Symmetries and Expressive Requirements for Learning General Policies

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Motivation & Contributions

- **General policies** are structures that encode action plans of infinite sized collections of classical planning problems $\mathcal{Q}$
- There exist **combinatorial** and **deep learning** approaches for learning general policies
- Two main issues:
  - **Scalability** in combinatorial setting
  - **Expressivity**, in both
- We introduce **abstractions** based on **state symmetries** (isomorphisms) for reducing the number of states in training
- We introduce a method for evaluating the **expressive requirements** of classes of classical planning problems

Symmetries & Abstractions

- Planning states are **relational structures**
- Two states s, s' are **isomorphic** $s \sim_{iso} s'$ iff their relational structures are **isomorphic**

  → Isomorphism is a bijective relationship preserving mapping between objects from s to s'

  → Isomorphic states represent the **same problem aspect**
- Find isomorphic states through graph isomorphism (GI) on undirected vertex colored graphs (Figure 1)
- Compute abstraction based on notion of isomorphic states induces abstractions (Figure 2)

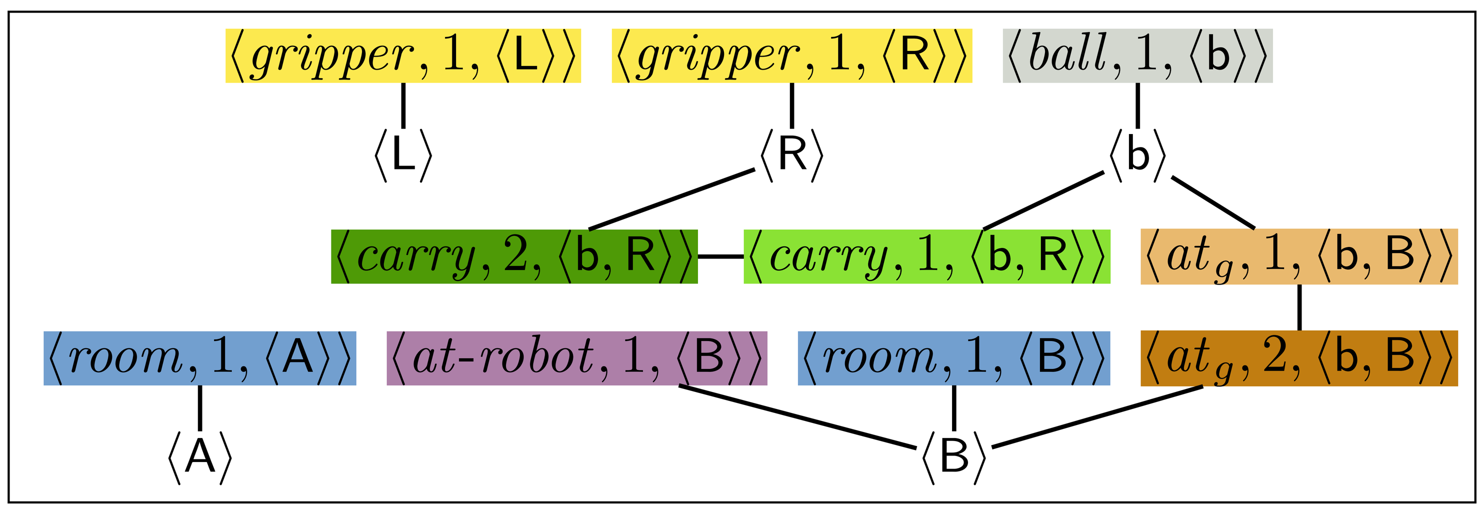

Figure 1: Graph $G(s)$ for a state in a problem from the Gripper domain.

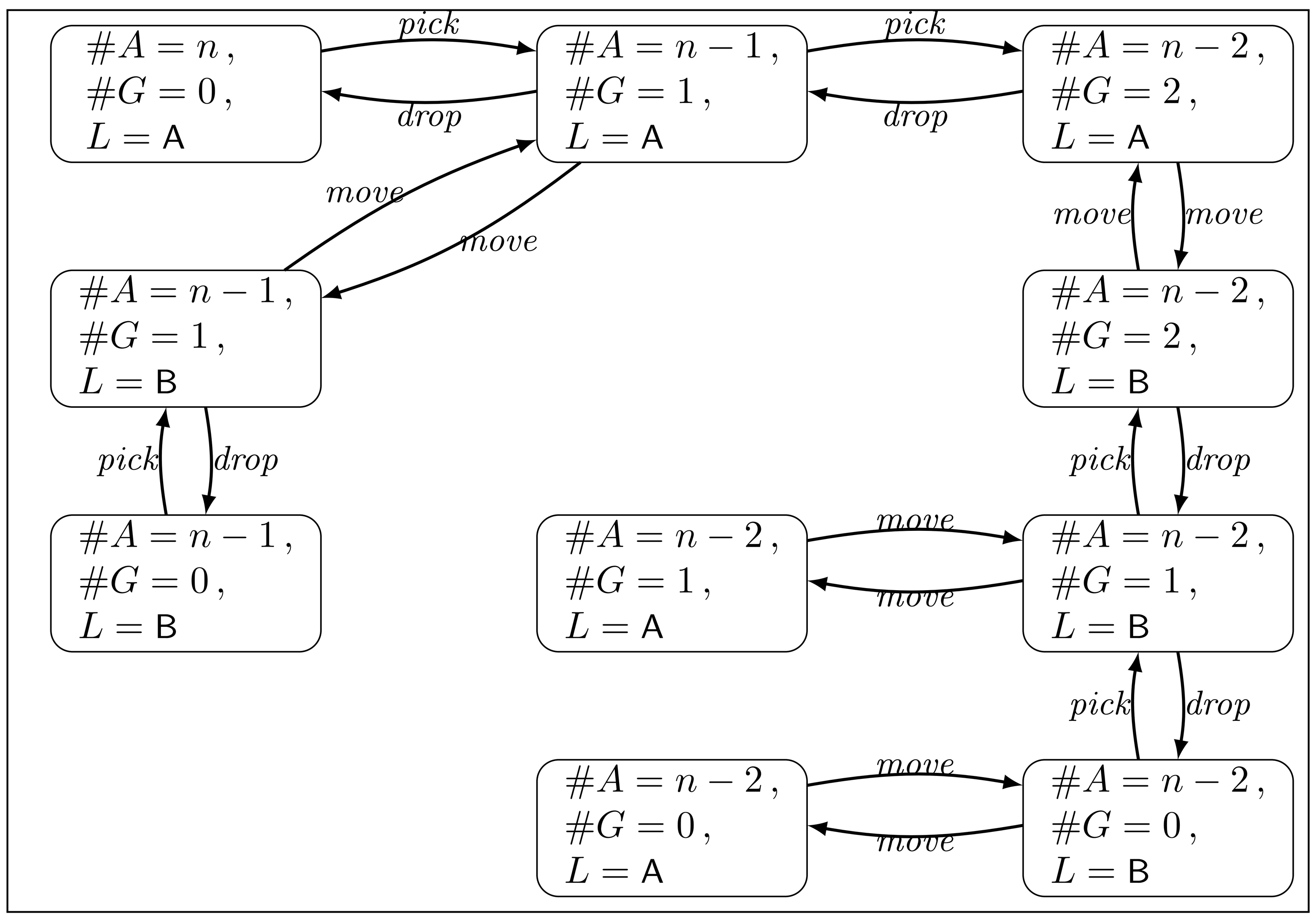

Figure 2: The abstraction of the state model is exponentially more compact with $6n$ states instead of 2^n states.

- **Reduced problems** $\tilde{\mathcal{Q}}$ contains one representative state from each class of isomorphic states
- **Theorem:** a general policy π solves $\mathcal{Q}$ iff π solves the **reduced problems** $\tilde{\mathcal{Q}}$

Expressive Learning Requirements

- GNN + RL for learning general policies (Ståhlberg et al. 2023)
- Nearly **perfect general policies** obtained in several domains (100%)

Domain	Coverage (%)	1-WL # conflicts
Delivery	100%	0
Gripper	100%	0
Logistics	36%	131
Grid	79%	42

- But interesting part is in the **failures**
  - **GNN expressivity not enough** (Logistics, Grid, Blocks)
  - Others: insufficient # network layers, sampling
- 1-WL, GNNs, C_2 have equivalent expressivity in distinguishing graphs (Cai et al. 1992, Grohe 2021)
- Indeed, 1-WL/GNNs can't distinguish pair of isomorphic states

				# Conflicts	
Domain	$\#\mathcal{Q}$	$\#\mathcal{S}$	$\#\mathcal{S}/{\sim_{iso}}$	1-WL	2-FWL
Barman	510	115 M	38 M	1,326	0
Blocks3ops	600	146 K	133 K	50	0
Blocks4ops	600	122 K	110 K	54	0
Blocks4ops-clear	120	31 K	3 K	0	0
Blocks4ops-on	150	31 K	8 K	0	0
Childsnack	30	58 K	5 K	0	0
Delivery	540	412 K	62 K	0	0
Ferry	180	8 K	4 K	36	0
Grid	1,799	438 K	370 K	42	0
Gripper	5	1 K	90	0	0
Hiking	720	44 M	5 M	0	0
Logistics	720	69 K	38 K	131	0
Miconic	360	32 K	22 K	0	0
Reward	240	14 K	11 K	0	0
Rovers	514	39 M	34 M	0	0
Satellite	960	14 M	8 M	5,304	0
Spanner	270	9 K	4 K	0	0
Visitall	660	3 M	2 M	0	0

Table 1: # conflicts in distinguishing isomorphic states.

- **Central takeaways:**
  - In some domains, 1-WL (and GNNs) are not expressive enough
  - **Most importantly**, 2-FWL, that *has the expressive power of C_3 appears to be sufficiently expressive in all domains*

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