Optimization-Based Path-Velocity Control for Time-Optimal Path Tracking under Uncertainties

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Abstract

This work addresses the **path-tracking problem** of time-optimal trajectories under **model uncertainties**, by proposing a real-time **predictive scaling** algorithm. The algorithm is formulated as a convex-optimization problem, designed to balance the trade-off between **improving feasibility** and **time optimality** of a trajectory. The predicted trajectory is scaled based on the presence of path segments particularly sensitive to model uncertainties, so-called critical and tangency points, within the prediction horizon.

Problem Formulation

The torque $\boldsymbol{\tau}$ that can actuate the robot to follow the reference path [1]

$$\boldsymbol{\tau} = \boldsymbol{\beta}_1(\boldsymbol{q}, s)\ddot{s} + \boldsymbol{\beta}_2(\boldsymbol{q}, \dot{\boldsymbol{q}}, s, \dot{s}) \tag{1}$$

and is subject to torque constraint

$$\boldsymbol{\tau}_{\min} \le \boldsymbol{\tau} \le \boldsymbol{\tau}_{\max} \tag{2}$$

The torque constraint (2) can be transformed to an **input** constraint

$$\ddot{s}_{\min}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \le \ddot{s} \le \ddot{s}_{\max}(\boldsymbol{\beta}_1, \boldsymbol{\beta}_2) \tag{3}$$

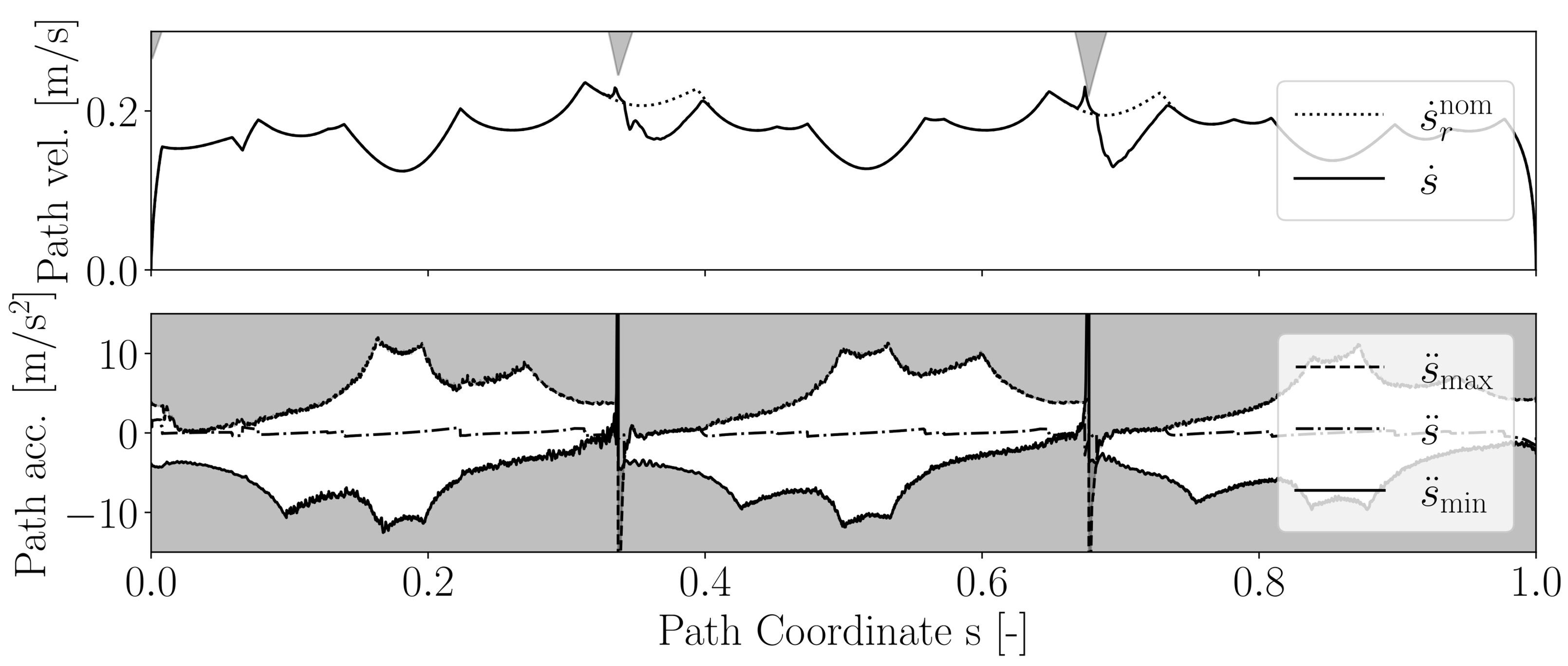
and implies a **state** constraint

$$\dot{s} \le \dot{s}_{\max} \tag{4}$$

Objective: constraint satisfaction of (2)–(4) during path tracking, while achieving as high path speed $\dot{s}$ as possible.

Observation

Feasibility of nominal time-optimal trajectory $\dot{s}^{\text{nom}}$ is sensitive to model uncertainties near the **critical** and **tangency** points [2]. The signal $\dot{s}_r^{\text{nom}}$ is not feasible at $s \approx 0.33$ and $s \approx 0.66$ because $\dot{s}_r^{\text{nom}} > \dot{s}_{\max}^{\text{true}}$.



Optimization-Based Trajectory Scaling (OPVC)

The torque (1) can be approximated as

$$\boldsymbol{\tau}_\gamma = \left(\boldsymbol{\Gamma}_1\ddot{s}_r + \boldsymbol{\Gamma}_2\dot{s}_r^2\right)\gamma^2 + \hat{\boldsymbol{G}} + \hat{\boldsymbol{B}}\Delta\boldsymbol{v} \tag{5}$$

Predictive scaling of the nominal trajectory with information about the location of the critical and tangency points [3]. Let $x_0 = \gamma^2$ and solve

$$\underset{x_0}{\text{maximize}} \quad x_0 - \frac{1}{N}\sum_{k=0}^{N-1} w(s_k)\left(x_{-1} - x_0\right)^2 \tag{6}$$

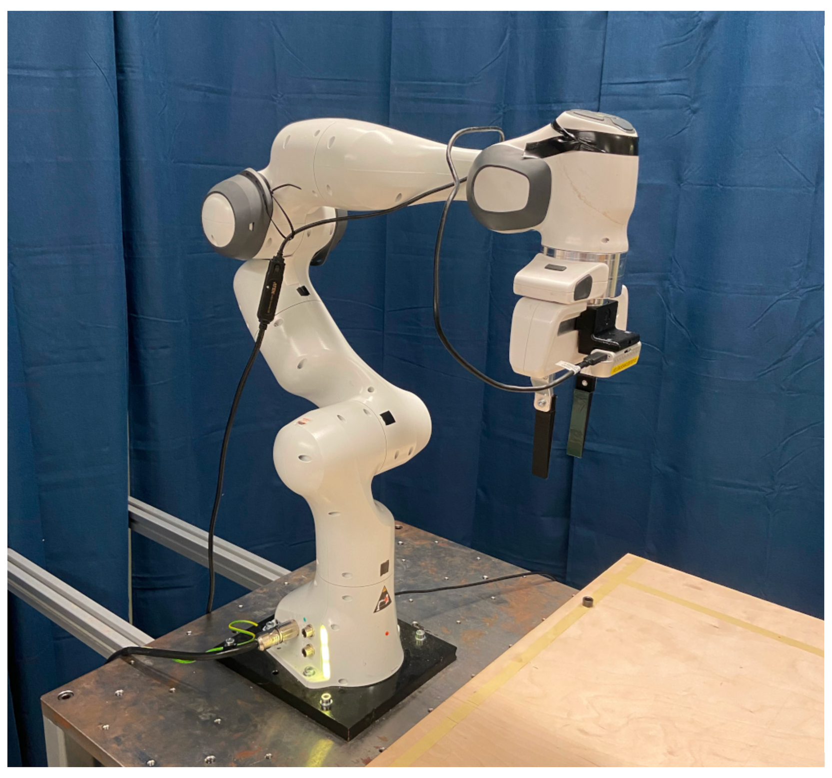
$$\text{subject to} \quad \boldsymbol{\tau}_k = \left(\boldsymbol{\Gamma}_1\ddot{s}_r + \boldsymbol{\Gamma}_2\dot{s}_r^2\right)_k x_0 + \hat{\boldsymbol{G}}_k + \hat{\boldsymbol{B}}_k\Delta\boldsymbol{v} \tag{7}$$

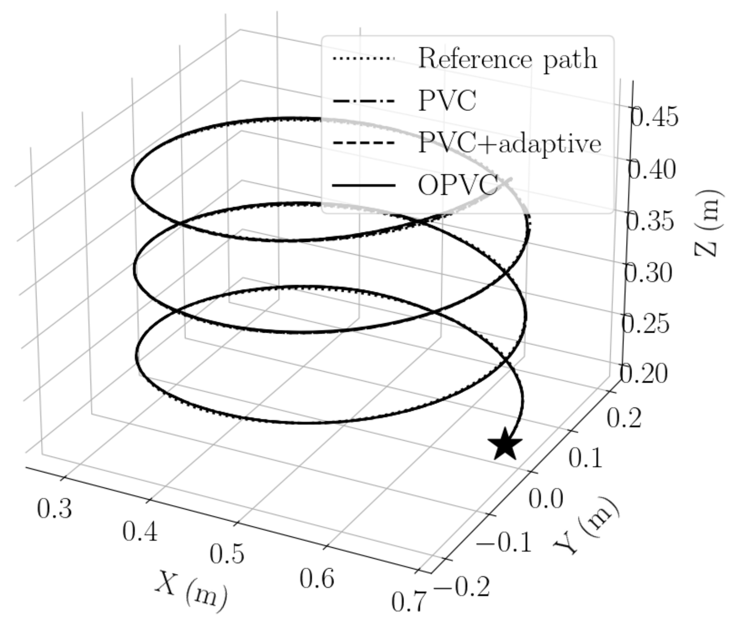
$$\boldsymbol{\tau}_{\min} \le \boldsymbol{\tau}_k \le \boldsymbol{\tau}_{\max}, \; k = 0, \ldots, N-1 \tag{8}$$

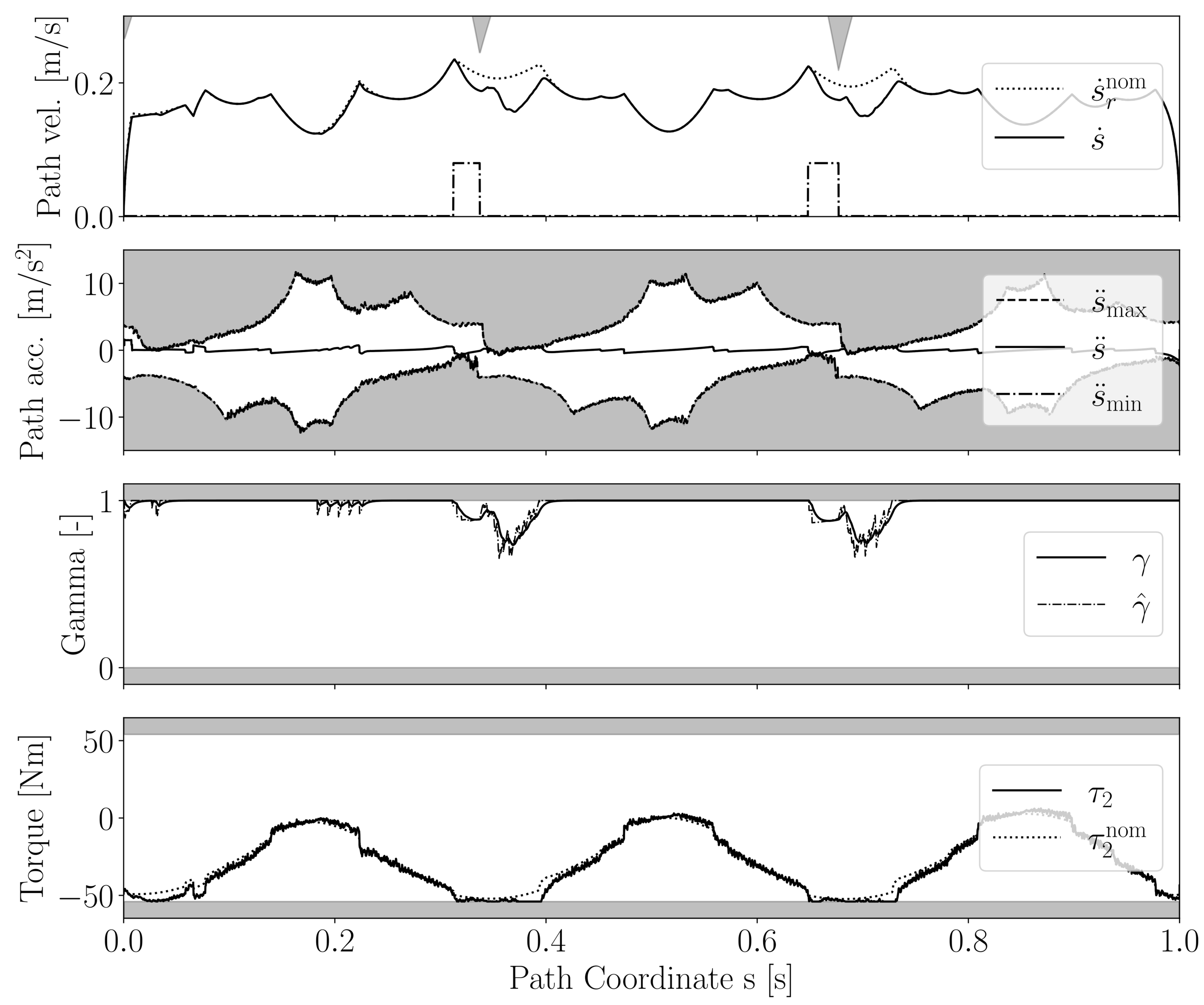
$$0 \le x_0 \le 1 \tag{9}$$

Results

Path tracking of a cylindrical path with a Franka Emika Panda Robot







The proposed OPVC addresses the problem [3]

- by circumventing the critical and tangency points (constraints (2)-(4) can be satisfied and trajectory feasibility is improved),
- by preserving time optimality whenever away from these particular points.

References

[1] Torque-Limited Path Following by Online Trajectory Time Scaling
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[3] Optimization-Based Path-Velocity Control for Time-Optimal Path Tracking under Uncertainties
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