Complexity Certification of Branch-and-Bound Methods for MILP and MIQP

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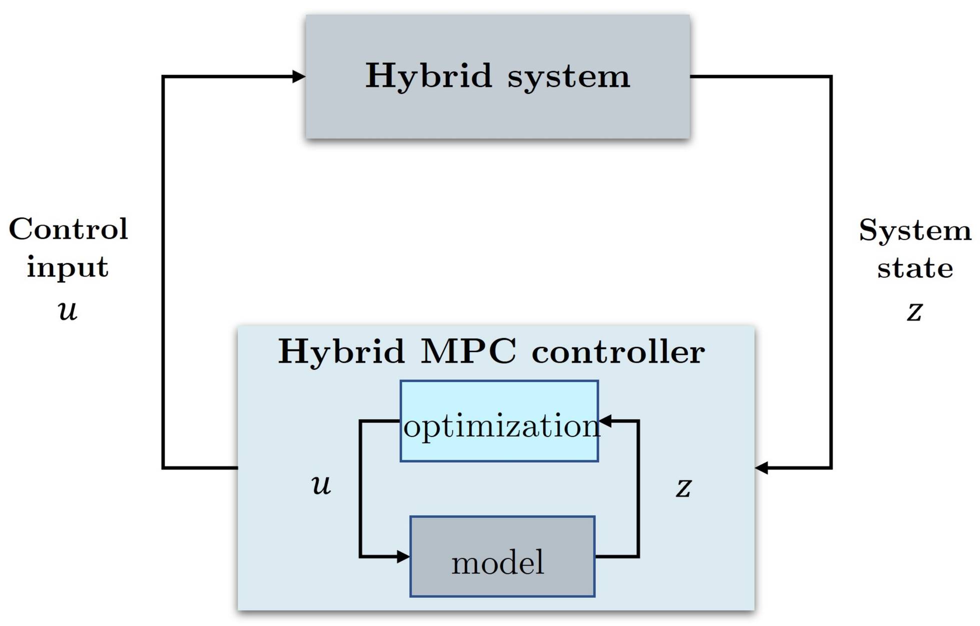
Motivation & Research Goals

When applying model predictive control (MPC) to linear hybrid systems with a linear or a quadratic performance measure, the resulting optimal control problem can be cast as a mixed-integer linear or quadratic programming (MILP/MIQP) problem that depends on parameters such as system states and reference signals. The aim of this work is to certify the computational complexity of standard branch-and-bound (B&B) methods for solving MILPs and MIQPs in terms of, e.g., the size of the search tree or the number of linear systems of equations (iterations) that are needed to be solved online to compute optimal solutions. In particular, this knowledge enables us to compute relevant worst-case complexity bounds for the B&B-based MILP and MIQP solvers, which has significant importance in, e.g., real-time hybrid MPC where hard real-time requirements have to be fulfilled.

Problem Formulation and Method

- **Application:** MPC for Hybrid Systems



- **Problem Formulation:** multi-parametric MILP and MIQP

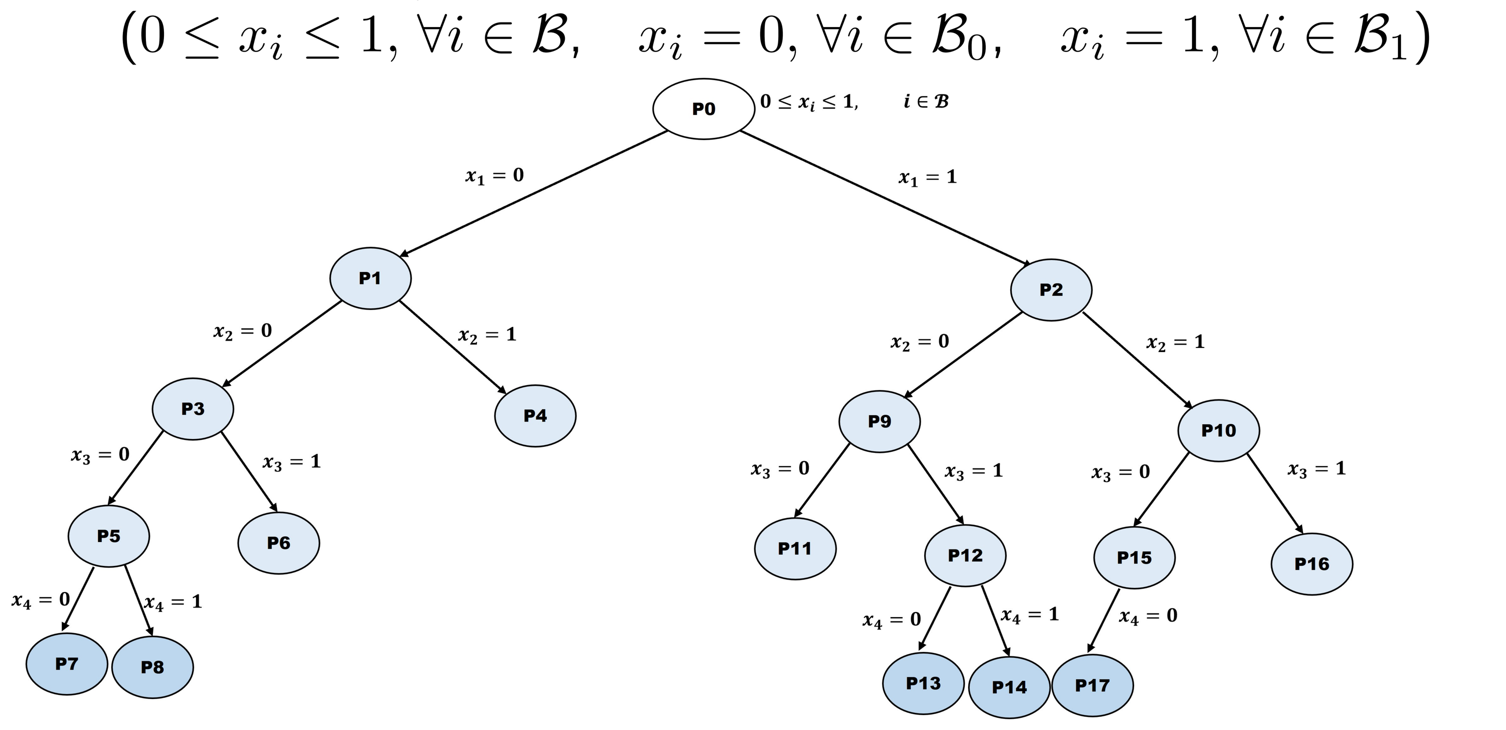
$$\boxed{\begin{aligned} \min_{x} \quad & J(x,\theta) \\ \text{s.t.} \quad & Ax \le b + W\theta, \\ & x_i \in \{0,1\}, \quad \forall i \in \mathcal{B}. \end{aligned}}$$

$x \in R^{n_c} \times \{0,1\}^{n_b}$: decision variable (related to u)

$\theta \in \Theta_0 \subset R^{n_\theta}$: parameter (related to z)

  - mp-MILP: $J(x,\theta) = c^T x$
  - mp-MIQP: $J(x,\theta) = \frac{1}{2}x^T H x + f^T x + \theta^T f_\theta^T x$

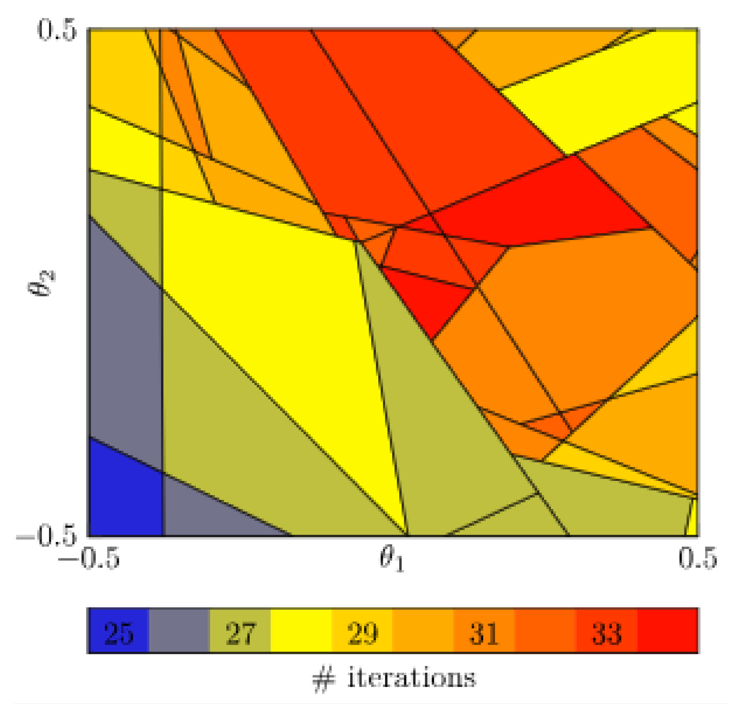
- **Method:** Branch and Bound (B&B)
  - To find the solution of a non-convex MILP/MIQP, a sequence of convex LP/QP relaxations is solved in a search tree ($0 \le x_i \le 1, \forall i \in \mathcal{B}, \quad x_i = 0, \forall i \in \mathcal{B}_0, \quad x_i = 1, \forall i \in \mathcal{B}_1$)



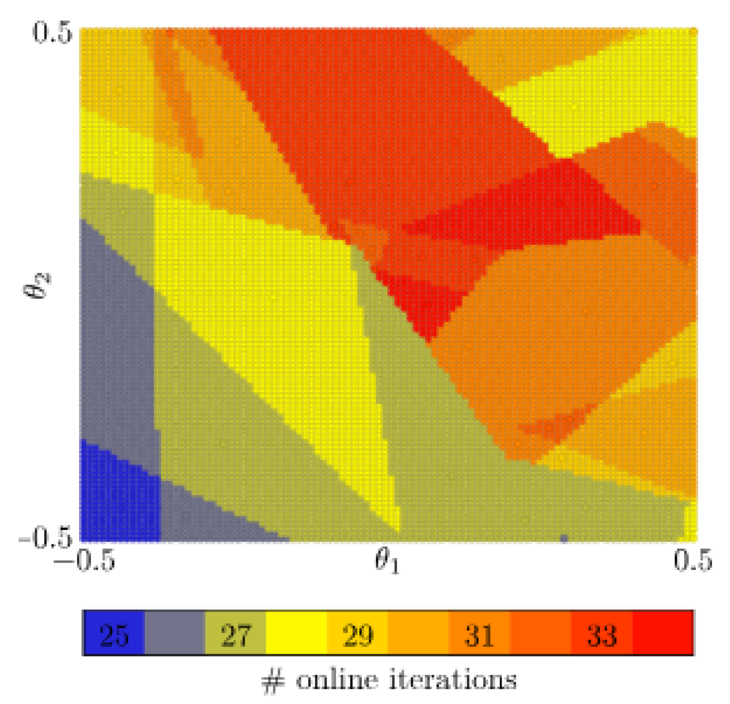
Contribution and Results

Contribution:
Computing the *exact worst-case* computational complexity for solving any MILP and *an upper bound of the worst-case* computational complexity for solving any MIQP that can arise from a specific parameter for a given mp-MILP/MIQP problem

Results: Partitioning the parameter space based on the total number of LP iterations for a random mp-MILP. Points with the same color share the same complexity measure.



Validating the results: Monte Carlo simulation (applying the online B&B to 10000 samples in the parameter space). The total number of LP iterations at each point is specified by *.



Results:
The complexity certification result completely coincides with the result obtained from the online algorithm in all sample points.

Ongoing & Future works:

- Unifying complexity certification framework for mixed-integer linear and quadratic programming
- Parallelize the proposed certification framework to be able to scale up the problem sizes
- Considering how to bound the computation time

References

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