Data-Driven model for discovery of intracellular calcium oscillations behaviour

Matthieu Barreau*, Marco Martens†, Kateryna Morozovska*, Özan Oktem, Liviana Palmisano, **Aurora Poggi**

KTH, Department of Mathematics, Stockholm, Sweden † Stony Brook, Department of Mathematics, New York, USA

* KTH, Department of Electrical Engineering, Stockholm, Sweden

AIM:

Estimate the dimension of the attractor for the flow dynamic, if possible characterize the attractor geometry.

Takens Embedding theory

Question:

What properties of the system one can reconstruct given a time series evolution?

Time delay embedding is based on a dicrete-time dynamical system $T: X \to X$ and an observable $h: X \to \mathbb{R}$. Consider a time series generated by a point $x \in X$ as $y_j = h(T^{j-1}x)$, so

$$(y_1, y_2, \dots, y_m) = (h(x), h(Tx), \dots, h(T^{m-1}x)).$$

A time delay embedding with delay-length k embeds the time series into $\mathbb{R}^k$ via the sliding-window procedure:

$$((y_1, y_2, \dots, y_k), (y_2, \dots, y_{k+1}), \dots, (y_{m-k+1}, \dots, y_m)).$$

Classical Takens theorem:

Let X be a compact manifold. If $k > 2\dim(X)$, then $\phi_{h,k}$ is injective for typical h, where $\phi_{h,k}(x) = (h(x), h(Tx), \dots, h(T^{k-1}x))$.

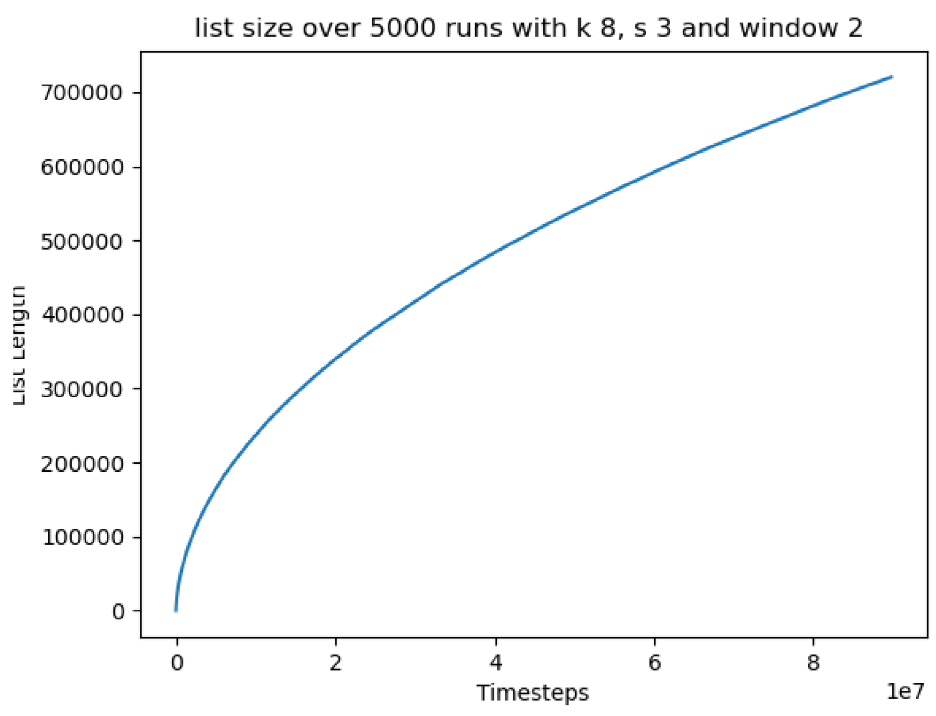
The minimum acceptable embedding dimension can be obtained looking at the behaviour of near neighbors under changes in the embedding dimension from d to $d+1$, via false neighbors [2].

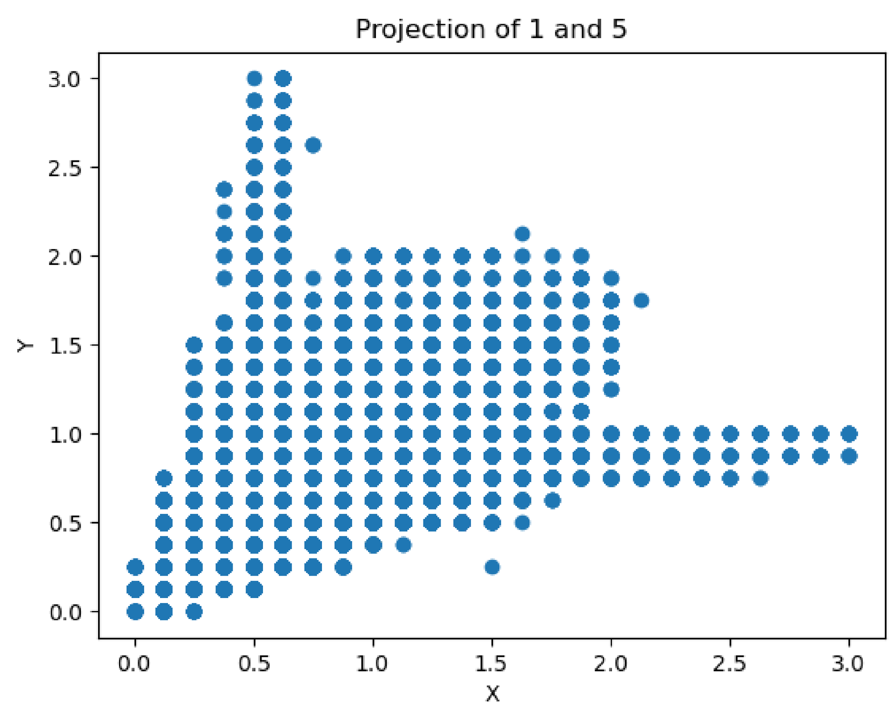
Fractal Geometry

The box counting dimension is defined on a non empty bounded subset E of $\mathbb{R}^n$, let $N_r(E)$ be the smallest number of sets of diameter r that can cover E [1]. Where the diameter of a set U is defined as $|U| = \sup\{|x-y| : x, y \in U\}$, that is the greatest distance apart of any pair of points in U. The Box counting dimension of E is

$$dim_B E = \lim_{r \to 0} \frac{\log N_r(E)}{-\log(r)}.$$

Main challenges: data shortages, that we address by taking a chosen time window and selecting pixel grids. Subsequently, we get additional data, which comprises specific sites of the attractor in a high-dimensional space.





AIM:

Recover the dynamics of the graph representation for the flow dynamics.

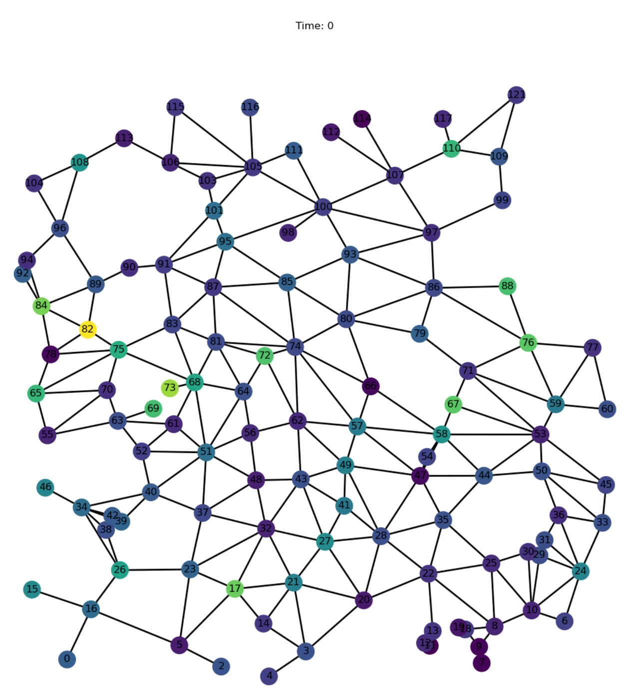
Control Theory

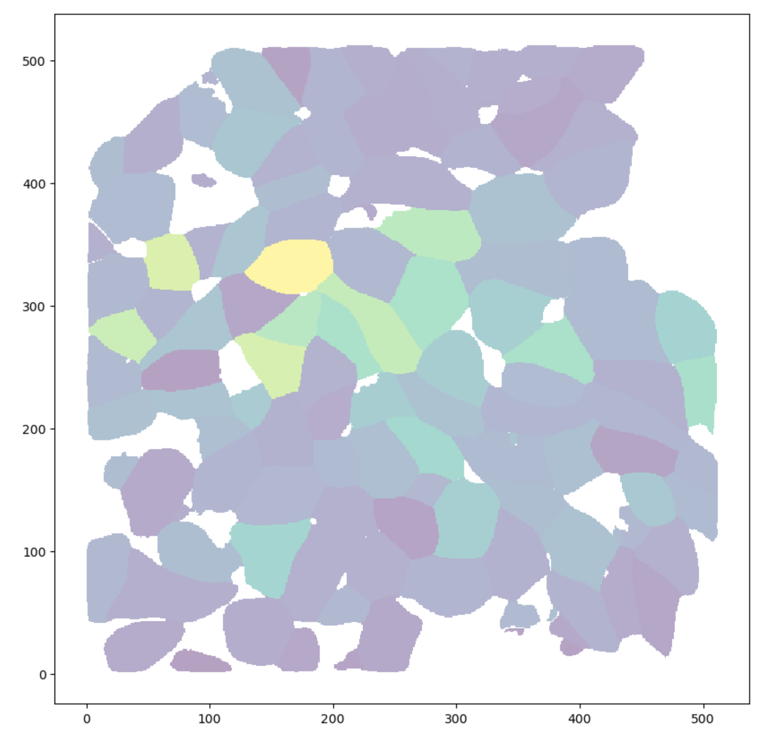
Setup: Given N agents, each agents moves with the same speed but different directions. Assuming the direction depends on the law

$$\theta_i(t+1) = \theta_i(t) + K(\theta_i(t) - \theta_j(t))u_i(t).$$

Consensus Problem:

Find $u_i(t)$ such that for $t \to \infty$ we have $\theta_1(t) = \theta_2(t) = \cdots = \theta_N(t)$, where each agent can only detect relative errors $x_j - x_i$ for j neighbour, i.e. $j \in N_i$.





Future work: First, to find out the best K, the problem is expressed as a least squares problem on K.
In order to improve the model's fit to the data, we intend to use a function trained by PINN or NeuralODE rather than a constant parameter K.

Previous approaches

- SINDy (Sparse identification of non linear dynamics).
- DMD (Dynamic Mode Decomposition).
- Symbolic Regression.
- Markov Chains.

References

[1] K. Falconer, Fractal Geometry: Mathematical Foundations and Applications, John Wiley & Sons, 2004.

[2] Determining embedding dimension for phase-space reconstruction using a geometrical construction, M.B. Kennel, R. Brown and H.D.I. Abarbanel, Phys. Rev. A 45, 1992

Funding

This work is supported by the Wallenberg AI, Autonomous Systems and Software Program (WASP) funded by the Knut and Alice Wallenberg Foundation.