Convex polygonal formation of multi-vehicle systems

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Abstract

In this project we study the convex polygonal formation control problem of multi-vehicle systems. The vehicles are described by a nonlinear unicycle model. The formation is achieved in an *intrinsic* way in the sense that final achieved polygon is only attributed to the initial conditions of the vehicle. Different initial conditions will result in polygons with different side lengths and interior angles. The control design consists of three phases. First, the vehicles are steered to form a circular formation, which is equivalent to the vehicles forming an inscribed polygon. Secondly, a consensus algorithm is applied to ensure all vehicles share the same heading. Finally, all vehicles maintain a polygonal formation while performing some prescribed motion, e.g., sinusoidal motion.

Problem Formulation

We aim to steer $n \geq 3$ mobile vehicles moving in a plane to form convex polygonal formation in a distributed manner. The achieved formation is not designated *a priori* in the controller. Instead, we aim to achieve formation in an intrinsic way, which means that the final formation patterns are only attributed to the inter-vehicle interaction network and the initial conditions of the vehicles. The dynamics of each vehicle can be described by a unicycle model:

$$\begin{aligned}\dot{x}_i &= v_i \cos\theta_i,\\ \dot{y}_i &= v_i \sin\theta_i,\\ \dot{\theta}_i &= \omega_i,\end{aligned}$$

where $z_i = [x_i \ y_i]^\top \in \mathbb{R}^2$ denotes the position of the vehicle $i \in \mathcal{V} = \{1, 2, \dots, n\}$, $\theta_i \in \mathbb{R}$ is its heading, $v_i \in \mathbb{R}_{\geq 0}$ and $\omega_i \in \mathbb{R}$ are its controlled translational and rotational velocities, respectively. We consider the scenario where the interaction network among the vehicles is a ring graph, i.e., vehicle i follows vehicle $i+1$ modulo n, one reason being that it requires the fewest communication links.
For analysis convenience, we consider *relative coordinates*. Let d_i denotes the distance between vehicle i and $i+1$, let α_i denotes the angle difference from the heading of vehicle i to the heading that would take it directly towards vehicle $i+1$, and let γ_i be the heading difference minus π. Then, the equations of motion in relative coordinates for vehicle i is described as

$$\begin{aligned}\dot{d}_i &= -v_i \cos\alpha_i - v_{i+1}\cos(\alpha_i + \gamma_i),\\ \dot{\alpha}_i &= \frac{1}{d_i}\left(v_i \sin\alpha_i + v_{i+1}\sin(\alpha_i+\gamma_i)\right) - \omega_i,\\ \dot{\gamma}_i &= \omega_i - \omega_{i+1}.\end{aligned}$$

The above formula is valid when $d_i \neq 0$. We assume that d_i is always positive. We suppose the following assumptions to achieve the control goal.

Assumption 1. *Each vehicle $i \in \mathcal{V}$ is equipped with sufficient sensors to measure d_i, α_i, and θ_i*[a].

Assumption 2. *Each vehicle $i \in \mathcal{V}$ have access to $\gamma_i := \theta_i - \theta_{i+1} - \pi$ by exchanging information with vehicle $i+1$.*

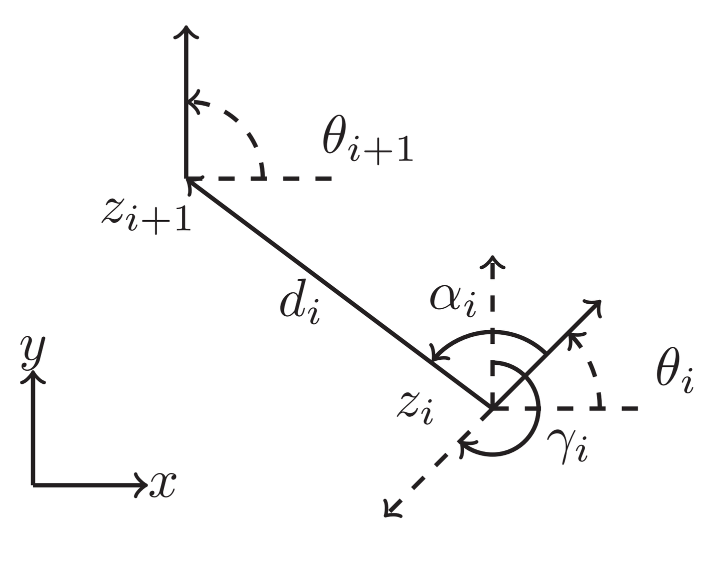

Figure 1: Illustration of relative coordinates.

[a] In practice, these measurements require the use of various sensors combined, include camera, IR, magnetometer, and many others. The key is that the vehicles have no access to absolute measurements/only have access to relative measurements.

Formation Transform

We focus on a special kind of polygon, inscribed polygon, a polygon that has all its vertices lying on the circumference of a circle. Therefore, the polygonal formation problem is transformed into a circular formation problem. Once a circular formation is achieved, it is able to apply switching control to ensure all vehicles to perform some certain motion while maintaining the polygonal formation. For each vehicle i, suppose its controls v_i and ω_i are designed using d_i, α_i, and γ_i. Let $\xi_i = [d_i \ \alpha_i \ \gamma_i]^\top$ and $\xi = [\xi_1^\top \ \xi_2^\top \ \dots \ \xi_n^\top]^\top$. Then we can view the complete multi-vehicle system as

$$\dot{\xi} = f(\xi).$$

Due to its high nonlinearity, classical linearization method fails to analyze the stability around the equilibrium points (i.e., circular formations), and more advanced stability analysis tools are used[a].

[a] The control design in detail and tedious mathematical analysis is omitted due to the space constraints.

Simulation

We simulate the scenarios when $n = 4$. For ease of tuning, we suppose all vehicles have identical and translational velocity, i.e., $v_i \equiv v \ \forall\, i \in \mathcal{V}$. Once the vehicles converge to a circular formation, we switch the control so that the vehicles perform a sinusoidal motion while maintaining a convex polygonal formation. It is shown that with different initial conditions of the vehicles, the vehicles converge to different convex polygonal formations.

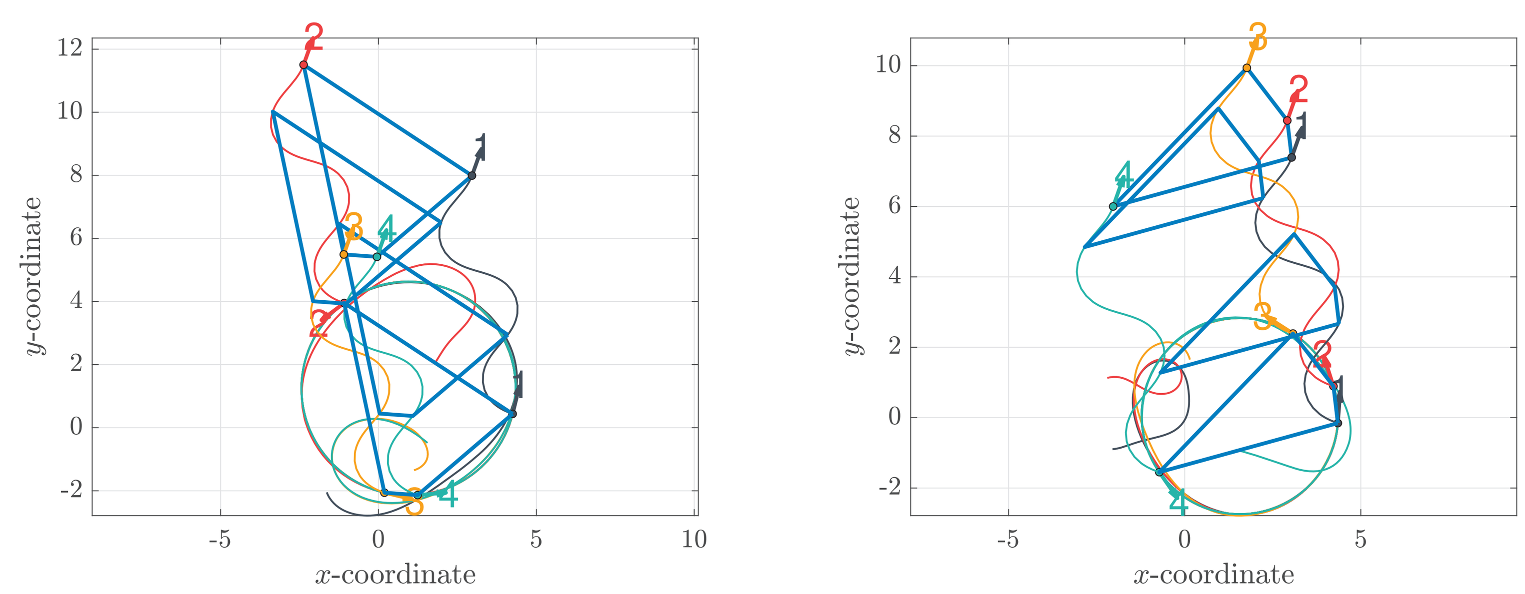

Figure 2: Vehicle trajectories visualization with different initial conditions

The switching control that we implement is quite simple. After the vehicles converge to and keep a stationary polygonal formation, we make all vehicles remain still ($v = 0$) while a consensus algorithm is applied for them to achieve agreement on their headings:

$$\dot{\theta}_i = -(\theta_i - \theta_{i+1}).$$

Then, after the vehicles reach an agreement on their headings, we rechoose a nonzero translational velocity $v > 0$ and for each vehicle $i \in \mathcal{V}$ we set its rotational velocity as $\omega_i = \sin t$.

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