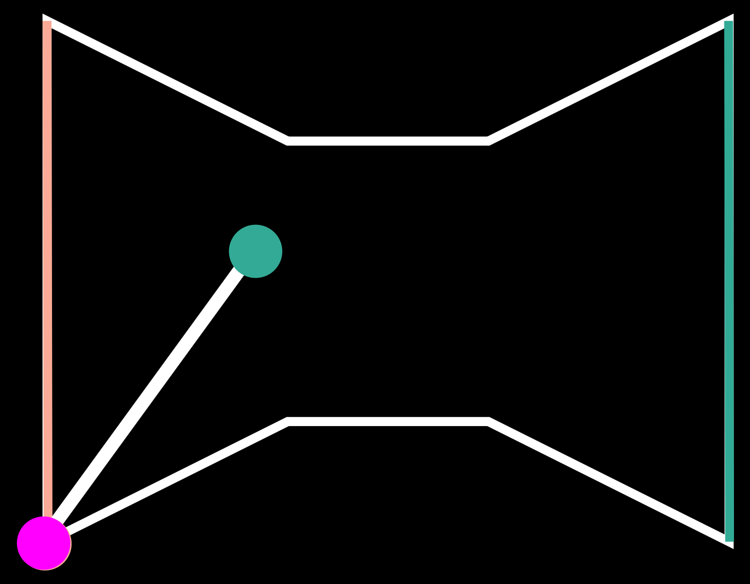

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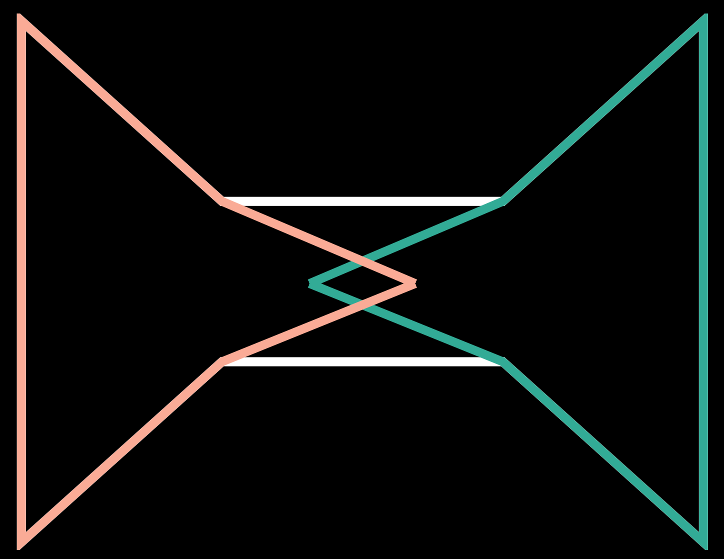

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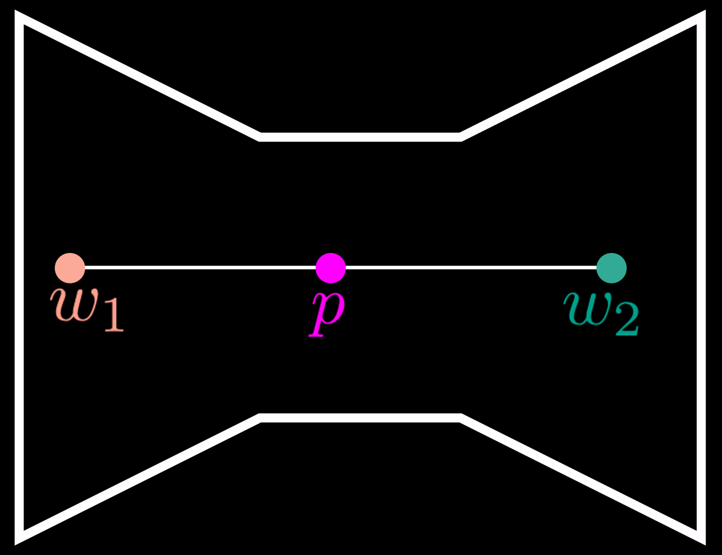

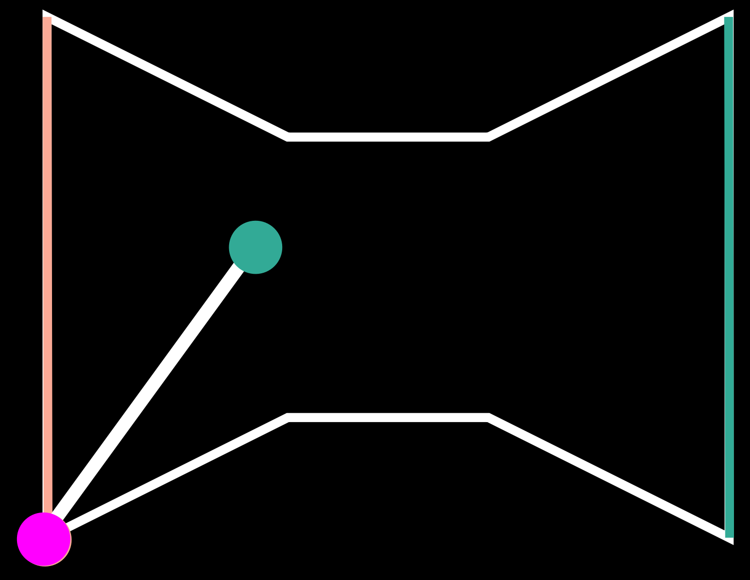

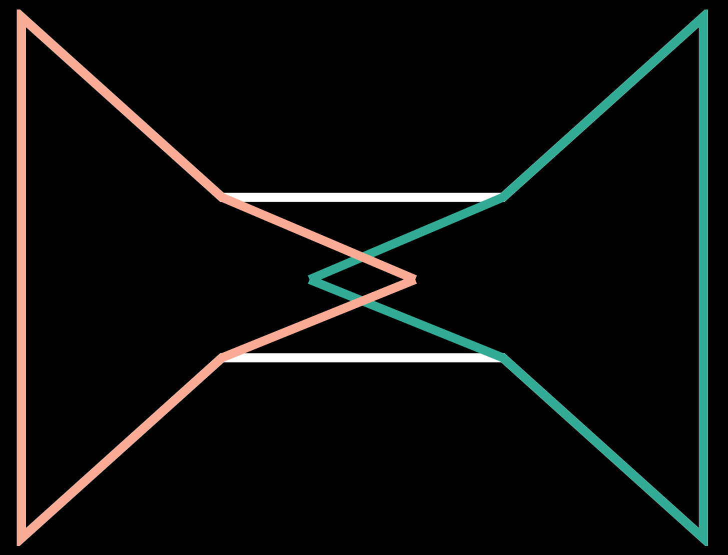

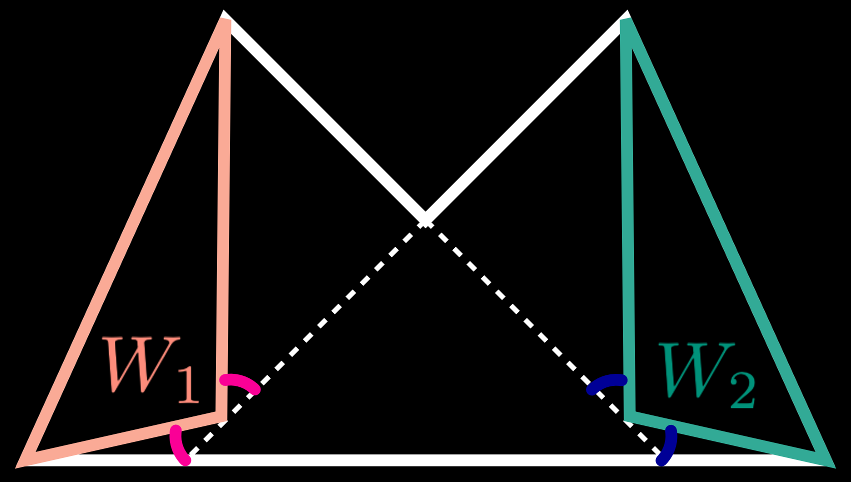

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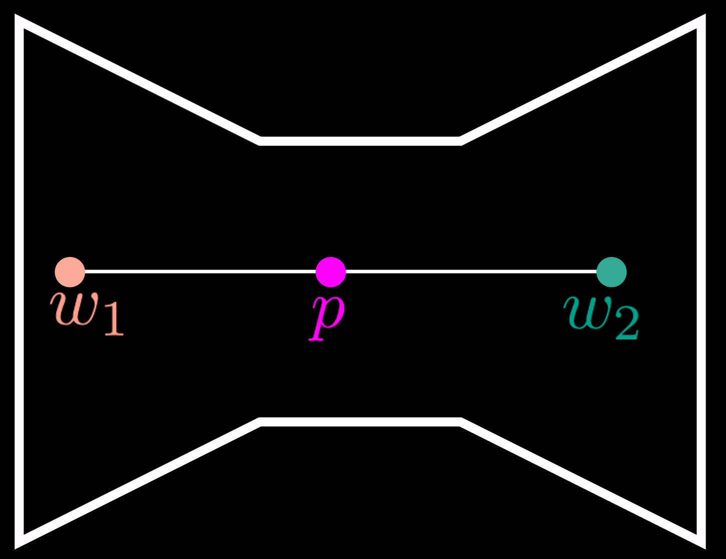

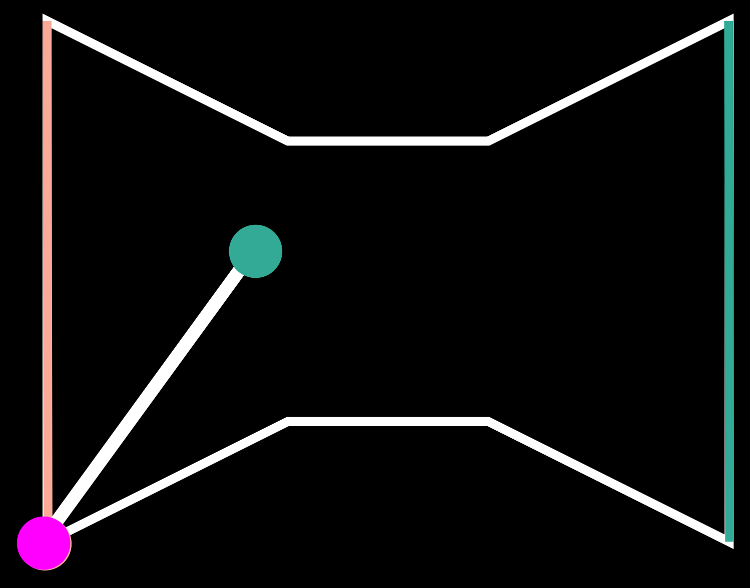

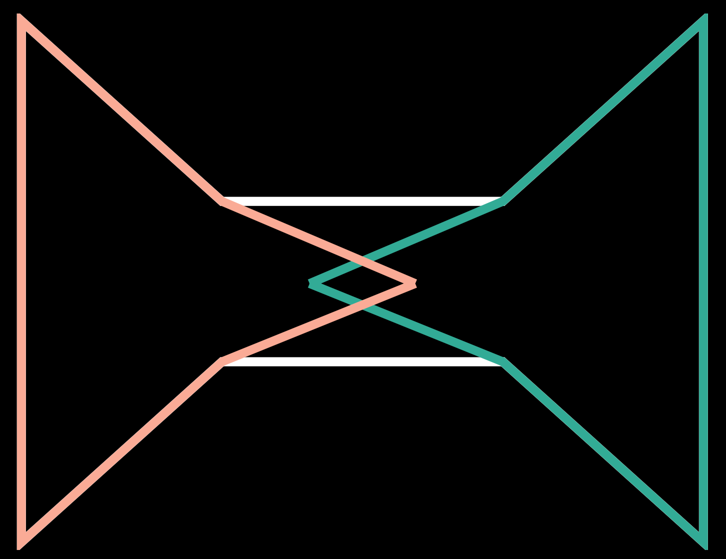

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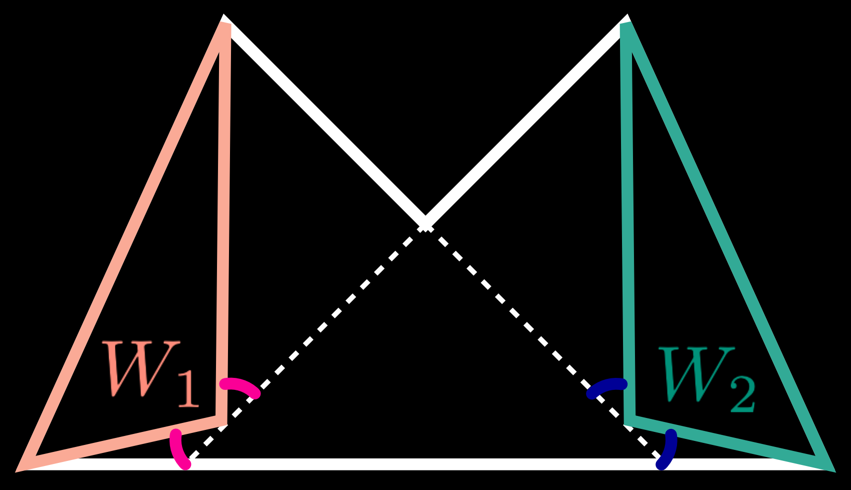

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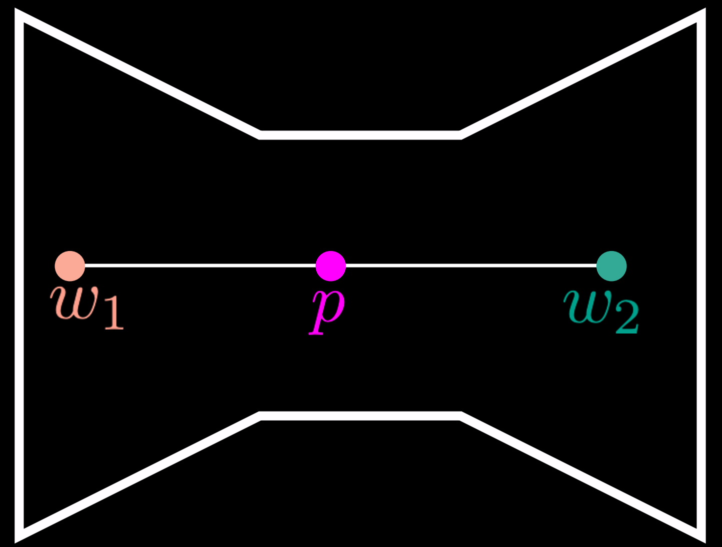
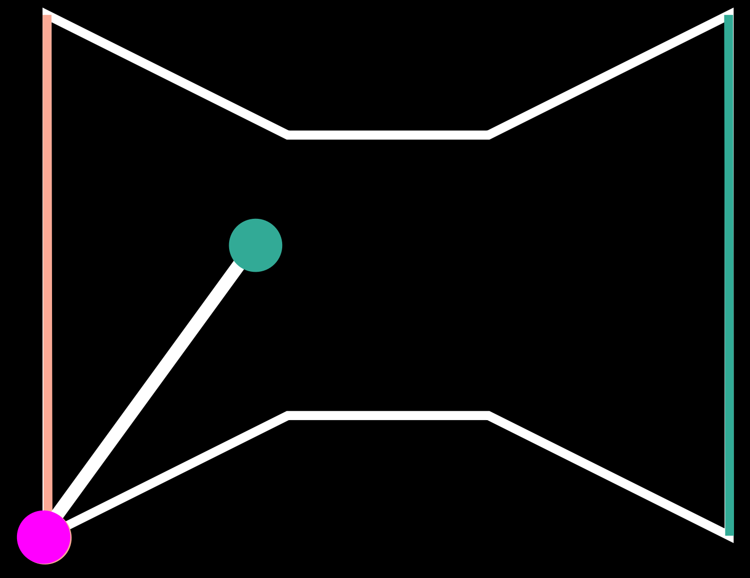
Objectives? Still min-max or min-sum





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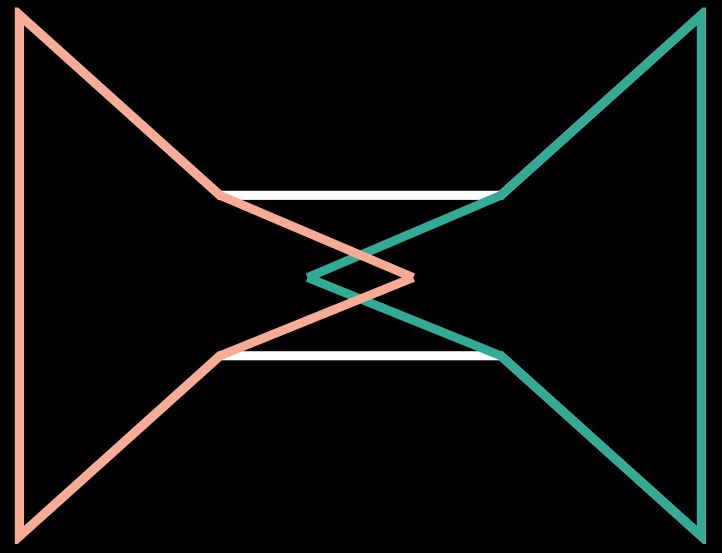

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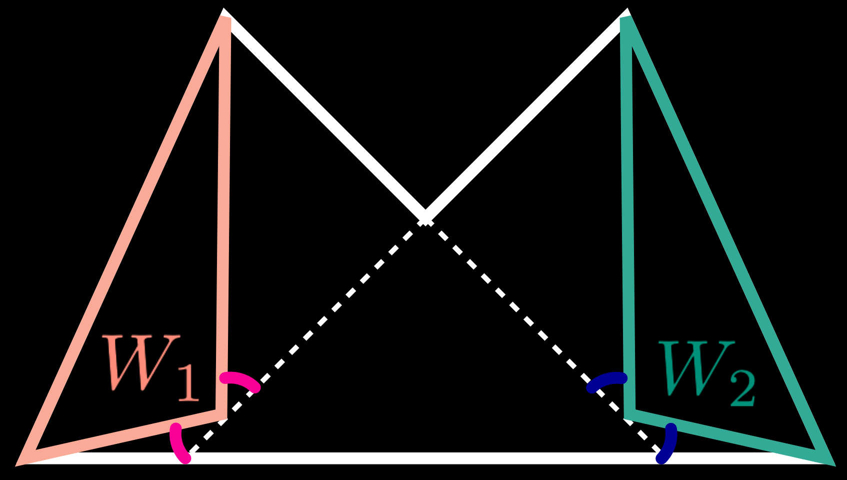

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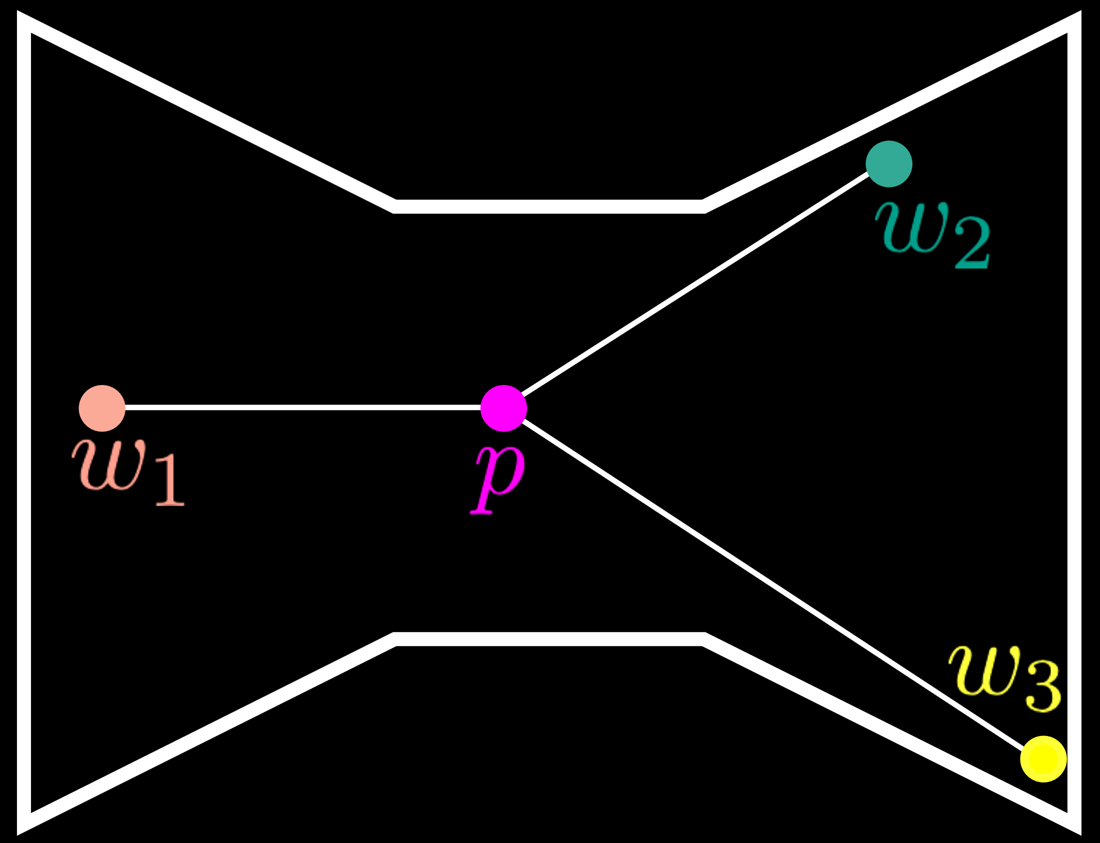

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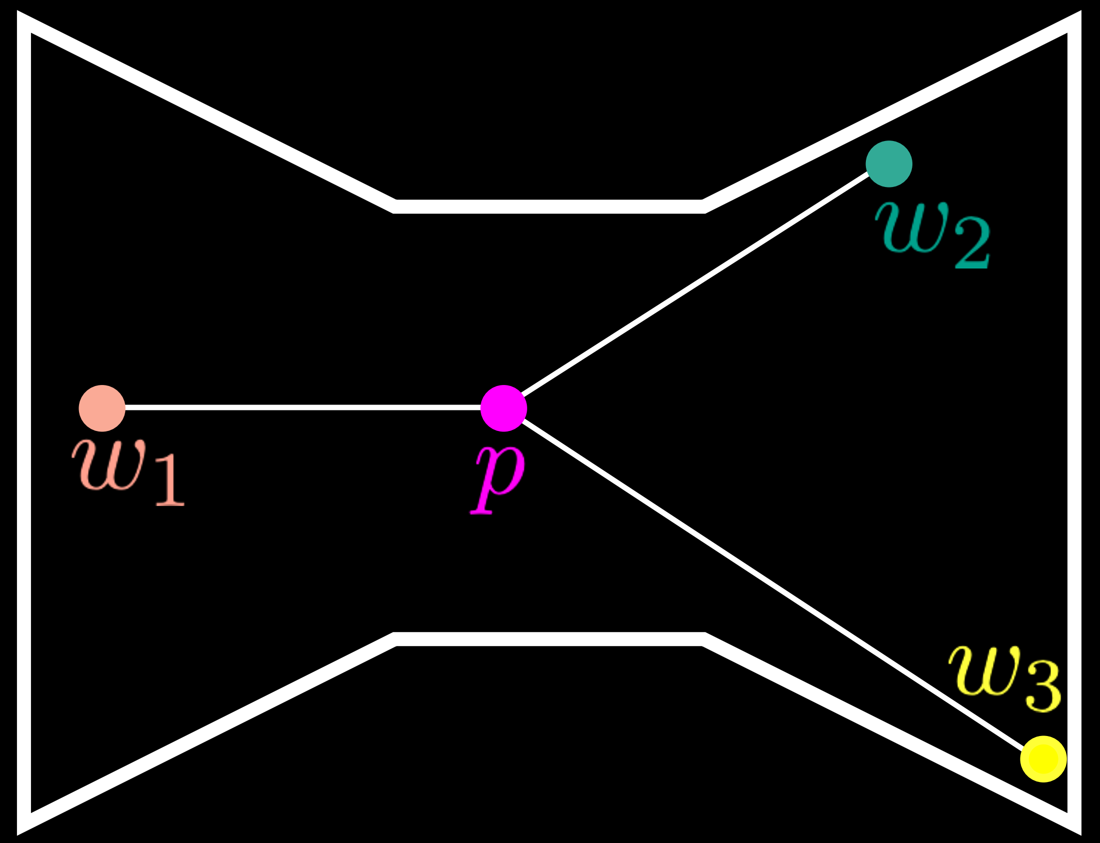
- k-gon-guarded points



Sufficient Conditions: The “Conditions Lemma”

*For any two points inside the region enclosed by the route, their shortest path is also contained within the enclosed region

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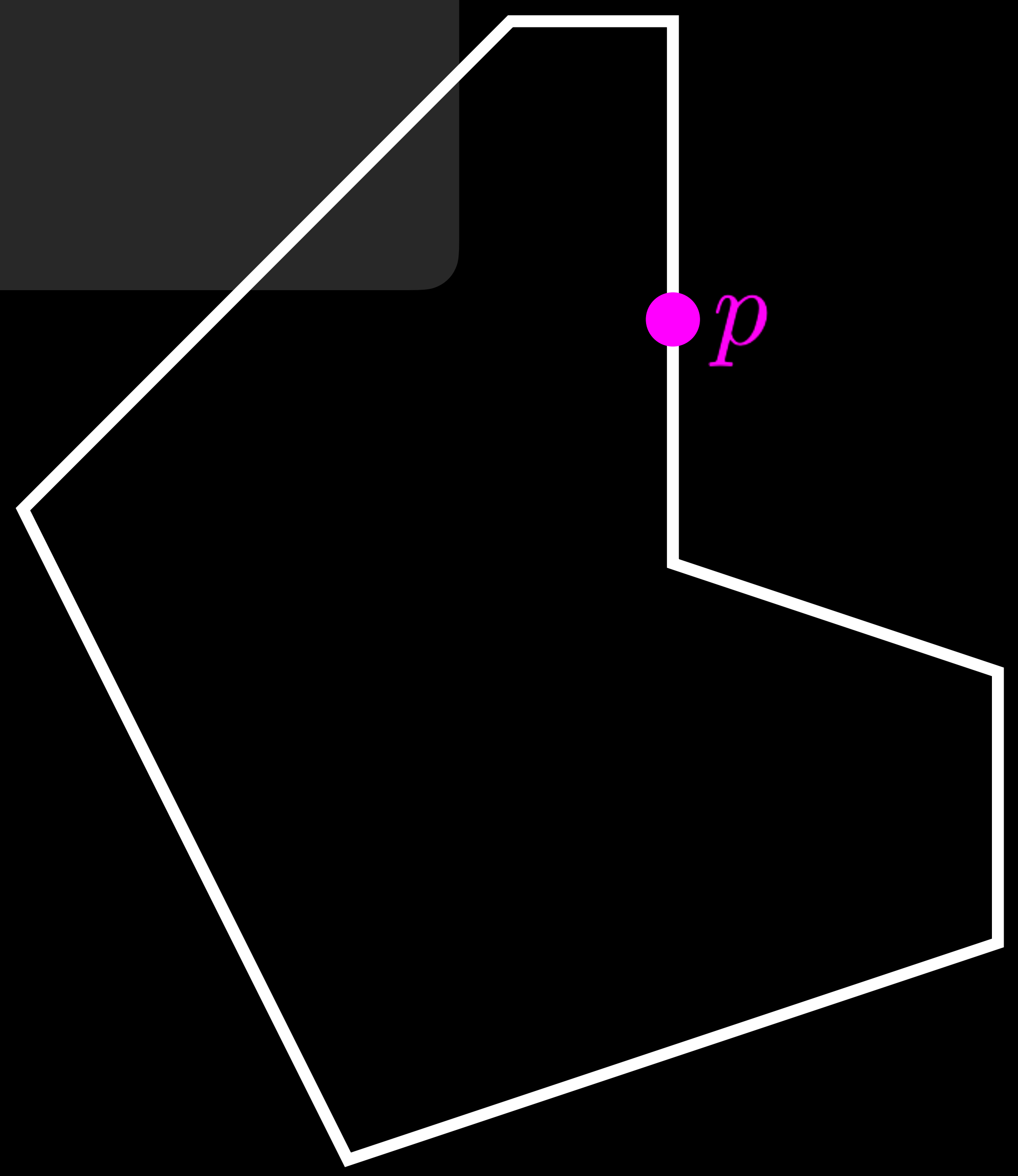

Proof: First show (2)$\Rightarrow$ W_1 and W_2 are watchman routes

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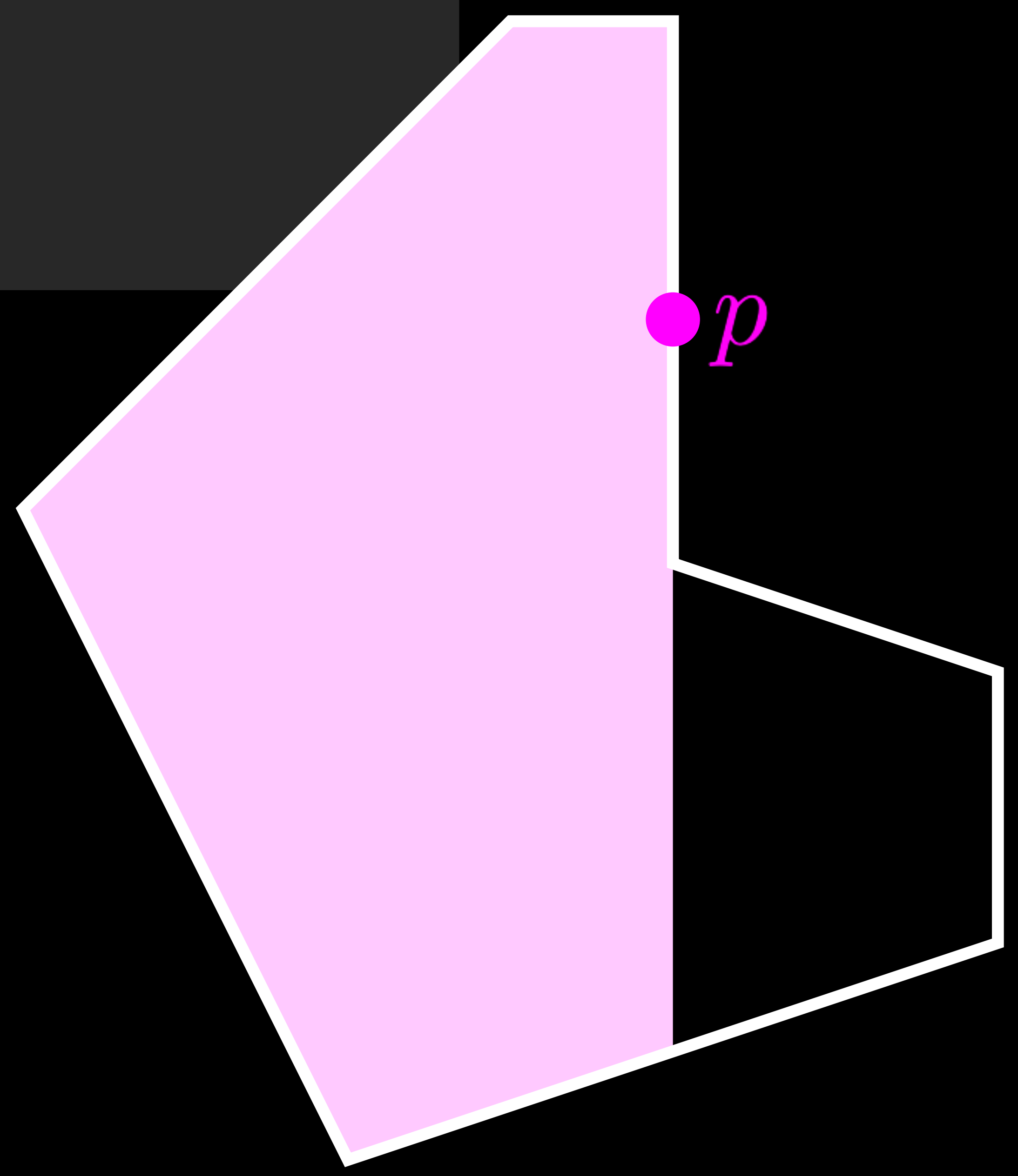

Proof: First show (2)⇒ W_1 and W_2 are watchman routes

Assume $p \in P$ is not seen by W_i

➜ W_i is fully contained in a pocket P' of p’s visibility polygon.

Subpolygon of P, no point visible from p

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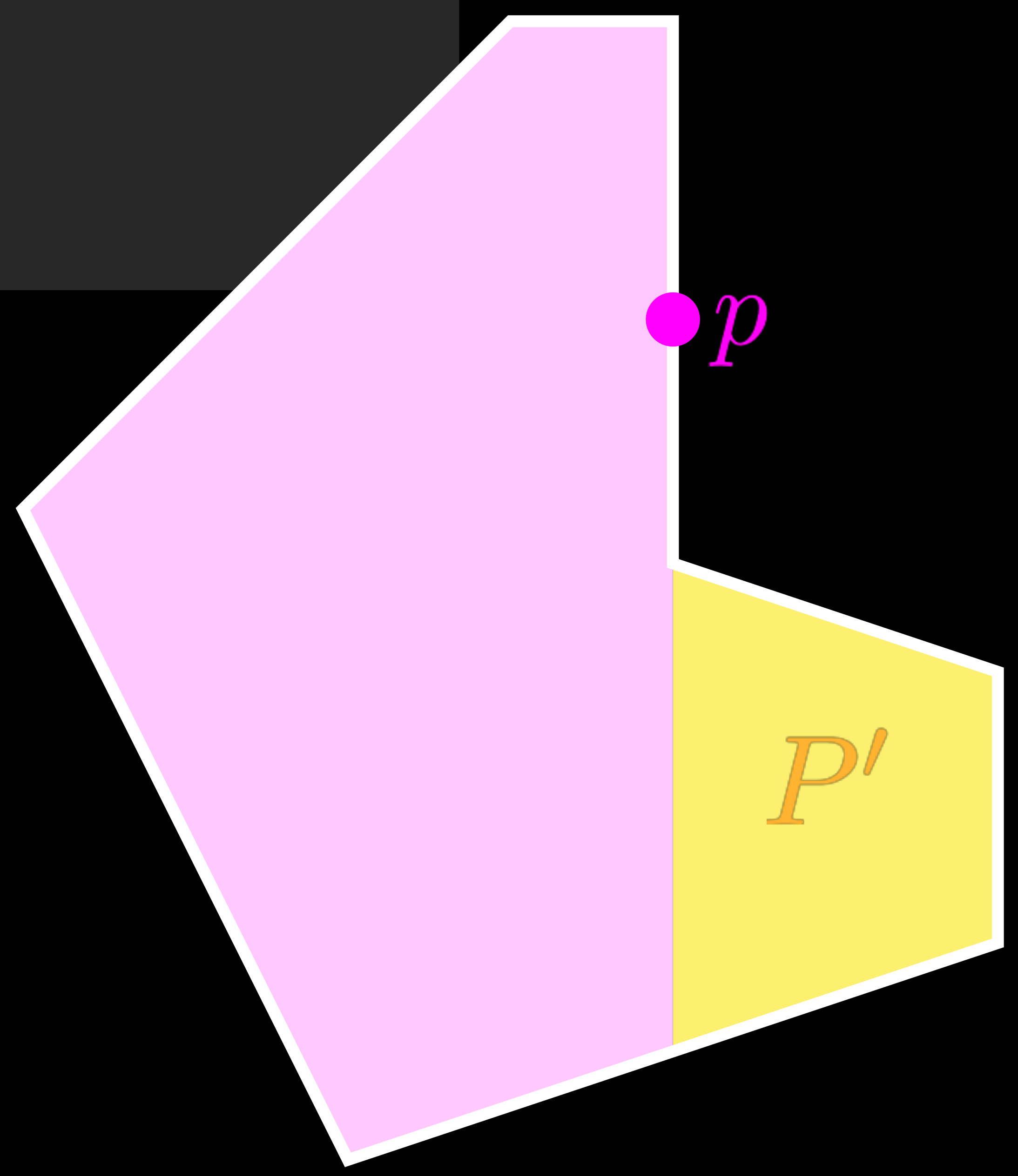

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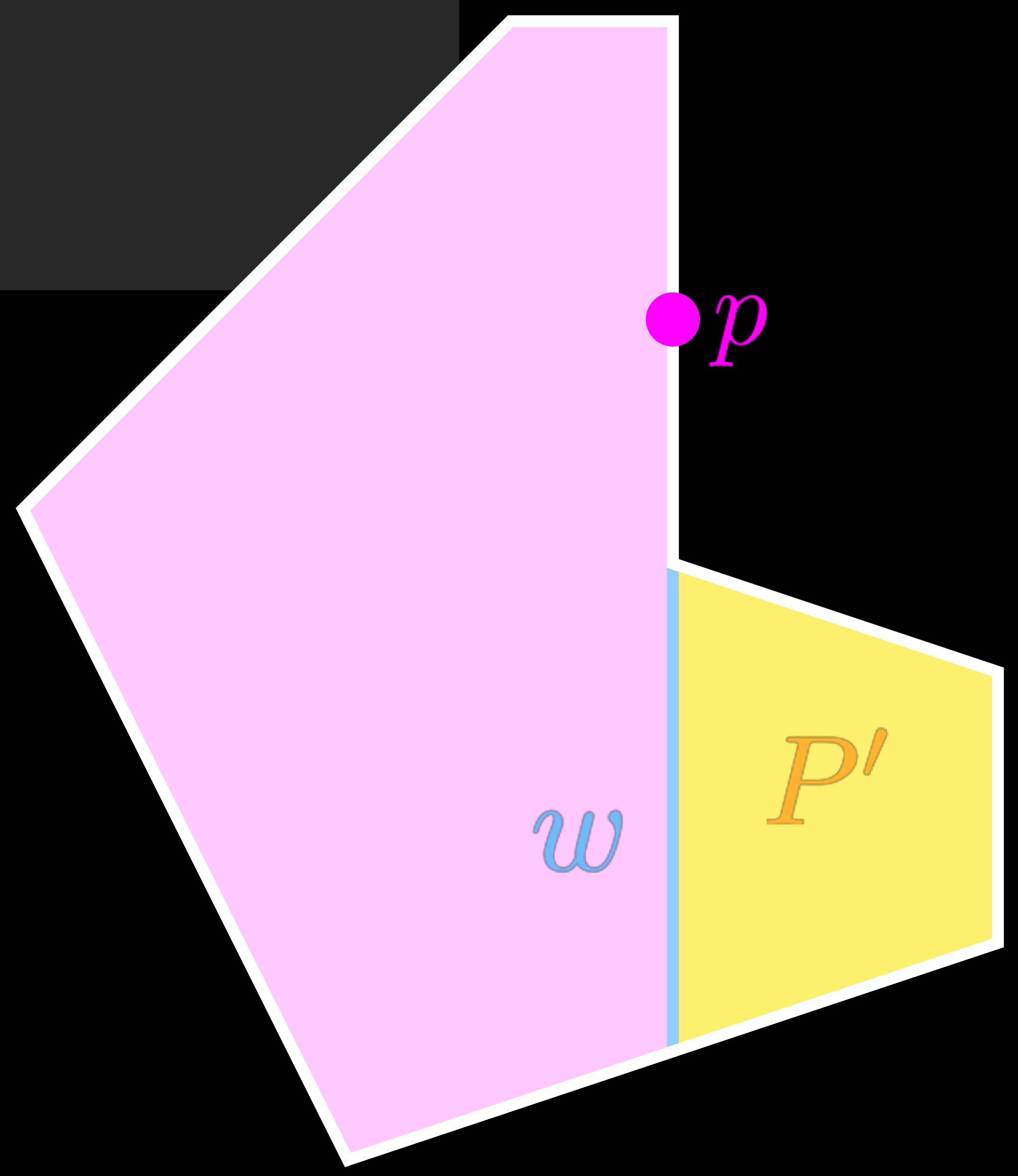
Extend window w of the pocket into a maximal line segment ℓ



Subpolygon of P, no point visible from p

Line segment separating P' and $P \setminus P'$

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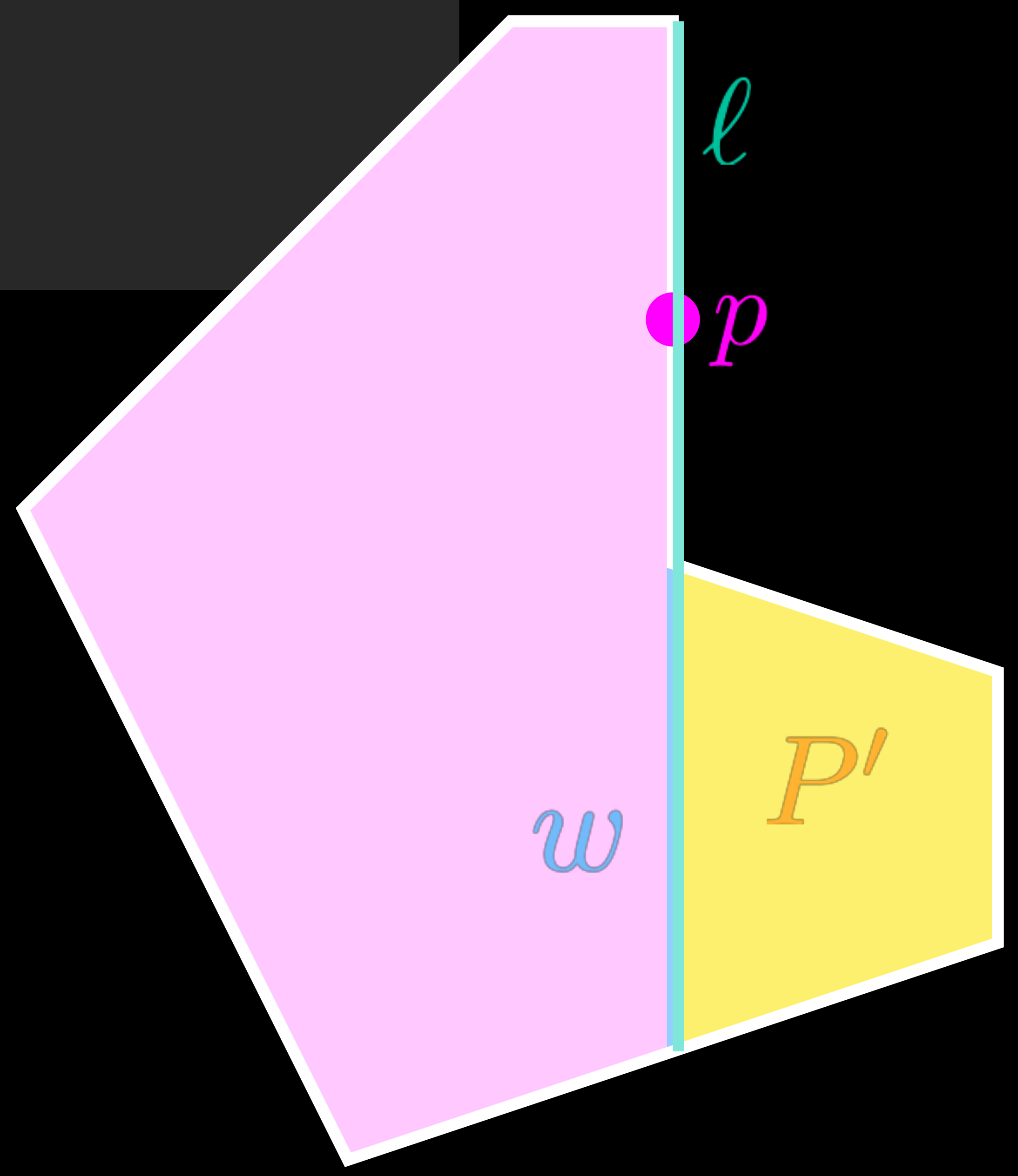

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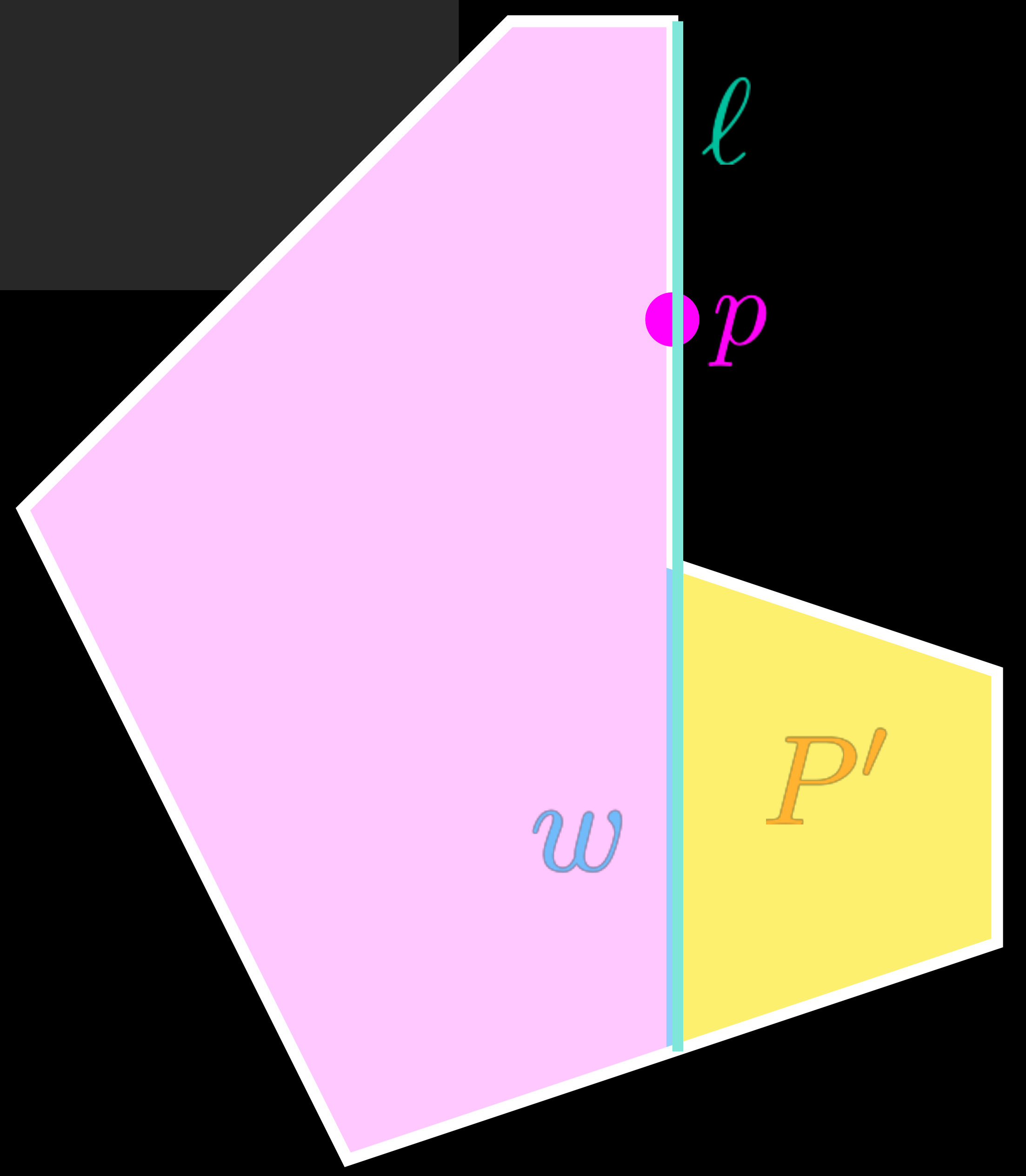

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We know: $p \in \ell$ ➜ Segment $\ell \backslash w$:

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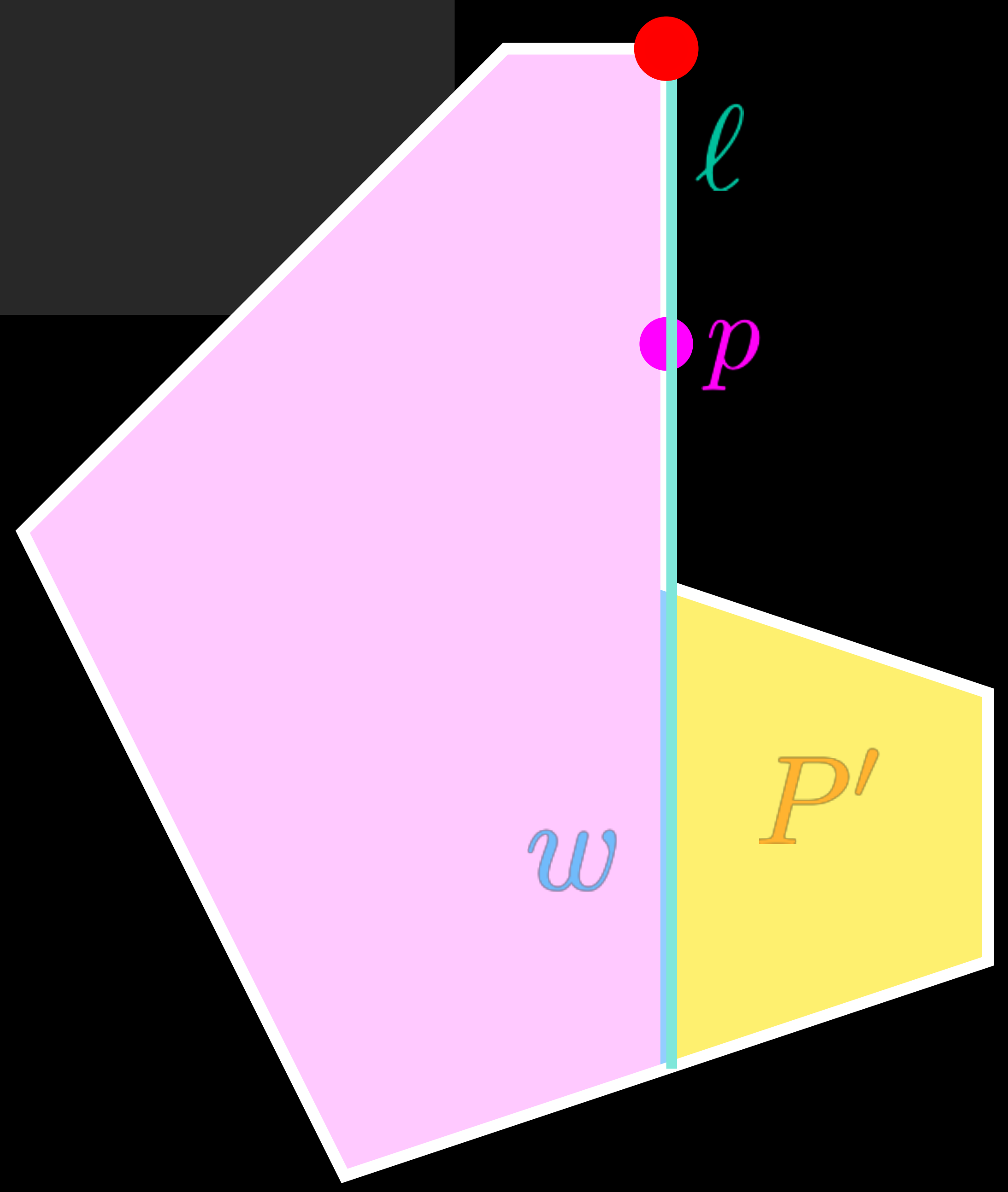

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We know: $p \in \ell$ ➜ Segment $\ell \setminus w$:

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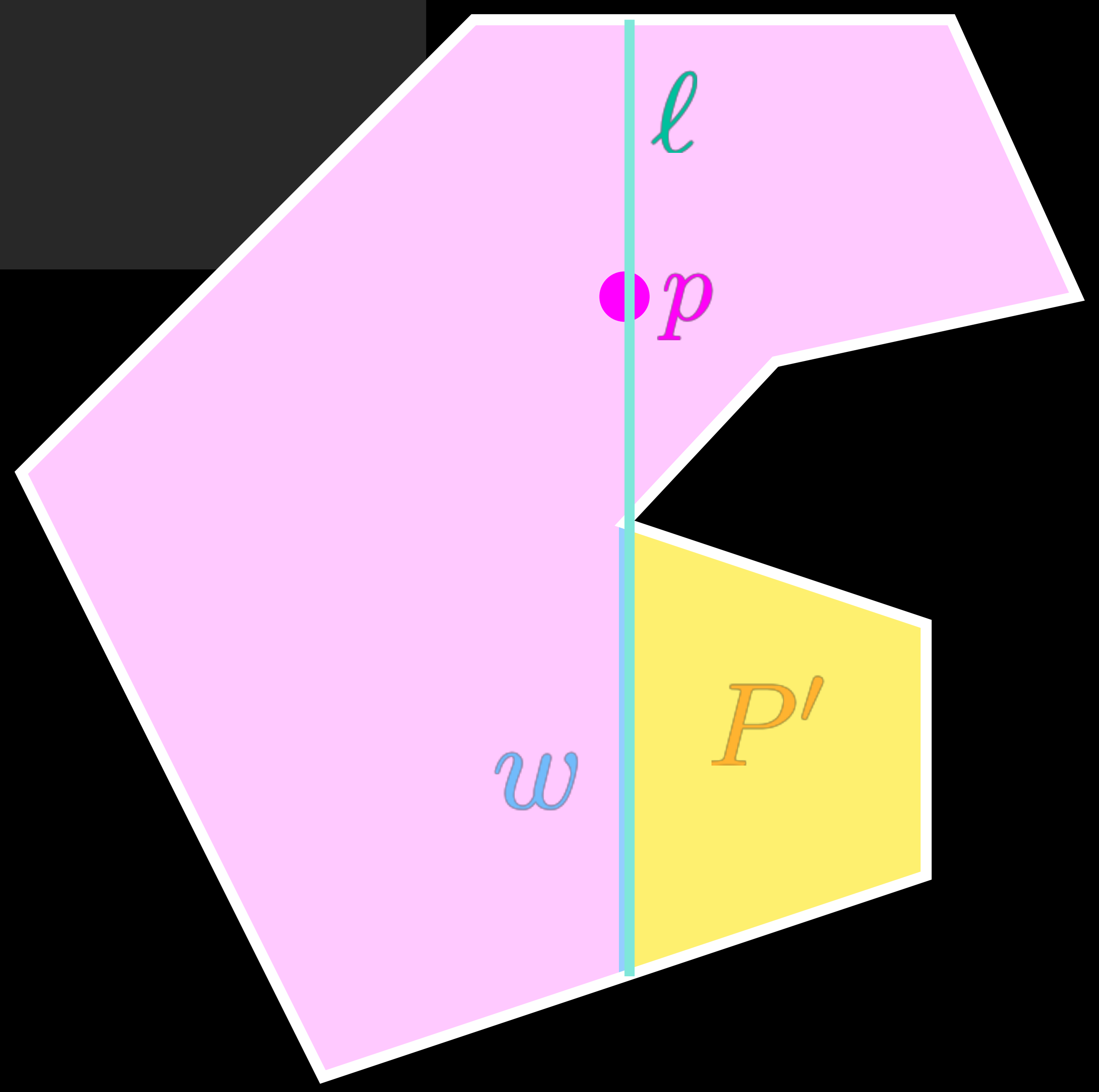

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- Splits P into at least two subpolygons.

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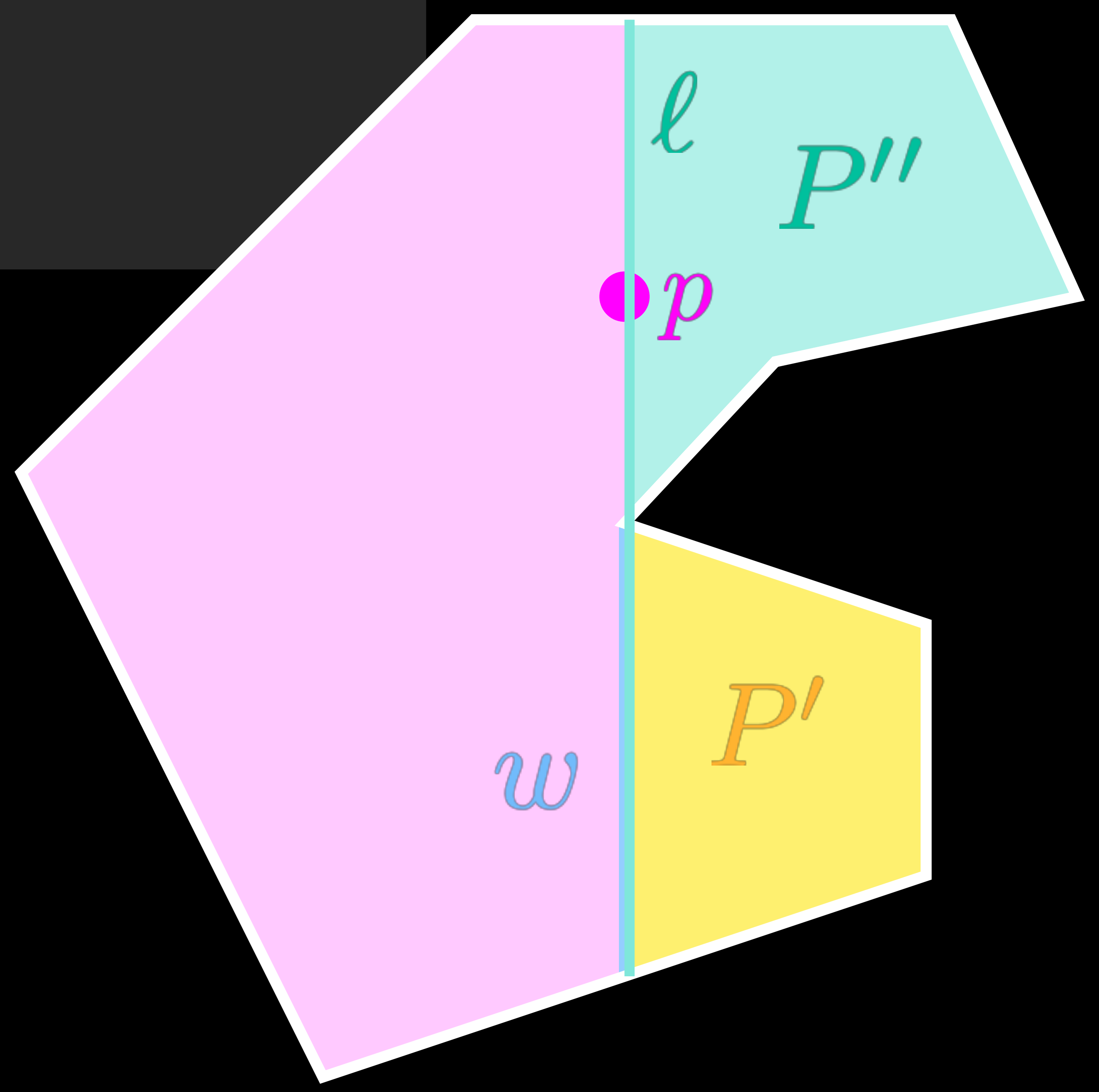

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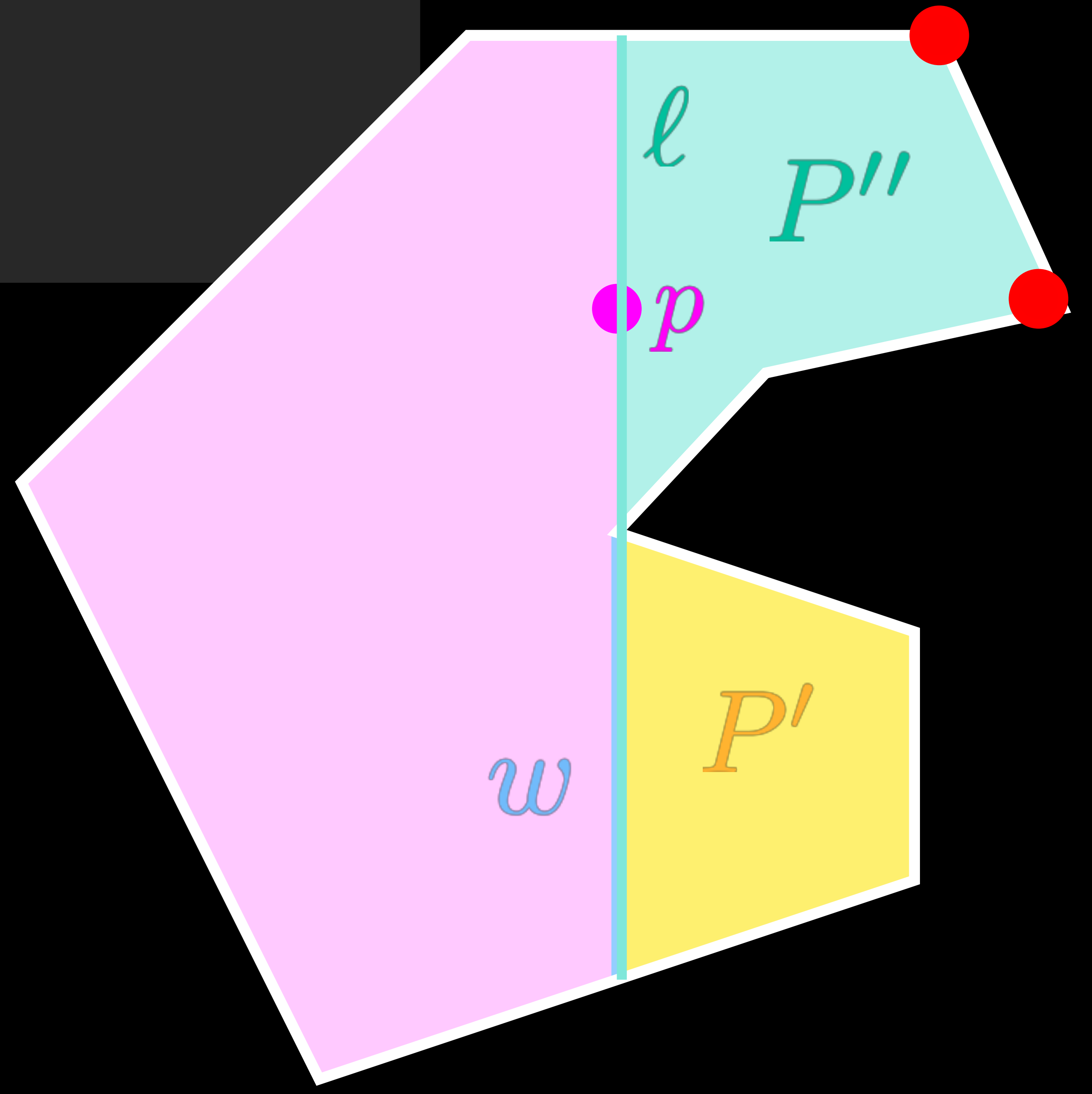

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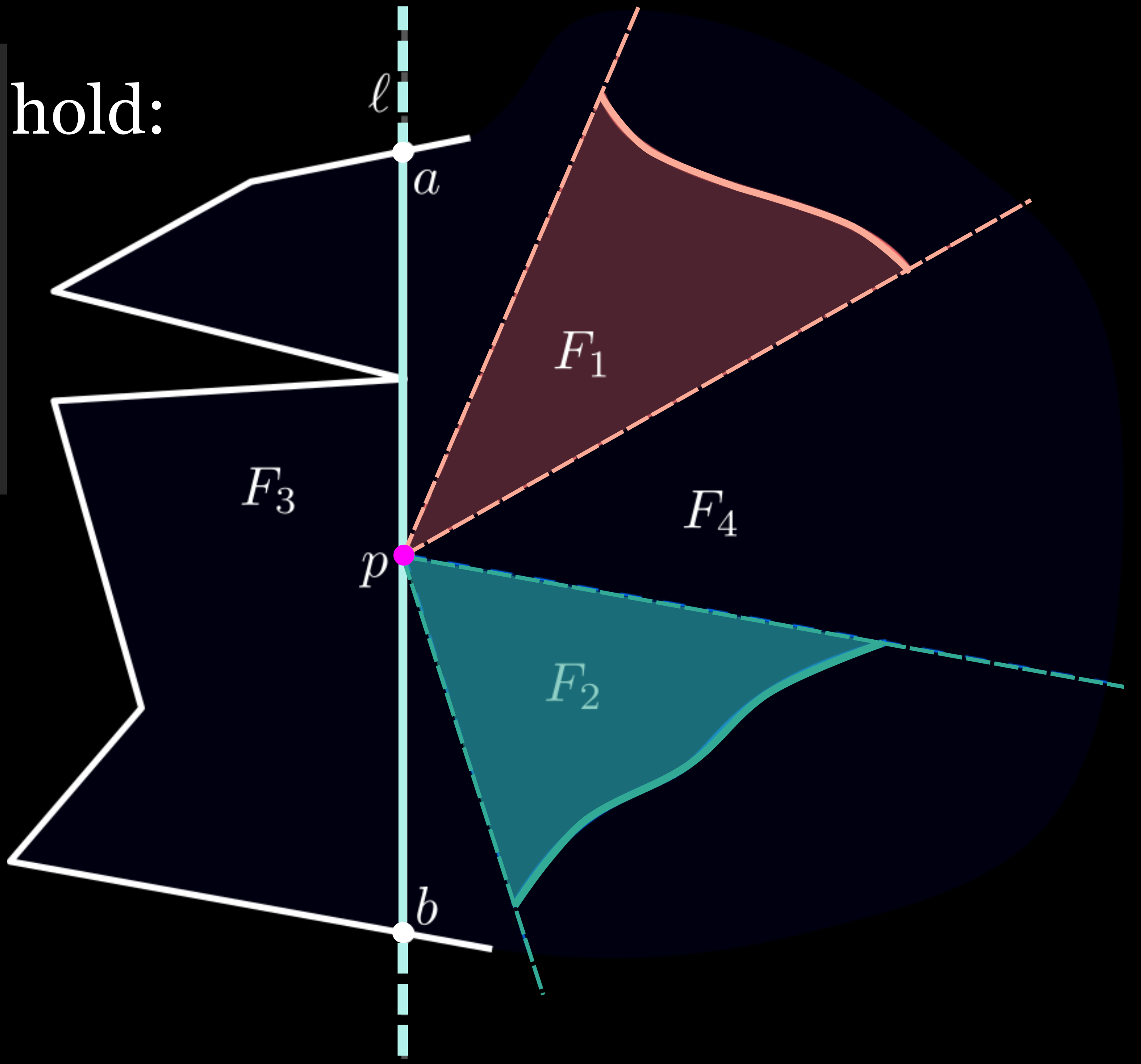

Proof ctd: (1)-(3) $\Rightarrow$ W_1 and W_2 are segment watchman routes

Consider $p \in P$

Both W_1 and W_2 are watchman routes

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Consider two maximal wedges defined by angles from which p views W_i— F_i

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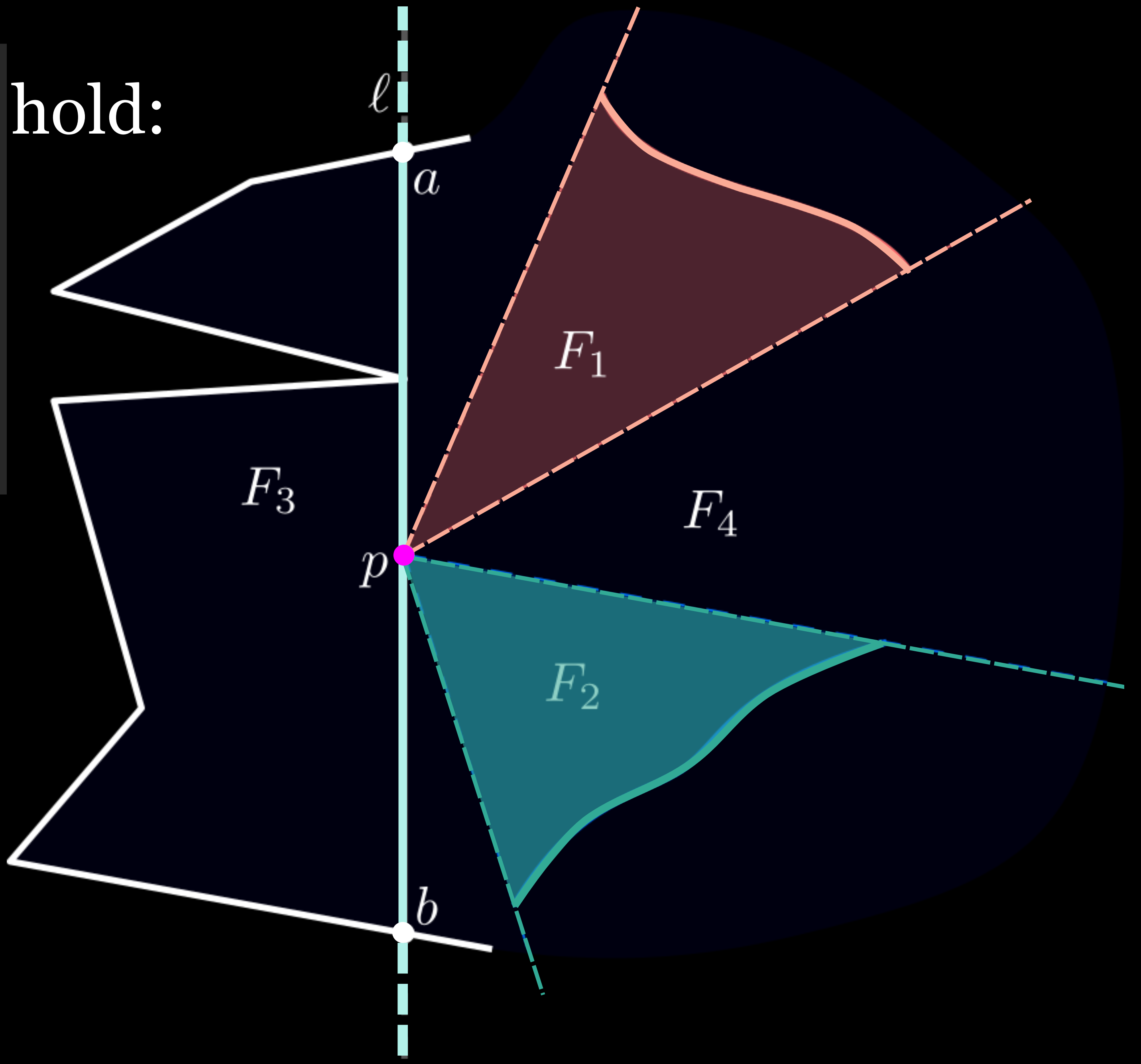

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Each of F_1 and F_2 covers either 360° or less than 180° (p within RCH and routes relatively convex):

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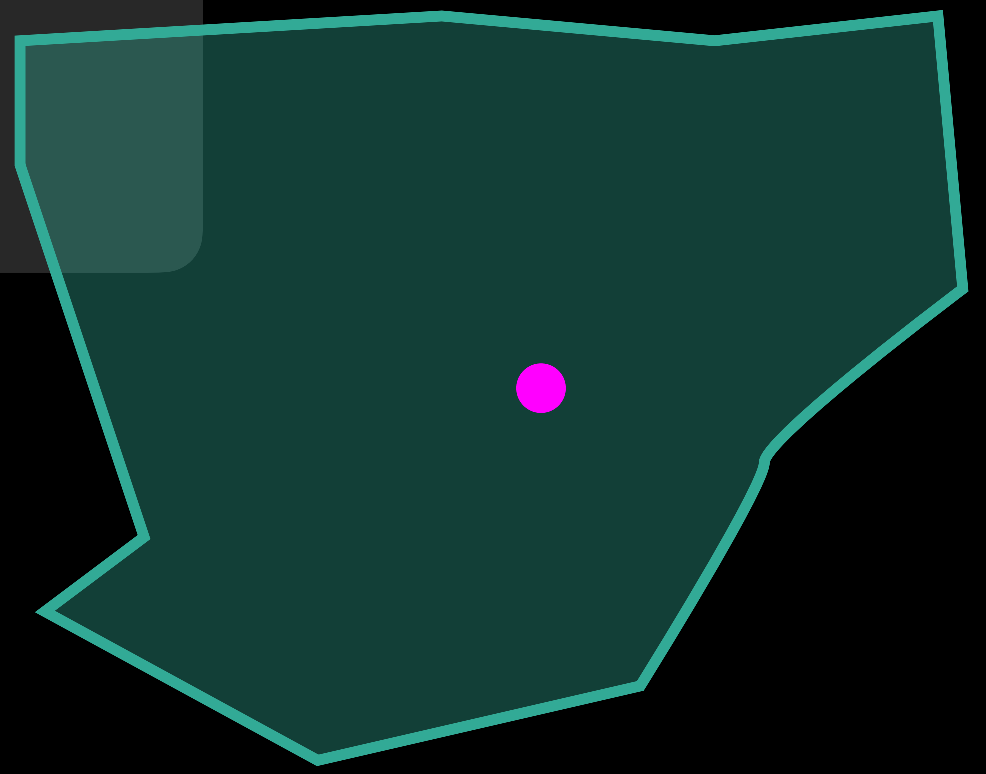

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- One (F_2) covers 360°: let $w_1 \in W_1$ be a point that sees p

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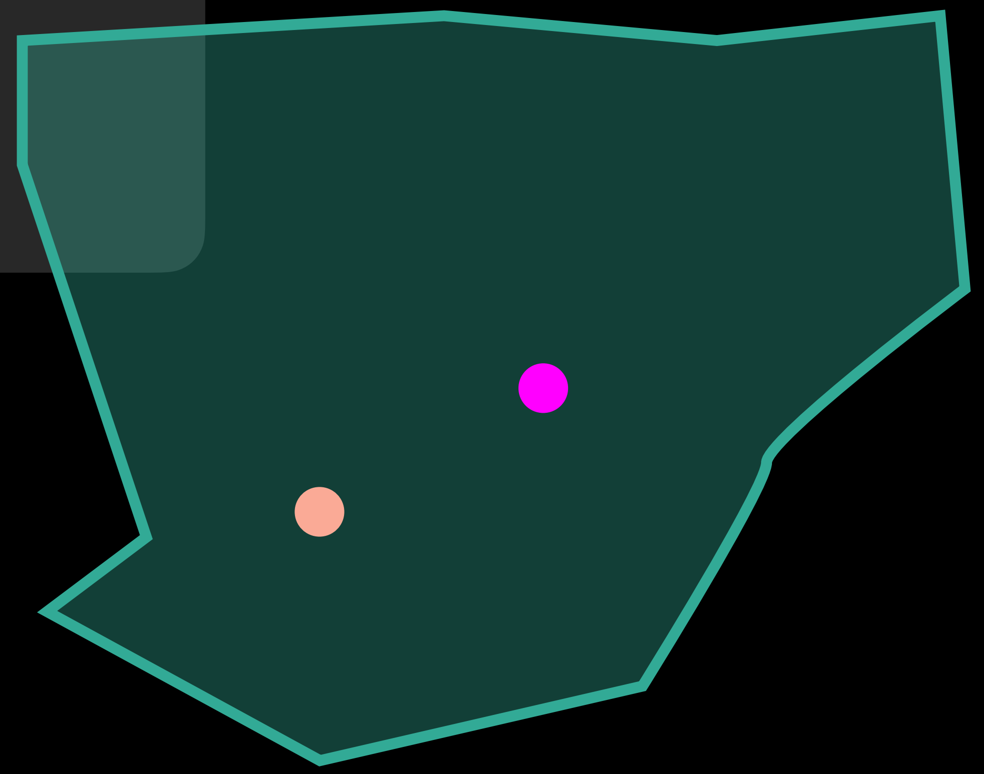

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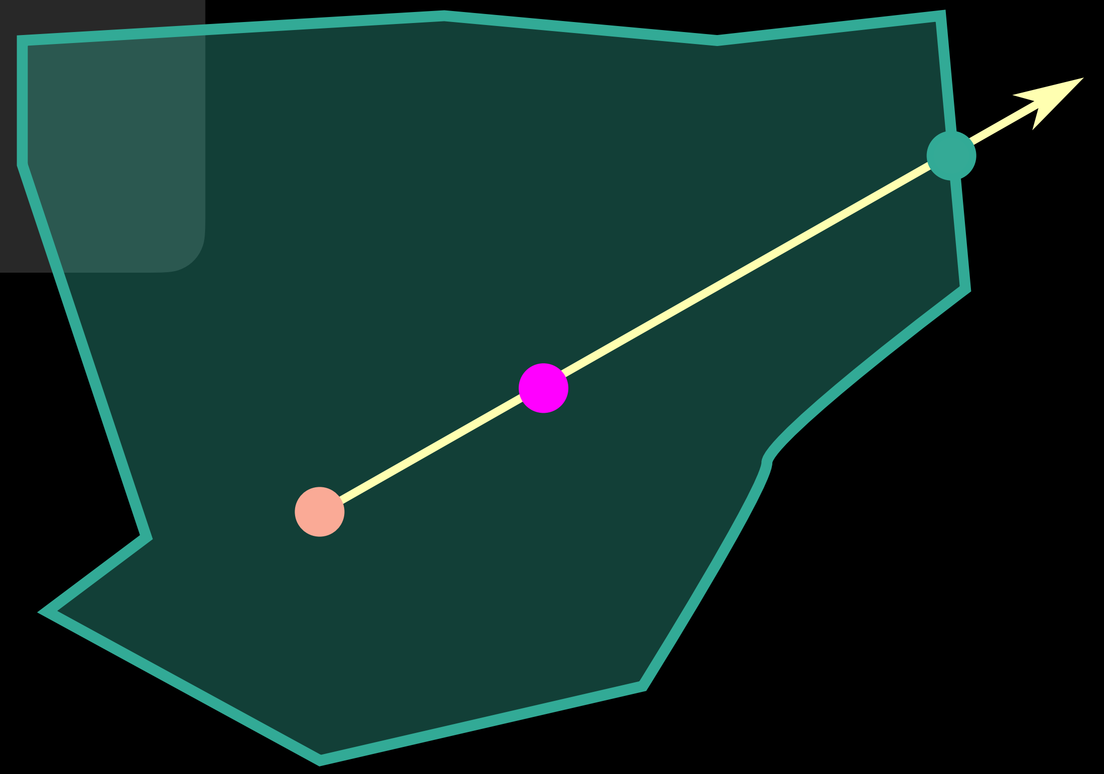

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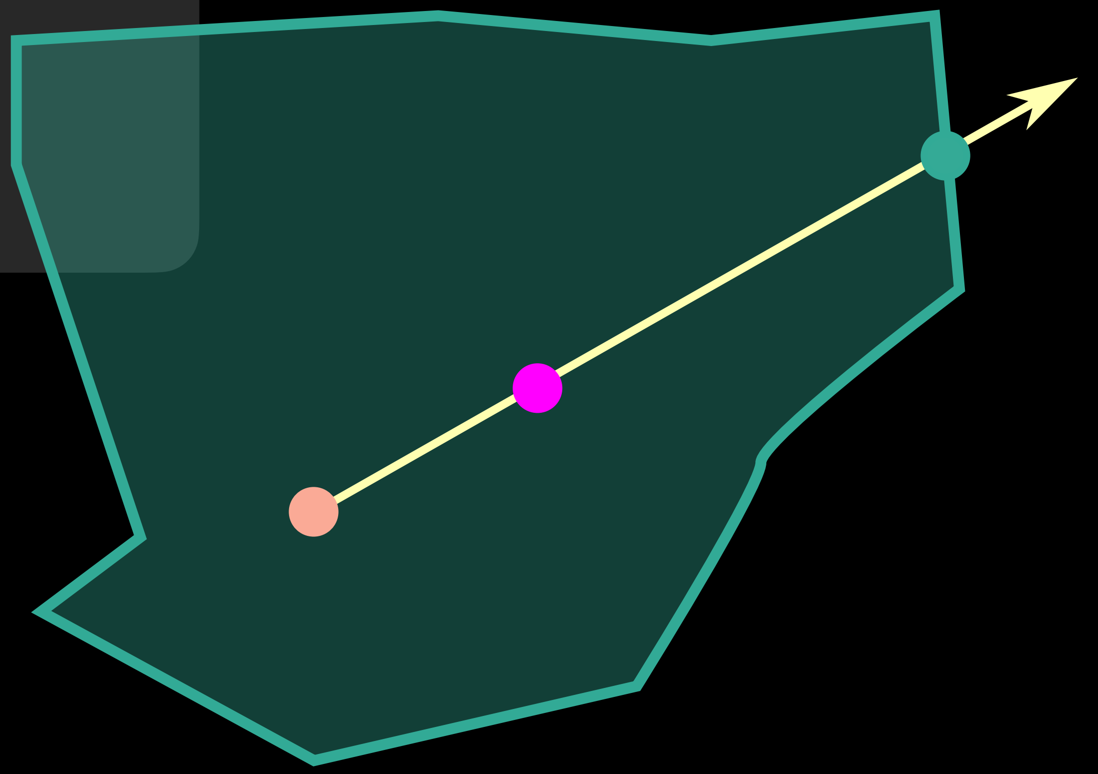

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➡ p is segment guarded by $\overline{w_1 w_2}$

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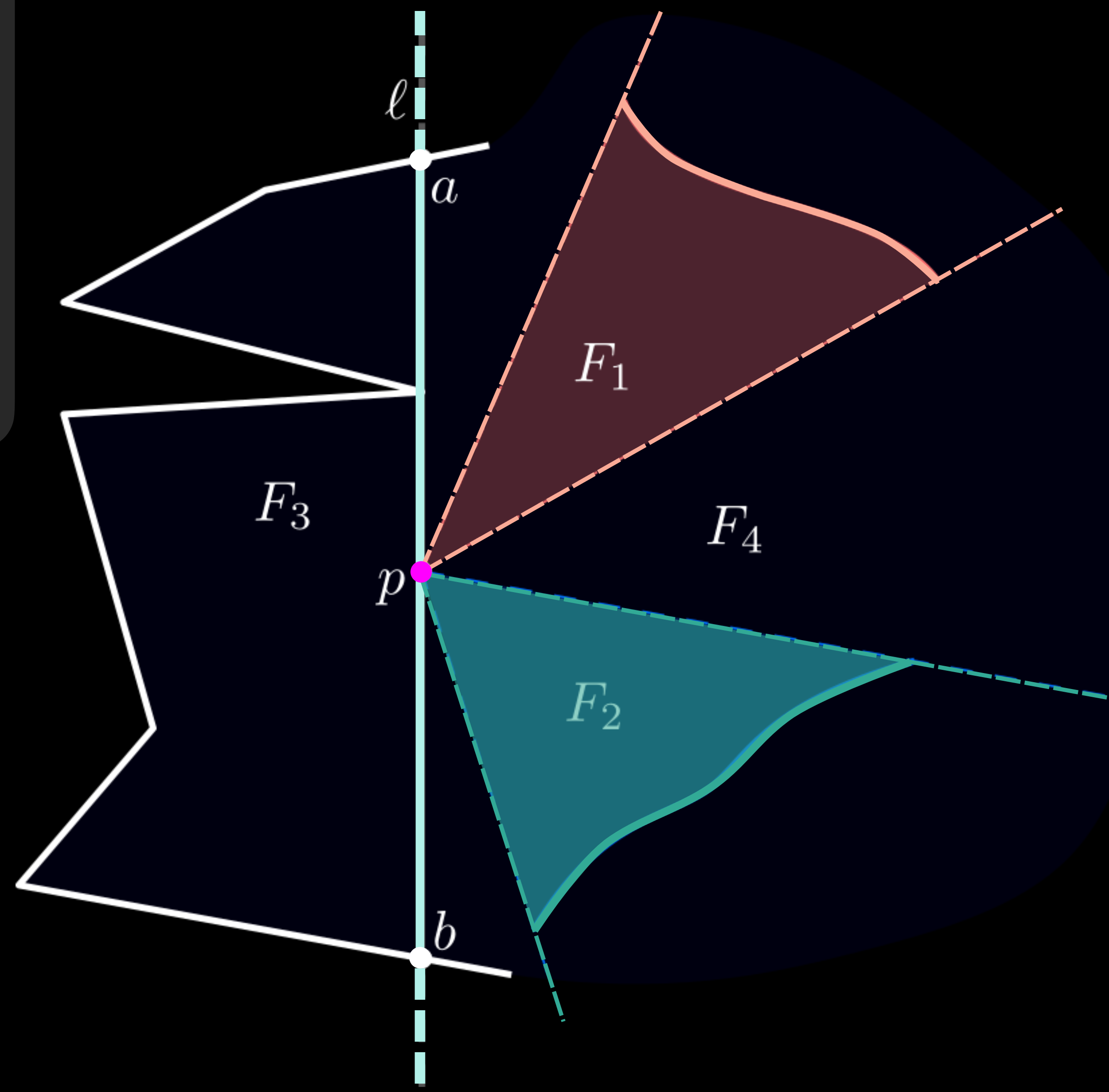

Proof ctd: (1)-(3) $\Rightarrow$ W_1 and W_2 are segment watchman routes

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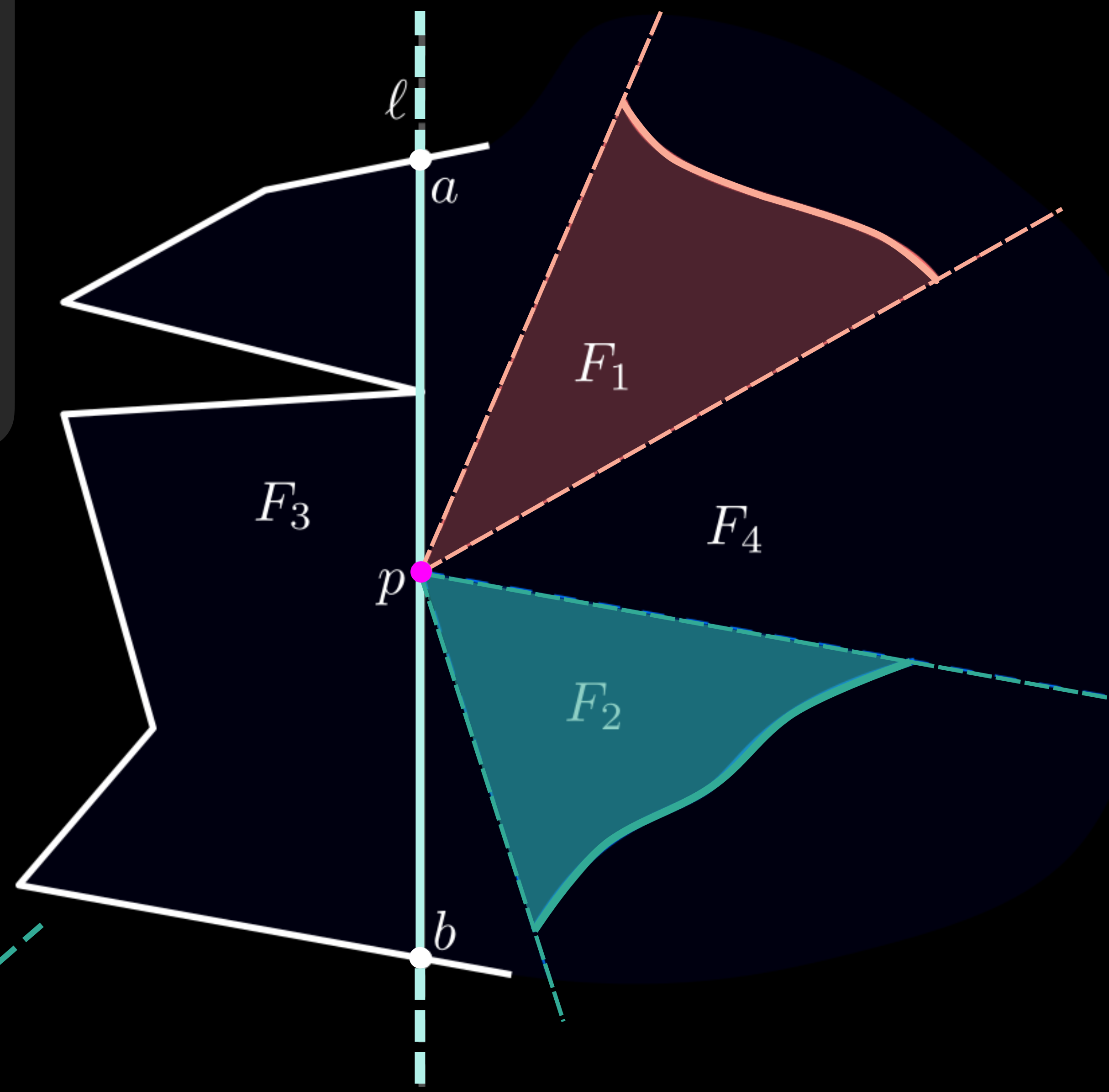

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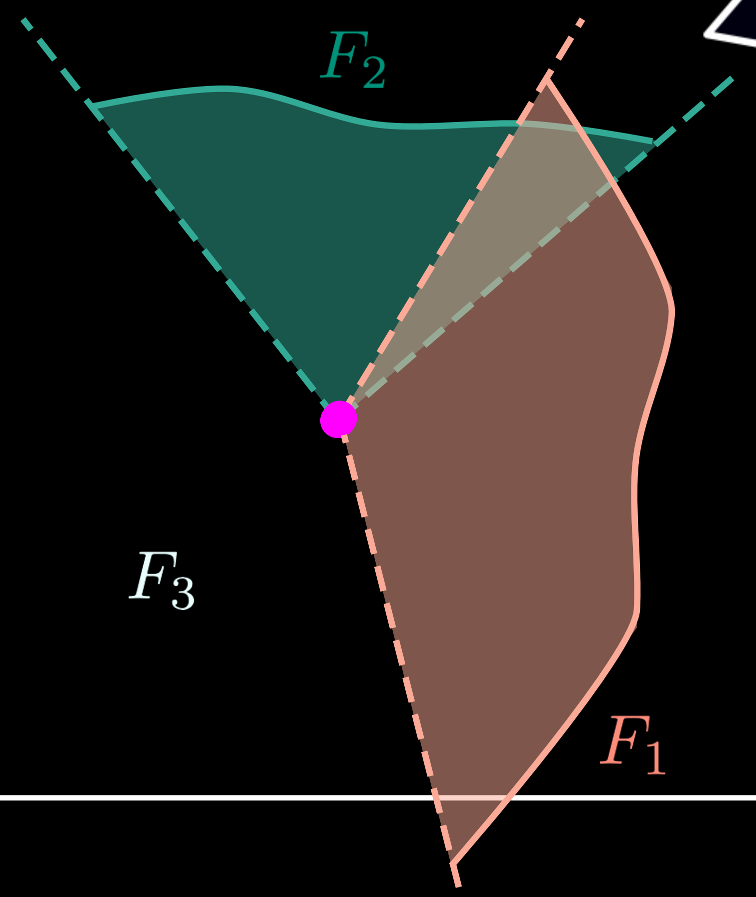

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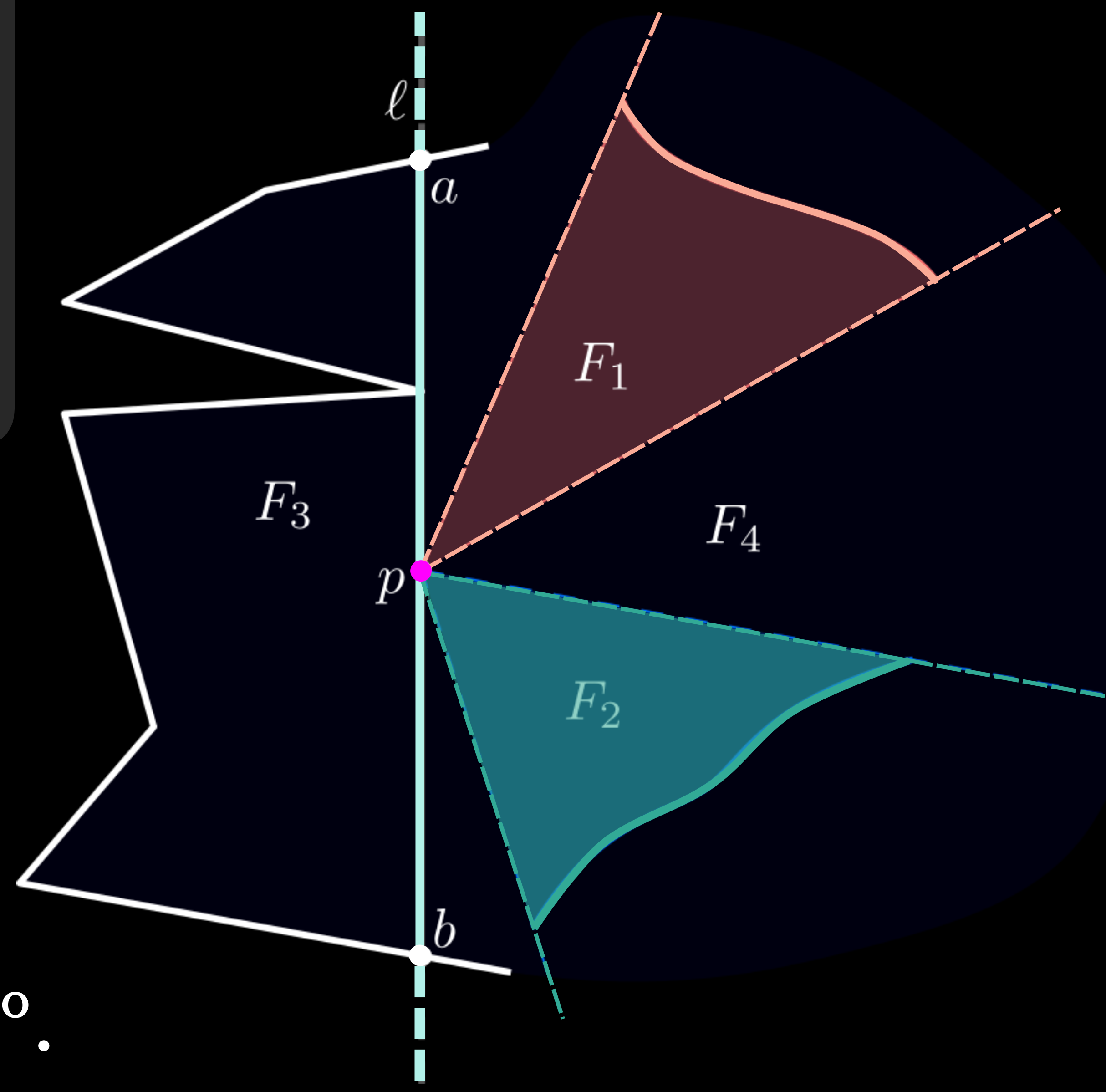

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Claim: F_3 or F_4 cannot cover more than 180°. W.l.o.g. assume F_3 covers more than 180°.

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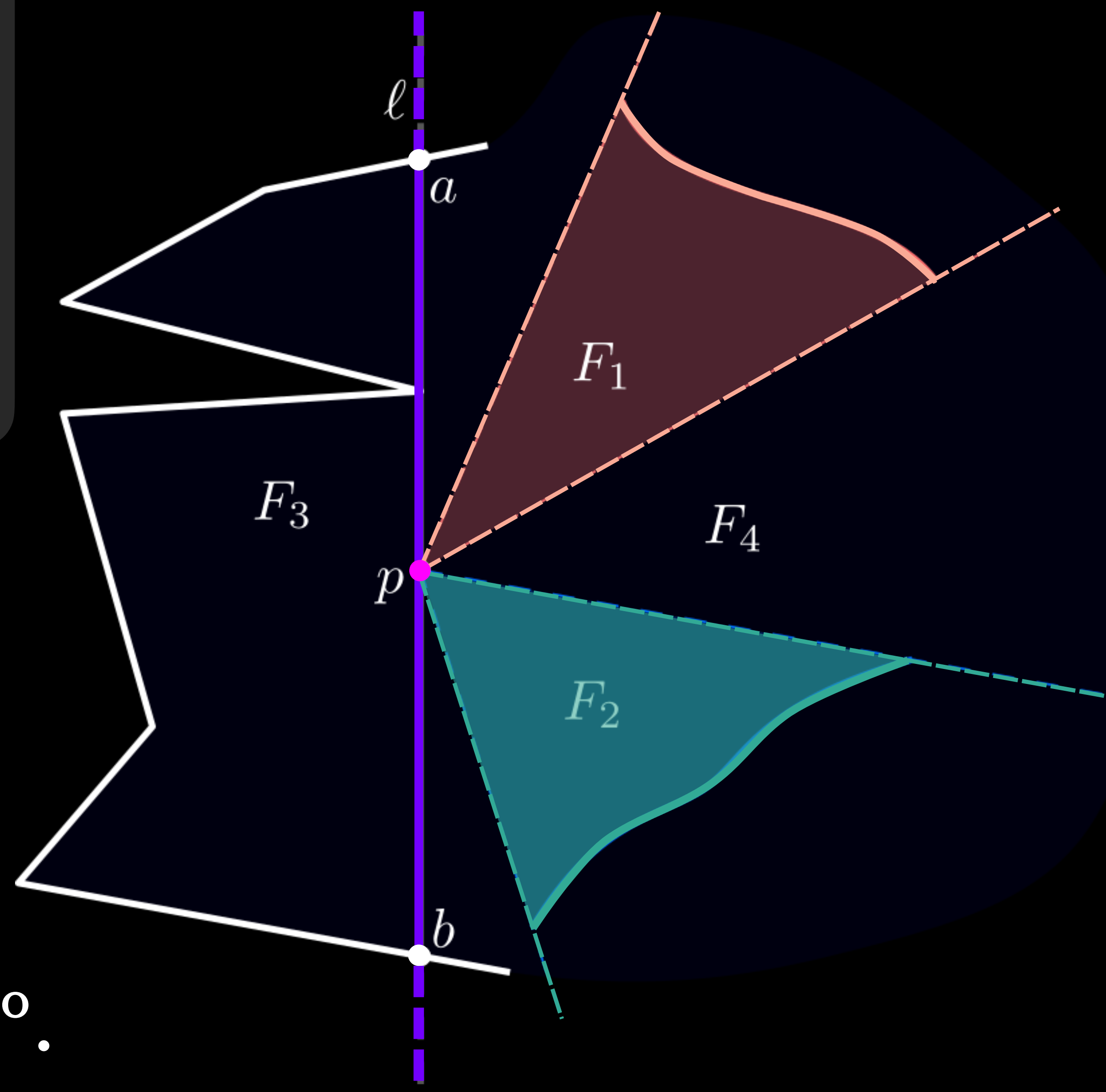

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  F_3 (and maybe F_4): maximal wedge(s), such that for each ray in F_3 (F_4), there is no point $w_1 \in W_1$ and $w_2 \in W_2$ in that direction that p sees

Claim: F_3 or F_4 cannot cover more than 180°. W.l.o.g. assume F_3 covers more than 180°.

Line ℓ through p in F_3 that does not contain edge of P, and F_1, F_2 on right side of ℓ

*For any two points inside the region enclosed by the route, their shortest path is also contained within the enclosed region

Sufficient Conditions: The "Conditions Lemma"

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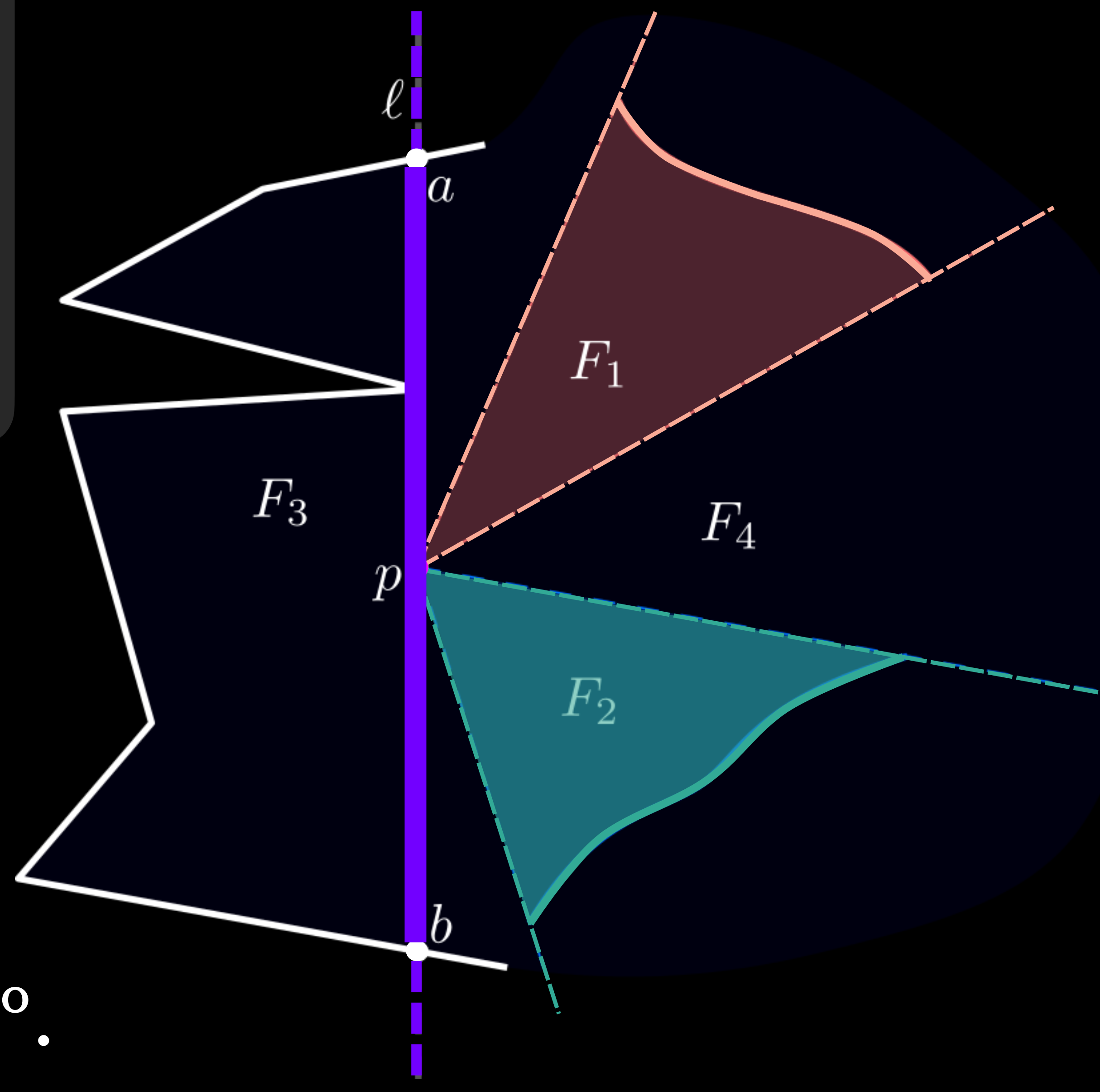

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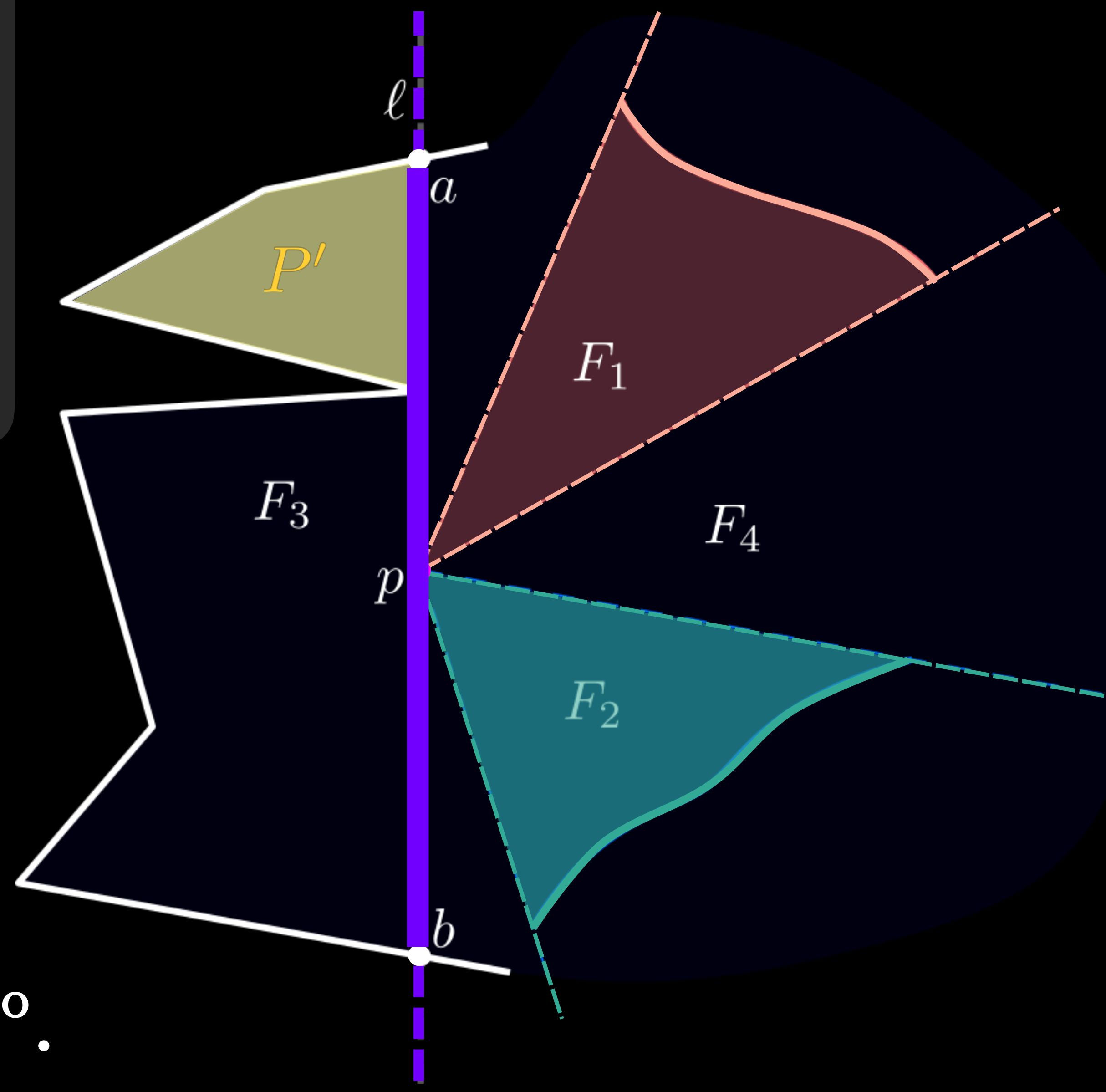

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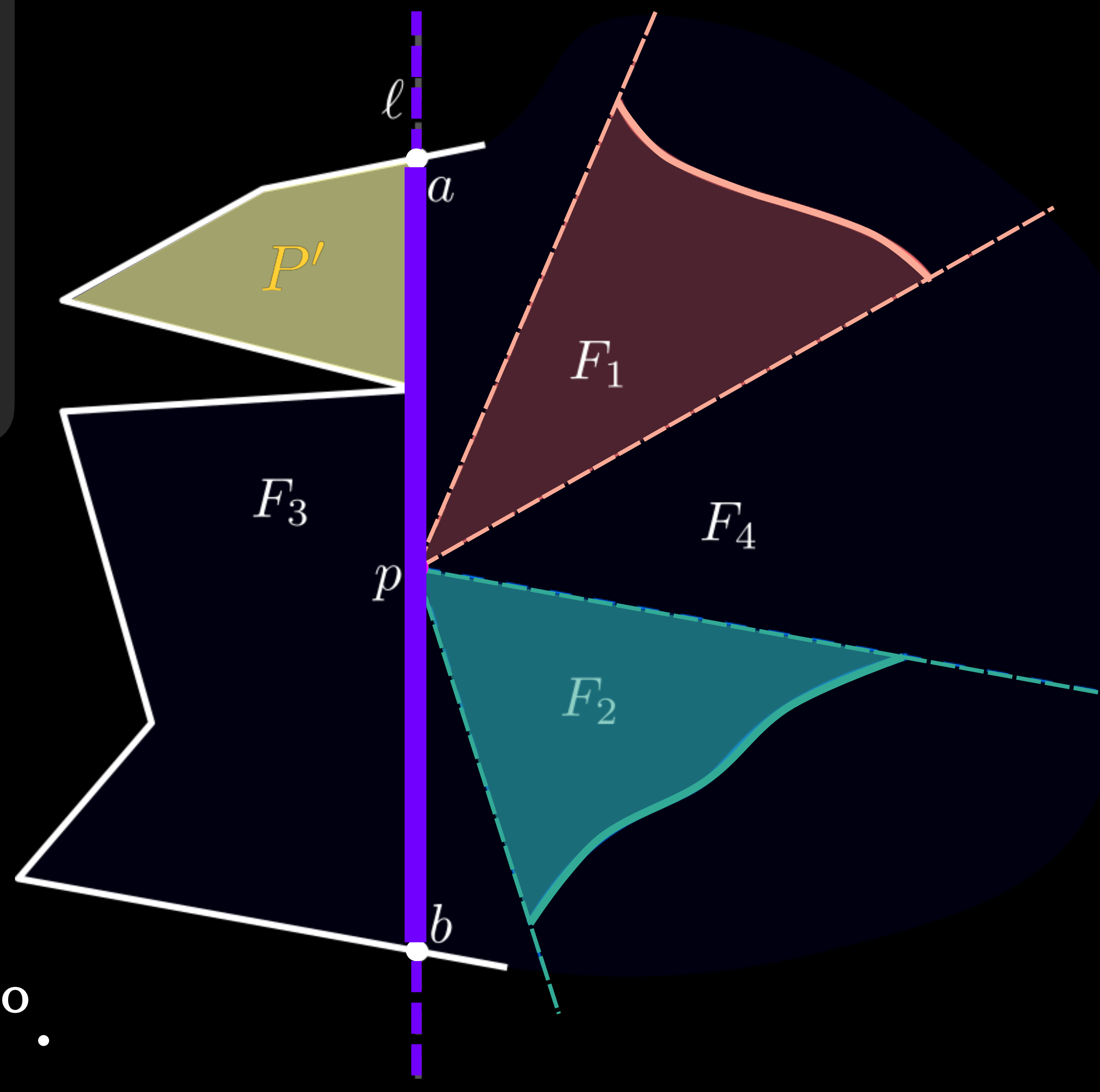

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P' must contain a convex vertex v, but no points of W_1 *and* W_2 in P' ↯

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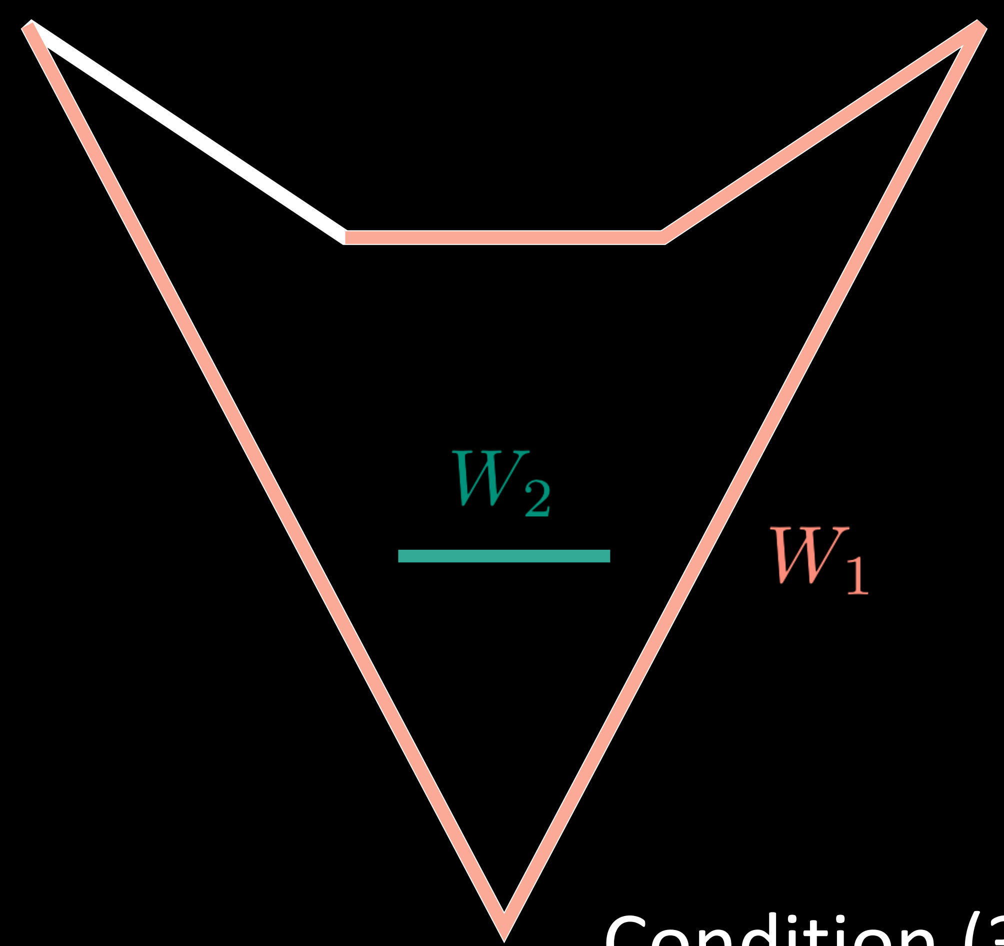
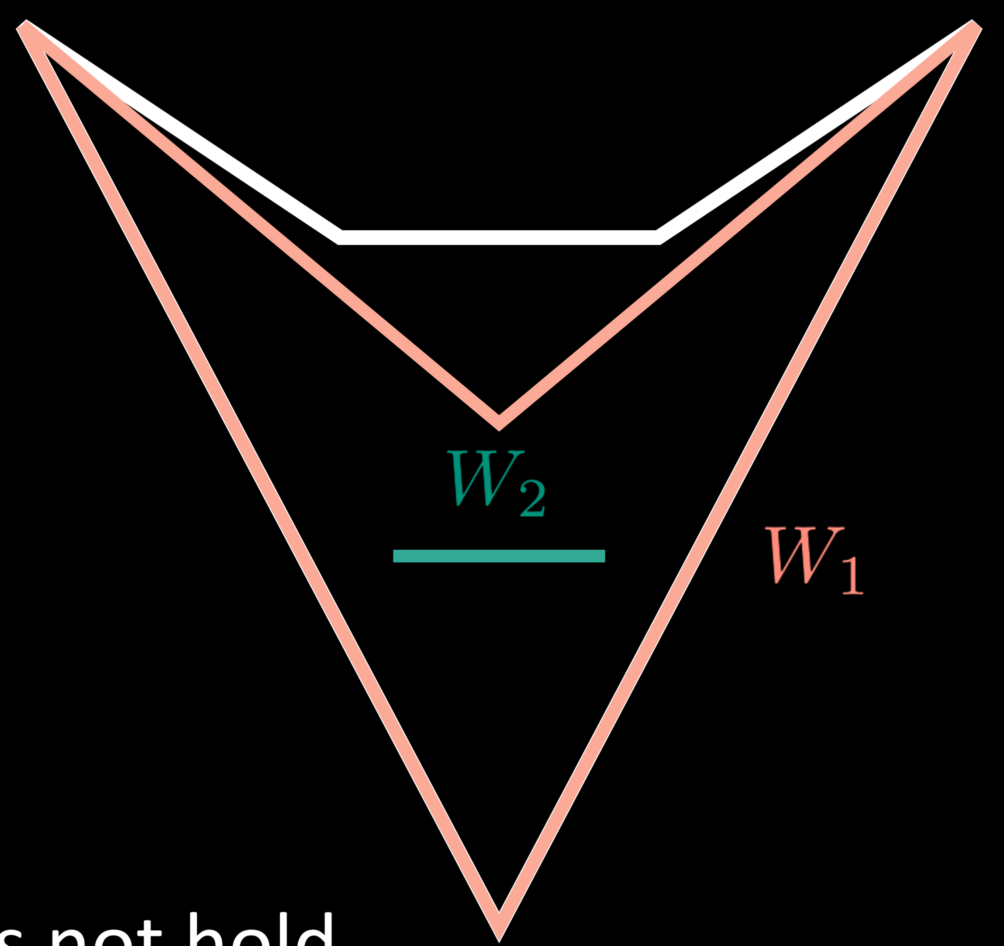
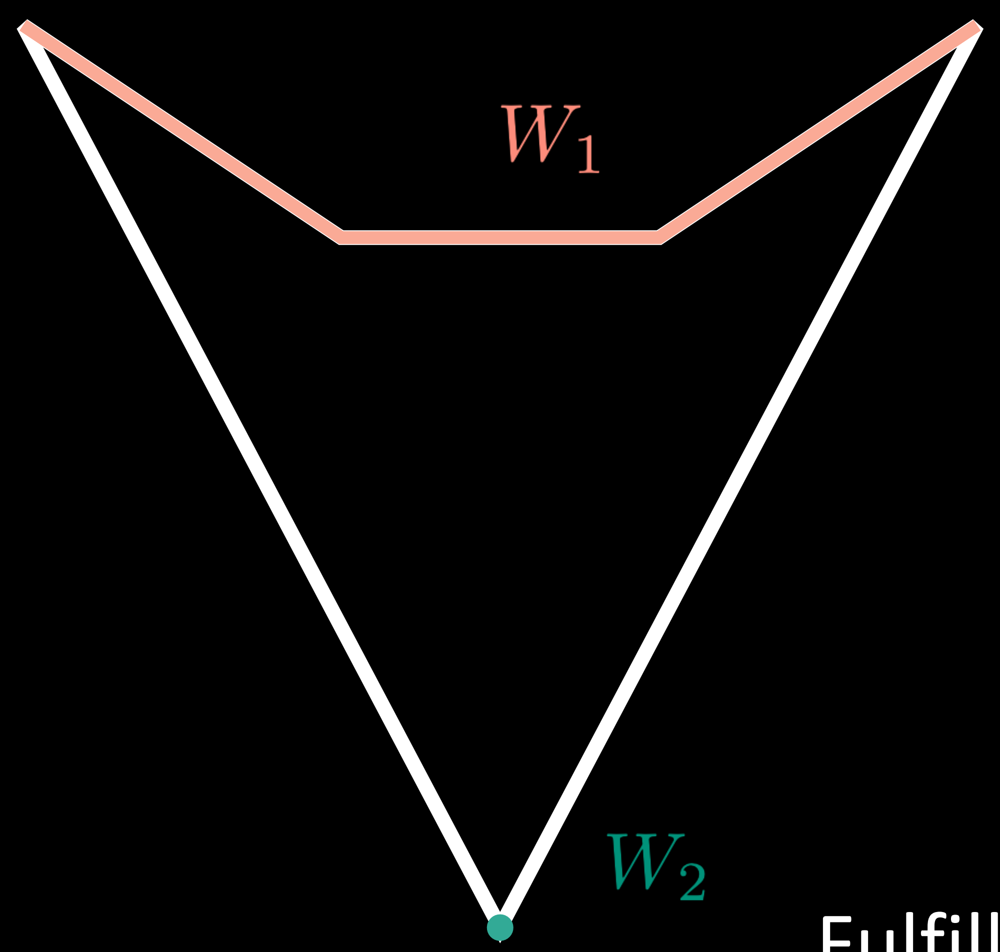
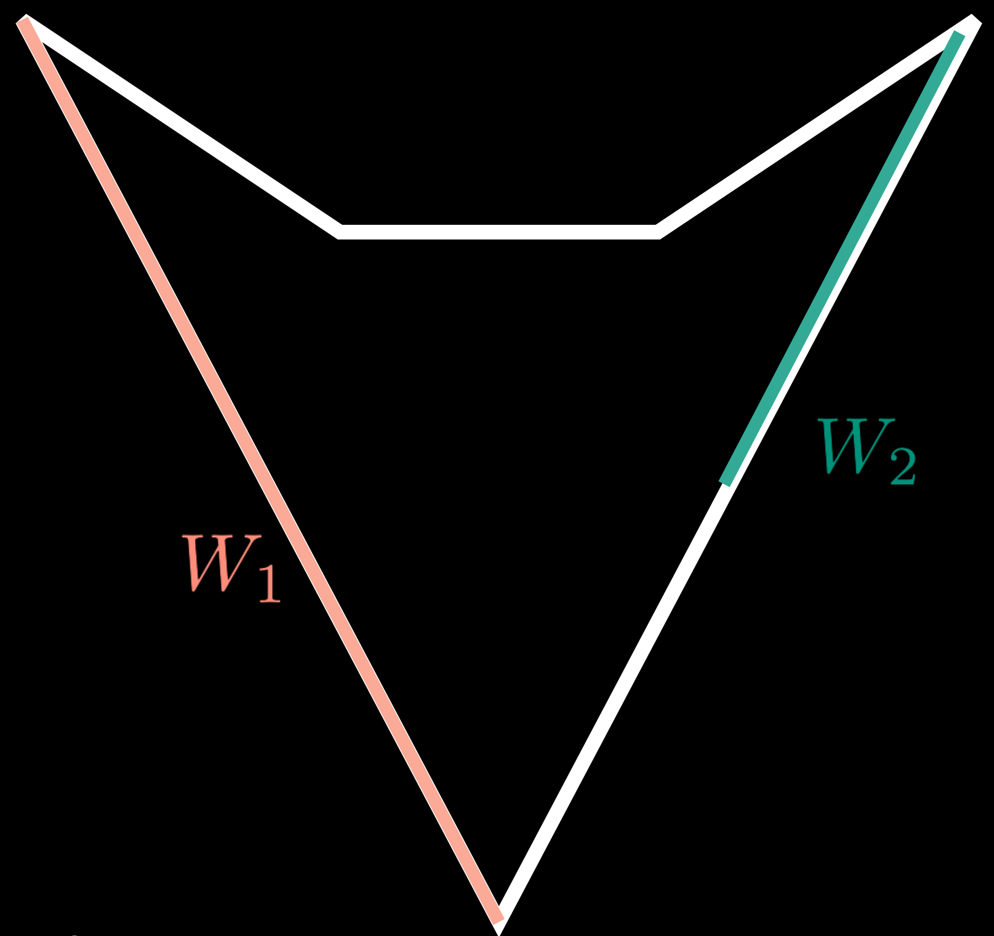




Condition (3) does not hold





Fulfill all conditions

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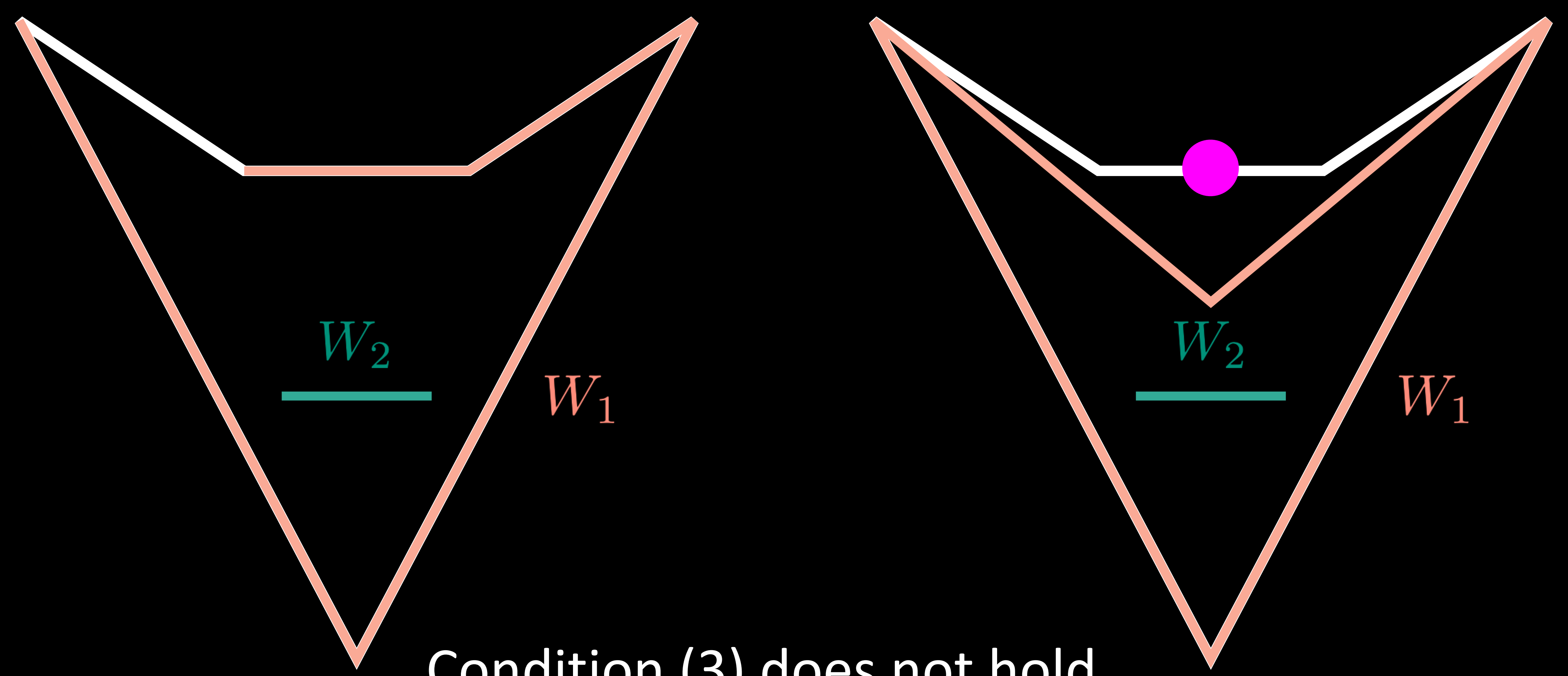

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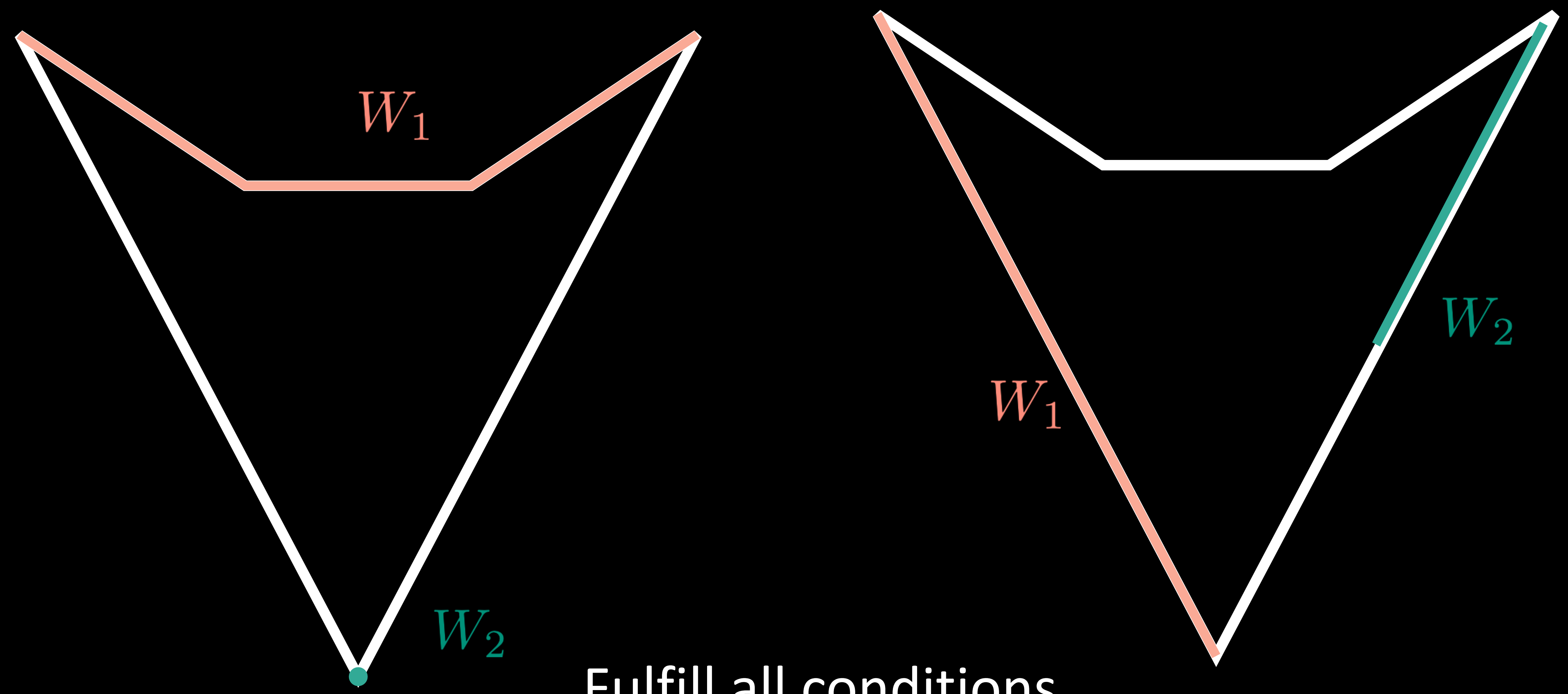

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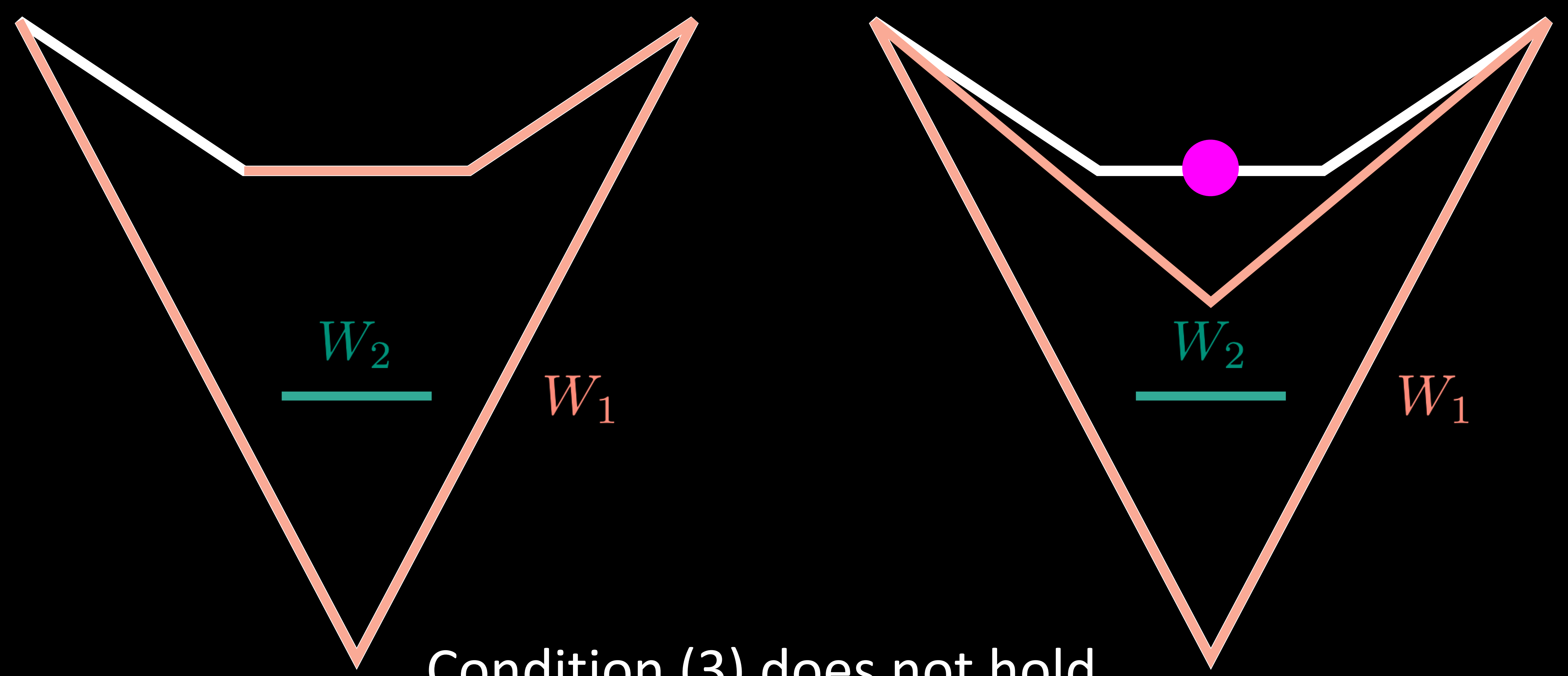
Two routes W_1 and W_2 are **optimal** segment watchman routes for P **if and only if** conditions of the lemma hold.



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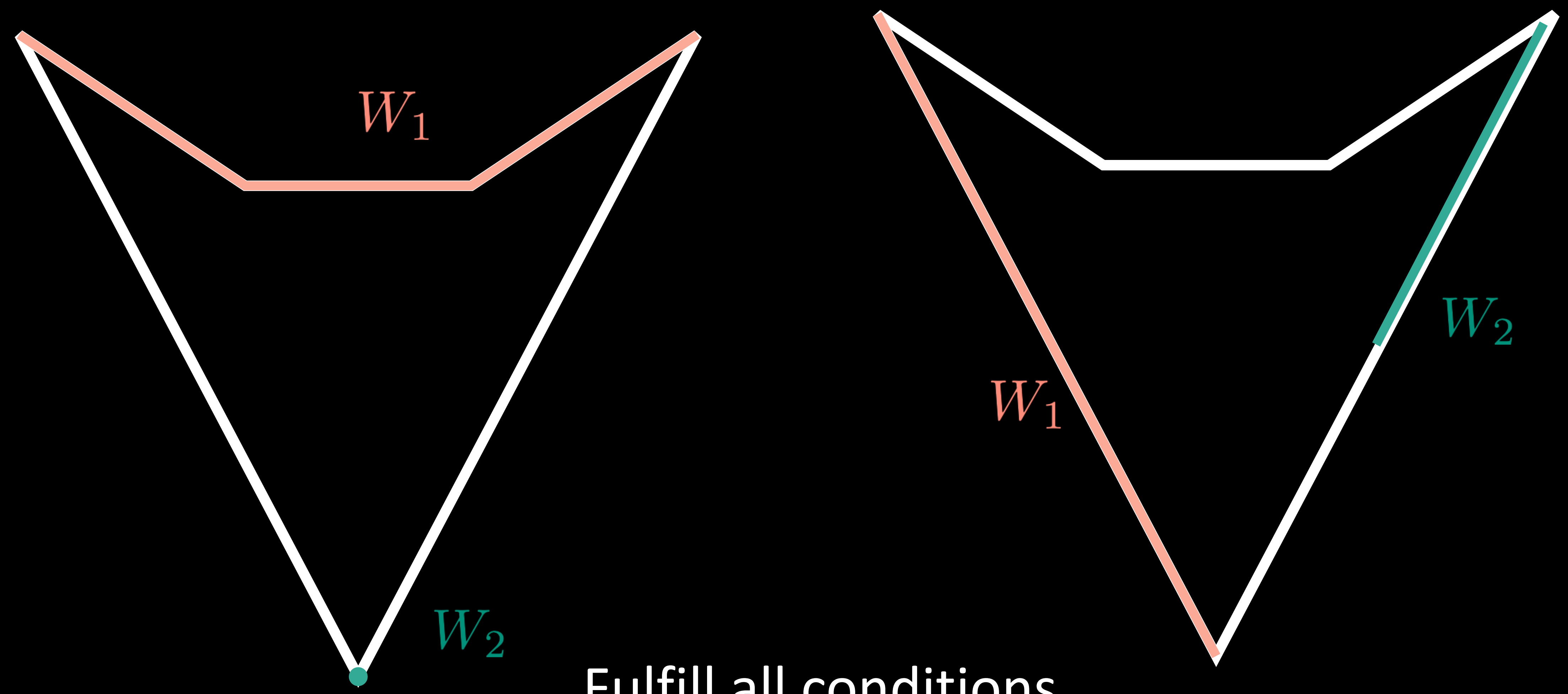

Fulfill all conditions

Our Results

Min-max objective:

- NP-hard even for simple polygons
- Polynomial-time 2-approximation algorithm
- For larger k: $(k+1)$-approximation algorithm

Min-sum objective:

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2-Approximation Algorithm

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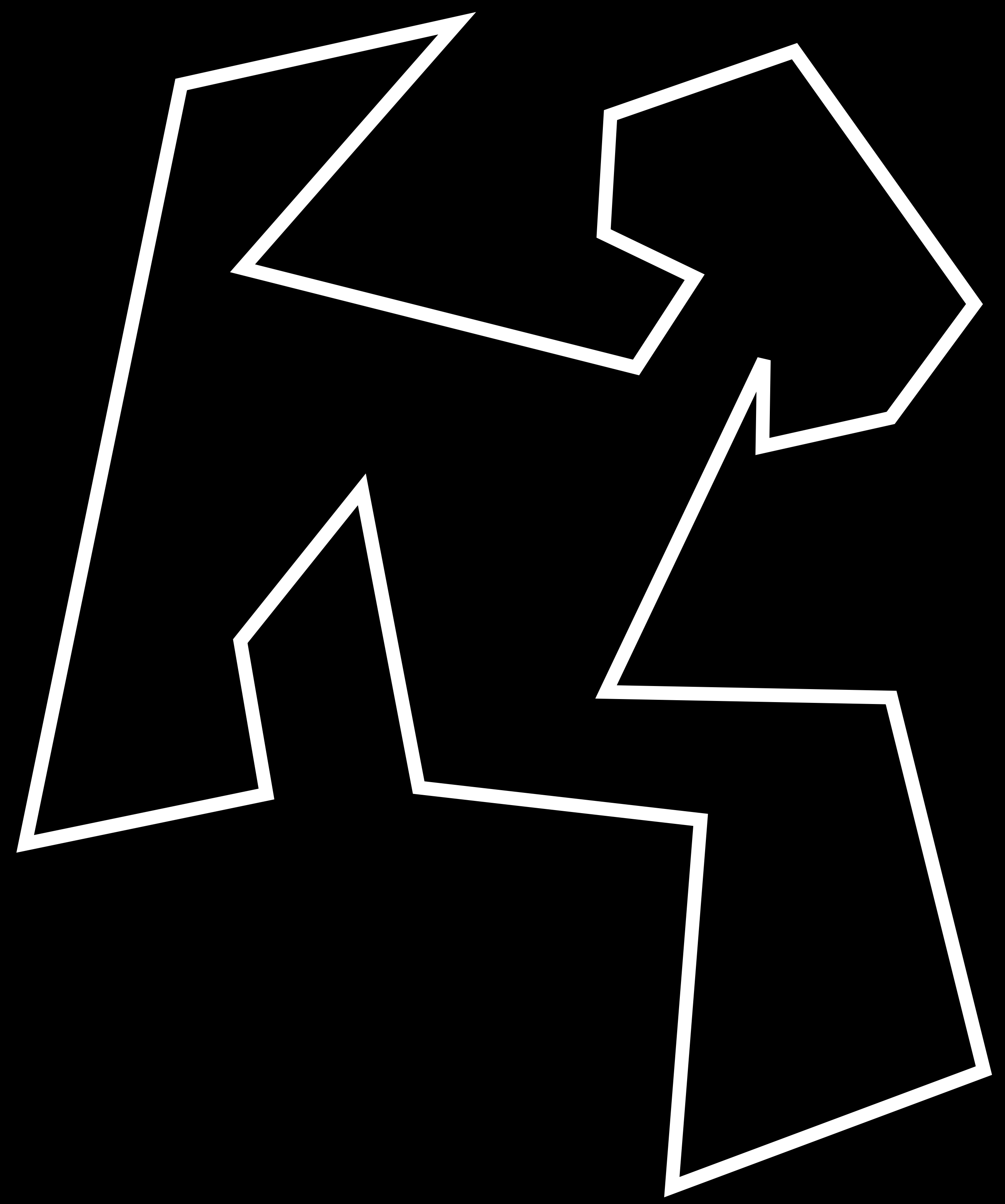
Idea:

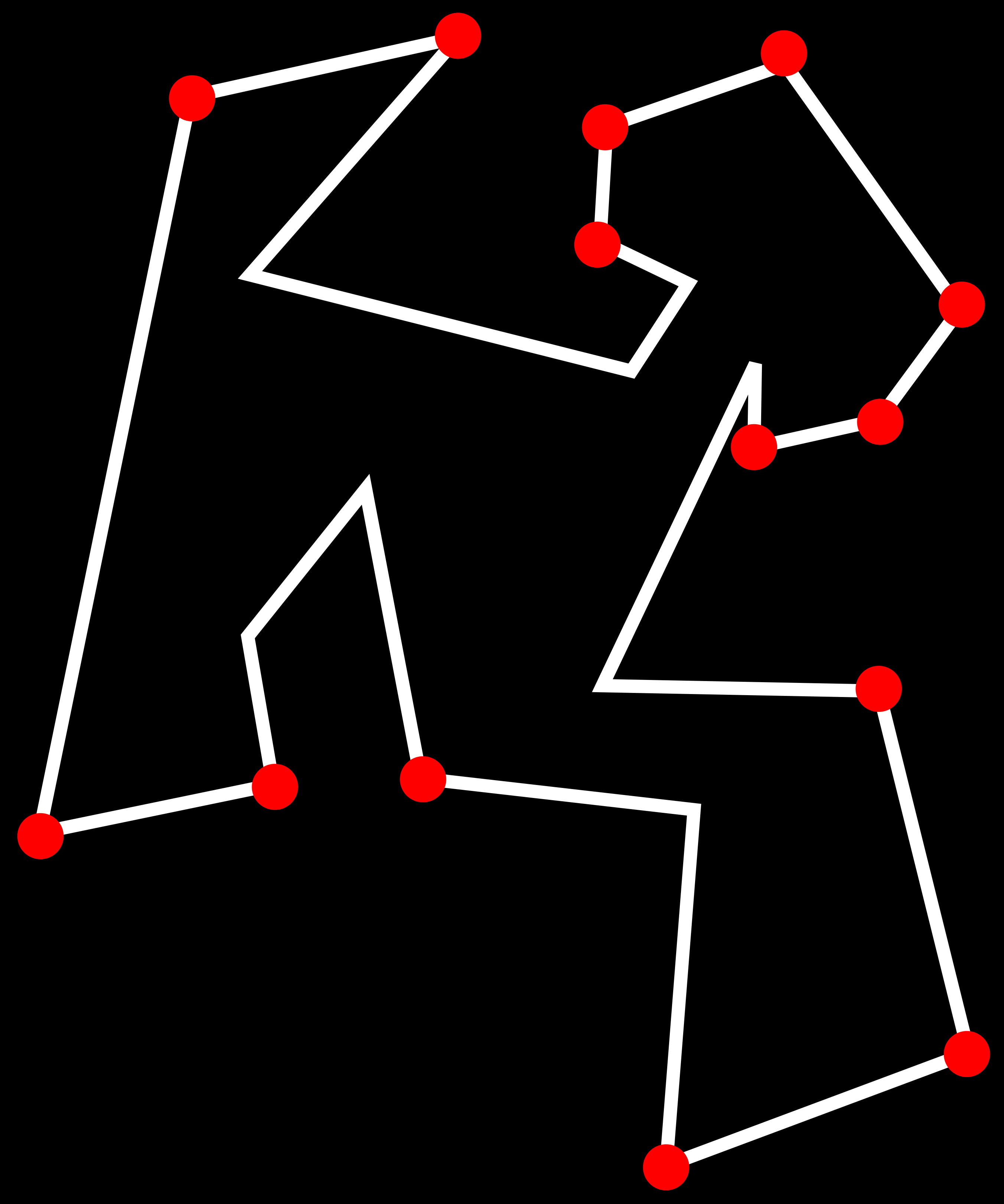


2-Approximation Algorithm

Idea:

Each route:

- Visits some convex vertices
- Sees all the other convex vertices



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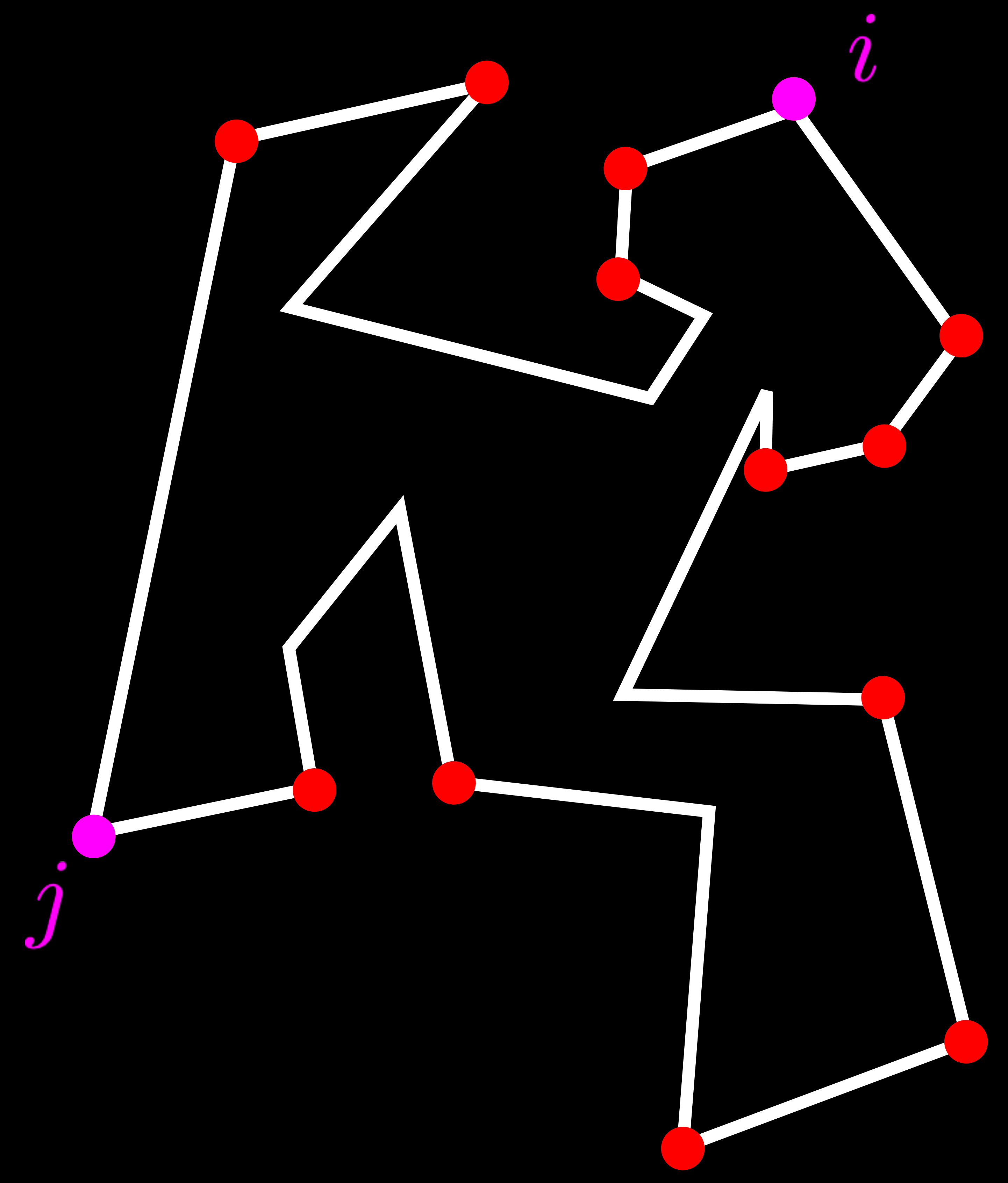

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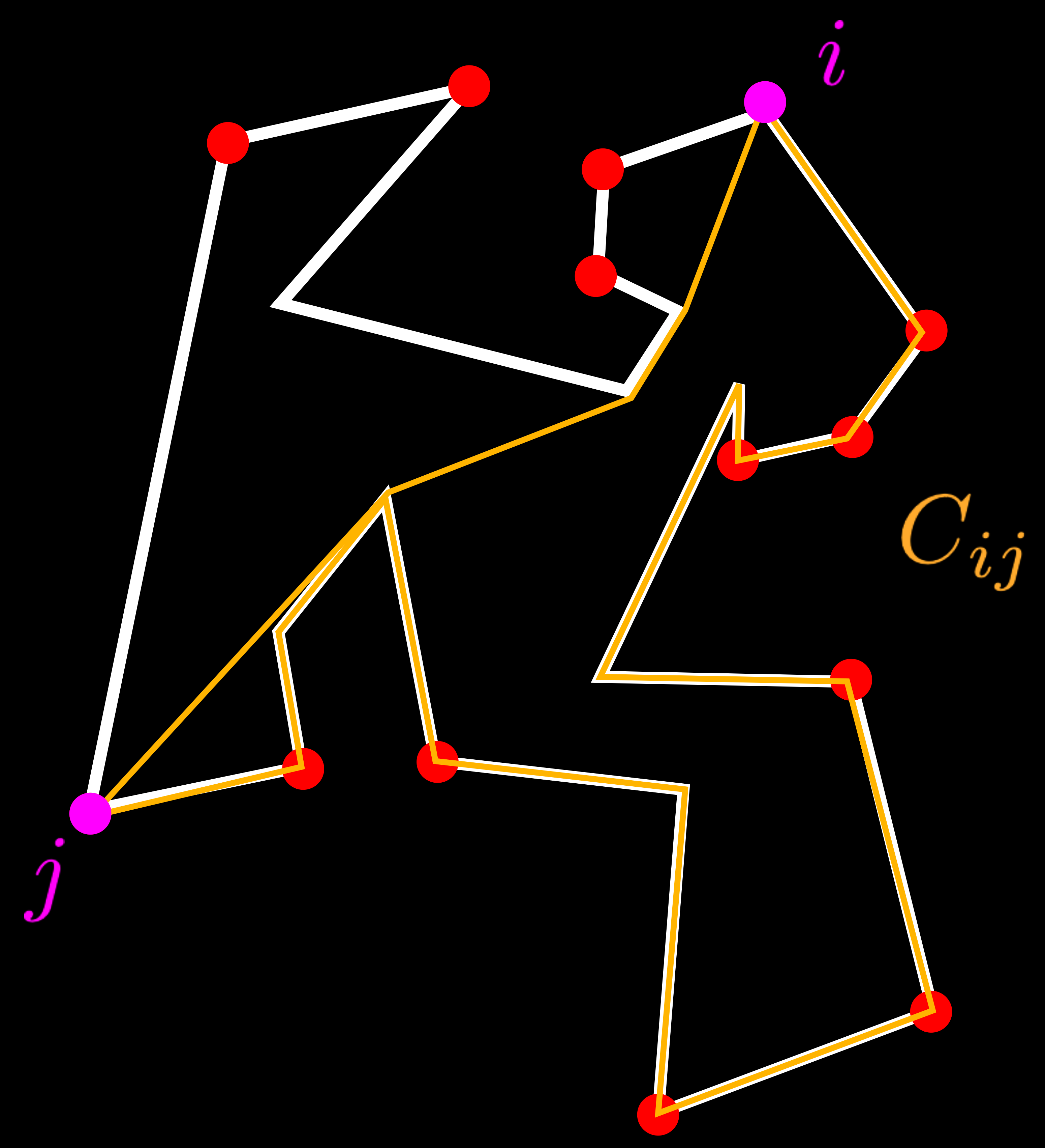

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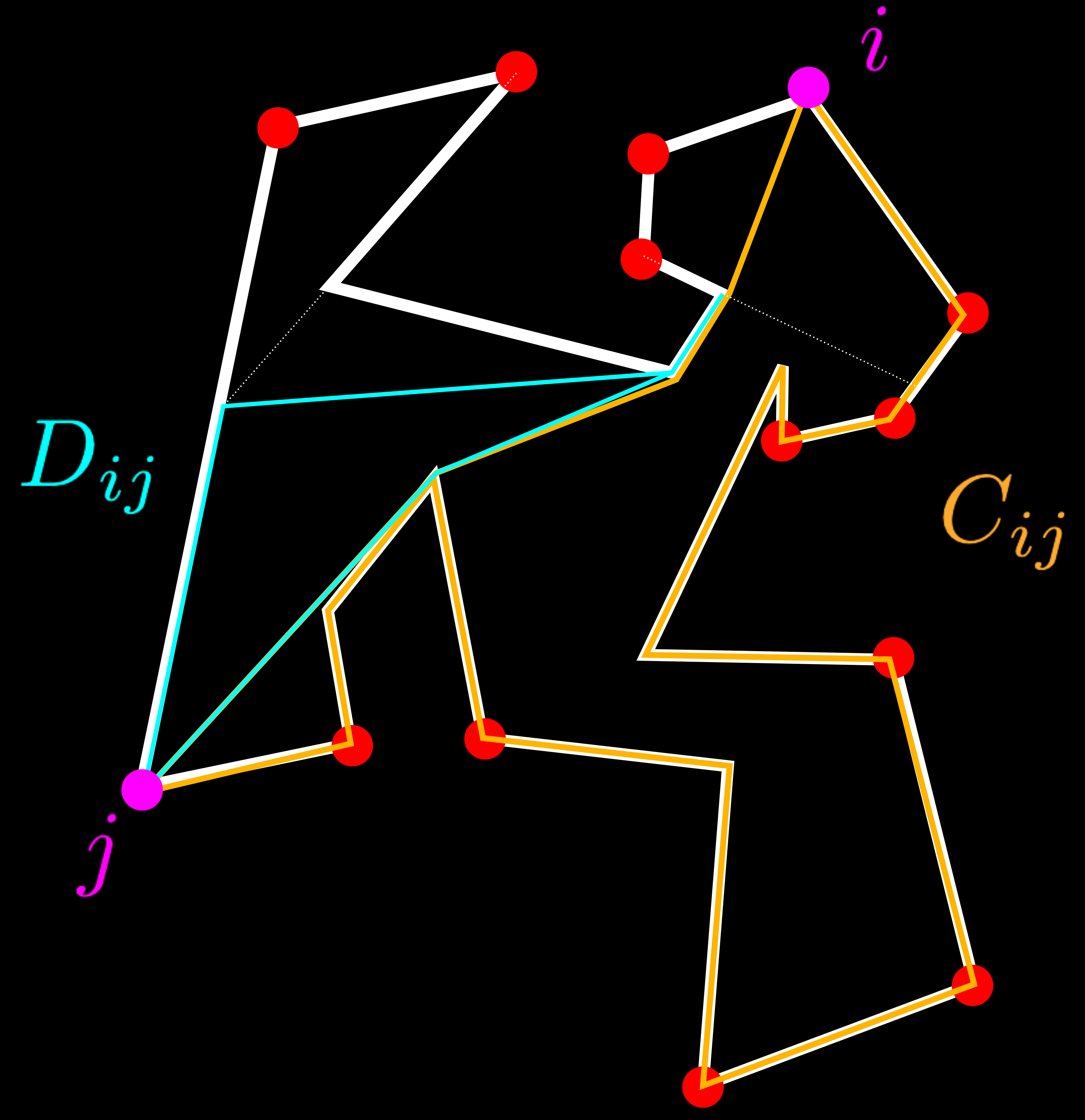

*The relative convex hull (RCH) of C_{ij} and D_{ij}: the minimal set that contains C_{ij} and D_{ij} and is closed under taking shortest paths

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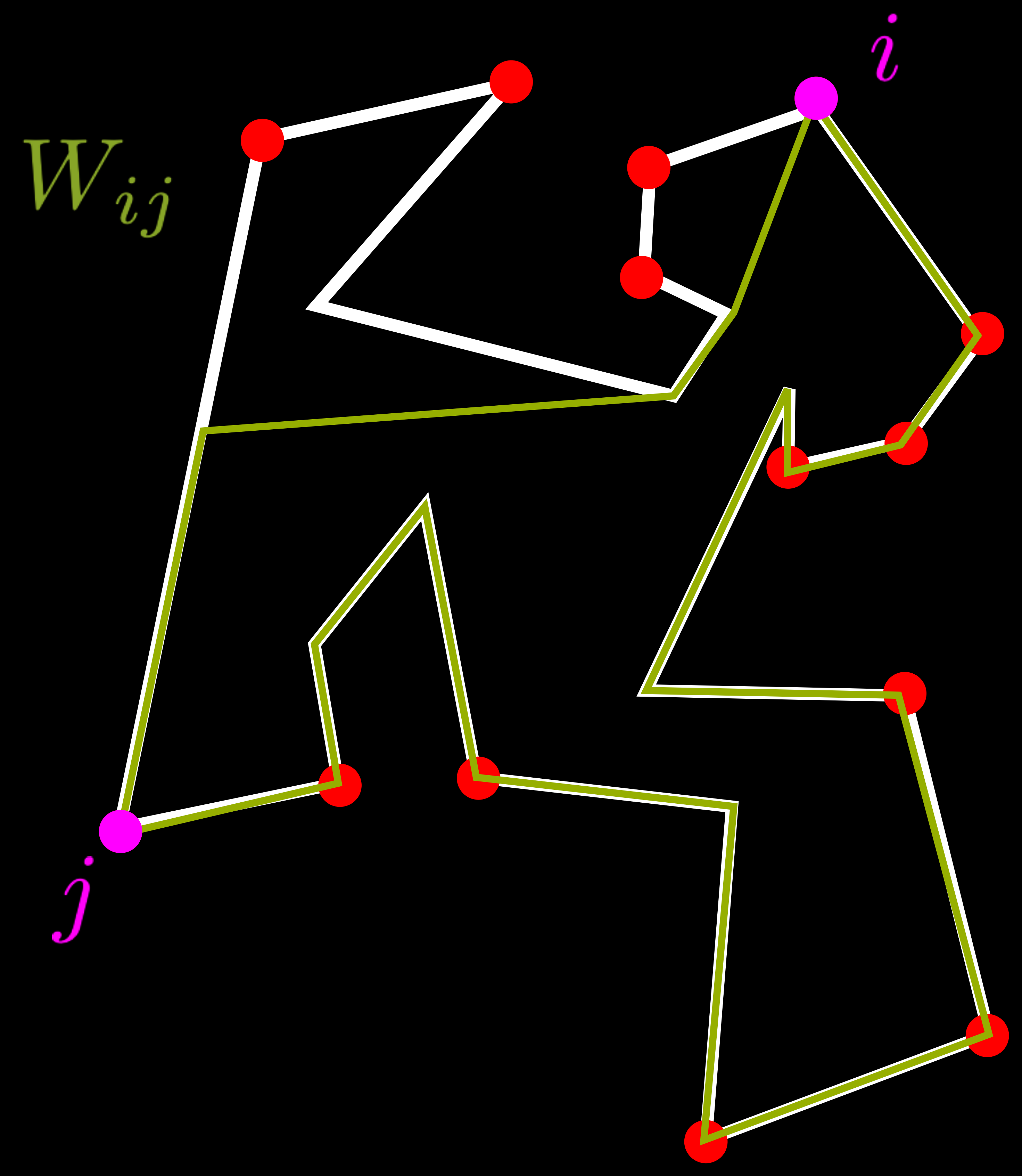

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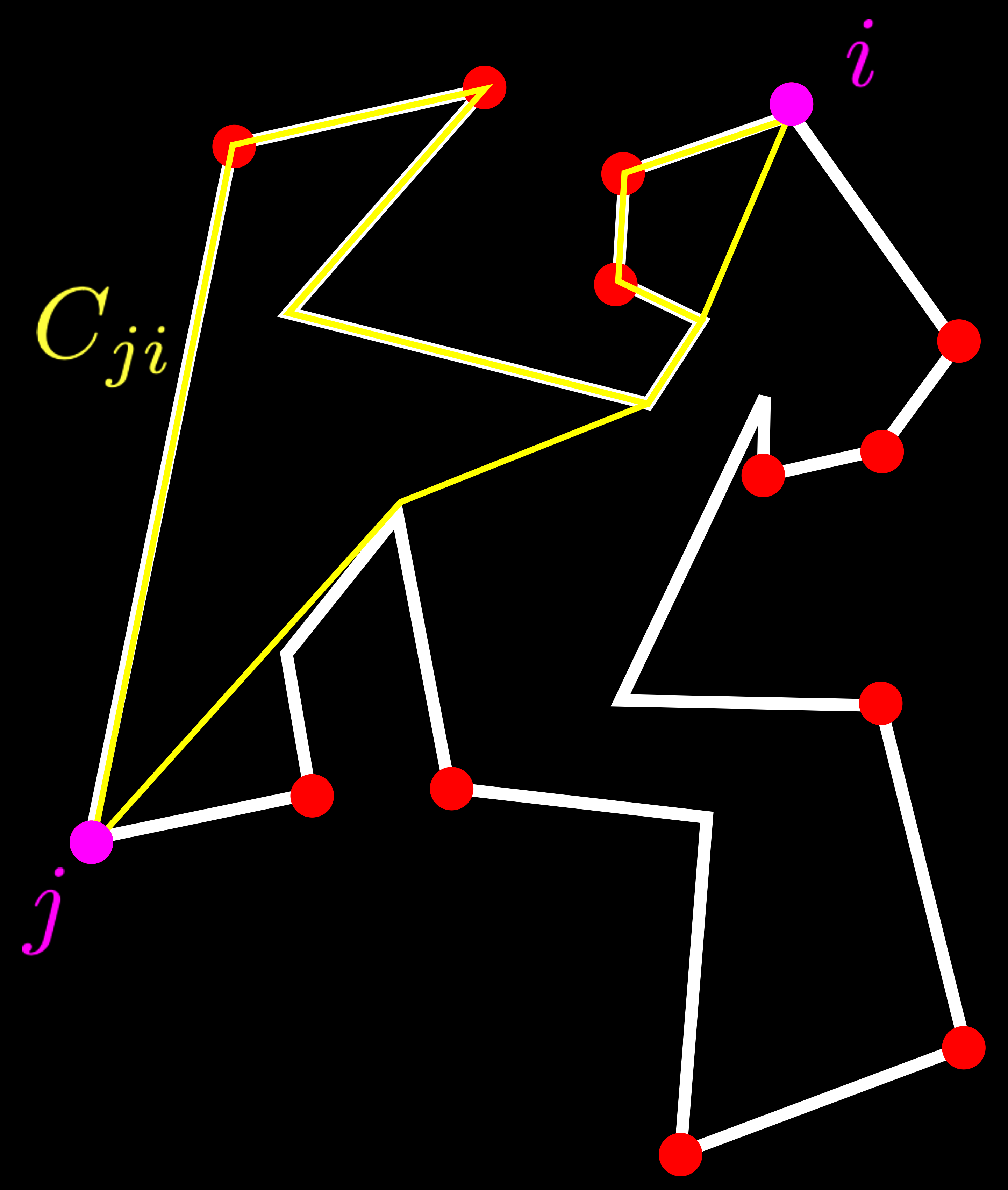

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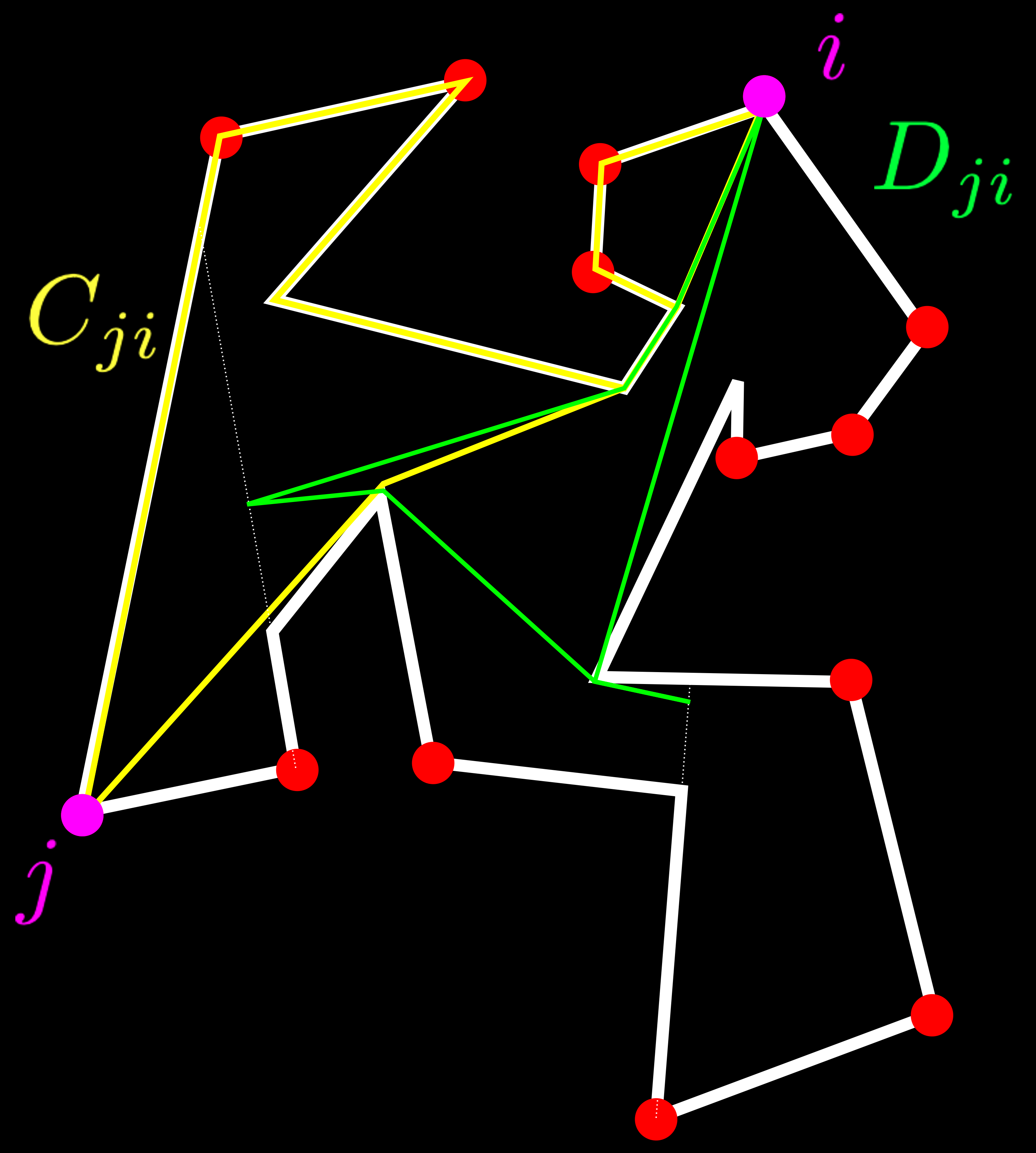

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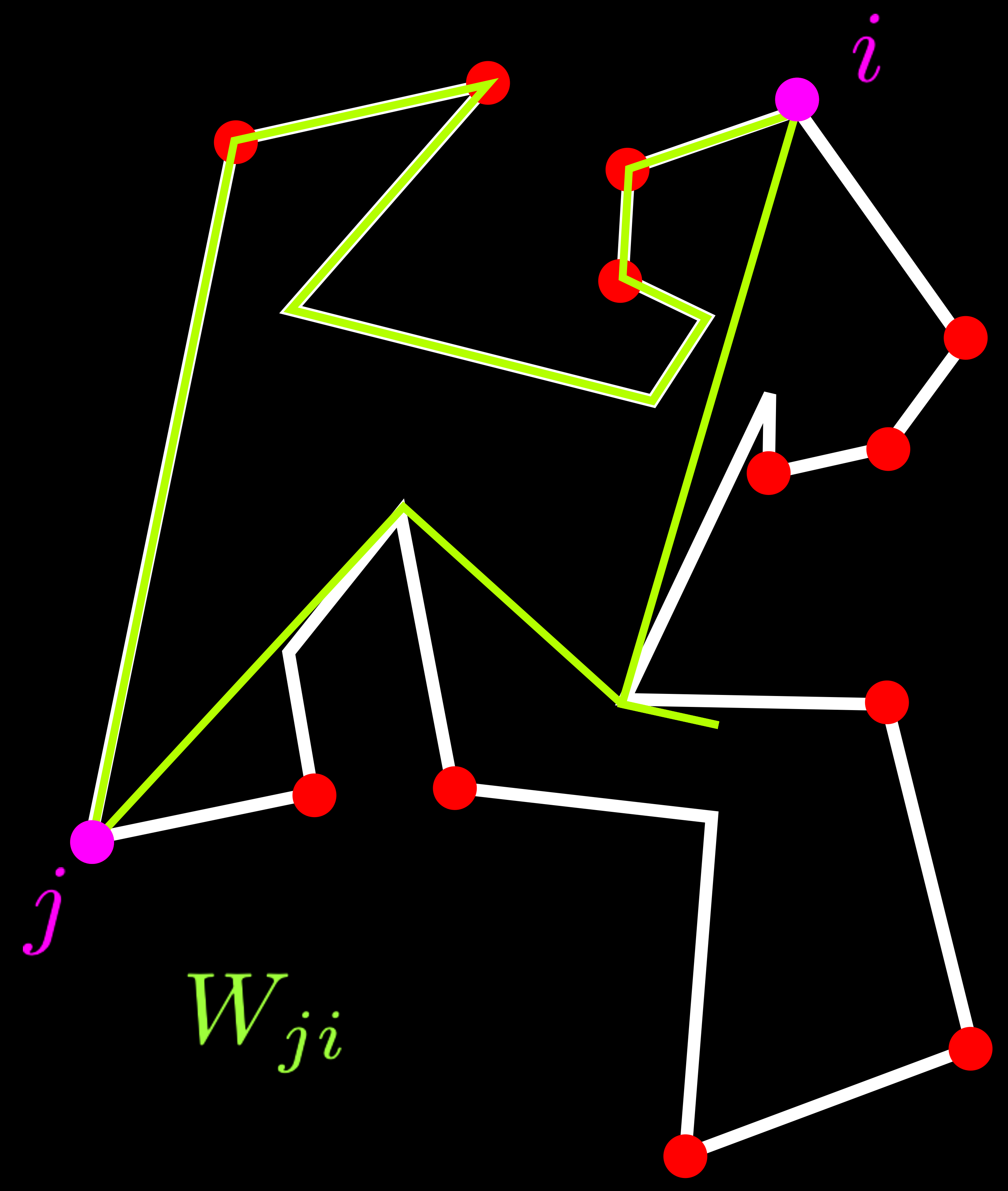
- Take RCH of yellow and green (someone needs to visit *i*)



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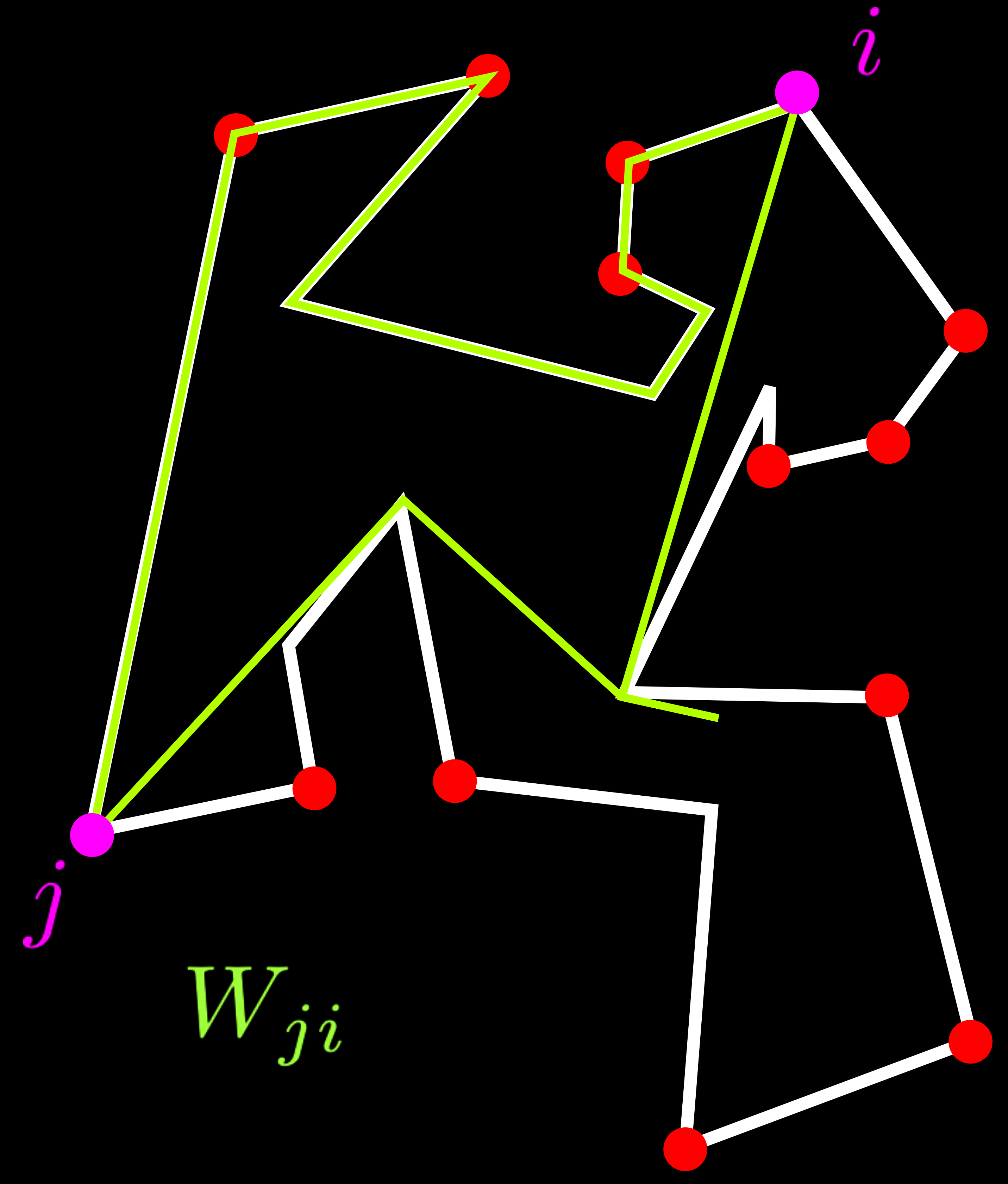
- C_P tour that **visits** all convex vertices



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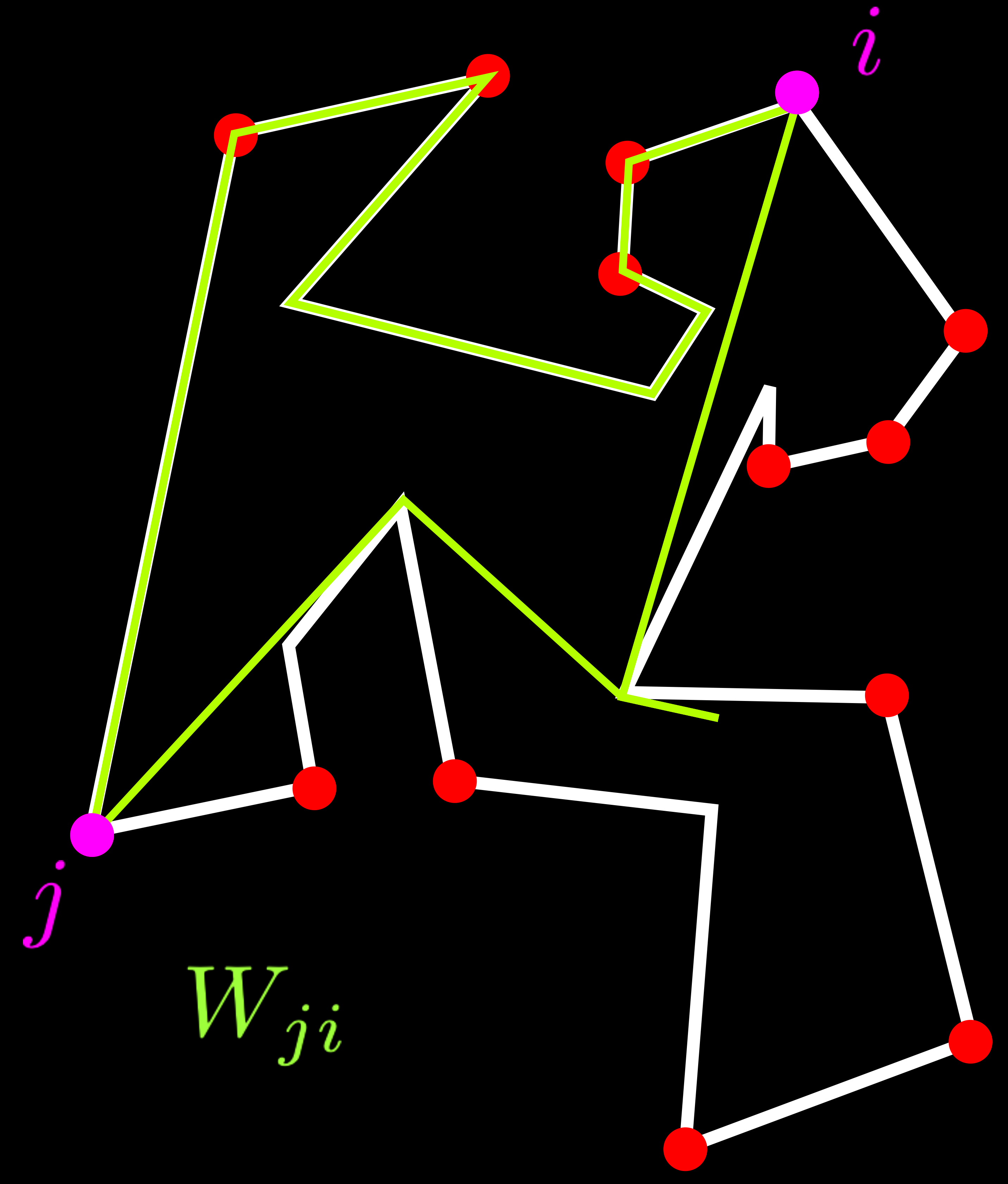
- C_P tour that **visits** all convex vertices
- D_P tour that **sees** all convex vertices



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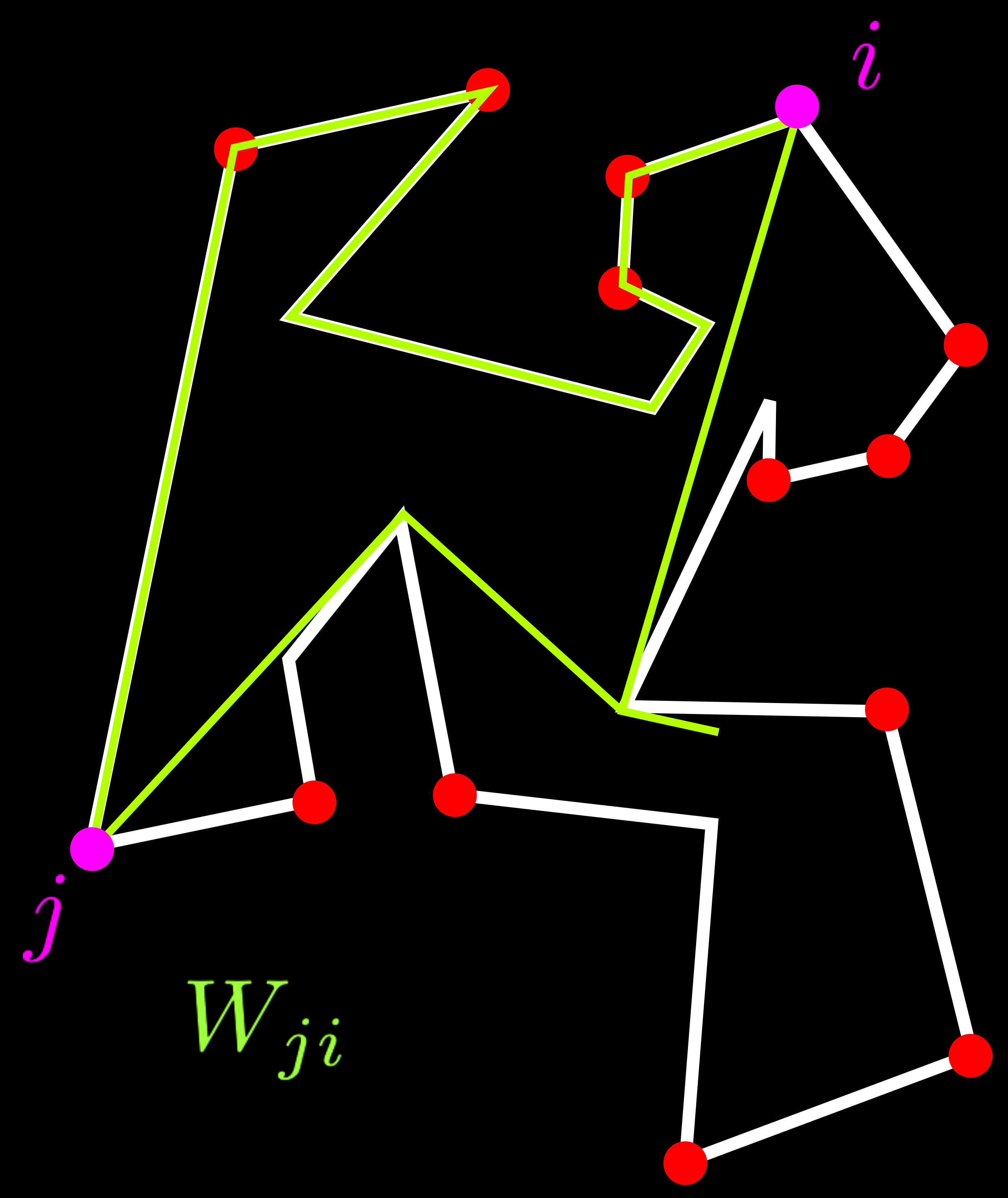
- C_P tour that **visits** all convex vertices
- D_P tour that **sees** all convex vertices
- Our approximation: $(W_1, W_2) = \arg\min_{i \neq j} \{\max\{|W_{ij}|,|W_{ji}|\}, \max\{|C_P|,|D_P|\}\}$

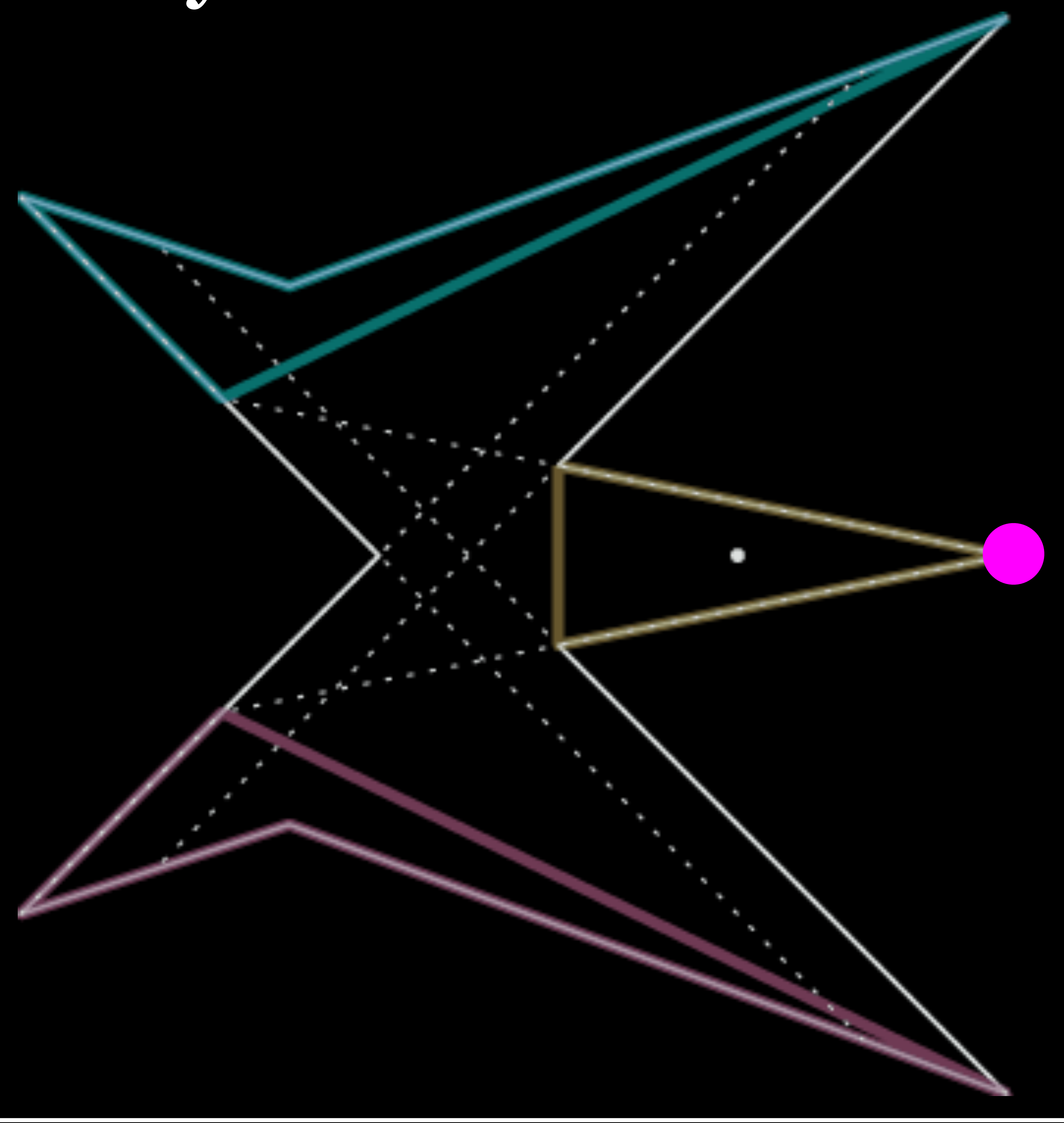


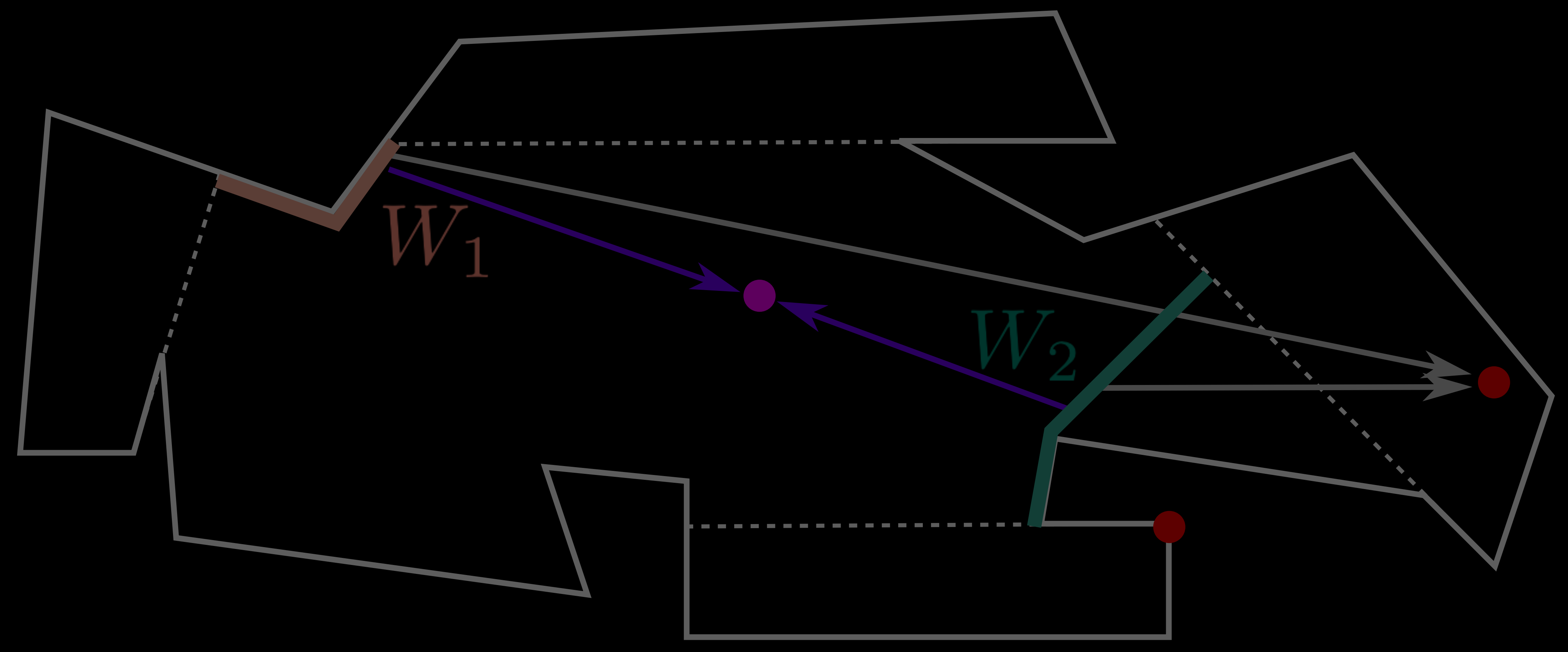
E.g., in case one of the optimal tours visits all convex vertices

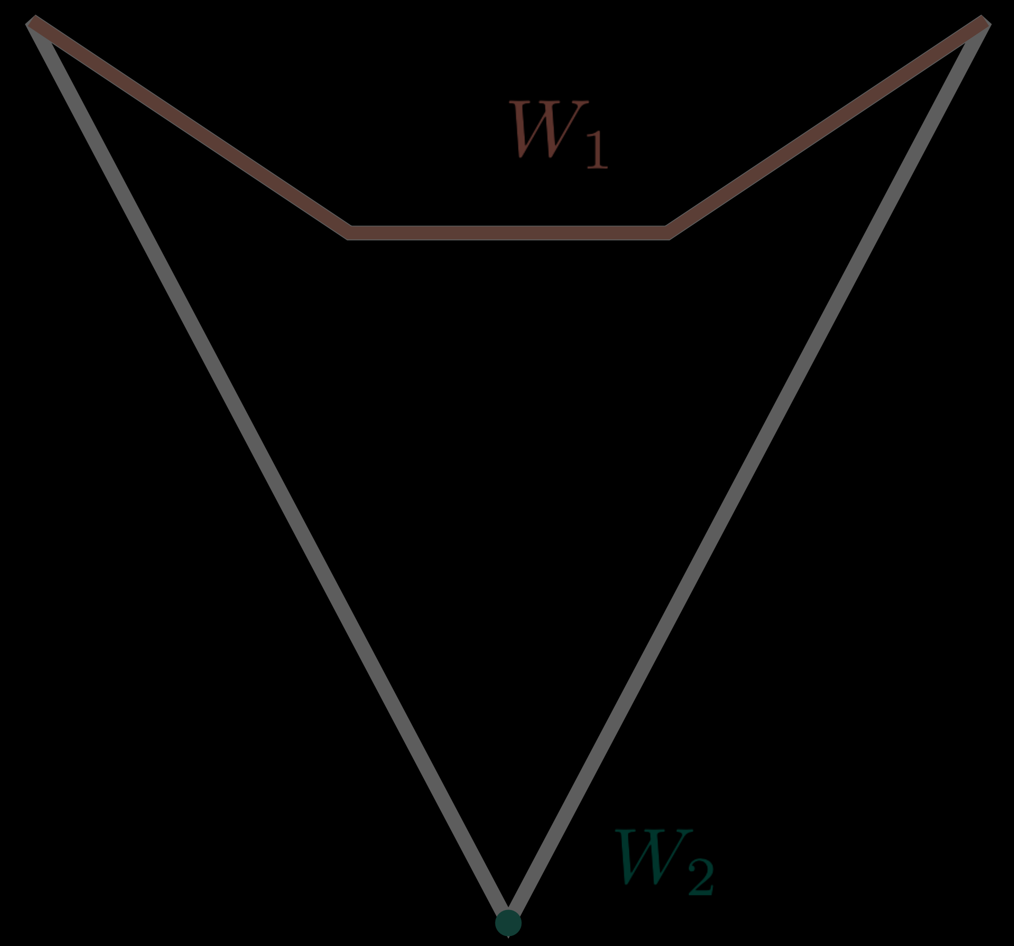
Outlook

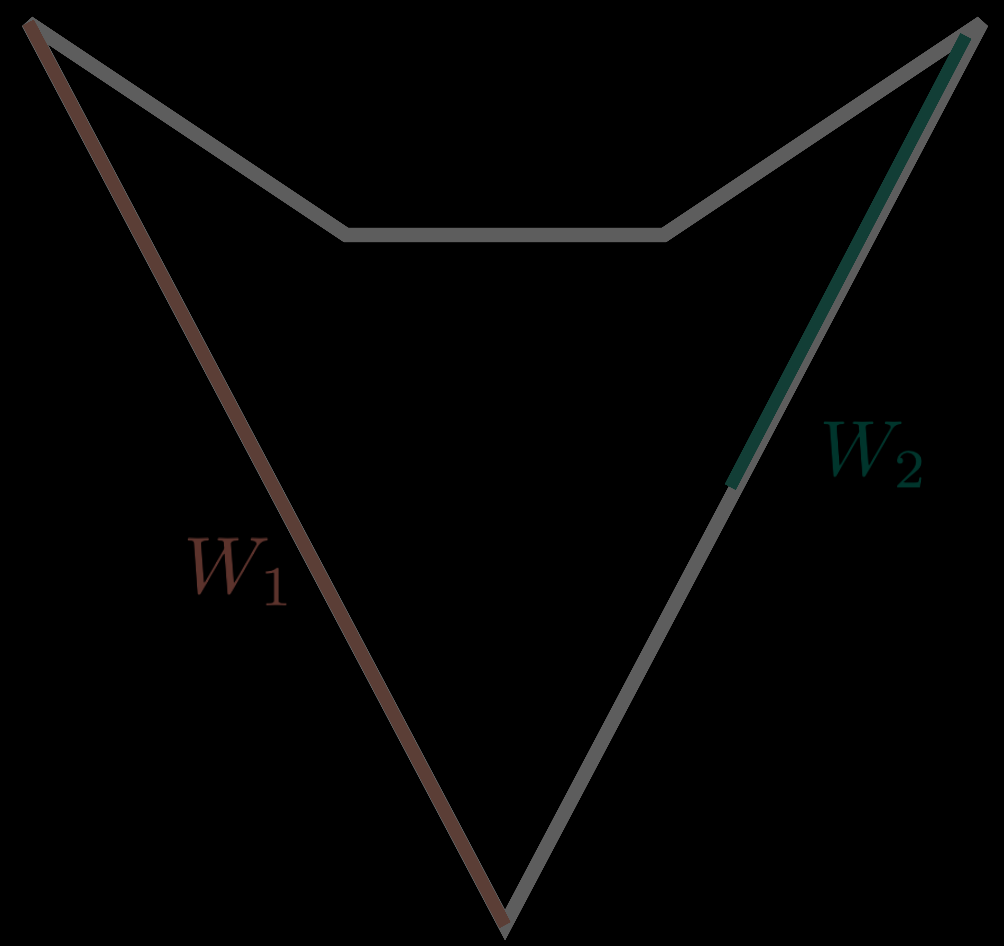
Outlook

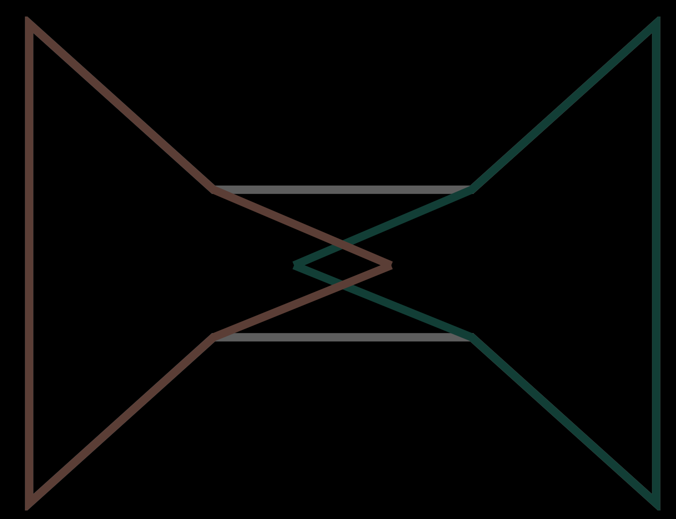
- Is the min-sum version NP-hard?
- Triangle-guarded points if the triangle must also be fully in P?

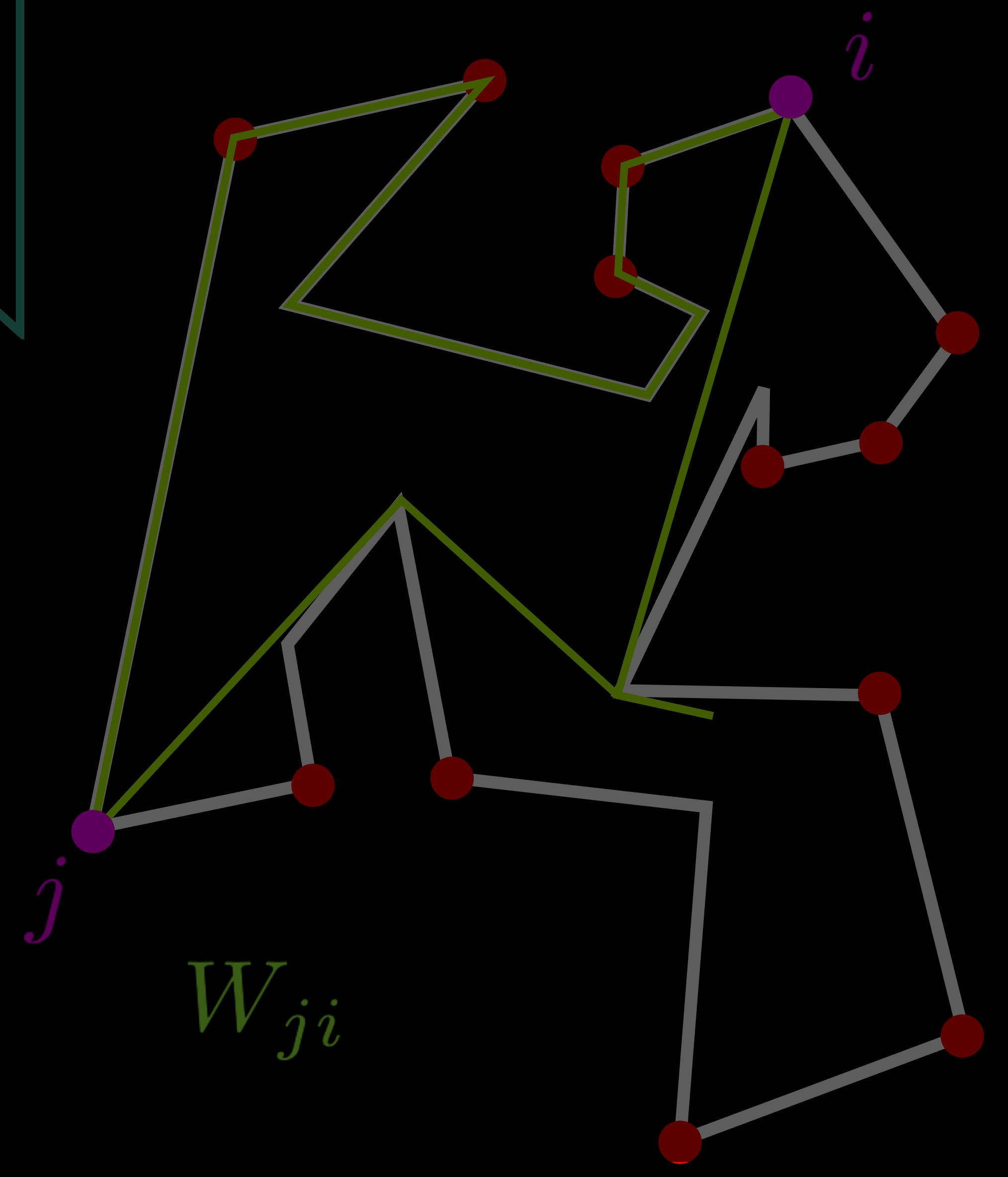




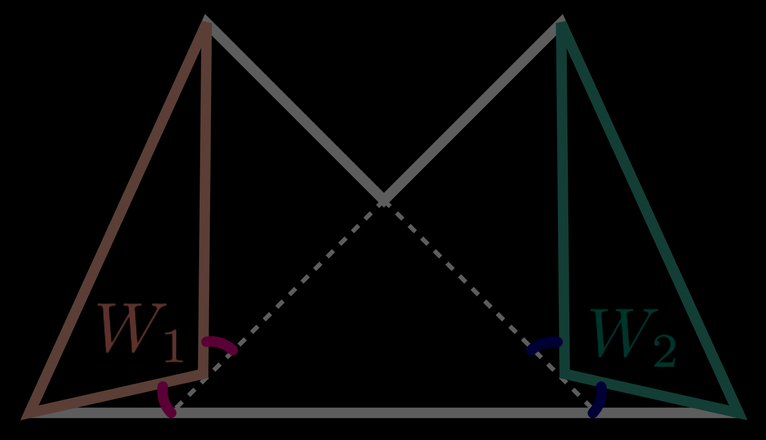


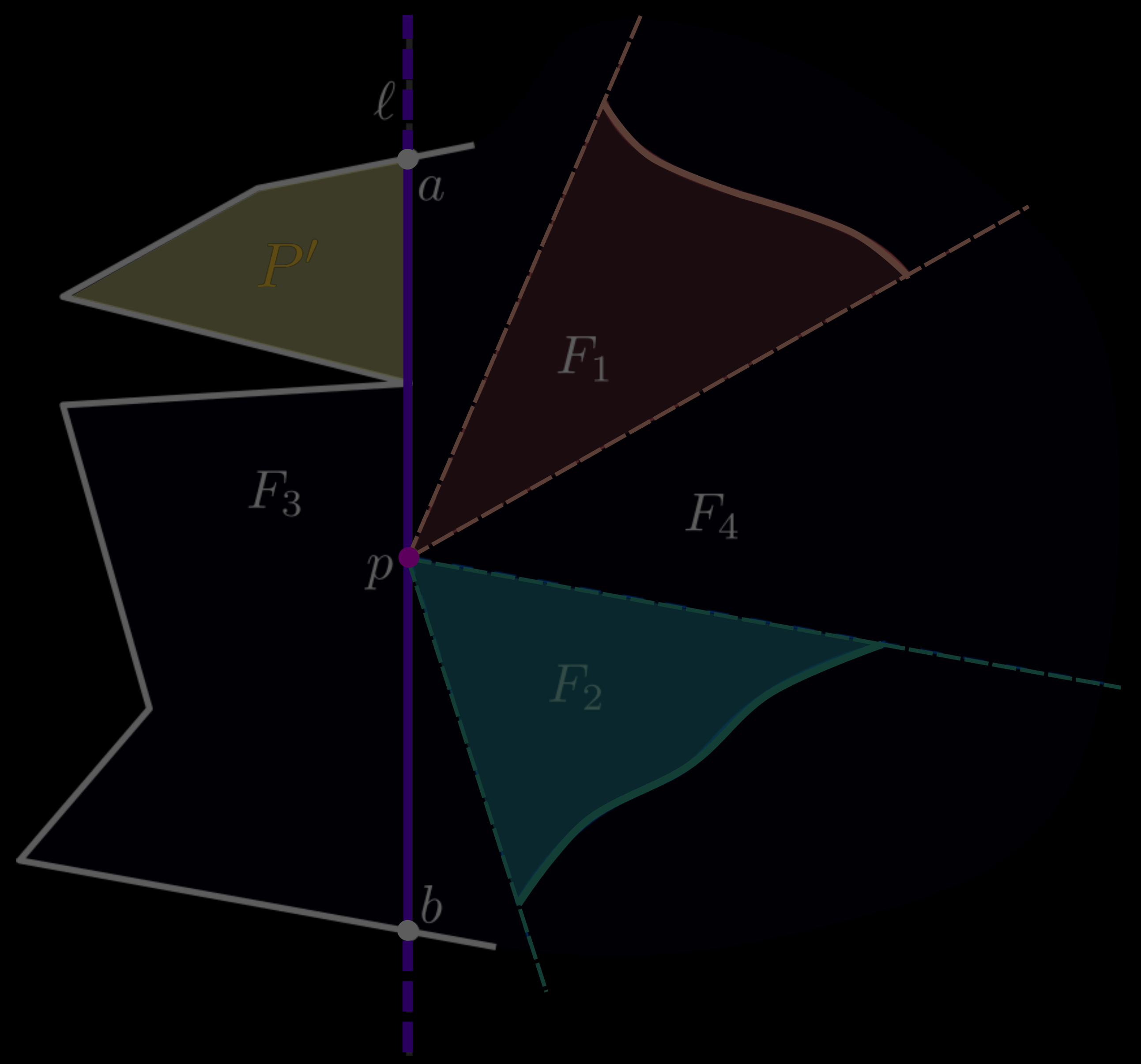






Thank you.





christiane.schmidt@liu.se
https://www.itn.liu.se/~chrsc91/