Exploring the Universality of Manifold Neural ODEs

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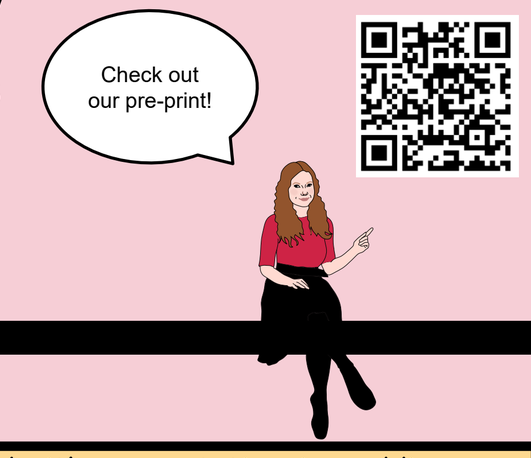
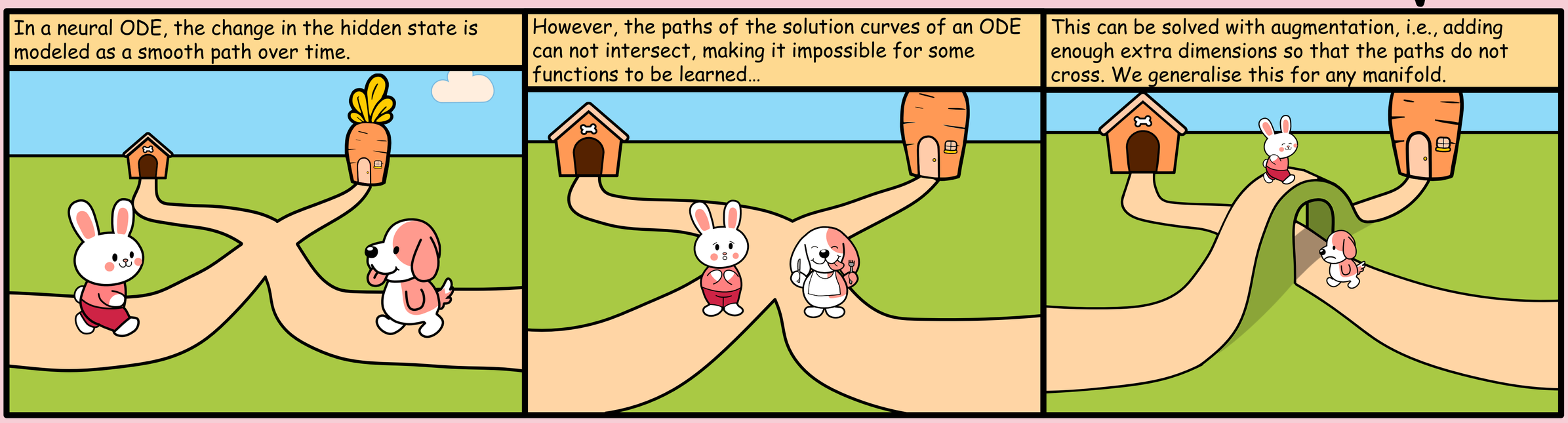


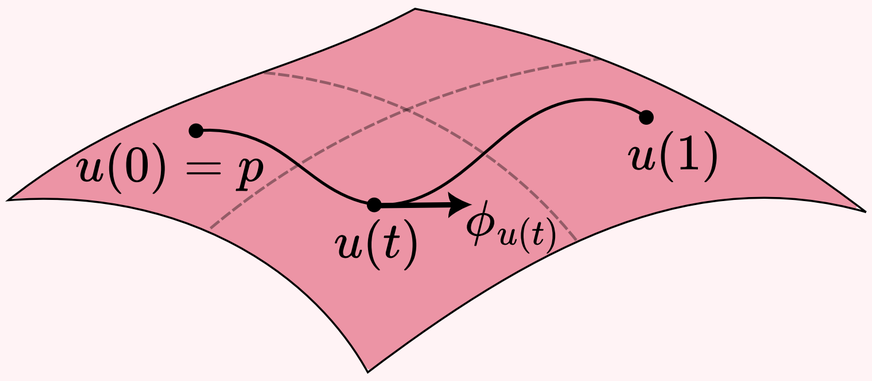


Manifold Neural ODEs

Manifold neural ODEs are a type of neural network models where the layers consist of a smooth curve u solving the Cauchy problem

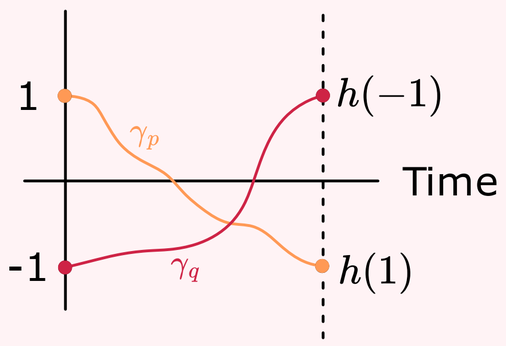
$$\frac{du(t)}{dt} = \phi_{u(t)}, \qquad u(0) = p,$$

where ϕ is a learnable vector field, p is the input and $u(1)$ is the output [1,2,3].



Problem: Intersecting solution curves

Not all diffeomorphisms can be learned by a neural ODE. The canonical example is $h(x) = -x$, which necessitates an intersection of the solution curves:

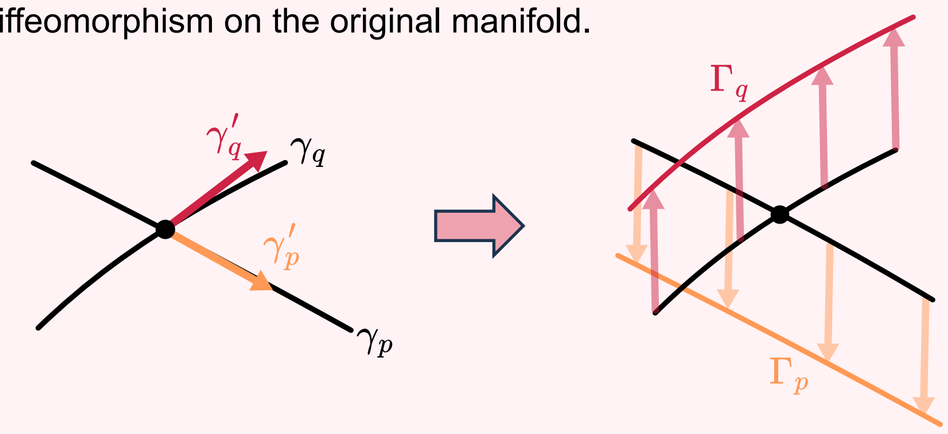


Solution: Augmentation

Neural ODEs can be augmented so that the solution curves are embedded in an ambient space [4,5].

Paths γ_p, with $\gamma_p(0) = p$ and $\gamma_p(1) = u(1)$, can be lifted via $\Gamma_p \coloneqq [\gamma_p, \gamma_p']$. The lifts don't intersect and define an embedding into the tangent bundle.

A manifold neural ODE with this lift can approximate *any* diffeomorphism on the original manifold.



References

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